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Conductivity of weakly disordered metals for an arbitrary, momentum-dependent, integrable scattering potential

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It is pointed out that the explicit form of the impurity scattering potential is not needed to derive the electrical conductivity of disordered metals in the quantum weakly localized regime. The derivation can be done for any momentum-dependent well-behaved potential, in terms of only the transport time and the elastic lifetime of the electrons. The specific form of the scattering potential is needed only for the computation of these two characteristic times. Comparison is made with previous results. The question is raised whether the one-parameter scaling remains valid in the general case of an arbitrary potential.

I. INTRODUCTION

The problem of the Anderson transition in disordered metals containing randomly spread nonmagnetic impurities has been extensively studied during the past decade.[1] The perturbative treatment in the weakly localized regime (for $\epsilon_F\tau \gg 1$, ϵ_F being the Fermi energy and τ the elastic lifetime) has proven to be quite useful in the case of free electrons, which is the case to which we will confine ourselves in the following (we will also assume neither inelastic nor spin-orbit scattering). Various studies have appeared, in general using a contact-type impurity scattering potential.[1] However, a few papers have dealt with "correlated disorder" models.[2] The case of interacting impurities has also been considered[3] when a small amount of local ordering between the impurities is taken into account. Recently,[4] in the independent-impurity case, a Yukawa-type of scattering potential with a variable range was examined. From these studies, it appears that, for integrable, well-behaved potentials belonging to the short-range universality class, the fixed point of the conductance, in $2+\epsilon$ dimensions, remains unchanged. On the other hand, except for the contact potential case and independent impurities, the transport time $\tau_{\rm tr}$ is different from the elastic lifetime τ, as it is affected differently by the specific scattering potential which is involved.

In the present paper, we point out that the precise form of the potential is not needed to derive, to first order in perturbation, the electrical conductivity σ in the weakly localized regime. The general form of σ is obtained in terms of $\tau_{\rm tr}$ and τ only, for the arbitrary dimension d. From such a derivation, it is particularly clear why the fixed point in the $2+\epsilon$ dimension remains unchanged (as long as one deals with an integrable, well-behaved potential belonging to the short-range universality class). From the general formula for σ, it is easy to verify that one recovers the usual contact-potential result for independent impurities.[1] We also discuss the result obtained for a Yukawa-type potential in Ref. 4, as well as the one for the contact-potential but interacting impurities of Ref. 3.

II. THE GENERAL FORMULA FOR THE CONDUCTIVITY

We consider an arbitrary, but integrable, well-behaved isotropic (to simplify) potential $V(r)$, whose Fourier transform $V(|\mathbf{k}-\mathbf{k}'|)$ depends on the transfer momentum $\mathbf{k}-\mathbf{k}'$ before and after collision (or on the scattering angle θ between $\mathbf{k}$ and $\mathbf{k}'$), in an *arbitrary* manner. Since the conduction electrons are close to the Fermi surface, we take advantage of the fact that $|\mathbf{k}|\approx|\mathbf{k}'|\approx k_F$, so that $V(|\mathbf{k}-\mathbf{k}'|)$ is only a function of $|\mathbf{k}-\mathbf{k}'| =\sqrt{2k_F^2(1-\cos\theta)}$, denoted as $[V(1-\cos\theta)]$. Standard techniques[5] tell us that the elastic lifetime τ is given by

$$\frac{1}{\tau}=2\pi N(0)n_I\int [v^2(1-\cos\theta)]\frac{d\Omega}{\int d\Omega}\ , \qquad (1)$$

where $N(0)$ is the density of states at the Fermi energy per spin direction, n_I is the impurity density, and the angular integral $\int d\Omega$ is

$$\int d\Omega=\frac{2\pi^{d/2}}{\Gamma\left(\dfrac{d}{2}\right)}\ . \qquad (2)$$

$\Gamma(d/2)$ is the gamma function[6] for the argument $d/2$. The transport time $\tau_{\rm tr}$ is given by

$$\frac{1}{\tau_{\rm tr}}=2\pi N(0)n_I\int (1-\cos\theta)[V^2(1-\cos\theta)]\frac{d\Omega}{\int d\Omega}\ . \qquad (3)$$

The Drude part of the conductivity, shown in Fig. 1(a), is obtained directly through the Einstein relation:

$$\sigma_{\rm Drude}=\frac{dn}{d\mu}D\ . \qquad (4)$$

D is the diffusion coefficient given in atomic units by

$$D=\frac{k_F^2\tau_{\rm tr}}{d}\ . \qquad (5)$$

$dn/d\mu$ is the derivative with respect to the chemical potential μ of the effective number of electrons per unit volume. We thus obtain, alternatively,

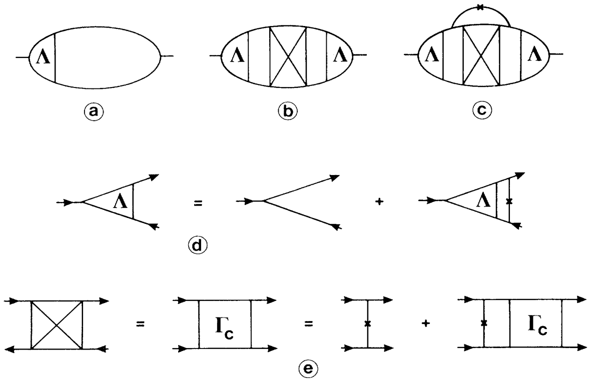

FIG. 1. (a), (b), and (c) are the diagrams contributing to the conductivity to first order in perturbation. (c) must be counted twice to account for the symmetric diagram with the extra single-impurity line dressing the bottom electron line. The vertex renormalization **Λ** obeys a Bethe-Salpeter equation diagrammatically shown in (d). The cooperon Γ_c, denoted by a square box with diagonals, obeys a Bethe-Salpeter equation shown in (e). In all these diagrams, the full lines are electron lines dressed by impurity scattering, and the ones with crosses single-impurity lines.

$$\sigma_{\text{Drude}}=2N(0)D\ . \tag{6}$$

On the other hand, one can calculate σ_{Drude} from the diagrams in Fig. 1(a), with the impurity vertex **Λ**, obeying the Bethe-Salpeter equation shown in Fig. 1(d) and calculated with the standard techniques[5] to be

$$\mathbf{\Lambda}=\mathbf{k}\cdot\lambda\ ,\qquad \lambda=\frac{\tau_{\text{tr}}}{\tau} \tag{7}$$

in the limit where the external Matsubara frequency $\omega_\nu \to 0$; **k** is the momentum of the electron lines in Fig. 1(a). Then, computing the diagram in Fig. 1(a), we obtain

$$\begin{aligned}\sigma_{(a)}&=\sigma_{\text{Drude}}\\ &=2\frac{\mathbf{k}\lambda\cdot\mathbf{k}}{d}N(0)\left[\int G(\mathbf{k},\tilde{\omega}_{n+\nu})G(\mathbf{k},\tilde{\omega}_n)\frac{d\xi}{2\pi}\right]_{\omega_\nu\to 0}\\ &=2\frac{k_F^2}{d}\frac{\tau_{\text{tr}}}{\tau}N(0)\tau=2N(0)D\end{aligned} \tag{8}$$

which identifies with (6). The factor 2 accounts for the spin degeneracy; $G(\mathbf{k},\tilde{\omega}_n)$ is the electron Green's function for the kinetic energy $\xi=k^2/2-\mu$ with momentum **k** and Matsubara frequency $\tilde{\omega}_n=\omega_n+1/(2\tau)\text{sgn}\omega_n$, $\omega_n=2\pi T(n+\frac{1}{2})$; however, ω_n does not appear in the result of the integral in (8).

The weak-localization correction comes from the conductivity diagrams shown in Figs. 1(b) and 1(c). By symmetry, diagram 1(c) has to be counted twice since the single-impurity line can dress the bottom electron line as well as the top one. Computing diagram 1(b) and taking care of momentum conservation, we obtain

$$\begin{aligned}\sigma_{(b)}=&\frac{2(\mathbf{k}\lambda)\cdot(-\mathbf{k}\lambda)}{d}\left[\int\frac{d^d\mathbf{Q}}{(2\pi)^d}\Gamma_c\right]\\ &\times N(0)\left[G^2(\mathbf{k},\tilde{\omega}_{n+\nu})G^2(\mathbf{k},\omega_n)\frac{d\xi}{2\pi}\right]_{\omega_\nu\to 0}\end{aligned} \tag{9}$$

where the cooperon Γ_c (with momentum **Q** and Matsubara frequency ω_ν) shown in Fig. 1(e) is known to have a diffusive pole due to the Ward identity, which insures the conservation of the total number of particles[7] as

$$(\Gamma_c)_{\omega_\nu=0}=\frac{1}{2\pi N(0)\tau^2}\frac{1}{DQ^2}\ ,\quad Q\to 0\ . \tag{10}$$

Therefore, we obtain

$$\sigma_{(b)}=-4\frac{k_F^2}{d}\frac{\tau_{\text{tr}}^2}{\tau^2}N(0)\tau^3\left[\int\frac{d^d\mathbf{Q}}{(2\pi)^d}\Gamma_c\right]\ . \tag{11}$$

For diagram 1(c), we also obtain

$$\begin{aligned}\Gamma_{(c)}&=4\int\frac{(-\mathbf{k}\lambda)\cdot(-\mathbf{k}'\lambda)}{d}n_I[V^2(1+\cos\theta)]d\theta\left[\int\frac{d^d\mathbf{Q}}{(2\pi)^d}\Gamma_c\right]N^2(0)\left[\int G^2(\mathbf{k},\tilde{\omega}_{n+\nu})G(\mathbf{k},\tilde{\omega}_n)d\xi\right]^2\\ &=-4\frac{k_f^2}{d}\frac{\tau_{\text{tr}}^2}{\tau^2}N^2(0)\tau^4\int\cos\theta n_I[V^2(1+\cos\theta)]d\theta\left[\int\frac{d^dQ}{(2\pi)^d}\Gamma_c\right]\ .\end{aligned} \tag{12}$$

Therefore, combining the localization terms, we have

$$\sigma_{(b)+(c)}=-4\frac{k_F^2}{d}\tau_{\text{tr}}^2N(0)\tau\left[\int\frac{d^d\mathbf{Q}}{(2\pi)^d}\Gamma_c\right]\left\{1+2\pi N(0)\tau\int\cos\theta n_I[V^2(1+\cos\theta)]\frac{d\theta}{2\pi}\right\}\ . \tag{13}$$

Indeed, a detailed examination of the momenta involved in diagram (c) of Fig. 1 easily shows that it is $[V^2(|\mathbf{k}+\mathbf{k}'|)]$ or $[V^2(1+\cos\theta)]$ which appears instead of $[V^2(|\mathbf{k}-\mathbf{k}'|)]$. However, one must take into account the following identity:

$$\begin{aligned}\frac{1}{\tau}+2\pi N(0)n_I\int\cos\theta[V^2(1+\cos\theta)]\frac{d\theta}{2\pi}&=2\pi N(0)n_I\left\{\int[V^2(1-\cos\theta)]\frac{d\theta}{2\pi}+\int\cos\theta[V^2(1+\cos\theta)]\frac{d\theta}{2\pi}\right\}\\ &\equiv 2\pi N(0)n_I\int(1-\cos\theta)[V^2(1-\cos\theta)]\frac{d\theta}{2\pi}=\frac{1}{\tau_{\text{tr}}}\ .\end{aligned} \tag{14}$$

Therefore,

$$\sigma_{(b)+(c)} = -\frac{4k_F^2}{d}\tau_{\rm tr}^2 N(0)\frac{\tau^2}{\tau_{\rm tr}}\left[\int \frac{d^d\mathbf{Q}}{(2\pi)^d}\Gamma_c\right] . \tag{15}$$

Using (2) and (10), we obtain

$$\begin{aligned}\int \frac{d^d\mathbf{Q}\Gamma_c}{(2\pi)^d} &= \frac{\int d\Omega}{(2\pi)^d}\int \frac{Q^{d-1}dQ}{2\pi N(0)\tau^2 DQ^2}\\ &= \frac{1}{\Gamma\left[\frac{d}{2}\right]2^d\pi^{(d/2+1)}}\frac{1}{N(0)\tau^2}\frac{d}{k_F^2\tau_{\rm tr}}\\ &\quad\times\int Q^{d-3}dQ ,\end{aligned} \tag{16}$$

so that

$$\sigma_{(b)+(c)} = -\frac{1}{\Gamma\left[\frac{d}{2}\right]2^{d-2}\pi^{(d/2+1)}}\int_{L^{-1}}^{Q_0} Q^{d-3}dQ . \tag{17}$$

The lower cutoff in the integral (when needed) is taken to be the inverse of the linear dimension of the system, in the absence of any other natural cutoff (we recall that we have assumed that we have no inelastic or spin-orbit scattering). The upper cutoff Q_0 will be discussed shortly. We note that $\tau_{\rm tr}$ *as well as* τ *has disappeared* from the coefficient in front of the integral over Q in (17), which is a pure number unaffected by the scattering potential. This tells us that, particularly in the $2+\epsilon$ dimension, the conductance fixed point remains the same, independent of the specific form of the potential.

In order to choose the appropriate cutoff Q_0, we ought to take into account the following conditions.

(i) ($k_F Q\tau<1$) is needed for the small-Q expansion of the single-particle Green's function to be valid.

(ii) On the other hand, the diffusive pole of the cooperon $\Gamma_c(Q,\omega_\nu)$ requires that $\omega_\nu\tau<1$ and $DQ^2\tau<1$. The last condition reads $(k_F Q\sqrt{\tau\tau_{\rm tr}})<1$.

It is therefore reasonable, on physical grounds, to choose Q_0 to be equal to $\{\min[(k_F\tau)^{-1},(k_F\sqrt{\tau_{\rm tr}\tau})^{-1}]\}$.

The common belief is that the small-Q range of the cooperon extends over $0<DQ^2<\tau_{\rm tr}^{-1}$, although this has never been proved microscopically. Instead, here, formula (10) follows from first-principles arguments given in Ref. 7, and has also been derived rigorously in Ref. 4 for the particular case of the Yukawa potential. (10) implies that $0<DQ^2<\tau^{-1}$, yielding condition (ii) above. When $\tau^{-1}<\tau_{\rm tr}^{-1}$ (a case encountered, for instance, in Ref. 3), there is no ambiguity, and Q_0 is equal to $(k_F\tau)^{-1}$. In the case where $\tau_{\rm tr}^{-1}<\tau^{-1}$, it may be argued that it takes the longest time, i.e., $\tau_{\rm tr}$, for the electron to come back to the point of origin after many collisions, while the time $\sqrt{\tau\tau_{\rm tr}}$, being shorter, allows for many less such events. If one could perform a calculation to higher orders in perturbation, in particular for the single-particle Green's-function self-energy, one could possibly imagine that in its renormalized lifetime, $\tau_{\rm tr}$ would replace τ, so that τ would disappear from the calculations and only $\tau_{\rm tr}$ would survive. However, to our knowledge, such a study has not been made. Therefore, to first order in perturbation theory, which is the case to which we confine ourselves here, it follows from the microscopic calculation that DQ^2 ought to be smaller than τ^{-1} so that $Q_0=(k_F\sqrt{\tau\tau_{\rm tr}})^{-1}$ when $\tau_{\rm tr}^{-1}<\tau^{-1}$.

Then, combining (8) and (17), we obtain

$$\sigma=\sigma_{(a)-(c)}=\frac{dn}{d\mu}D-\frac{1}{\Gamma\left[\frac{d}{2}\right]2^{d-2}\pi^{(d/2+1)}}\int_{L^{-1}}^{\min[(k_F\tau)^{-1},(k_F\sqrt{\tau\tau_{\rm tr}})^{-1}]}Q^{d-3}dQ \tag{18}$$

with D given by (5) with (3). Thanks to identity (14), formula (18) has been derived *without any knowledge of the specific form of the potential* , which will be needed only to calculate explicitly $\tau_{\rm tr}$ and τ. This will be done in the following section for some particular cases.

III. APPLICATIONS OF THE GENERAL FORMULA (18) TO KNOWN CASES

In this section, we examine various scattering potential forms.

A. The contact potential with independent impurities, $d=3,2$

This case has been extensively studied in the literature.[1] Here $V^2(|\mathbf{k}-\mathbf{k}'|)$ is a constant V_0^2. Then,

$$\frac{1}{\tau_{\rm tr}}=\frac{1}{\tau}=2\pi N(0)n_I V_0^2 . \tag{19}$$

Thus (18) yields [with (Ref. 6) $\Gamma(3/2)=\sqrt{\pi}/2$ and $\Gamma(1)=1$]

$$\begin{aligned}\sigma&=\frac{k_F^3\tau}{3\pi^2}-\frac{1}{\pi^3}\left[\frac{1}{k_F\tau}-\frac{1}{L}\right], \quad d=3\\ \sigma&=\frac{k_F^2\tau}{2\pi}-\frac{1}{\pi^2}\ln\left[\frac{L}{k_F\tau}\right], \quad d=2 ,\end{aligned} \tag{20}$$

which are well-known results.[1]

B. The Yukawa potential for variable range and independent impurities for $d=2$

In Ref. 4, a Yukawa type of scattering potential was studied, $V(r)=\alpha[k_F/(2\pi)]\exp(-\gamma k_F r)/r$ (where α and

γ are numbers), whose Fourier transform yielded $d=2$:

$$V(|\mathbf{k}-\mathbf{k}'|)=\alpha/(\gamma^2+|\mathbf{k}-\mathbf{k}'|^2/k_F^2)^{1/2}\ .$$

Then

$$n_I[V^2(1-\cos\theta)]=[n_I\alpha^2/2](\mathrm{ch}\varphi-\cos\theta)^{-1}\ ,$$

with $\mathrm{ch}\varphi=(\gamma^2/2)+1$. It was noted in Ref. 4 that if the constant α is chosen to be equal to γ and let $\rightarrow\infty$, one recovers the contact-potential case. But if $\alpha=\mathrm{const}\ (=\sqrt{2})$ and γ varies, one finds, from (1) and (3) [and with $N(0)=1/(2\pi)$ in $2d$],

$$\frac{1}{\tau}=n_I\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{1}{\mathrm{ch}\varphi-\cos\theta}=\frac{n_I}{\mathrm{sh}\varphi}\ , \tag{21}$$

$$\frac{1}{\tau_{\mathrm{tr}}}=n_I\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{1-\cos\theta}{\mathrm{ch}\varphi-\cos\theta}=\frac{n_I}{\mathrm{sh}\varphi}(1-e^{-\varphi})=\frac{2n_I}{e^{\varphi}+1}\ . \tag{22}$$

Thus, with τ^{-1} given by (21), and taking into account *only* the condition (i) of Sec. II [i.e., if Q_0 is chosen to be equal to $(k_F\tau)^{-1}$], (18) would read

$$\sigma=\frac{k_F^2}{2\pi}\frac{e^{\varphi}+1}{2n_I}-\frac{1}{\pi^2}\ln\left[\frac{L}{k_F\tau}\right]\ . \tag{23a}$$

(23a) identifies with formulas (36) and (37) in Ref. 4. However, since according to (21) and (22), $\tau_{\mathrm{tr}}^{-1}<\tau^{-1}$, then, if we combine conditions (i) and (ii) of Sec. II, the above formula should read

$$\sigma=\frac{k_F^2}{2\pi}\tau_{\mathrm{tr}}-\frac{1}{\pi^2}\ln\left[\frac{L}{k_F\sqrt{\tau_{\mathrm{tr}}\tau}}\right]\ . \tag{23b}$$

In this case the two characteristic times τ_{tr} and τ explicitly appear in σ. Next, we will see another example where this is so.

C. The contact potential with interacting impurities

In Ref. 3, a contact potential was used. However, the local positional ordering between the impurities (always present even if it is usually ignored) was shown to yield an "effective" potential depending on the scattering angle. If we divide σ, as written in formula (18) above, by σ_0 (the conductivity for independent impurities) we obtain

$$\begin{aligned}\frac{\sigma}{\sigma_0}&=\frac{l_{\mathrm{tr}}}{l_0}-\frac{1}{\sigma_0\Gamma\left[\frac{d}{2}\right]2^{d-2}\pi^{(d/2+1)}}\int_{L^{-1}}^{l^{-1}}Q^{d-3}dQ\\&=\frac{l_{\mathrm{tr}}}{l_0}\left[1-\frac{d}{k_F\frac{dn}{d\mu}l_{\mathrm{tr}}}\frac{1}{\Gamma\left[\frac{d}{2}\right]2^{d-2}\pi^{(d/2+1)}}\int_{L^{-1}}^{l^{-1}}Q^{d-3}dQ\right]\end{aligned} \tag{24a}$$

(with $l_{\mathrm{tr}}=k_F\tau_{\mathrm{tr}}$ and $l=k_F\tau$), when Q_0 is again chosen as to fulfill *only* condition (i) of Sec. II. (24a) identifies with formula (17) of Ref. 3. However, taking into account (i) *and* (ii) of Sec. II, we must instead write

$$\frac{\sigma}{\sigma_0}=\frac{l_{\mathrm{tr}}}{l_0}\left[1-\frac{d}{k_F\frac{dn}{d\mu}l_{\mathrm{tr}}}\frac{1}{\Gamma\left[\frac{d}{2}\right]2^{d-2}\pi^{(d/2+1)}}\int_{L_{-1}}^{\min[l^{-l},(l_{\mathrm{tr}}l)^{-1/2}]}Q^{d-3}dQ\right]\ , \tag{24b}$$

where $\{\min[l^{-1},(l_{\mathrm{tr}})^{-1/2}]\}$ will depend on the sign of the impurity interactions involved in the problem. Indeed, according to Ref. 3, $\tau_{\mathrm{tr}}^{-1}\lessgtr\tau^{-1}$, depending on whether the impurities attract or repel each other. Therefore, in this case as well, σ depends on *both* τ_{tr} and τ.

IV. CONCLUSION

We have studied the conductivity of weakly disordered metals for an arbitrary scattering potential to first order in perturbation. When $\tau_{\mathrm{tr}}\neq\tau$, the correct perturbative parameter is actually $(k_F\tau_{\mathrm{tr}})^{-1}$ instead of $(k_F\tau)$. We have shown that the conductivity can be expressed in terms of *two* characteristic times τ_{tr} and τ_1 without any knowledge of the precise angular dependence of the scattering potential. Then, with identity (14) we have proved that the coefficient of the integral $\int Q^{d-3}dQ$ is a pure number depending only on d. Therefore, whatever the potential is, in particular whether it has a zero or a finite range, the above coefficient is the same for a given dimensionality. Particularly in the $2+\epsilon$ dimension, the fixed point of the theory is insensitive to the particular shape of the potential or its range. However, the explicit formulas for τ_{tr} and τ require the specific expression of the potential. Therefore it is necessary to deal with a well-behaved, integrable potential. On the other hand, the fact that, *in general,* two characteristic times, or alternatively, two characteristic lengths $l_{\mathrm{tr}}=k_F\tau_{\mathrm{tr}}$ and $l=k_F\tau$ are involved in σ appears to render questionable the "one-parameter scaling" argument, especially when one takes into account the positional local ordering between the impurities. As already mentioned, such a local ordering is, indeed, always present, since the experimental samples are always quenched from a finite temperature (only an infinite quenching temperature would allow to consider the impurities to be independent). τ, when it explicitly appears, does so only via the cutoff Q_0. This is not a minor point, especially when dealing with small or mesoscopic samples; then the precise value of Q_0 will become essential when compared to L^{-1}.

Note added. After this paper was submitted for publication, we became aware of a paper by R. N. Bhatt, P. Wolfle, and T. V. Ramakrishnan, Phys. Rev. B **32**, 569 (1985), dealing with an anisotropic scattering in two dimensions. They show that the effect of the anisotropy only results in an anisotropic diffusion constant. Compared to the Bhatt, Wolfle, and Ramakrishnan derivation, ours is much simpler since it uses only first-principles relations and yields a result valid for an arbitrary dimension. Furthermore, we discuss the question of one or two characteristic lengths, which is an important question to raise and which was not addressed in the Bhatt, Wolfle, and Ramakrishnan paper.

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[3] M. T. Béal-Monod and G. Forgacs, Phys. Rev. B **45**, 3971 (1992) and earlier references therein. A couple of misprints have appeared in that paper: in formula (10), the exponent (-1) should affect the overall bracket and not the inside one; the last expression in formula (13) must be divided by the dimensionality.

[4] M. T. Béal-Monod, A. Theumann, and G. Forgacs, Phys. Rev. B (to be published).

[5] See, for instance, A. A. Abrikosov, L. P. Gorkov, and I. Ye. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon, Oxford, 1965).

[6] See, for instance, M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1970).

[7] See, for instance, S. V. Maleev and B. T. Toperverg, Zh. Eksp. Teor. Fiz. **69**, 1440 (1975) [Sov. Phys. JETP **42**, 734 (1976)].