Resonant tunneling in a Luttinger liquid

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The resonant tunneling through a double-barrier structure in a one-dimensional system of interacting spinless fermions is studied. The conductance is calculated as a function of gate voltage and temperature. It is shown that even in Luttinger liquids the line shape of resonances is almost the same as that of the noninteracting Fermi liquid provided that the temperature is not too low. The *temperature dependence*, on the other hand, is drastically changed by the interaction, and the height of the conductance peak is a nonmonotonic function of temperature for weak repulsive interaction.

The importance of the electron correlation in quantum transport phenomena in mesoscopic systems has been revealed by recent theoretical and experimental studies. A well-explored example is the Coulomb blockade in ultrasmall tunnel junctions.[1] Recently a very narrow, quasi-one-dimensional quantum wire with double barriers has also been fabricated. The conductance of this system was measured as a function of gate voltage V_g and temperature T, and a nonmonotonic temperature dependence of the height of the conductance peaks was found.[2] Meir, Wingreen, and Lee analyzed the Coulomb blockade oscillations treating the Coulomb interaction in terms of the Hartree-type approximation.[3,4] They proposed two characteristic energy scales, i.e., the Coulomb charging energy U and the quantization energy $\Delta\epsilon$ of the quantum dot, which they estimated as $\Delta\epsilon \sim 0.05\,\text{meV}$ and $U \sim 0.5\,\text{meV}$, respectively, and discussed that the conductance behaves differently for $T \ll \Delta\epsilon \ll U$ and $\Delta\epsilon \ll T \ll U$. On the other hand, in contrast to the previous works where the Coulomb interaction is considered only near the tunnel junction or as the on-site repulsion U, Kane and Fisher[5] recently treated the interaction between electrons in *single-channel leads* by using the bosonization method.[6] They put a single barrier in a spinless Luttinger liquid, and found that the electrons are completely reflected by the barrier for repulsive interaction while they are perfectly transmitted through it for attractive interaction at $T = 0\,\text{K}$. Another related problem is the tunneling of the Wigner crystal through the pinning barrier.[7]

The aim of this paper is to investigate the effect of the electron correlation on the resonant tunneling in a one-dimensional (1D) single-channel system. The model we employ is the 1D spinless Tomonaga-Luttinger model with two barriers at $x = -R/2$ and $R/2$. A similar model has recently been studied by Kane and Fisher[8] by a different method in the low-temperature limit. Although some of the results are in agreement, there are several differences, in addition to our description of the crossover around $T \sim v_F/R$ given below. We use the units such that $\hbar = k_B = 1$.

Using the standard bosonization method,[6] we write the partition function at temperature $T = 1/\beta$ as[9]

$$Z = \int \mathcal{D}\theta \exp\Bigg(- \int_0^\beta d\tau \int dx \left[\frac{1}{8\pi v\eta}[\partial_\tau \theta(x,\tau)]^2 + \frac{v}{8\pi\eta}[\partial_x\theta(x,\tau)]^2 \right] + \frac{1}{\pi\alpha}\int_0^\beta d\tau \,\{V_1\cos[\theta(-R/2,\tau) - k_F R] + V_2 \cos[\theta(R/2,\tau) + k_F R]\} \Bigg)\,, \tag{1}$$

where α is a cutoff of the order of the lattice constant. v and η are expressed, in terms of a forward-scattering matrix element g_2, as

$$v = \left[v_F^2 - \left(\frac{g_2}{2\pi}\right)^2 \right]^{1/2} \quad \text{and} \quad \eta = \left(\frac{2\pi v_F - g_2}{2\pi v_F + g_2} \right)^{1/2},$$

where v_F is the Fermi velocity. For repulsive interactions η is smaller than unity, while $\eta > 1$ for attractive interactions. We introduce a high-frequency cutoff $\Lambda \sim v_F/\alpha$ to avoid ultraviolet divergences.

We shall consider a symmetric double-barrier model in which the strength of the potentials is the same: $|V_1| = |V_2| = V_0$. Without loss of generality we can take $V_1 = V_2 = V_0$. The effect of asymmetry will be briefly discussed later. Integrating out the continuum phase field $\theta(x,\tau)$ except $\theta(R/2)$ and $\theta(-R/2)$, we obtain the effective action as

$$S_{\rm eff} = \frac{1}{2\pi\eta\beta}\sum_{\omega_n}\frac{|\omega_n|}{1+\exp(-|\omega_n|/\Delta\epsilon)}|\bar\theta(i\omega_n)|^2 + \frac{1}{8\pi\eta\beta}\sum_{\omega_n}\frac{|\omega_n|}{1-\exp(-|\omega_n|/\Delta\epsilon)}|\tilde\theta(i\omega_n)|^2 + \frac{U}{(2\pi)^2}\int_0^\beta d\tau[\tilde\theta(\tau)]^2 - \frac{eV}{2\pi}\int_0^\beta d\tau\,\bar\theta(\tau) - \frac{eV_g}{2\pi}\int_0^\beta d\tau\,\tilde\theta(\tau) - \frac{2V_0}{\pi\alpha}\int_0^\beta d\tau\cos\bar\theta(\tau)\cos[\tfrac{1}{2}\tilde\theta(\tau)+k_FR], \quad (2)$$

where $\omega_n = 2\pi n/\beta$ (n is an integer) is the Matsubara frequency and $\Delta\epsilon = v/R$. The average phase $\bar\theta = \frac{1}{2}[\theta(R/2)+\theta(-R/2)]$ is related to the current J through the two barriers by $J = -(e/2\pi)(d\bar\theta/dt)$, while the phase difference $\tilde\theta = \theta(R/2)-\theta(-R/2)$ is related to the excess charge Q accumulated in the confined region between two barriers by $Q = -e\tilde\theta/2\pi$. In order to take account of the long-range part of the Coulomb interaction which is not fully incorporated in the bosonization method, we have introduced in Eq. (2) the charging energy $Q^2/2C = U(\tilde\theta/2\pi)^2$ where C is the capacitance; the repulsive energy $U = e^2/2C$ is assumed to be larger than $\Delta\epsilon$. We have included also the energy coming from the difference of the chemical potential across the barriers:

$$-\frac{1}{2\pi}\left[\int_{-\infty}^{-R/2}\mu_L\partial_x\theta\,dx + \int_{-R/2}^{R/2}\mu_I\partial_x\theta\,dx + \int_{R/2}^{\infty}\mu_R\partial_x\theta\,dx\right] = -\frac{1}{2\pi}e(V\bar\theta + V_g\tilde\theta)$$

with $V = (\mu_R-\mu_L)/e$ being the voltage difference between the right- and left-hand sides of the barrier region, and the gate voltage $V_g = [\mu_I - \frac{1}{2}(\mu_R+\mu_L)]/e$. The resonance is achieved by controlling V_g. As can be easily seen in Eq. (2), the charge fluctuation $\tilde\theta$ in the confined region has a mass gap of the order of U while the average phase $\bar\theta$ remains massless and is subject to the dissipation[10,11] whose strength crosses over from $1/2\pi\eta$ for $|\omega_n| \gg \Delta\epsilon$ to $1/4\pi\eta$ for $|\omega_n| \ll \Delta\epsilon$. This decrease in the dissipation as the temperature is lowered across $\Delta\epsilon$ results in the nonmonotonic temperature dependence of the peak height of the conductance resonances as described below.

We first examine the weak potential limit $V_0 \ll \alpha\Lambda$, where a naive picture holds that free-electron propagation is slightly disturbed by barrier potentials. Since the $\tilde\theta$ field has the mass gap, we can safely integrate it out perturbatively in powers of V_0 to get a cumulant expansion for the effective potential of the phase $\bar\theta$. The first-order cumulant is then obtained as $-\frac{2V_1}{\pi\alpha}\cos\bar\theta(\tau)$, where

$$V_1 = V_0\cos(\varphi/2)\exp(-\gamma) \quad (3)$$

with

$$\varphi = \frac{2\pi eV_g}{\frac{\pi}{\eta}\Delta\epsilon + 2U} + 2k_FR, \quad (4a)$$

$$\gamma = \frac{\pi^2}{4\beta}\sum_{\omega_n}\left(U + \frac{\pi}{2\eta}\frac{|\omega_n|}{1-\exp(-|\omega_n|/\Delta\epsilon)}\right)^{-1}. \quad (4b)$$

Thus V_1 vanishes when $\cos(\varphi/2) = 0$, which is just the resonance condition. In the same way, it is easily shown that V_{2m+1} vanishes on resonance, i.e., $\varphi = (2n+1)\pi$ (n is an integer). The second-order cumulant is also obtained as $-\frac{V_2}{\pi\alpha}\cos^2\bar\theta(\tau)$, in which V_2 is given by

$$V_2 \propto \frac{V_0^2}{\pi\alpha U}(1-e^{-2\gamma})(1-e^{-2\gamma}\cos\varphi), \quad (5)$$

which shows that V_2 never vanishes even on resonance, since the exponential factor of $\cos\varphi$ is always smaller than unity. Hence away from (on)resonance the transport through the barriers depends on whether or not V_1 (V_2) is a relevant perturbation. By a renormalization-group method, we obtain the lowest-order flow equations as $dV_1/dl = (1-\eta)V_1$ and $dV_2/dl = (1-4\eta)V_2$ where $l = -\ln\Lambda$. Thus V_1 (V_2) becomes relevant when $\eta < 1$ (1/4). Hence in this weak potential limit, the phase boundary lies at $\eta = 1$ in the off-resonance case while at $\eta = 1/4$ in the on-resonance case, in agreement with the results by Kane and Fisher.[8]

The conductance G as a function of the temperature T and the gate voltage V_g is calculated perturbatively with respect to the potential V_0 up to the fourth order:

$$G = \frac{e^2\eta}{2\pi} - a_1e^2\left(\frac{V_0}{\alpha\Lambda}\right)^2\left(\frac{2\eta U}{\Delta\epsilon}\right)^\eta\left(\frac{\pi T}{\Lambda}\right)^{2(\eta-1)}\times(1+\cos\varphi) - b_1e^2\left(\frac{V_0}{\alpha\Lambda}\right)^4\left(\frac{\pi T}{\Lambda}\right)^{2(2\eta-1)}\left(\frac{\pi T}{\Delta\epsilon}\right)^{4\eta}\left(\frac{\Lambda}{U}\right)^2\times\left[1+\left(\frac{2\eta U}{\pi\Lambda}\right)^{2\eta}\cos\varphi\right]^2 + \cdots \quad (6a)$$

for $T \ll \Delta\epsilon$ and

$$G = \frac{e^2\eta}{2\pi} - a_2e^2\left(\frac{V_0}{\alpha\Lambda}\right)^2\left(\frac{2\eta U}{\pi\Lambda}\right)^\eta\left(\frac{\pi T}{\Lambda}\right)^{\eta-2}\times(1+\cos\varphi) - b_2e^2\left(\frac{V_0}{\alpha\Lambda}\right)^4\left(\frac{\pi T}{\Lambda}\right)^{2(2\eta-1)}\left(\frac{\Lambda}{U}\right)^2\times\left[1+\left(\frac{2\eta U}{\pi\Lambda}\right)^{2\eta}\cos\varphi\right]^2 + \cdots \quad (6b)$$

for $\Delta\epsilon \ll T \ll U$, where a_i and b_i are dimensionless numbers of order 1.[12] Just on resonance the second terms vanish ($\cos\varphi = -1$). Since at low temperatures $G - \frac{e^2\eta}{2\pi}$ is proportional to $T^{2(\eta-1)}$ away from resonance and to $T^{2(4\eta-1)}$ on resonance, the expansion is valid down to $T = 0$ if $\eta > 1$ and $\eta > \frac{1}{4}$, for respective cases. Otherwise, the above expansion is justified only above the temperature $\widetilde{T}$ at which the temperature-dependent correction becomes comparable to the first term $e^2\eta/2\pi$. Below $\widetilde{T}$ the conductance G will scale to zero as $T \to 0$.

Next we shall consider the strong potential limit $V_0 \gg \alpha\Lambda$. The electron transport in this limit can be viewed as the tunneling between minima of the cosine potential. For simplicity we assume $k_F R \equiv 0 \pmod{2\pi}$ and $-U - \frac{\pi}{2\eta}\Delta\epsilon \leq eV_g \leq 0$. Then the potential minima are $(\bar{\theta}, \tilde{\theta}) = (2\pi l, 2\pi m)$ with l and m being integers [Fig. 1(a)]. Since the $\tilde{\theta}$ field has the mass gap, possible configurations of the phase fields may be restricted for $T \ll U + \frac{\pi}{2\eta}\Delta\epsilon$ to the filled circles neighboring the dashed line, $\tilde{\theta} = 2\pi eV_g/(\frac{\pi}{\eta}\Delta\epsilon + 2U)$, in Fig. 1(a). Thus the problem reduces to a 1D tight-binding model with a hopping matrix element t and an off-resonance energy $\varepsilon = eV_g + U + \frac{\pi}{2\eta}\Delta\epsilon$ [Fig. 1(b)]. By introducing a heat bath consisting of harmonic oscillators linearly coupled with the phases $\bar{\theta}$ and $\tilde{\theta}$, we can construct an equivalent model to Eq. (2) as Caldeira and Leggett did.[10] The tunneling conductance G is calculated from the second-order hopping (tunneling) process as

$$G = \beta e^2 t^4 \int_{-\infty}^{\infty} dt_0 \int_0^{\infty} dt_1 \int_0^{\infty} dt_2 \langle e^{iH_i(t_0-t_1+t_2)} e^{iH_m t_1} e^{-iH_f t_0} e^{-iH_m t_2} \rangle_i e^{-\Gamma(t_1+t_2)}, \tag{7}$$

where $\langle Q \rangle_i = \mathrm{Tr}\, Q e^{-\beta H_i}/\mathrm{Tr}\, e^{-\beta H_i}$. The three Hamiltonians H_i, H_m, and H_f are those for the harmonic oscillators with shifted origins: $H_i = H(0,0)$, $H_m = H(\pi, -2\pi)$, and $H_f = H(2\pi, 0)$ where

$$H(\bar{\theta}, \tilde{\theta}) = \sum_\alpha \left(\frac{P_\alpha^2}{2M_\alpha} + \tfrac{1}{2} M_\alpha \Omega_\alpha^2 X_\alpha^2 + \bar{\lambda}_\alpha \bar{\theta} X_\alpha + \frac{\bar{\lambda}_\alpha^2 \bar{\theta}^2}{2M_\alpha \Omega_\alpha^2} \right) + \sum_\beta \left(\frac{p_\beta^2}{2m_\beta} + \tfrac{1}{2} m_\beta \omega_\beta^2 x_\beta^2 + \tilde{\lambda}_\beta \tilde{\theta} x_\beta + \frac{\tilde{\lambda}_\beta^2 \tilde{\theta}^2}{2m_\beta \omega_\beta^2} \right)$$
$$+ \frac{1}{8\pi^2}\left(U + \frac{\pi v}{\eta R} \right) \tilde{\theta}^2 - \frac{eV}{2\pi}\bar{\theta} - \frac{eV_g}{2\pi}\tilde{\theta} + \frac{2V_0}{\pi\alpha}(1 - \cos\bar{\theta}\cos\tilde{\theta}) \tag{8}$$

with

$$\sum_\alpha \frac{\pi \bar{\lambda}_\alpha^2}{2M_\alpha \Omega_\alpha} \delta(\omega - \Omega_\alpha) = \frac{\omega}{\pi\eta}\left[\frac{1}{2} + \pi\Delta\epsilon \sum_{n=1}^{\infty} \delta[\omega - \pi\Delta\epsilon(2n-1)] \right], \tag{9a}$$

$$\sum_\beta \frac{\pi \tilde{\lambda}_\beta^2}{2m_\beta \omega_\beta} \delta(\omega - \omega_\beta) = \frac{\omega}{4\pi\eta}\left[\frac{1}{2} + \pi\Delta\epsilon \sum_{n=1}^{\infty} \delta(\omega - 2\pi n \Delta\epsilon) \right]. \tag{9b}$$

The lifetime effect of the intermediate state m is taken into account by the exponential factor $e^{-\Gamma(t_1+t_2)}$ in Eq. (7), which is equivalent to the Weisskopf-Wigner theory of the resonant scattering. The escape rate Γ from the state m via the first-order tunneling process is given by

$$\Gamma = 2t^2 \int_{-\infty}^{\infty} dt_0 \langle e^{-iH_m t_0} e^{iH_i t_0} \rangle_i \approx \begin{cases} c_1 \Delta\epsilon \left(\frac{t}{\Lambda} \right)^2 \left(\frac{\pi\Delta\epsilon T}{\Lambda^2} \right)^{\frac{1}{\eta}-1}, & T \ll \Delta\epsilon \\ c_2 T \left(\frac{t}{\Lambda} \right)^2 \left(\frac{T}{\Lambda} \right)^{2(\frac{1}{\eta}-1)}, & \Delta\epsilon \ll T \ll U, \end{cases} \tag{10}$$

where the c_i's are dimensionless numbers. After some manipulations Eq. (7) can be transformed to

$$G = \frac{e^2\beta}{8\pi} \int_{-\infty}^{\infty} dE \frac{\Gamma^2}{(E-\varepsilon)^2 + \Gamma^2} \left| \frac{\Gamma\left(\frac{1}{2\eta} + i\frac{\beta E}{2\pi} \right)}{\Gamma\left(\frac{1}{2\eta} \right)} \right|^4, \tag{11}$$

where $\Gamma(x)$ is the gamma function. Note that for the noninteracting case ($\eta = 1$) Eq. (11) reproduces the correct result, $G = \frac{e^2}{2\pi} \int g(E-\varepsilon)[-f'(E)]dE$, where $f(E)$ is the Fermi distribution function and $g(E)$ is the transmission probability $\Gamma^2/(E^2+\Gamma^2)$. For general interacting systems (Luttinger liquids), however, Eq. (11) can be justified only when $T \gg \varepsilon, \Gamma$ or $\epsilon \gg T, \Gamma$. In other words, Eq. (11) is valid as long as the conductance is much smaller than $e^2\eta/2\pi$. In the case of $T \gg \varepsilon, \Gamma$ Eq. (11) is approximated as

$$G \approx \frac{e^2}{8}\beta\Gamma \left| \frac{\Gamma\left(\frac{1}{2\eta} + i\frac{\beta\varepsilon}{2\pi} \right)}{\Gamma\left(\frac{1}{2\eta} \right)} \right|^4, \tag{12}$$

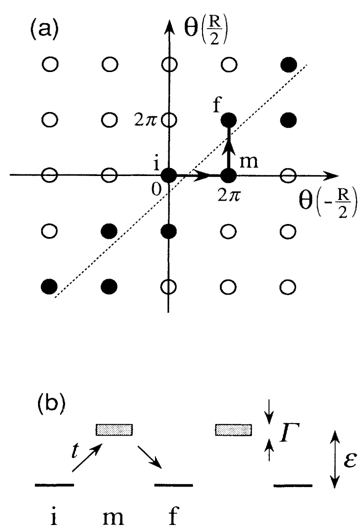

FIG. 1. (a) Illustration of possible configurations of the phase fields for $T \ll U$; the dashed line is $\tilde{\theta} = 2\pi eV_g/(\frac{\pi}{\eta}\Delta\epsilon + 2U)$. (b) Schematic of the tight-binding model.

which is proportional to $e^2(\beta\varepsilon)^{\frac{2}{\eta}-2}e^{-\beta\varepsilon}\beta\Gamma$ for $\beta\varepsilon \gg 1$. In this case the tunneling can be thought of as the thermally activated sequential tunnelings via a real transition to the intermediate state m. In the other case of $\varepsilon \gg T, \Gamma$ Eq. (11) is approximated as

$$G \approx e^2 \frac{\Gamma^2}{\varepsilon^2}. \tag{13}$$

This conductance originates from a tunneling via a virtual transition to the intermediate state.

The line shape of the resonance peaks, i.e., ε dependence of G, is now ready for discussion for $T \gg \Gamma$, which is most relevant in experiments. In this temperature range the ε dependence is given by Eq. (12) for $\varepsilon \lesssim T$ and by Eq. (13) for $\varepsilon \gg T$ (far away from resonance). We plot $|\Gamma(\frac{1}{2\eta} + i\frac{\beta\varepsilon}{2\pi})/\Gamma(\frac{1}{2\eta})|^4$ in Fig. 2. One can see that for $T \gg \Gamma$ the line shape is very similar to that for the noninteracting Fermi liquid $-f'(\varepsilon)$, when the temperature in $f(\varepsilon)$ is properly adjusted. Therefore the temperature obtained from the experimental data by applying the noninteracting electron formula is higher (lower) than the true temperature when the interaction is repulsive (attractive). The line shape in the low-temperature regime ($T \ll \Gamma$) will be briefly discussed later.

The T dependence of the resonance peak height or width, on the other hand, is quite different from the Fermi liquid due to the renormalization of Γ. In contrast to the noninteracting case ($\eta = 1$ and $U = 0$), Γ in Eq. (10) in general depends on T. Note that the interaction is taken into account through U as well as η different from 1. From Eq. (10) both Γ and Γ/T depend on T and behave as $\Gamma \sim T^{\frac{2}{\eta}-1}$, $\Gamma/T \sim T^{\frac{2}{\eta}-2}$ for $T \gg \Delta\epsilon$, and $\Gamma \sim T^{\frac{1}{\eta}-1}$, $\Gamma/T \sim T^{\frac{1}{\eta}-2}$ for $T \ll \Delta\epsilon$. When $\eta < 1/2$, both Γ and Γ/T monotonically decrease as T is lowered. When $1/2 < \eta < 1$, Γ keeps decreasing while Γ/T decreases for $T \geq \Delta\epsilon$ but turns to increase below $\Delta\epsilon$. When $1 < \eta < 2$, Γ decreases for $T \geq \Delta\epsilon$ and turns to increase below $\Delta\epsilon$ while Γ/T increases monotonically with decreasing temperature. When $\eta > 2$, both Γ and Γ/T always increase. Thus Γ or Γ/T is a nonmonotonic function of T for $1/2 < \eta < 2$. For $T \leq \Gamma$ the temperature dependence of Γ is directly observable as the width of the resonance peak, while for $T \geq \Gamma$ it is reflected not in the width $\sim T$ but in the peak height $\sim e^2\Gamma/T$. For moderate repulsive interaction ($1/2 < \eta < 1$), which we consider to be the most relevant case to experiments, the typical T dependence of the conductance line shape is summarized in Fig. 3. Starting with the temperature T ($> \Gamma$ and $> \Delta\epsilon$), as the temperature is lowered, both the peak height and the width decrease first as $T^{\frac{2}{\eta}-2}$ and T, respectively. Around $T \approx \Delta\epsilon$, however, the peak height has a minimum and turns to increase as $T^{\frac{1}{\eta}-2}$ below $\Delta\epsilon$, while the width continues to decrease as T. Further lowering the temperature to some temperature T^* ($< \Delta\epsilon$), Γ will become comparable with T, i.e., the peak height $\sim e^2$. Below this temperature T^*, the peak height saturates to be $\sim e^2$ while the width decreases as $T^{\frac{1}{\eta}-1}$.

Combining the above results for $V_0 \ll \alpha\Lambda$ and $V_0 \gg \alpha\Lambda$, we arrive at the following schematic phase diagram at $T = 0$, which was first proposed by Kane and Fisher.[8] There are three phases A, B, and C separated by two lines $\eta = 1$ and $\eta = \eta^*(V_0)$, where $\eta^*(V_0)$ continuously changes from $\eta^*(0) = \frac{1}{4}$ to $\eta^*(\infty) = \frac{1}{2}$. In the A phase ($\eta > 1$) the barrier potentials are irrelevant perturbations, and the conductance at $T = 0$ is always $\frac{e^2\eta}{2\pi}$ irrespective to V_g. In this phase the perturbative calculation in powers of $V_0/\alpha\Lambda$ works well. In the C phase ($0 < \eta < \eta^*$), on the other hand, the potentials are relevant and the conductance is always zero at $T = 0$. Lastly, in the B phase ($\eta^* < \eta < 1$), the conductance at $T = 0$ is $\frac{e^2\eta}{2\pi}$ precisely on resonance and zero otherwise. The line shape of the resonance peak in this phase is shown for $T^* < T < \Delta\epsilon$ in Fig. 2; the peak grows higher as $T^{\frac{1}{\eta}-2}$. The noninteracting Fermi liquid ($\eta = 1$) locates on the phase boundary between A and B so that, as is well known, the line shape of the resonance peak is $-f'(\varepsilon)$ at high temperature and Lorentzian with T-independent linewidth at low temperature; with decreasing temperature the peak height first increases as T^{-1}, and eventually reaches $e^2/2\pi$ at zero temperature.

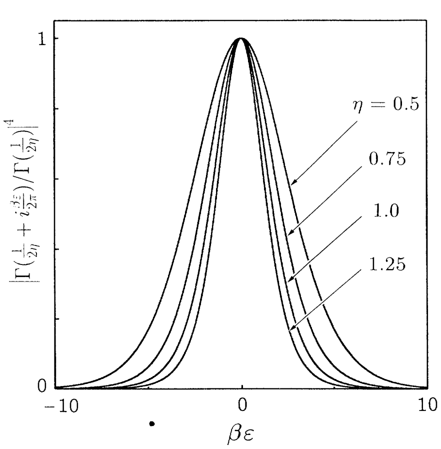

FIG. 2. The normalized line shape of the conductance G for $T \gg \Gamma$ in Eq. (12), i.e., $|\Gamma(\frac{1}{2\eta} + i\frac{\beta\varepsilon}{2\pi})/\Gamma(\frac{1}{2\eta})|^4$ as a function of ε. For $\eta = 1$ it is equal to $-4f'(\varepsilon) = \mathrm{sech}^2\frac{\beta\varepsilon}{2}$.

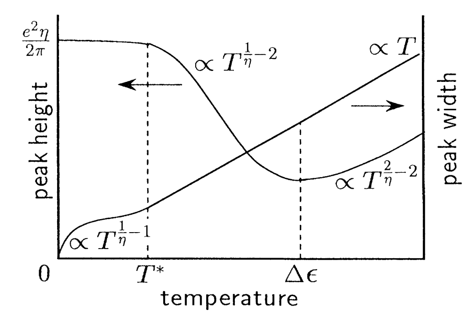

FIG. 3. The temperature dependence of the height and the width of the conductance peak for $1/2 < \eta < 1$. The peak height has a minimum around $T \approx \Delta\epsilon$. The width eventually vanishes as $T \to 0$.

Now we comment on the low-temperature line shape of the conductance resonances in the B phase. At low temperature and just on resonance the perturbative treatment with respect to $V_0/\alpha\Lambda$ is valid, and the peak value of the conductance is given by Eq. (6b) with $\cos\varphi = -1$; $G \approx e^2\eta/2\pi$. Far away from resonance, on the other hand, the perturbative calculation in powers of t becomes appropriate, and the conductance is obtained from Eq. (11) as $G \approx e^2\Gamma^2/\varepsilon^2$. It is not easy to calculate the conductance for the intermediate regime between the two limits, just on resonance and far away from resonance, since one must sum up all the contributions from the higher-order tunneling processes. Very recently Kane and Fisher[8] have argued by using the renormalization group that $G \approx e^2(c\varepsilon/T^{1-\eta})^{-\frac{2}{\eta}}$ in this regime, where c is a dimensionful constant. Therefore one can expect that as ε is increased, the conductance decreases, from the maximum value of order $e^2\eta/2\pi$, first as $\varepsilon^{-\frac{2}{\eta}}$ and then as ε^{-2}. Thus there must be a characteristic crossover energy around which the exponent changes from $-\frac{2}{\eta}$ to -2. In our theory this energy scale is implicitly assumed to be of order Γ. However, further studies on this point are needed.

Finally we discuss the effect of asymmetry of barrier structures. In asymmetric structures ($|V_1| \neq |V_2|$), the resonance cannot be achieved both in the weak-potential limit and in the strong-potential limit. In the former limit, the lowest-order cumulant of the double-barrier potential is proportional to $(V_1 + V_2)\cos\bar{\theta}\cos\frac{\varphi}{2} + (V_1 - V_2)\sin\bar{\theta}\sin\frac{\varphi}{2}$, which cannot be identical to zero unless $|V_1| = |V_2|$. Also in the opposite limit it is shown by a renormalization-group argument[8] that the true resonance cannot be achieved.

In summary, we have studied the resonant tunneling through a double-barrier structure in a 1D interacting spinless fermion system. We have calculated perturbatively the conductance as a function of T as well as of V_g or ε. It is found that for $T \gg \Gamma$ the line shape of the conductance peaks, the ε dependence, for Luttinger liquids is very similar to that for the noninteracting system. It is the T dependence of the peak height or width that is dramatically changed by the interaction, and the nonmonotonic temperature dependence of the peak height is predicted for the moderate repulsive interaction. Although further detailed analysis is needed, this mechanism may give a possible explanation of the puzzling temperature dependence of the conductance observed experimentally[2] because the relevant temperature ($\sim$0.5 K) is of the same order of the discretization energy $\Delta\epsilon \sim 0.05\,\text{meV}$ for $R \sim 1000\,\text{nm}$.

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[12]Equation (6a) is obtained under the assumption that $U \gg v/R$. The results for the case of $U = 0$ can be obtained by replacing U by $\pi v/2\eta R$ in Eq. (6a). Note also that only the leading term in each order is written in Eq. (6a); the second leading term of order V_0^2 is in fact $T^{2(\eta-1)} \times T^2 = T^{2\eta}$, which may dominate the next leading term, $V_0^4 T^{2(4\eta-1)}$.