Energy-loss rate of hot carriers in semiconductors with nonequilibrium phonon distribution in the extreme quantum limit at low temperatures

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A theoretical model has been developed to investigate the effect of nonequilibrium phonons or hot phonons on the energy-loss rate of hot carriers in semiconductors in the extreme quantum limit at low temperatures. The acoustic-phonon scattering via the deformation potential and piezoelectric scattering are assumed to be the dominant scattering mechanisms at low temperatures. The model includes band nonparabolicity, energy nonequipartition of phonons, Landau-level broadening, and classical free-carrier screening. The energy-loss rates of hot electrons with nonequilibrium phonons as well as the thermal phonon distribution have been calculated using the above-mentioned model for n-type InSb. These theoretical results have also been compared with experimental results for n-type InSb at $B=3$ T and $T_L=4.2$ K. The energy-loss rate with the thermal phonon distribution is found to be much higher than the experimental result. The energy-loss rate of hot electrons calculated with use of the nonequilibrium phonon distribution with phonon boundary scattering is reduced compared with the values of the energy-loss rate obtained with use of the equilibrium phonon distribution. The incorporation of nonequilibrium phonons brings the theoretical results into agreement with the experimental data, giving a reasonable value for the phonon lifetime. However, the phonon lifetime required to fit the experimental data is found to be much higher than the values obtained from phonon boundary scattering. This discrepancy may be attributed to the acoustic-phonon mismatch factor.

I. INTRODUCTION

Hot-electron transport in semiconductors has attracted many authors[1] for investigations in view of the applications in solid-state devices. The application of a quantizing magnetic field to a semiconductor makes hot-electron transport more interesting because of the quantum nature of the electron gas. Yao, Inagaki, and Maekawa[2] made an experimental investigation on energy relaxation time in n-type InSb in the extreme quantum limit at $B=3$ T and $T_L=4.2$ K. A theoretical analysis of the energy-loss rate in n-type InSb would be interesting for understanding the basic energy-loss mechanisms controlling hot carrier kinetics in this technologically important material. In the presence of a strong electric field, the electrons gain energy from the electric field which exceeds the thermal energy of the electrons. Thus the carrier system becomes disturbed from equilibrium and may be characterized by a temperature different from the lattice temperature. The energy gain is mainly dissipated via phonon emission and nonequilibrium phonon distribution is obtained as a result of energy transfer to the lattice when the electron system is far from equilibrium. The modification of phonon distribution will depend on the rates at which carriers supply energy to the phonons compared with the rate at which phonons lose the excess energy to the thermal bath. The modified phonon distribution will be different from the Bose-Einstein (BE) distribution and it will affect the transport properties of hot electrons significantly. The effect due to the modified phonon distribution is also called the hot-phonon effect. In this paper, a theoretical model has been developed to calculate the energy-loss rate of hot electrons in narrow gap semiconductors in the extreme quantum limit assuming an electric-field-dependent perturbed phonon distribution, i.e., a nonequilibrium phonon distribution due to the presence of the high electric field.

The model is used to study the effect of a nonequilibrium phonon distribution on the energy-loss rate in n-type InSb at low temperatures. Acoustic-phonon scattering via the deformation potential and piezoelectric coupling are considered as the dominant scattering mechanisms for the present calculation. The model also includes further complexities such as band nonparabolicity, Landau-level broadening due to electron-impurity interactions, and nonequipartition of phonons. The effect of classical free carrier screening has also been included in the analysis. It has been shown[4] that the free carrier screening in the presence of a quantizing magnetic field is not given by the classical Debye screening length. The effect of a quantizing magnetic field is to modify the classical free carrier screening into a magnetic-field-dependent anisotropic screening parameter. However, it is found that the modified quantum screening due to magnetic quantization does not affect the hot-electron energy-loss rate significantly.[5] Hence, the classical screening has been included in the model. Since the lattice temperature is not well defined in the presence of a nonequilibrium phonon distribution, the temperature T_L mentioned in the model is the ambient temperature.

The model has been used to calculate the theoretical energy-loss rate of hot electrons with nonequilibrium phonons as well as the thermal equilibrium phonon distribution in n-type InSb. These results have also been com-

pared with the experimental energy-loss rates obtained by Yao, Inagaki, and Maekawa[2] at $B=3$ T and $T_L=4.2$ K. The calculation based on the present model has been applied in the low-temperature region because of the availability of the experimental results at low temperatures.[2] The effect of longitudinal-optical (LO) phonons has not been included in the analysis due to low temperatures. However, the model can also be applied in the high-temperature region including the LO-phonon scattering in a straightforward manner in Eq. (3). The present analysis of the energy-loss rate of hot electrons in the extreme quantum limit has been performed due to the simplicity of the results because of the absence of oscillatory effects. The oscillatory effect may arise because of inter-Landau-subband scattering.

The theoretical results with the thermal equilibrium phonons are found to be much higher than the experimental energy-loss rate. We consider that this discrepancy is due to the perturbed phonon distribution or nonequilibrium phonon distribution. When we take into consideration the hot-phonon effect or nonequilibrium phonon effect with the phonon scattering lifetime for acoustic phonons given by the boundary phonon scattering,[6] the theoretical values of the energy-loss rate are reduced compared with the thermal phonon values.

At very low temperatures, the phonon mean free path becomes comparable to the sample dimensions and the phonons are scattered from the boundaries of the sample in a time $\tau_p=L/u$, where L is the shortest dimension of the sample and u is the acoustic sound velocity. The quantity τ_p is also called the phonon lifetime. There are other phonon scattering processes contributing to phonon relaxation processes, such as phonon-phonon scattering in addition to phonon boundary scattering. A relaxation-time approximation for the phonon scattering is assumed in the present model.[7–9] It is found that the incorporation of nonequilibrium acoustic phonons brings the theoretical results on the energy-loss rate into agreement with the experimental data, giving a reasonable value for the phonon lifetime. However, the phonon lifetime obtained from the boundary scattering is found to be less than the value required to fit the experimental data. The difference in the phonon lifetime to fit the experimental results and the value obtained from the phonon boundary condition may be attributed to the acoustic-phonon mismatch factor.[7] The acoustic-phonon mismatch factor in the boundary scattering originates from the boundary losses.[7,9] It is primarily determined from the reflections and transmissions of elastic waves at the boundary between the crystal and the surrounding medium.

We present the theoretical formulation in Sec. II and the results and conclusions in Sec. III.

II. THEORETICAL FORMULATION

We consider a nonparabolic, spherically symmetric conduction-band semiconductor. We assume a strong quantizing magnetic field B in the z direction so that this field quantizes the energy levels and causes the electrons to occupy the lowest Landau-level subband. This is the extreme quantum limit condition. We also assume a heating electric field applied in the z direction, i.e., the direction of **B**. The energy dispersion relation for these electrons in the conduction band can be written as[10]

$$E=-\frac{E_g}{2}+\frac{E_g\alpha_0}{2}+\frac{\hbar^2k_z^2}{2m^*\alpha_0}\ , \tag{1}$$

where E is the electron energy, E_g the band-gap energy, $\hbar$ Planck's constant divided by 2π, k_z the z component of the electron wave vector, m^* the band-edge effective mass, and

$$\alpha_0=\left[1+\frac{2\hbar\omega_c}{E_g}\left[1-|g|\frac{m^*}{2m_0}\right]\right]^{1/2} \tag{2}$$

the nonparabolicity factor which tends to unity for the parabolic band semiconductor. In Eq. (2) $\omega_c=eB/m^*$, e being the electronic charge, m_0 the free-electron mass, and $|g|$ the spin-split g factor.

The electrons in the conduction band gain energy continuously from the applied electric field. But this energy gain is balanced by the energy loss of the electrons due to scattering with the lattice in the steady state. We assume that the electron gas behaves as a nondegenerate electron gas which obeys the displaced Maxwellian distribution with electron temperature T_e in the presence of a heating electric field. Thus the energy-loss rate per electron due to the scattering with the lattice can be written as[11]

$$P=\frac{(m^*\alpha_0)^{1/2}\omega_0}{\pi(2\pi k_BT_e)^{1/2}t_l}\exp\left[-\frac{m^*\alpha_0u^2}{2k_BT_e}\right]\int_0^\infty q_\perp dq_\perp\int_0^\infty\frac{dq_z}{q_z}C_i|f_i(q)|^2\exp\left[-\frac{l^2q_\perp^2}{2}-\frac{\hbar^2q_z^2}{8m^*\alpha_0k_BT_e}-\frac{m^*\alpha_0u^2}{2k_BT_e}\frac{q_\perp^2}{q_z^2}\right]$$
$$\times[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)]\ , \tag{3}$$

where k_B is the Boltzmann constant, u is the sound velocity, $l=(\hbar/eB)^{1/2}$ is the Landau radius, q_z and $q_\perp$ are the longitudinal and transverse components of the phonon wave vector q, respectively, $\nu_e=\hbar\omega_0/2k_BT_e$, $C_i|f_i(q)|^2$ is the coupling constant due to electron-phonon interaction which depends on the free carrier screening, and $\hbar\omega_0=\hbar uq=huq_\perp(1+q_z^2/q_\perp^2)^{1/2}$ is the acoustic-phonon energy. The term $q_z^2/q_\perp^2$ in the expression of $\hbar\omega_0$ is very small and its effect on the energy-loss rate calculation is negligible, so it is neglected. Hence, $\hbar\omega_0=\hbar uq_\perp=\hbar u/l^5$ [because $q_\perp\sim1/l$]. In Eq. (3), N_R represents the phonon occupation number in the respective cases of thermal and

nonequilibrium phonon distributions.

The coupling constant for the acoustic-phonon scattering via deformation potential with classical screening is given by[12]

$$C_i|f_i(q)|^2=\frac{C_{\rm ac}q}{[1+q_s^2/q^2]^2}\ , \tag{4}$$

where $C_{\rm ac}=E_1^2\hbar/2\rho u_{\rm ac}$, q_s is the inverse Debye screening length, E_1 is the deformation-potential constant, ρ is the mass density, and $u_{\rm ac}$ is the acoustic sound velocity.

Similarly, the coupling constant for the piezoelectric scattering with classical screening can be expressed as

$$C_i|f_i(q)|^2=\frac{C_{\rm pz}}{q[1+q_s^2/q^2]^2}\ , \tag{5}$$

where

$$C_{\rm pz}=\hbar^2e^2e_{14}^2/2\rho u_p\epsilon_s^2\epsilon_0^2\ ,$$

e_{14} is the piezoelectric modulus, u_p is the piezoelectric sound velocity, and ϵ_s is the dielectric constant of the material.

Now, we use Eqs. (4) and (5) in Eq. (3) and make the following substitutions:

$$\nu=\frac{\hbar^2u^2}{8k_B^2T_e^2l^2},\quad u=\tfrac{1}{2}l^2q_\perp^2\ ,\quad \alpha_s=q_s^2l^2\ ,$$

$$\beta=\frac{8m^*a_0l^2k_BT_e}{\hbar^2},\quad \upsilon=\frac{\hbar^2q_z^2}{8m^*a_0k_BT_e}\ .$$

Then the energy-loss rates per electron for acoustic scattering via the deformation potential and piezoelectric coupling are given by

$$P_{\rm ac}=[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)]W_{\rm ac}\times\int_0^\infty f_{\rm ac}(\upsilon)\exp(-\upsilon)\frac{d\upsilon}{\upsilon} \tag{6}$$

and

$$P_{\rm pz}=[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)]W_{\rm pz}\times\int_0^\infty f_{\rm pz}(\upsilon)\exp(-\upsilon)\frac{d\upsilon}{\upsilon}\ , \tag{7}$$

where

$$W_{\rm ac}=\frac{(m^*a_0)^{1/2}E_1^2\omega_0}{8\pi(k_BT_e)^{1/2}l^3\rho u_{\rm ac}}\exp\left[-\frac{m^*a_0u_{\rm ac}^2}{2k_BT_e}\right]\ , \tag{8}$$

$$f_{\rm ac}(\upsilon)=(1+\beta\upsilon/u_0)^{5/2}(1+\nu/\upsilon)^{-3/2}\times[1+(\beta\upsilon+\alpha_s)/u_0]^{-2} \tag{9}$$

and

$$W_{\rm pz}=\frac{(m^*a_0)^{1/2}e^2e_{14}^2\omega_0}{8\pi(k_BT_e)^{1/2}\rho u_p\epsilon_s^2\epsilon_0^2l}\exp\left[-\frac{m^*a_0u_p^2}{2k_BT_e}\right]\ , \tag{10}$$

$$f_{\rm pz}(\upsilon)=(1+\beta\upsilon/u_0)^{3/2}(1+\nu/\upsilon)^{-1/2}\times[1+(\beta\upsilon+\alpha_s)/u_0]^{-2}\ . \tag{11}$$

In obtaining Eqs. (6) and (7) from Eq. (3), the integrals over u are simplified by assuming the functions $(1+\beta\upsilon/u_0)^{5/2}[1+(\beta\upsilon+\alpha_s)/u_0]^{-2}$ and $(1+\beta\upsilon/u_0)^{3/2}[1+(\beta\upsilon+\alpha_s)/u_0]^{-2}$ to be slowly varying functions of u, and taken outside the integral by putting $u=u_0/2=\frac{1}{2}l^2q_\perp^2\approx1$. This approximation can be justified for the extreme quantum limit condition.[7] The functions $f_{\rm ac}(\upsilon)$ and $f_{\rm pz}(\upsilon)$ in Eqs. (9) and (10) are also slowly varying functions of υ, so they can be taken outside the integral by taking the maximum value of υ as unity. The energy-loss rates $P_{\rm ac}$ and $P_{\rm pz}$ show a divergence at $\upsilon=0$ or $q_z=0$. This divergence may be overcome if we consider the effect of Landau-level broadening due to electron-impurity interactions. This corresponds to taking the lower limit of the integral as $E_c/4k_BT_e$, where E_c is the cutoff parameter and it measures the Landau-level broadening. The cutoff parameter E_c is obtained from the measurements of the damping constant of Shubnikov–de Haas oscillations of magnetoresistance and it depends on the magnetic flux density B as $B^{2/3}$.[13] Thus the general expression of the energy-loss rate can be expressed in the following manner:

$$P=\alpha[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)]\ . \tag{12}$$

The corresponding energy-loss rates due to acoustic-phonon scattering via the deformation potential and piezoelectric coupling can be written as

$$P_{\rm ac}=\alpha_{\rm ac}[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)] \tag{13}$$

and

$$P_{\rm pz}=\alpha_{\rm pz}[(N_R+1)\exp(-\nu_e)-N_R\exp(\nu_e)]\ , \tag{14}$$

where

$$\alpha_{\rm ac}=W_{\rm ac}f_{\rm ac}\ln\left[\frac{4k_BT_e}{E_c}\exp(-C)\right]$$

and

$$\alpha_{\rm pz}=W_{\rm pz}f_{\rm pz}\ln\left[\frac{4k_BT_e}{E_c}\exp(-C)\right]\ .$$

Here C is Euler's constant and $f_{\rm ac}(\upsilon)$ and $f_{\rm pz}(\upsilon)$ are calculated at $\upsilon=1$ and Eqs. (13) and (14) are the expressions of the energy-loss rates both for the thermal and the nonequilibrium phonon distributions. To get the expressions of the energy-loss rates for the nonequilibrium phonon as well as for thermal phonon distribution, we must obtain the phonon occupation number N_R in the respective cases.

A. Nonequilibrium phonons

First we consider the case of a nonequilibrium phonon distribution In this case the phonon occupation number can be obtained in the following manner.

For a nonequilibrium phonon distribution the energy-loss rate per electron can be written as

$$P=-\frac{1}{n_0 V}\sum_q \hbar\omega_0\left[\frac{\partial N_R}{\partial t}\right]_e$$
$$=\frac{\hbar\omega_0}{4\pi^2 n_0}\left[\frac{\partial N_R}{\partial t}\right]_e\int_0^{q_{\perp m}} q_\perp dq_\perp \int_0^{q_{zm}} dq_z \ , \quad (15)$$

where n_0 is the carrier density, V is the crystal volume, $q_{\perp m}^2=1/l^2$, and $\hbar^2 q_{zm}^2/2m^*\alpha_0=4k_B T_e$. The term $(\partial N_R/\partial t)_e$ in Eq. (15) represents the rate of generation of phonons due to hot electrons in the steady state, and it can be written as[3]

$$\left[\frac{\partial N_R}{\partial t}\right]_e=\frac{N_R-N_0}{\tau_p} \ , \quad (16)$$

where N_R and N_0 are the nonequilibrium and thermal phonon occupation numbers, respectively. The right-hand side of Eq. (16) represents the rate of loss of phonons with phonon lifetime τ_p.

Hence, from (15) to (16), we obtain

$$\frac{N_R-N_0}{\tau_p}=\frac{2\sqrt{2}\pi^2 n_0 l^2}{\omega_0(m^*\alpha_0)^{1/2}(k_B T_e)^{1/2}}P \ . \quad (17)$$

Combining Eqs. (12) and (17) we get

$$N_R=\frac{a\alpha\tau_p\exp(-\nu_e)+N_0}{a\alpha\tau_p[\exp(\nu_e)-\exp(-\nu_e)]+1} \ , \quad (18)$$

where

$$a=\frac{2\sqrt{2}\pi^2 n_0 l^2}{\omega_0(m^*\alpha_0 k_B T_e)^{1/2}} \ .$$

Equation (18) represents the phonon occupation number for nonequilibrium phonon distribution where τ_p is the phonon relaxation time given by the phonon boundary scattering. Substituting $\alpha=\alpha_{ac}$ and $\alpha=\alpha_{pz}$ in Eq. (18), where α_{ac} and α_{pz} are constants dependent on material parameters given in Eqs. (13) and (14), we obtain the phonon occupation numbers for the acoustic-phonon scattering via the deformation-potential and piezoelectric coupling, respectively. Now, we substitute the corresponding phonon occupation numbers in Eqs. (13) and (14) to get the energy-loss rates for acoustic-phonon and piezoelectric scattering.

B. Thermal phonons

When we consider the case of thermal phonons, the phonon occupation number N_R is obtained from the BE statistics, i.e., $N_R=[\exp(\hbar\omega_0/k_B T_L)-1]^{-1}$. For thermal phonon distribution, the phonon occupation number is obtained from Eq. (17) in the limiting value of $\tau_p\to 0$. So, the energy-loss rates due to acoustic-phonon scattering via the deformation potential and piezoelectric scattering with thermal phonons are obtained by putting

$$N_R=N_0=\left[\exp\left[\frac{\hbar\omega_0}{k_B T_L}\right]-1\right]^{-1}$$

in Eqs. (13) and (14).

III. RESULTS

The numerical results on the energy-loss rate of hot electrons are obtained for n-type InSb at $B=3$ T and $T_L=4.2$ K. The material parameters for n-type InSb (Refs. 14 and 15) are given below:

$m^*=0.014m_0$, $E_g=0.24$ eV, $\rho=5.8\times10^3$ kg m^{-3},
$|g|=47$, $u_{ac}=3.74\times10^3$ ms^{-1}, $E_1=7.2$ eV,
$e_{14}=0.06$ cm^{-2}, $u_p=2.8\times10^3$ ms^{-1}, $\epsilon_s=15.68$,
$n_0=1.07\times10^{14}$ cm^{-3}, $E_c=0.1$ meV.

The cutoff energy E_c required for the present calculation is not known accurately. However, it must be comparable to the thermal energy and is taken to be 0.1 meV at $B=3$ T and $T_L=4.2$ K in agreement with other measurements.[16,17] The energy-loss rate [Eqs. (13) and (14)] depends on E_c logarithmically. So the calculated loss rates are insensitive to the uncertainty in the value of E_c.

The theoretical results for energy-loss rate with the nonequilibrium phonon distribution are presented along with the theoretical results for thermal phonons. These results are also compared with the experimental results and shown in Fig. 1. The theoretical results in Fig. 1 assume the combined effects of acoustic-phonon scattering via the deformation potential and piezoelectric coupling and the boundary phonon scattering for phonon relaxation in the case of the nonequilibrium phonon distribution. It is found from Fig. 1 that the theoretical results for thermal phonons are much larger than the experimental loss rates. However, the inclusion of nonequilibrium phonons lowers the theoretical value and brings it closer

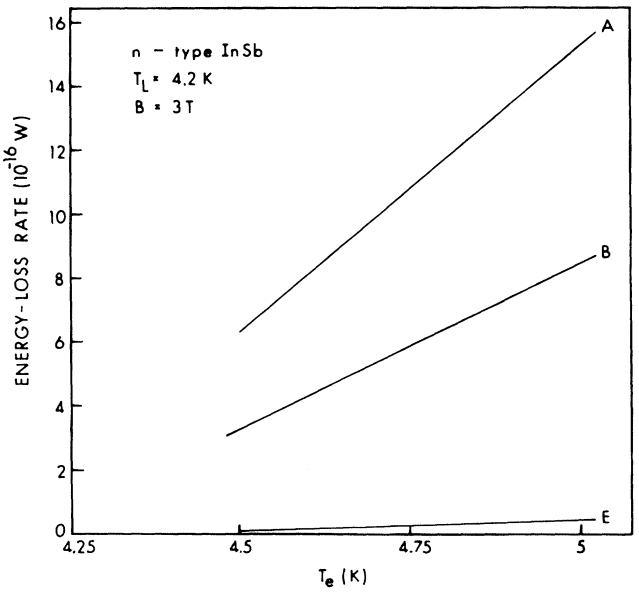

FIG. 1. Variation of the energy-loss rates in n-type InSb at $B=3$ T and $T_L=4.2$ K as a function of electron temperature. Curves A and B represent the theoretical results with thermal and nonequilibrium phonon distributions, respectively, while curve E represents the experimental results.

to the experimental results. The phonon lifetime is obtained assuming L to be the shortest dimension of the sample, which is 0.052 cm. The average phonon lifetime required to fit the experimental data is found to be 1614 ns, which is quite large compared to the phonon lifetime obtained from the phonon boundary scattering. This discrepancy in the phonon lifetime may be attributed to the acoustic-phonon mismatch factor[7] in semiconductors and the uncertainty in the material parameters. In addition there may be other phonon relaxation processes.[6]

Furthermore, we have also investigated the effect of individual scattering mechanisms on the energy-loss rate of hot electrons with thermal and nonequilibrium phonon distributions. The energy-loss rates due to acoustic-phonon scattering via the deformation potential and piezoelectric coupling assuming the phonon boundary scattering with the shortest dimension of the sample are presented in Fig. 2 for the thermal phonons and nonequilibrium phonons, respectively. These theoretical results are also compared with the experimental results in Fig. 2. It is observed that the energy-loss rates due to piezoelectric scattering show better agreement with the experimental results in comparison with the acoustic-phonon scattering via the deformation potential. The phonon lifetime in the case of piezoelectric phonon scattering obtained from the boundary scattering is 184.7 ns, which is slightly higher than the phonon lifetime for acoustic-phonon scattering via the deformation potential.

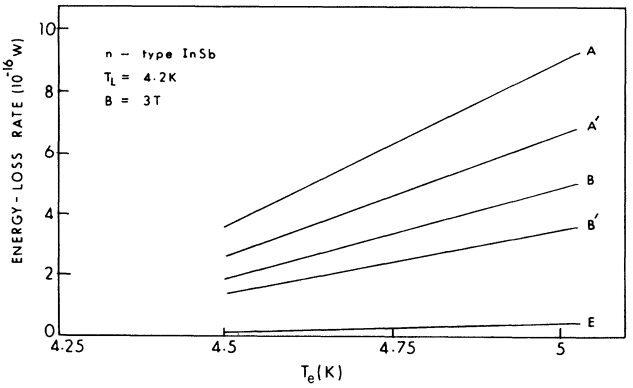

FIG. 2. Variation of the energy-loss rates in n-type InSb at $B=3$ T and $T_L=4.2$ K as a function of electron temperature. Curves A and B represent the energy-loss rates due to acoustic-phonon scattering via the deformation potential with thermal and nonequilibrium phonon distributions, respectively, while curves A' and B' represent the same due to piezoelectric scattering. Curve E represents the experimental loss rates.

The present work also suggests that measurements of the energy-loss rate in semiconductors may be useful in investigating phonon relaxation processes in semiconductors in addition to electron-phonon interactions in semiconductors.

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