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Gauge theories of high-T_c superconductors

Boris Blok and Hartmut Monien
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
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It is widely believed that the t-J model contains some of the key physics of the high-temperature superconductors. It was proposed that the low-energy physics of the t-J model can be described by a gauge theory of fermions and bosons coupled to a gauge field. The normal state of these fermions was assumed to be a Fermi liquid. We show that perturbation theory around the Fermi-liquid state breaks down at low temperature in a wide class of gauge theories of high-temperature superconductors due to the interaction with an overdamped singular mode arising from the gauge field. This breakdown is present even if the coupling to the gauge field is arbitrary small.

Recently gauge theories of high-temperature (HTC) superconductors attracted a lot of interest. These theories were able to explain a number of the normal state properties of high T_c superconductors.[1-4] Although the high-temperature properties of these theories are intensively discussed at present, much less is known about their ground state. The particular interesting question is whether their ground state is a normal Fermi liquid or not.

The gauge field theories are thought to describe the low-energy physics of the two-dimensional t-J model in terms of fermions (spinons) and bosons (holons) coupled to a U(1) Abelian gauge field. The effective Lagrangian has the form

$$\begin{aligned} L(\mathbf{r},\tau) = & \sum_\sigma f^*_\sigma(\mathbf{r},\tau)\left[\partial_\tau - a_0 - \mu_F + \frac{1}{2m}(-i\nabla - \mathbf{a})^2\right] f_\sigma(\mathbf{r},\tau) \\ & + b^*(\mathbf{r},\tau)\left[\partial\tau - a_0 - \mu_B + \frac{1}{2m_b}(-i\nabla - \mathbf{a})^2\right] b(\mathbf{r},\tau) + S_{\text{gauge}} \; . \end{aligned} \tag{1}$$

Here b is the holon field, f is the spinon field. The parameters μ_F and μ_B are the chemical potentials of fermions and bosons, respectively. The holon and spinon masses are denoted as m_b and m. The action of the gauge field Sgauge is given by[1]

$$S_{\text{gauge}} = \sum_{\mathbf{q},\omega_n}\left[\chi_d q^2 + i\pi\frac{|\omega_n|}{qv_F}\right]\left[\delta_{\alpha\beta} - \frac{q_\alpha q_\alpha}{q^2}\right] \times a_\alpha(\mathbf{q},\omega_n) a_\beta(-\mathbf{q},-\omega_n) \; , \tag{2}$$

where $\chi_d = \chi_F + \chi_B$ and χ_F and χ_B are the diamagnetic susceptibilities of the fermions and the bosons and $a(\mathbf{q},\omega_n)$ is the gauge field. The gauge field coupling constant is $g^2 = m\chi_d^{-1}$.

For definiteness we will work here in a slave-boson formalism. We assume that the bosons do not condense. The key to the description of the normal state, and the common property of this class of theories, is the existence of an overdamped singular mode, i.e., a mode with a dispersion law $\omega \sim ik^3$.[5-7] The importance of this mode for properties of the HTC superconductors was first recognized by Lee.[8] The physical origin of this mode is the transverse part or magnetic part of the gauge field which is not screened statically at large distances. The cutoff is provided by dynamical screening or Landau damping, for details see, e.g., Ref. 9. Formally it arises here once we integrate out of the lower spinon band in the t-J model.[1] This mode is not unique to the gauge theories of the t-J model. It arises in any U(1) gauge theory if the transverse, i.e., magnetic part, of the interaction is not screened.

The question of the influence of the overdamped singular mode on a Fermi-liquid ground state for the three-dimensional relativistic quark gluon plasma and the relativistic electron gas was discussed by Baym *et al.*[9,10] In this case the overdamped singular model arises due to the radiative corrections to the gauge field propagator. These theories describe the fermions with the Lagrangian

$$L(\mathbf{r},\tau) = \sum_\sigma f^*_\sigma(\mathbf{r},\tau)\left[\partial_\tau - a_0 - \mu_F + \frac{1}{2m}(-i\nabla - \mathbf{a})^2\right] f_\sigma(\mathbf{r},\tau) + S_{\text{gauge}} \; . \tag{3}$$

S_{gauge} is given again by Eq. (2). All the fields now live in $3+1$ dimensions however. Reizer noted that the interaction with the overdamped singular mode leads to the creation of the nonphysical resonant state near the Fermi surface.[7] The influence of the overdamped singular mode on the ground state was studied in detail in Refs. 9 and 10. It was shown that this interaction leads to the logarithmic correction to the self-energy of fermions. The dispersion law of the low-lying excitations near the Fermi surface has the form

$$\epsilon \sim v_F(p-p_F) - g^2 v_F(p-p_F)\ln|p-p_F| \ . \tag{4}$$

Here g^2 is the dimensionless gauge coupling constant. The dispersion law Eq. (4) implies the break down of the Fermi-liquid picture. The dispersion relation, Eq. (4), is only valid for small couplings and momenta not too close to the Fermi momentum, since we used perturbation theory to derive it. Nevertheless we can say that the perturbation around a normal Fermi-liquid ground state breaks down already in the second order. A new ground state is needed if we want to be able to use perturbation theory.

The analysis of the three-dimensional case[7,10,9] shows that the interaction with the singular mode plays a major role in the study of the ground state. It is natural to ask what will be the effect of the singular overdamped mode in two dimensions in the case of the gauge theories that describe high-temperature superconductors. In this paper we consider the influence of the singular mode of a gauge field on the Fermi-liquid ground state in two dimensions for the gauge theories described by the Lagrangian Eq. (1). We show that this interaction leads to severe infrared problems in these theories. In particular, the perturbation theory around a normal Fermi-liquid state of spinons breaks down no matter how small is the coupling to a gauge field. This means that the perturbation theory cannot be applied in a straightforward way for the study of the ground state properties of these theories. A summation of all orders of perturbation theory must be carried out or a new ground state must be chosen in order to make these theories self-consistent.

All our calculations have been carried out at zero temperature. The effects discussed here become irrelevant at high temperatures where the infrared divergencies are smeared out. Thus our results do not apply at high temperatures where the calculations of the transport properties in these theories have been done. We believe that coulombic interactions due to the longitudinal part of the gauge field does not play a role since they are Debye screened.

We consider the theory with a Lagrangian Eq. (1).[1,2,4] We restrict ourselves to the case of a spinon spectrum. In particular we shall calculate the spinon dispersion law. We assume that there is no Bose condensation of holons and that the spinons are in a Fermi-liquid ground state.

We calculate the spinon self-energy to lowest order in g^2. Then we find the spectrum of low-lying excitations near the spinon Fermi surface by looking at the poles of the single particle propagator:

$$G^{-1}(\mathbf{p}) = G_0^{-1}(\mathbf{p},\epsilon) + \Sigma(\mathbf{p},\epsilon) = 0 \ . \tag{5}$$

Here $G(\mathbf{p},\epsilon)$ is the full spinon propagator, $G_0(\mathbf{p},\epsilon)$ is the noninteracting spinon propagator and $\Sigma(\mathbf{p},\epsilon)$ is the self-energy of the spinons arising from the interaction with the gauge field.

The analytic evaluation of the self-energy gives the following expression valid near the Fermi surface (i.e., $\epsilon \sim 0$; $p \sim p_F$), for details see, e.g., Ref. 5:

$$\mathrm{Im}\Sigma^R(\mathbf{p},\epsilon) = g^2 \int_0^{\epsilon} d\epsilon' \int_{|p(\epsilon)-p|}^{2p_F} dq \, \mathrm{Im}\, D^R(\epsilon'-\epsilon, q) \ . \tag{6}$$

Here

$$\mathrm{Im}\, D^R(\epsilon'-\epsilon, q) = \frac{(\epsilon'-\epsilon)k}{\chi_d q^3 + N(0)\dfrac{\epsilon'-\epsilon}{v_F}} \tag{7}$$

is the spectrum function for the gauge field.[4] The momentum $p(\epsilon)$ is defined as $p(\epsilon) = \sqrt{2m(\epsilon+\mu_F)}$ where μ_F is the spinon Fermi energy. An explicit calculation shows that there are two regimes for the energy:

$$\begin{aligned} \mathrm{Im}\Sigma^R(\epsilon,\mathbf{p}) &\sim \left[\frac{\epsilon}{\mu_F}\right]^2 \quad \left[\frac{\epsilon}{\mu_F}\right] \ll \left[\frac{p-p_F}{p_F}\right]^3 , \\ \mathrm{Im}\Sigma^R(\epsilon,\mathbf{p}) &\sim \left[\frac{\epsilon}{\mu_F}\right]^{2/3} \quad \left[\frac{\epsilon}{\mu_F}\right] \gg \left[\frac{p-p_F}{p_F}\right]^3 . \end{aligned} \tag{8}$$

In particular for $p = p_F$ we find in agreement with[8]

$$\mathrm{Im}\Sigma^R(\epsilon, p=p_F) = g^2 \mu_F^{1/3} \epsilon^{2/3} \ . \tag{9}$$

The real part can be found from the dispersion relations that can be conveniently written in the form

$$\frac{\partial\, \mathrm{Re}\Sigma}{\partial \epsilon'} \sim \int \frac{d\epsilon}{\pi} \left[\frac{1}{\epsilon'-\epsilon} + \frac{1}{\epsilon'+\epsilon}\right] \frac{\partial\, \mathrm{Im}\Sigma}{\partial \epsilon} \ . \tag{10}$$

Using Eq. (10) we see that $\mathrm{Re}\Sigma$ has the same structure as $\mathrm{Im}\Sigma$. For the two regimes we find

$$\begin{aligned} \mathrm{Re}\Sigma^R(\epsilon,\mathbf{p}) &\sim \left[\frac{\epsilon}{\mu_F}\right] \quad \left[\frac{\epsilon}{\mu_F}\right] \ll \left[\frac{p-p_F}{p_F}\right]^3 , \\ \mathrm{Re}\Sigma^R(\epsilon,\mathbf{p}) &\sim \left[\frac{\epsilon}{\mu_F}\right]^{2/3} \quad \left[\frac{\epsilon}{\mu_F}\right] \gg \left[\frac{p-p_F}{p_F}\right]^3 . \end{aligned} \tag{11}$$

Particular at $p = p_F$ we have

$$\mathrm{Re}\Sigma \sim g^2 \mu_F^{1/3} \epsilon^{2/3} \ . \tag{12}$$

Let us now find the dispersion law for low-lying excitations. To order g^2 the dispersion relation is given by

$$\epsilon = v_F(p-p_F) + g^2 \mathrm{Re}(\epsilon = v_F(p-p_F), p-p_F) \ . \tag{13}$$

Using the explicit form of $\mathrm{Re}\Sigma$, Eq. (11) it is easy to see that

$$\mathrm{Re}\Sigma(\epsilon = v_F(p-p_F), p-p_F) \sim (p-p_F)^{2/3} p_F^{1/3} \ . \tag{14}$$

Indeed, this is consistent with Eq. (11)

$$\frac{\epsilon}{\mu_F} \sim \frac{p-p_F}{p_F} \gg \left[\frac{p-p_F}{p_F}\right]^3 . \tag{15}$$

Since $(p-p_F)/p_F$ is small in our case. We can use then

Eq. (9). Hence we obtain

$$\epsilon \sim v_F(p-p_F)-g^2(p-p_F)^{2/3}p_F^{4/3}/m \ . \tag{16}$$

In other words, the velocity for the low-lying excitations diverges at the Fermi momentum:

$$v_p=\frac{\partial\epsilon}{\partial p}=v_F\{1-g^2[p_F/(p-p_F)]^{1/3}\} \ . \tag{17}$$

We see from the latter expression that the perturbation theory breaks down since the effective parameter in the perturbation theory is not g^2, but

$$g_{\text{eff}}^2 \sim g^2\left[\frac{p-p_F}{p_F}\right]^{-1/3} \tag{18}$$

or, since $\epsilon \sim v_F(p-p_F)$,

$$g_{\text{eff}}^2 \sim g^2\left[\frac{\epsilon}{\mu_F}\right]^{-1/3} \ . \tag{19}$$

The singularity is even more severe than in the three-dimensional case where the velocity has a logarithmic singularity, Eq. (4). Here the singularity has a power-law behavior.

The effective mass m_{eff} for spinons goes to infinity once $p \sim p_F$. This means that there is a break down of the Fermi-liquid picture. But this result was derived using perturbation theory and is indicating that using perturbation theory is not valid in this regime. The breakdown of the perturbation theory happens for effective coupling strengths $g_{\text{eff}} \sim 1$. In other words, at an energy scale of

$$\epsilon \sim g^6\mu_F \ . \tag{20}$$

This energy scale defines a "crossover" temperature when the effects considered here become important:

$$T_K \sim g^6\mu_F \ . \tag{21}$$

If we assume that the coupling constant g is small and therefore using perturbation theory is allowed, this temperature is of course very small. This means that the effects considered above have no influence on the calculation of the transport coefficients at high temperatures.[2–4] However we must note here that in the case of a coupling constant g of the order of 1 that probably occurs in real materials, we cannot exclude the possibility that T_K becomes large. This follows from Eq. (21), although the more sophisticated analysis is certainly needed in order to get a reliable estimate of T_K for strong coupling. If T_K is indeed large, the condition $T \geq T_K$ will be an additional constraint on the validity of the calculations of the transport coefficients.

Another branch of the dispersion relation can be found by setting $p=p_F$ in the Dyson equation, Eq. (5). Then using Eq. (13) we see that Eq. (5) has two roots: $\epsilon=0$ and $\epsilon=g^6\mu_F$. The latter is a broad resonant state near the Fermi surface. It is very wide ($\text{Im}\Sigma \sim g^6\mu_F$). This is the two-dimensional analog of the bound state Reizer found in his study of the three-dimensional electron gas.[7] Its existence also must be an artifact of the perturbation theory.

We see that no matter how small (but nonzero) the gauge coupling is the interaction with an overdamped mode leads to the breakdown of the perturbation theory in the spinon sector at low temperatures. The reason is that effective coupling constant is not g^2, but $g^2/[(p-p_F)/p_F]^{1/3}$ and becomes strong near the Fermi surface. It would be interesting to investigate the bosonic sector. In this case the situation is much more complicated because there are no small parameters like ϵ/μ_F and p/p_F.

There are several possible ways out of the difficulties mentioned above. First, we could try to sum higher orders in perturbation theory and look whether taking into account the higher corrections will cure the theory. Second, some other mechanism, present in the real material can provide the infrared cutoff. For example, the bosons can condense or the fermions can form Cooper pairs above the crossover temperature Eq. (21) although the situation is still unclear and requires further studies. Another possibility is that in the real material scattering by impurities can smear the singularity. Finally, the breakdown of the perturbation theory can signal that a normal Fermi liquid is not the ground state.[11] It is important to know which mechanism really provides the infrared cutoff if we want to know the true ground state of the theory.

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