Disorder effects on tunneling through one-dimensional double-barrier quantum-well structures: A coherent-potential approximation

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We present a formalism for studying the disorder effects on electron tunneling through one-dimensional double-barrier quantum-well structures based on the coherent-potential approximation. This formalism enables us to calculate the configuration-averaged transmission coefficients in a nonperturbative way. It is shown that elastic scattering reduces and broadens the resonance peak, and also destroys its Lorentzian behavior. For sharp resonance structures, the transmission coefficient directly reflects the structures in the density of states within the well. The result obtained using this method agrees well with direct numerical simulation.

I. INTRODUCTION

The so-called peak-to-valley ratio of a tunneling double-barrier quantum-well (DBQW) structure is one of the most important parameters for its device applications. Since the first experimental realization of resonant tunneling through DBQW structure by Chang, Esaki, and Tsu,[1] it has been found that the peak-to-valley ratios measured for real devices are much smaller than those predicted from simple theory. Quantitatively understanding the measured peak-to-valley ratio raises a challenging problem to both the experimentalist and the theoretician.

During the tunneling processes, electrons may be elastically scattered by impurities, interface roughness, and compositional disorder, and may also suffer inelastic collisions, such as phonon-electron and electron-electron scattering processes. In a series of recent experiments, Gueret *et al.*[2] found that the longitudinal-optical (LO) phonons only appear beyond the valley voltage, and that the I-V characteristic within the negative-differential-conductance (NDC) region is temperature independent. This suggests that inelastic scattering is not important in the resonant tunneling process, and the reduction of the peak-to-valley ratio is mainly due to the elastic scattering caused by various disorders within the structure. Therefore, it is most desirable to have a theory that enables us to calculate the effects of elastic scattering on resonant tunneling through DBQW structures.

Recently, a number of publications on this problem have appeared in the literature.[3–7] In most of these studies, the disorder potential is treated as a perturbation. Fertig, He, and Das Sarma[4] calculated the lowest-order correction of the transmission coefficient due to disorder within the well region using a tight-binding model. Leo and MacDonald[5] proposed a systematic theory based on an extended basis introduced by Duke, Kleiman, and Stakelon.[8] They obtained the leading-order correction to the transmission coefficient due to interface roughness scattering. Rudberg[6] and Vinter and Chevoir[7] have also used the Fermi golden rule to calculate the interface-roughness effects on tunneling. All these perturbation theories successfully show that the disorder scattering tends to reduce the peak-to-valley ratio.

Perturbation theories have a crucial disadvantage in that they only work well for narrow barrier structures. For DBQW structures with thick barrier widths, the perturbation theory may give unphysical results (for example, the transmission coefficient may be larger than unity). The reason is that, as the barrier width increases, the density of states within the well region becomes sharply peaked at the resonant energy. The disorder scattering is then very strong near the resonant energy even if the disorder potential is quite weak, which causes a breakdown of the perturbation theory. Since in most experimental studies, structures with quite thick barriers have been used, one cannot expect to explain experimental results by using perturbation theory.

In this paper, we present a formalism for calculating the disorder effect on tunneling through one-dimensional DBQW structures based on the coherent-potential approximation (CPA). In this formalism, we use the CPA effective medium as the basis of our approach, and the scattering due to disorder fluctuation from the CPA effective medium is evaluated explicitly. This method allows us to study disorder effects on resonance without limitation with respect to the shape of the structures or the strength of the disorder.

As an application of the method, we have calculated, within effective-mass theory, the transmission coefficients of GaAs/Al_xGa_{1-x}As DBQW systems. We take a binary random-alloy model to mimic the compositional disorder in the $Al_xGa_{1-x}As$ barriers and the disorder introduced by impurities within the GaAs quantum well. To examine the validity of the method, we have computed the transmission coefficient for a large number of disorder configurations by direct numerical simulation. The total density of states (DOS) in the well region of the disordered DBQW structure is also calculated within CPA, and the relationship between the tunneling transmission and the DOS is discussed.

II. THE MODEL

Figure 1 shows an example of a disorder-free double-barrier quantum-well structure. The Hamiltonian of the system is

$$h_0(x)=\begin{cases} -\dfrac{\hbar^2}{2m^*}\dfrac{d^2}{dx^2}+V_{\text{off}}, & x\in \text{barriers} \\ -\dfrac{\hbar^2}{2m^*}\dfrac{d^2}{dx^2}, & x\notin \text{barriers} . \end{cases} \tag{2.1}$$

where, for simplicity we assume that an electron has the same effective mass m^* in both the barrier and well regions and V_{off} is the height of the barrier set by the conduction-band offset between the barrier and well materials. For a more realistic DBQW structure, the Hamiltonian is given by

$$H(x)=h_0(x)+\Delta(x) , \tag{2.2}$$

where $\Delta(x)$ describes the disorder caused by interface roughness, alloy disorder, and impurities. In the present work, we assume that $\Delta(x)$ has the form

$$\Delta(x)=\begin{cases} \displaystyle\sum_i^N v_i\theta\left[\frac{a_i}{2}-|x_i-x|\right] & \text{for } 0<x<l , \\ 0 \quad \text{otherwise} , \end{cases} \tag{2.3}$$

where N is the number of monolayers in the DBQW structure of length l, x_i is the central position of the ith monolayer, a_i is the width of the ith monolayer, and v_i is the deviation of the potential of the ith monolayer from that of the perfect structure. In effect, the disorder potential is averaged over the plane perpendicular to the growth direction to maintain a one-dimensional model system.

As long as we are only interested in the tunneling through the lowest resonance state, the wavelengths of the states considered are much larger than one-monolayer width. As a good approximation, therefore, we can write $\Delta(x)$ in a simplified form,

$$\Delta(x)=\begin{cases} \displaystyle\sum_i^N v_i a_i\delta(x-x_i) & \text{for } 0<x<l , \\ 0 \quad \text{otherwise} . \end{cases} \tag{2.4}$$

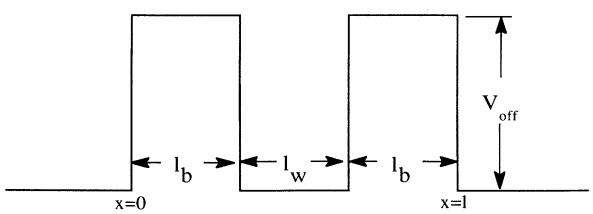

FIG. 1. A schematic diagram of a DBQW structure.

III. THEORY

In order to study tunneling through disordered DBQW structures, we consider the disorder-free structure first. Following Duke, Kleiman, and Stakelon[8] and Leo and MacDonald,[5] we choose a linearly independent set of wave functions of the perfect structure as follows:

$$\psi_l^{(0)}(E,x)=\begin{cases} e^{ikx}+r_0e^{-ikx} & \text{for } x<0 \\ c_l^+u_+(x)+c_l^-u_-(x) & \text{for } 0<x<l \\ \eta_0e^{ikx} & \text{for } x>l \end{cases} \tag{3.1}$$

$$\psi_r^{(0)}(E,x)=\begin{cases} \eta_0e^{-ikx} & \text{for } x<0 \\ c_r^+u_+(x)+c_r^-u_-(x) & \text{for } 0<x<l \\ e^{-ikx}+r_0e^{ikx} & \text{for } x>l \end{cases} \tag{3.2}$$

$$k=\left[\frac{2m^*E}{\hbar^2}\right]^{1/2} \tag{3.3}$$

In the above equations, $u_+(x)$ [$u_-(x)$] is the decaying (growing) solution within the barrier and well regions, η_0 and r_0 are the transmission and reflection amplitudes of the disorder-free structure, respectively. $\psi_l(E,x)$ [$\psi_r(E,x)$] is the wave function of an electron incident from the left (right) with energy E scattered by the perfect structure only. If we take into account the scattering by the disorder potential $\Delta(x)$, the wave function in the disorder system of an electron incident from the left is given by the Lippman-Schwinger equation

$$\psi_l(x)=\psi_l^{(0)}(x)+\int\int G_0(x,x')T(x',x'')\psi_l^{(0)}(x'')dx'dx'' , \tag{3.4}$$

where $G_0(x,x')$ is the Green's function of the perfect structure defined by

$$[E-h_0(x)]G_0(x,x')=\delta(x-x') \tag{3.5}$$

and $T(x,x')$ is the so-called T matrix defined as

$$T(x,x')=\Delta(x)\delta(x-x')+\int\Delta(x)G_0(x,x'')T(x'',x')dx'' . \tag{3.6}$$

$G_0(x,x')$ can be derived by solving the differential equation (3.5) directly to give

$$G_0(x,x')=\begin{cases} \dfrac{m^*}{i\hbar^2k\eta_0}\psi_l^{(0)}(x')\psi_r^{(0)}(x), & x<x' , \\ \dfrac{m^*}{i\hbar^2k\eta_0}\psi_l^{(0)}(x)\psi_r^{(0)}(x'), & x>x' . \end{cases} \tag{3.7}$$

For the disorder potential defined in Eq. (2.3), we can easily show that

$$T(x,x')=\sum_{i,j}T_{i,j}\delta(x-x_i)\delta(x'-x_j) , \tag{3.8}$$

with

$$T_{i,j}=v_ia_i\delta_{i,j}+v_ia_i\sum_k G_{0i,k}T_{k,j} , \tag{3.9}$$

where we have used the notation $G_{0i,j}=G_0(x_i,x_j)$.

The transmission coefficient of the state described by the wave function in Eq. (3.4) is given by

$$\eta=\frac{1}{k}\mathrm{Im}\left[\psi_l^*(x)\frac{d\psi_l(x)}{dx}\right] . \tag{3.10}$$

By substituting Eqs. (3.7) and (3.8) into Eq. (3.4), and then Eq. (3.4) into Eq. (3.10), we obtain the following expression for the configuration-averaged transmission coefficient:

$$\langle\eta\rangle=|\eta_0|^2(1+I_1+I_2) , \tag{3.11}$$

where η_0 is the transmission amplitude of the perfect system defined in Eqs. (3.1) and (3.2), and

$$I_1=\frac{2m^*}{2\hbar^2k\eta_0}\mathrm{Re}\left\{\sum_{i,j}\psi_r^{(0)}(x_i)\psi_l^{(0)}(x_j)\langle T_{i,j}\rangle\right\} , \tag{3.12}$$

$$I_2=\sum_{i,j,k,l}\left[\frac{m^*}{\hbar^2k\eta_0}\right]^2\psi_r^{*(0)}(x_i)\psi_r^{(0)}(x_k)\times\psi_l^{*(0)}(x_j)\psi_l^{(0)}(x_l)\langle T_{i,j}^*T_{k,l}\rangle . \tag{3.13}$$

The complex nature of the T matrix makes it impossible to calculate $\langle T_{i,j}\rangle$ and $\langle T_{i,j}^*T_{k,l}\rangle$ exactly. For the case where the disorder is weak and the resonance is quite broad (i.e., for the thinner and lower barrier case), perturbation theory, as applied by Leo and MacDonald,[5] can give quite good results. However, for the sharp resonance structure, multiple scattering becomes very important. In such a case, perturbation theory becomes invalid.

We have applied the CPA to deal with this problem. As the best single-site approximation for a disordered system, the CPA self-energy renormalized transmission coefficient gives a good first-order estimate. The two terms in Eq. (3.11) containing $\langle T_{i,j}\rangle$ and $\langle T_{i,j}^*T_{k,l}\rangle$, caused by the disorder potential *deviating* from the CPA effective medium, are, therefore, much smaller than those in the perfect structure basis, and can be easily calculated within CPA.

We define an effective disorder-free structure described by the Hamiltonian

$$h^{\mathrm{CPA}}(x)=h_0(x)+\Sigma^{\mathrm{CPA}}(x) . \tag{3.14}$$

Here $\Sigma^{\mathrm{CPA}}(x)$ is the position-dependent self-energy that satisfies the single-site CPA condition

$$\langle t_i^{\mathrm{CPA}}\rangle=0 , \tag{3.15}$$

where

$$t_i^{\mathrm{CPA}}=\frac{[v_i-\Sigma^{\mathrm{CPA}}(x_i)]a_i}{1-[v_i-\Sigma^{\mathrm{CPA}}(x_i)]a_iG_{0i,i}} . \tag{3.16}$$

Using this effective disorder-free Hamiltonian determined by CPA as our starting point, we have $\langle T_{i,j}^{\mathrm{CPA}}\rangle\approx 0$. Then the configuration-averaged transmission coefficient of the disordered system may be written as

$$\langle\eta\rangle=|\eta_0^{\mathrm{CPA}}|^2+|\eta_0^{\mathrm{CPA}}|^2\sum_{i,j,k,l}\left[\frac{m^*}{\hbar^2k\eta_0^{\mathrm{CPA}}}\right]^2\psi_r^{\mathrm{CPA}*}(x_i)\times\psi_r^{\mathrm{CPA}}(x_k)\psi_l^{\mathrm{CPA}*}(x_j)\times\psi_l^{\mathrm{CPA}}(x_l)\times\langle T_{i,j}^{\mathrm{CPA}*}T_{k,l}^{\mathrm{CPA}}\rangle , \tag{3.17}$$

with

$$T_{i,j}^{\mathrm{CPA}}=[v_i-\Sigma^{\mathrm{CPA}}(x_i)]a_i\delta_{i,j}+[v_i-\Sigma^{\mathrm{CPA}}(x_i)]a_i\times\sum_k G_{i,k}^{\mathrm{CPA}}T_{k,j}^{\mathrm{CPA}} . \tag{3.18}$$

Here, $\psi_{r(l)}^{\mathrm{CPA}}(x)$ and G^{CPA} are the wave function and Green's function of $h^{\mathrm{CPA}}(x)$, respectively, and η_0^{CPA} is the transmission amplitude of the structure defined by $h^{\mathrm{CPA}}(x)$. In Eq. (3.17), $|\eta_0^{\mathrm{CPA}}|^2$ gives the CPA self-energy renormalized transmission coefficient of the disordered structure; the second term on the right-hand side contains all other correction terms beyond the CPA self-energy approximation.

To evaluate $\langle T_{i,j}^{\mathrm{CPA}*}T_{k,l}^{\mathrm{CPA}}\rangle$ in Eq. (3.17), we write the T matrix as in Ref. 9 (hereafter, all matrices are double underlined):

$$\underline{\underline{T}}^{\mathrm{CPA}}=\sum_i\underline{\underline{Q}}_i^{\mathrm{CPA}} \tag{3.19}$$

The matrix $\underline{\underline{Q}}_i^{\mathrm{CPA}}$ satisfies the following equation:

$$\underline{\underline{Q}}_i^{\mathrm{CPA}}=\underline{\underline{T}}_i^{\mathrm{CPA}}\left[\underline{\underline{1}}+\underline{\underline{G}}^{\mathrm{CPA}}\sum_{j\neq i}\underline{\underline{Q}}_j^{\mathrm{CPA}}\right] , \tag{3.20}$$

where $\underline{\underline{T}}_i^{\mathrm{CPA}}$ is the T matrix for a single site given by

$$T_{i_{l,m}}^{\mathrm{CPA}}=t_i^{\mathrm{CPA}}\delta_{i,l}\delta_{i,m} , \tag{3.21}$$

where t_i^{CPA} is defined by Eq. (3.16). Using Eq. (3.19), we obtain

$$\langle T_{i,j}^{\mathrm{CPA}*}T_{k,l}^{\mathrm{CPA}}\rangle=\left\langle t_i^{\mathrm{CPA}*}\left[\delta_{i,j}+\sum_{\substack{s\\ n\neq i}}G_{i,s}^{\mathrm{CPA}*}Q_{n_{s,j}}^{\mathrm{CPA}*}\right]\times t_k^{\mathrm{CPA}}\left[\delta_{k,l}+\sum_{\substack{r\\ m\neq k}}G_{k,r}^{\mathrm{CPA}}Q_{m_{r,l}}^{\mathrm{CPA}}\right]\right\rangle . \tag{3.22}$$

Following the standard practice within CPA (Refs. 9 and 11), we replace the average of the product by a product of averages as follows:

$$\langle T_{i,j}^{\mathrm{CPA}*}T_{k,l}^{\mathrm{CPA}}\rangle=\langle t_i^{\mathrm{CPA}*}t_k^{\mathrm{CPA}}\rangle\times\left\langle\left[\delta_{i,j}+\sum_{\substack{s\\ n\neq i}}G_{i,s}^{\mathrm{CPA}*}Q_{n_{s,j}}^{\mathrm{CPA}*}\right]\times\left[\delta_{k,l}+\sum_{\substack{r\\ m\neq k}}G_{k,r}^{\mathrm{CPA}}Q_{m_{r,l}}^{\mathrm{CPA}}\right]\right\rangle . \tag{3.23}$$

Noting that t_i^{CPA} are independent random variables with zero mean, we see that $\langle T_{i,j}^{\mathrm{CPA}*} T_{k,l}^{\mathrm{CPA}} \rangle$ is nonzero only when $i=k$. Similarly, we can also show that $\langle T_{i,j}^{\mathrm{CPA}*} T_{k,l}^{\mathrm{CPA}} \rangle$ is nonzero only when $i=j=k=l$, i.e.,

$$\langle T_{i,j}^{\mathrm{CPA}*} T_{k,l}^{\mathrm{CPA}} \rangle \approx \gamma_i \delta_{i,j} \delta_{i,k} \delta_{i,l} , \tag{3.24}$$

where

$$\gamma_i \approx \langle |t_i^{\mathrm{CPA}}|^2 \rangle \left[1 + \sum_{\substack{g,h \\ n \neq i, m \neq i}} G_{i,g}^{\mathrm{CPA}*} G_{i,h}^{\mathrm{CPA}} \times \langle Q_{n_{g,i}}^{\mathrm{CPA}*} Q_{m_{h,i}}^{\mathrm{CPA}} \rangle \right] . \tag{3.25}$$

Substituting Eq. (3.20) into (3.25), and taking the same approximation as that used in obtaining Eq. (3.24), we find

$$\gamma_i \approx \langle |t_i^{\mathrm{CPA}}|^2 \rangle \left[1 + \sum_{n \neq i} |G_{i,n}^{\mathrm{CPA}}|^2 \langle |Q_{n_{n,i}}^{\mathrm{CPA}}|^2 \rangle \right] . \tag{3.26}$$

We then define a new matrix with elements

$$\Gamma_{i,j} = \langle |Q_{i_{i,j}}^{\mathrm{CPA}}|^2 \rangle . \tag{3.27}$$

Using Eq. (3.20) and once again taking the approximation used in obtaining Eqs. (3.24) and (3.26), we show that

$$\Gamma_{i,j} \approx \langle |t_i^{\mathrm{CPA}}|^2 \rangle \left[\delta_{i,j} + \sum_{n \neq i} |G_{i,n}^{\mathrm{CPA}}|^2 \langle |Q_{n_{n,j}}^{\mathrm{CPA}}|^2 \rangle \right] . \tag{3.28}$$

Comparing Eq. (3.26) with (3.28), we see that

$$\gamma_i = \Gamma_{i,i} . \tag{3.29}$$

Equation (3.28) gives a closed relation for $\Gamma_{i,j}$, hence we can obtain all γ_i's by solving this equation.

IV. RESULTS

In this section, we present some results obtained with the method used in this paper. To make sensible contact with experiment, we choose the parameters of the structures used for our calculation to be similar to that used in the experimental study by Gueret *et al.*[2] The structure consists of two $Al_xGa_{1-x}As$ ($x \approx 0.15$) barriers of identical width separated by a GaAs quantum well and sandwiched between two GaAs electrodes. The well is 24 monolayer (~7 nm) thick, and the $Al_xGa_{1-x}As$ barriers are 120 meV high. The effective mass of the electron is taken to be 0.067 in units of free-electron mass. We take the width of the barriers to be a variable. A simple binary random alloy model is used to mimic the compositional disorder in the barriers and the disorder caused by impurities or other kinds of defects in the well. We assume that v_i is a random variable in the disorder region, which is described by a distribution function

$$P(v_i) = \begin{cases} P_A, & i = A \\ 1-P_A, & i = B \end{cases} \tag{4.1}$$

and satisfies

$$\langle v_i \rangle = P_A v_A + (1-P_A) v_B = 0 . \tag{4.2}$$

We first consider a structure with only compositional disorder in its two barriers. The width of each of the two barriers is 39 monolayers. The transmission coefficients calculated for different degrees of disorder in the structure are presented in Fig. 2.

In addition to the results of our theory, Figs. 2(a)–2(d) also show the transmission coefficients obtained by direct numerical simulation. The procedure of the direct numerical simulation is as follows. We generate a large number of different sets of $\{v_i\}$ according to the distribution statistics described in Eqs. (4.1) and (4.2). Each set of $\{v_i\}$ gives one configuration of the disordered structure. The Hamiltonian of a given configuration is defined by Eq. (2.2). The transmission coefficient for each disorder configuration is calculated by solving the Schrodinger equation using the transfer-matrix method.[10] The final results are given by the average of the results for these different configurations. We have used 200 configurations to obtain the results shown in Fig. 2.

From Fig. 2, we see that the resonance transmission peaks are lowered and broadened by disorder scattering. As the disorder strength increases, the transmission coefficient curve becomes less smooth and some new features appear. Compared with the results from numerical simulation, the new theory gives physically reasonable results for arbitrary degrees of disorder. For the weak-disorder case, as shown in Fig. 2(a), the agreement between the result of the method used here and the numerical simulation is excellent. For strong disorder cases shown in Figs. 2(b)–2(d), although this method cannot reproduce the detailed structures appearing in the numerical-configuration average transmission curve, it still gives correct width and height for the main peak.

The structures in the configurational-averaged transmission-coefficient curve correspond to similar structures in the density of states. It is well known that the rich structures in the density of states result from compositional fluctuations.[11–13] The effect of compositional fluctuations is strongest in one-dimensional binary-alloy model. It is not surprising, therefore, that our theory cannot reproduce these detailed structures, since the CPA is only a single-site approximation, in which the compositional fluctuations are not considered. To include the compositional fluctuations, one may use the so-called cluster-CPA,[13] in which the short-range compositional fluctuations within the cluster are treated exactly. The compositional fluctuations become weaker with increasing system dimensionality. Hence, one can expect that a generalization of the present theory to the three-dimensional case will give much better results.

Figure 3(a) shows the transmission coefficients calculated for four structures with different barrier widths and alloy disorder in the barriers. The disorder strengths are the same in each case. The disorder effect on the transmission coefficient is very weak for structures with narrow barriers. For off-resonant transmission, disorder scattering only slightly increases the transmission

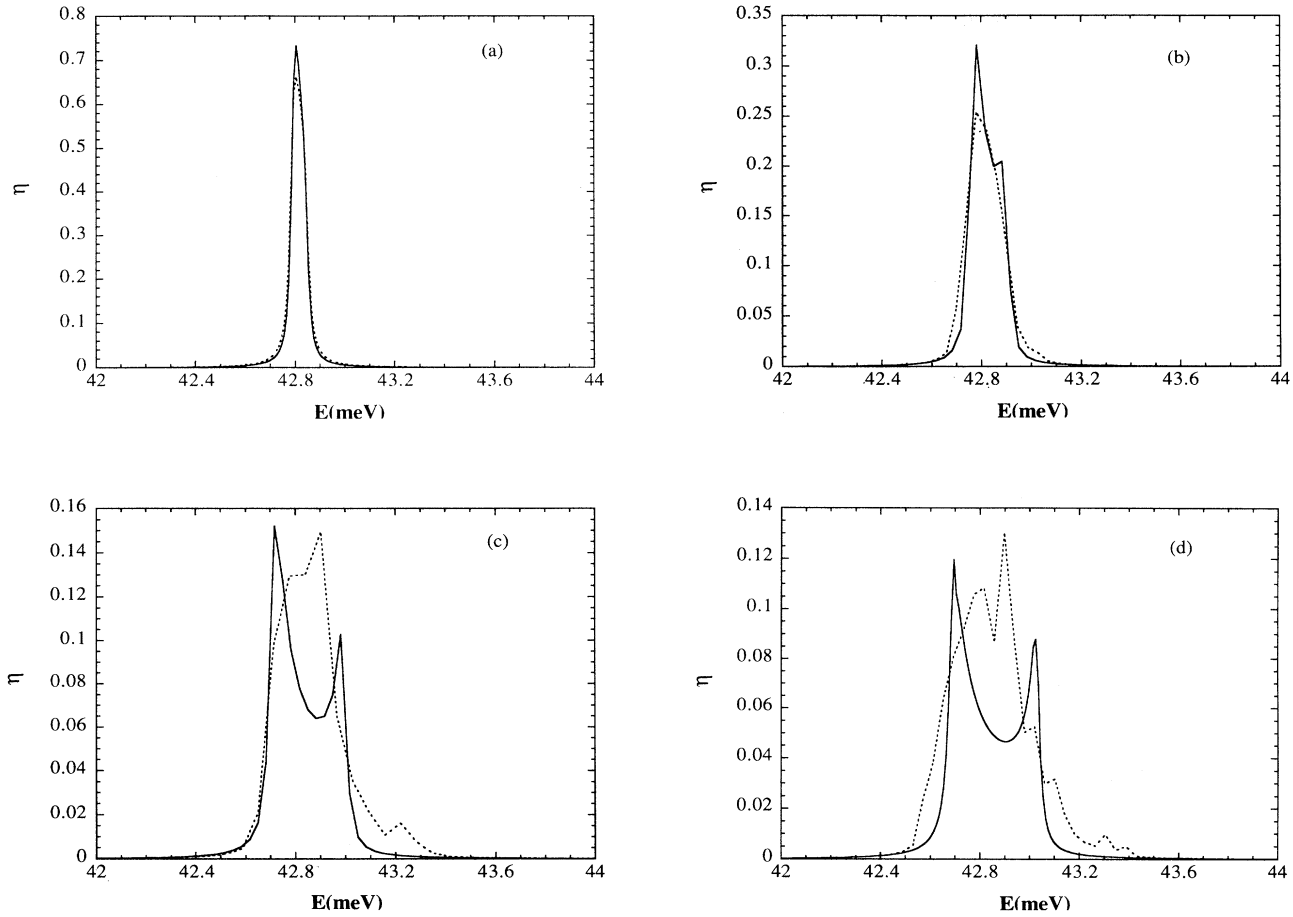

FIG. 2. Transmission coefficients calculated from the CPA method (solid line) compared with numerical simulation (dotted line) for DBQW structures with different disorder strength in the barriers: $P_A = 0.15$, $\nu_A =$ (a) 1.7 meV, (b) 4.25 meV, (c) 8.5 meV, and (d) 10.2 meV. Structural parameters: $V_{\text{off}} = 120$ meV, the width of the barriers and the well are 39 and 24 (in units of monolayer width), respectively.

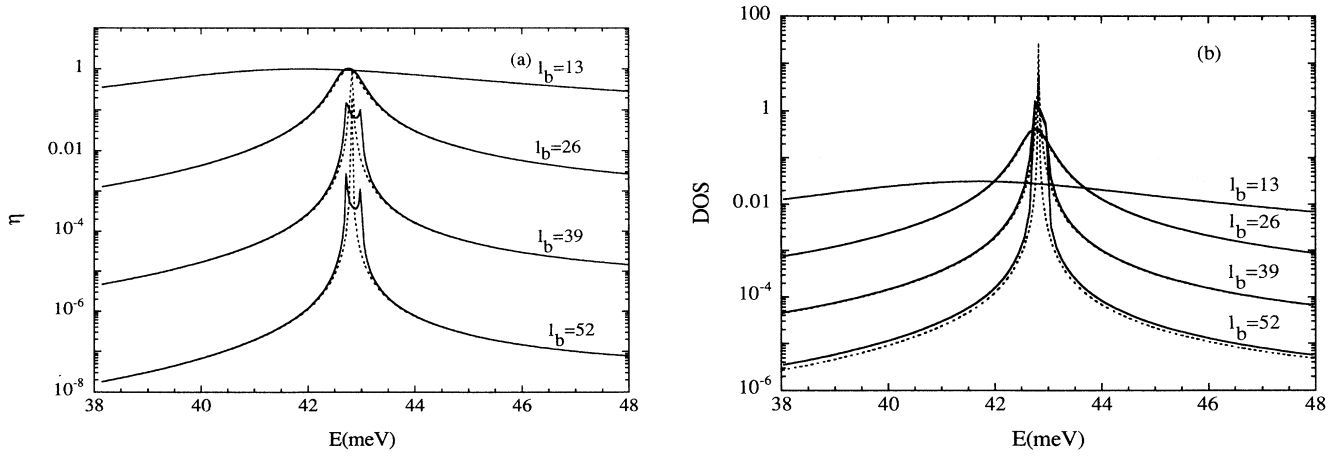

FIG. 3. Effect of alloy disorder on (a) the barriers on the transmission coefficients and (b) the DOS in the well region for structures with the same disorder strength ($P_A = 0.15$, $\nu_A = 8.5$ meV, $V_{\text{off}} = 120$ meV) and the same well width ($l_w = 24$ monolayers) but different barrier widths, $l_b = 13$, 26, 39, 52 monolayers from top to bottom. The results for disorder-free structures are also plotted with the dotted line for comparison.

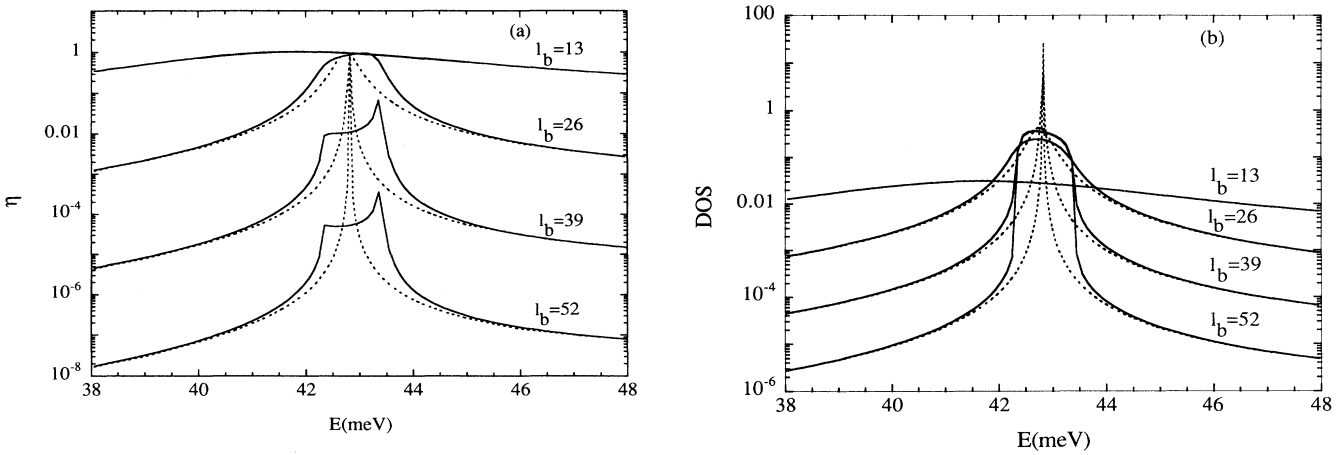

FIG. 4. Effect of alloy disorder inside the well on (a) the transmission coefficients and (b) the DOS in the well region for structures with the same disorder strength ($P_A=0.15$, $v_A=8.5$ meV, $V_{\text{off}}=120$ meV) and the same well width ($l_w=24$ monolayers) but different barrier widths, $l_b=13$, 26, 39, 52 monolayers from top to bottom. The results for disorder-free structures are also plotted with the dotted line for comparison.

coefficient. However, with increasing the barrier width, the transmission coefficient curve near the resonant energy is drastically changed by the disorder scattering. For structures with thick enough barriers, the disorder scattering reduces the resonant peak exponentially with increasing the barrier width, and broadens the resonant peak to a fixed width. This is in complete contrast to perfect DBQW structures, where the resonance width decreases exponentially with increasing barrier width and the resonant peak is constant at unity.

In order to understand the calculated results for the transmission coefficient, we also present the total density of states in the well region calculated within CPA in Fig. 3(b). In CPA, the total density of states in the well region is given by

$$\rho(E)=-\frac{1}{\pi}\mathrm{Im}\left[\sum_i G_{i,i}^{\mathrm{CPA}}(E)\right], \quad i\in \text{ well region} . \tag{4.3}$$

Comparing Figs. 3(a) and 3(b), we can see that the width of the resonant transmission is directly related to the range of density of state in the well. As the barrier width increases, the electronic state in the well becomes increasingly localized, the density of state tends to a fixed distribution, and consequently the width of resonant transmission tends to a fixed value.

We have also considered the case where there exists a disorder potential in the well region. Figure 4 shows our calculated results for both the transmission coefficients and densities of states. In order to make easy comparison, all the parameters used in Fig. 4 are the same as in Fig. 3 except that the disorder is in the well region. We find that the effects of disorder on transmission and density of states in the well region are similar to those of disorder in the barriers. However, since the wave function is concentrated in the well region near resonance, the effects of disorder in the well are much stronger than those of disorder in the barriers. Hence, disorder within the well, arising from interface roughness or impurities, will have the most important effect on resonant tunneling, especially for sharp resonant structures.

V. SUMMARY

We have described a theory for studying disorder effects on resonant tunneling through a DBQW structure based on the coherent-potential approximation. The advantage of this theory is that it enables us to calculate the configuration-averaged transmission coefficient in a nonperturbative way, and there is no limitation with respect to the disorder strength and the structure parameters.

Our numerical calculation for a $GaAs/Al_xGa_{1-x}As$ DBQW structure shows that the effects of disorder scattering are mainly determined by the disorder strength and the resonant width of the disorder-free structure. Disorder scattering not only reduces and broadens the resonance peak, but also destroys its Lorentzian behavior, which is quite different from the inelastic scattering case.[14,15] For sharp resonant structures, the transmission coefficients directly reflect the structures of the density of states within the well.

Finally we want to point out that, although a one-dimensional model has been used in this paper, the present work provides a basis for the investigation of real DBQW structures, since the theory can be generalized to the three-dimensional case. On the other hand, in view of the increasing interest in tunneling through quasi-one-dimensional lateral semiconductor structures[16] and polymeric superlattices,[17] the study of the one-dimensional system presented in this paper will become more and more important.

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