Hopping conductivity of a hierarchical lattice

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The dynamic conductivity of a one-dimensional hopping system with hierarchically distributed transition rates is calculated at all frequencies of the driving electric field using a real-space renormalization-group approach. It is found that the conductivity of this system can display quite different kinds of low- and high-frequency behavior as the hierarchy parameter R is varied.

I. INTRODUCTION

In recent years considerable effort has been devoted to the theoretical investigation of the dynamical properties of hierarchical structures since these structures are believed to arise in various physical contexts (see Ref. 1 for a review). Much attention has been focused on the transport[2–5] and the electronic and vibrational properties[6–9] of these systems. In the case of diffusion,[2–5] it has been shown that a hierarchical system can undergo a dynamical transition from anomalous to ordinary diffusion, as well as display anomalous diffusion behavior when the hierarchy parameter R is varied. In the electronic and vibrational problems,[6–9] the energy spectra of the hierarchical system are found to be zero-measure Cantor sets and the system possesses eigenfunctions that are self-similar and critical. Similar to the diffusion problem, different types of scaling behavior of the spectra are obtained in electronic or vibrational spectra depending on the value of R. However, up to now, the directly measurable quantities of this system have not yet been studied much. In this paper we consider the ac hopping conductivity of a one-dimensional (1D) hierarchical lattice.

Studies concerning the hopping-transport properties on 1D systems have shown that the distribution of the transition rates has great influence on the qualitative behavior of the response to external electric fields. The expression for the low-frequency ac conductivity, for example, is regular for periodic chains and becomes nonanalytic when the transition rates are distributed randomly.[10,11]

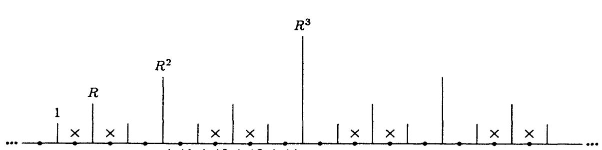

FIG. 1. Schematic representation of the hierarchical lattice with a hierarchy of transition rates. The dots stand for the isoenergetic atomic sites and the vertical segments represent the transition rates between two adjacent sites. Sites with crosses are decimated during the present renormalization scheme.

The intermediate cases, represented by some deterministic aperiodic systems, were shown to display different frequency dependences.[12] So a natural question is whether or not the hopping conductivity of a hierarchical lattice, which is another type of deterministic aperiodic system, will show different types of frequency behavior.

The model treated here is similar to that of Aldea and Dulea (AD).[12] Isoenergetic sites or identical atomic centers along a straight line are joined by a regular bifurcating hierarchical array of links (see Fig. 1). The transition rate W_n between the sites n and $n+1$ is given by

$$W_n=\begin{cases}1, & n=2l+1\ ,\\ R^k, & n=2^k(2l+1)\ ,\end{cases} \tag{1}$$

where R is the hierarchical parameter. As we study the influence of the hierarchical distribution of the transition rates on the conductivity, we limit ourselves at this stage to the discussion of the case of equidistant sites, with the spacings between the adjacent sites d_n being set to be 1. When a spatially constant external electric field $E=E_0e^{i\omega t}$ is applied along the line of the hierarchical lattice, the hopping conductivity, determined by taking the spatial average of the current flowing between pairs of adjacent sites, can be written as[12,13]

$$\sigma=\frac{1}{EL}\sum_n I_n\ , \tag{2}$$

where L is the length of the chain whereas the "elementary currents" I_n, representing the thermally averaged rate at which charge is transferred between the nth site and the $(n+1)$th or the current flowing between the sites n and $n+1$, are the solution of the following Miller-Abrahams (MA) equations

$$\left[\frac{i\omega}{W_n}+2\right]I_n=I_{n+1}+I_{n-1}+i\omega E\ . \tag{3}$$

II. RENORMALIZATION-GROUP APPROACH

Owing to the self-similarity or "inflation symmetry" of the system, we may expect the renormalization-group approach to be applicable. Following Newman and Stinchcombe,[13] we divide the hierarchical lattice into two sublattices consisting of the odd number sites and the even number sites, respectively. Then the MA equations take the more general form

$$\begin{aligned} \epsilon_1 I_n &= \gamma I_{n-1} + I_{n+1} + i\omega E h_1 \quad \text{for odd } n ,\\ \epsilon_n I_n &= I_{n-1} + \gamma I_{n+1} + i\omega E h_2 \quad \text{for even } n , \end{aligned} \tag{4}$$

which can be cast into the same form before and after the decimation procedure. Clearly, the original set of the MA equations (3) is a special case of Eq. (4) with

$$\epsilon_n = 2 + \frac{i\omega}{W_n}, \quad h_1 = h_2 = 1, \quad \gamma = 1 . \tag{5}$$

In terms of the generalized MA equations (4), the conductivity (2) for the system becomes

$$\sigma = \frac{1}{EL} \sum_n I_n h_n , \tag{6}$$

where h_n takes either h_1 or h_2, depending on whether n is odd or even. After decimating all $4n+2$ and $4n+3$ sites (marked by crosses in Fig. 1) and eliminating from the equations the subset of the "elementary currents" I_n on these sites, we are left with a new set of equations linking the remaining currents. By relabeling the remaining sites we can recast the new set of equations in the same form as the old one, except that we have the renormalized parameters

$$\begin{aligned} \epsilon'_{n-1} &= \frac{\epsilon_n \gamma' - \epsilon_2}{\gamma} \quad n > 2, \quad \epsilon'_1 = \frac{\epsilon_1 \gamma' - \epsilon_1}{\gamma} ,\\ h'_1 &= \left[1 + \frac{\gamma'}{\gamma} \right] h_1 + \frac{\epsilon_1}{\gamma} h_2 ,\\ h'_2 &= \left[1 + \frac{\gamma'}{\gamma} \right] h_2 + \frac{\epsilon_2}{\gamma} h_1 ,\\ \gamma' &= \epsilon_1 \epsilon_2 - \gamma^2 . \end{aligned} \tag{7}$$

Using Eqs. (6)–(7) and after some algebra, we can show that the conductivity σ', defined by the value of Eq. (6) on the renormalized chain, is given by

$$\sigma' = \frac{2\gamma'(h_1 + h_2)}{\gamma(h'_1 + h'_2)} \sigma - \frac{i\omega h_1^2 x}{\gamma(h'_1 + h'_2)} , \tag{8}$$

where

$$x = \epsilon_2 + \frac{2h_2}{h_1}\gamma + \left[\frac{h_2}{h_1} \right]^2 \epsilon_1 . \tag{9}$$

Thus, a straightforward iterative procedure yields

$$\sigma = \frac{h_1^{(N-1)} + h_2^{(N-1)}}{2^N \gamma^{(N-1)}} \sigma^{(N-1)} + \frac{i\omega}{4} \sum_{i=0}^{N-2} \frac{x^{(i)} (h_1^{(i)})^2}{2^i \gamma^{(i)} \gamma^{(i+1)}} , \tag{10}$$

where the parameters $h_1^{(i)}$, $h_2^{(i)}$, and $\gamma^{(i)}$ denote the values of h_1, h_2, and γ after i iterations of Eq. (7) with the initial values given by Eq. (5), whereas $x^{(i)}$ is obtained by replacing h_j, ϵ_j ($j=1,2$), and γ by $h_j^{(i)}$, $\epsilon_j^{(i)}$, and $\gamma^{(i)}$, respectively, in Eq. (9). In the practical calculation, we study the infinite chain consisting of the periodic repetition of an N-order hierarchical chain with length 2^N. Such an N-order approximant to the real hierarchical chain is indeed obtained by setting the transition rates (1) with $k \geq N$ to be of the following form:

$$W_n = R^N, \; n = 2^k(2l+1) \quad \text{as } k \geq N . \tag{11}$$

The real hierarchical lattice itself is regarded as the limit of this N-order approximant as $N \to \infty$. If we start with such an N-order approximant of period 2^N, then after decimating $N-1$ times, we are left with a simple periodic chain composed of only two types of links. The MA equations for such a final chain are given by

$$\begin{aligned} \epsilon_1^{(N-1)} I_n &= \gamma^{(N-1)} I_{n-1} + I_{n+1} + i\omega E h_1^{(N-1)} , \quad \text{for odd } n ,\\ \epsilon_2^{(N-1)} I_n &= I_{n-1} + \gamma^{(N-1)} I_{n+1} + i\omega E h_2^{(N-1)} , \quad \text{for even } n . \end{aligned} \tag{12}$$

From Eq. (12) it is not difficult to derive

$$\begin{aligned} \sigma^{(N-1)} = \frac{i\omega}{\Delta} [& (h_1^{(N-1)})^2 \epsilon_2^{(N-1)} + (h_2^{(N-1)})^2 \epsilon_1^{(N-1)} \\ & + 2h_1^{(N-1)} h_2^{(N-1)} (1 + \gamma^{(N-1)})] , \end{aligned} \tag{13}$$

with

$$\Delta = (h_1^{(N-1)} + h_2^{(N-1)}) [\epsilon_1^{(N-1)} \epsilon_2^{(N-1)} - (1 + \gamma^{(N-1)})^2] . \tag{14}$$

So by iterating Eqs. (7) with the initial values (5) and substituting $h_j^{(i)}$, $\epsilon_j^{(i)}$, and $\gamma^{(i)}$ into Eqs. (9)–(10) and (13), we may obtain the conductivity of an arbitrarily high-order approximant to the hierarchical lattice. The results for the real hierarchical lattice corresponding to the limit of $N \to \infty$ can be obtained by an extrapolation.

III. NUMERICAL RESULTS AND DISCUSSION

The first result of our numerical calculation is that when $\omega \to 0$, we have exactly $\mathrm{Re}(\sigma) \to \sigma_0$ with

$$\sigma_0 = 1/m_{-1} ,$$

$$m_{-1} = \frac{1}{2^N} \sum_n W_n^{-1} = \frac{R}{2R-1} + \frac{R-1}{2R-1} \left[\frac{1}{2R} \right]^N . \tag{15}$$

Obviously, when $R > \frac{1}{2}$, $\mathrm{Re}(\sigma)$ tends to a finite limit $\sigma_0 = (2R-1)/R$ as $\omega \to 0$ and $N \to \infty$, whereas for $R < \frac{1}{2}$, we find $\mathrm{Re}(\sigma) \to 0$. In order to attest this result, we have calculated $\mathrm{Re}(\sigma)$ at frequencies as low as 10^{-60} in units of W_1, where our numerical results still corroborate the analytical result (15) derived from a formal fluctuation expansion.[12] This result differs from that of the Fibonacci chain, where Newman and Stinchcombe[13] found $\mathrm{Re}(\sigma)=0$ when $\omega=0$, which is contrary to the prediction by AD that $\mathrm{Re}(\sigma)$ would be finite as $\omega \to 0$.[12] The difference, we believe, lies in the appearance of the factor

γ'/γ before σ in Eq. (8). For the Fibonacci chain,[13] the coefficient before σ is L/L', with L' the length of the renormalized chain, while for our model, we have the factor γ'/γ in addition to the simple rate of lengths L/L'.

Next, we have studied the low-frequency behavior of $\mathrm{Re}(\sigma)-\sigma_0$ and $\mathrm{Im}(\sigma)$ for a variety of values of R. Since our practical numerical calculations are proceeded with finite N, we have calculated the conductivity for quite a few values of N to extrapolate the frequency dependence of the real hierarchical system. Figures 2–4 show the hopping conductivity for the chains with $R=0.3$, 1.8, and 2.5, respectively, and $N=20$, 30, and 40. For $R=0.3$, the ordinary low-frequency dependences $\mathrm{Re}(\sigma)-\sigma_0\sim\omega^2$ and $\mathrm{Im}(\sigma)\sim\omega$ are believed to arise from the period effect (see Fig. 2). Similar cases hold for other values of R. Eliminating the ordinary period effect by an extrapolation, we observe the power-law behavior of the conductivity at low frequencies for various values of R, i.e.,

$$\mathrm{Re}(\sigma)-\sigma_0\sim\omega^{\delta},\quad \mathrm{Im}(\sigma)\sim\omega^{\delta'}\ , \tag{16}$$

where the power-law exponents δ and δ' depend on R. To be specific, we shall distinguish the following cases.

(1) For $R\geq 2$, we have $\delta=2$ and $\delta'=1$, the conductivity possesses the same low-frequency behavior as in the ordinary periodic case[10]

$$\mathrm{Re}(\sigma)-\sigma_0\sim\omega^2\ ,\quad \mathrm{Im}(\sigma)\sim\omega\ . \tag{17}$$

(2) For $1<R<2$, the low-frequency behavior of $\mathrm{Im}(\sigma)$ remains ordinary as in the case of $R\geq 2$, while the exponent for the real part of the conductivity $\delta=R$, i.e.,

$$\mathrm{Re}(\sigma)-\sigma_0\sim\omega^{R}\ ,\quad \mathrm{Im}(\sigma)\sim\omega\ . \tag{18}$$

Thus, the crossover for a dynamical transition from ordinary to anomalous low-frequency dependence of $\mathrm{Re}(\sigma)-\sigma_0$ is observed at $R=2$.

(3) For $\frac{1}{2}<R<1$, we have $\delta\simeq\delta'=2R-1$, i.e.,

$$\mathrm{Re}(\sigma)-\sigma_0\sim\omega^{2R-1},\quad \mathrm{Im}(\sigma)\sim\omega^{2R-1}\ . \tag{19}$$

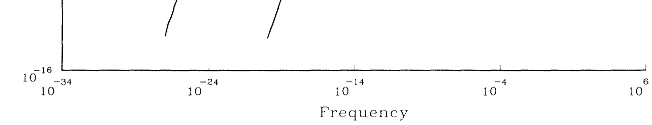

FIG. 2. $\mathrm{Re}(\sigma)-\sigma_0$ (solid line) and $\mathrm{Im}(\sigma)$ (dashed line) as functions of frequency for $R=0.3$, $N=20$, 30, 40. The spacings between the adjacent sites $d_n\equiv 1$ and the transition rates are given by Eq. (1). The frequency is in units of W_1 and the conductivity in units of $W_1 d_n$. It is easily seen that the ordinary low-frequency dependences $\mathrm{Re}(\sigma)-\sigma_0\sim\omega^2$ and $\mathrm{Im}(\sigma)\sim\omega$ arise from the period effect.

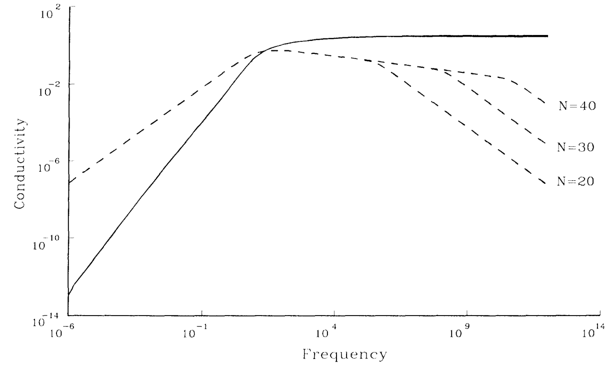

FIG. 3. The same as in Fig. 2 but for $R=1.8$. Note that the curves for $\mathrm{Re}(\sigma)-\sigma_0$ against ω for $N=20$, 30, 40 coincide and the high-frequency behavior $\mathrm{Im}(\sigma)\sim\omega^{-1}$ comes from the period effect.

Similarly, a transition of the power-law exponent for $\mathrm{Im}\sigma(\omega)$ occurs at $R=1$, with $R=1$ corresponding to $W_n\equiv 1$ and thus $\mathrm{Re}(\sigma)=\sigma_0=1$; $\mathrm{Im}(\sigma)=0$.

(4) For $0<R<\frac{1}{2}$, we have $\sigma_0\to 0$ as $N\to\infty$ from Eq. (15). Our numerical results show $\delta=\delta'$ (see, e.g., Fig. 2). Although we cannot give an explicit expression for the exponents δ or δ' in the context of this paper, we find $d\delta/dR<0$, which is obviously contrary to the case of $R>\frac{1}{2}$.

(5) For $R=\frac{1}{2}$, we have $\delta\sim\delta'\sim 0$. However, our numerical results show evidence that a logarithmic singularity may exist in the low-frequency behavior, which is similar to the Fibonacci chain.[12]

We would like to point out that some of the above low-frequency dependences of the conductivity can be inferred analytically. According to AD,[12] the dynamic hopping conductivity can be expressed as a formal fluctuation expansion

$$\sigma(\omega)=\sigma_0+\frac{i\omega}{m_{-1}^2}\lim_{N\to\infty}\sum_{q\neq 0}\frac{|S_N(q)|^2}{2(1-\cos q)+i\omega m_{-1}}\ , \tag{20}$$

where m_{-1} and σ_0 are given by Eq. (15) whereas $S_N(q)$ is the Fourier transform of the fluctuations

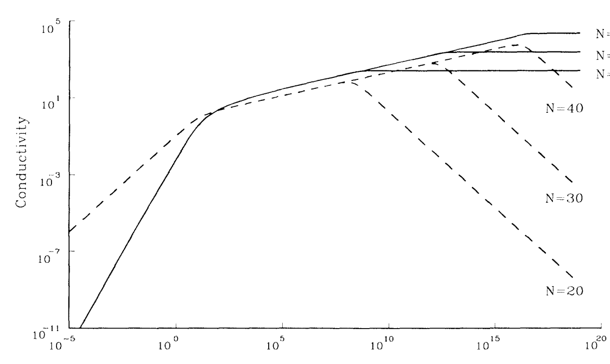

FIG. 4. The same as in Fig. 2 except $R=2.5$. Clearly, $\mathrm{Re}(\sigma)-\sigma_0\to$const. and $\mathrm{Im}(\sigma)\sim\omega^{-1}$ as $\omega\to\infty$ are due to the period effect.

$\delta_n = W_n^{-1} - m_{-1}$:

$$S_N(q) = \frac{1}{2^N}\sum_n e^{inq}\delta_n \ . \tag{21}$$

For the hierarchical model, it is easy to derive

$$|S_N(q)|^2 = \sum_{n=0}^{N-1} Q_n^2 \sum_{k=0}^{2^n-1} \delta_{q,(2k+1)\pi/2^n} \ , \tag{22}$$

where

$$Q_n = \frac{[1-(2R)^{N-n}]}{(2R)^N} A \ , \tag{23}$$

with $A=(R-1)/(2R-1)$. Substituting Eq. (22) into (20) yields

$$\sigma(\omega) = \sigma_0 + i\omega \lim_{N\to\infty} \frac{g_N}{m_{-1}^2} \ , \tag{24}$$

with

$$g_N = \sum_{n=0}^{N-1} Q_n^2 \sum_{k=0}^{2^n-1} \frac{1}{4\sin^2[(2k+1)\pi/2^{n+1}] + i\omega m_{-1}} \ . \tag{25}$$

Taking into account the mathematical formula[14]

$$\sum_{k=0}^{N-1} \frac{1}{\sin^2[(2k+1)\pi/4N] + \sinh^2(x/2)} = \frac{2N\tanh(Nx)}{\sinh x} \ , \tag{26}$$

we obtain

$$g_N = \sum_{n=0}^{N-1} \frac{A^2}{(2R)^{2N}} \frac{2^{n-1}\tanh(2^n y)}{\sinh(2y)} - \sum_{n=0}^{N-1} \frac{A^2}{(2R)^{N+n}} \frac{2^n\tanh(2^n y)}{\sinh(2y)} + \sum_{n=0}^{N-1} \frac{A^2}{(2R)^{2n}} \frac{2^{n-1}\tanh(2^n y)}{\sinh(2y)} \ , \tag{27}$$

with $\sinh^2 y = im_{-1}\omega/4$. Retaining the dominant terms as $\omega\to 0$, we have

$$\frac{\tanh(2^n y)}{\sinh(2y)} \sim 2^{n-1}\sum_k C_k 4^{nk}(im_{-1}\omega)^k \ , \tag{28}$$

where the real coefficients C_k are independent of R and n. Combining Eqs. (27) and (28),

$$g_N = \left[\tilde{\alpha}_0 + \sum_{k=0}^{\infty} \alpha_k \left[\frac{2\times 4^k}{2R}\right]^{2N} \omega^{2k}\right] + i\omega\left[\tilde{\beta}_0 + \sum_{k=0}^{\infty} \beta_k \left[\frac{2\times 4^k}{R}\right]^{2N} \omega^{2k}\right] \ , \tag{29}$$

where $\tilde{\alpha}_0$, $\tilde{\beta}_0$, α_k, and β_k are real coefficients dependent on R but independent of N. Although we cannot determine the ω dependence of σ from Eq. (29), combining Eqs. (24) and (29) we can have the following inferences: If we have $\omega^{-2}[\mathrm{Re}(\sigma)-\sigma_0] \sim \omega^{\delta_1(R)}$ for $R>2$, then for $(2R)>2$ or $R>1$, we should have $\omega^{-1}\mathrm{Im}(\sigma) \sim \omega^{\delta_1(2R)}$. Furthermore, if we find $\omega^{-2}[\mathrm{Re}(\sigma)-\sigma_0] \sim \omega^{\delta_2(R)}$ for $1<R<2$, then for $1<2R<2$, we have $\omega^{-1}\mathrm{Im}(\sigma) \sim \omega^{\delta_2(2R)}$. Our numerical results [Eqs. (17)–(19)] agree with these inferences quite well.

Now we turn to the high-frequency behavior of $\mathrm{Re}(\sigma)$ and $\mathrm{Im}(\sigma)$. As in the low-frequency case, we also find the ordinary frequency dependences $\mathrm{Re}(\sigma)\to$const. and $\mathrm{Im}(\sigma)\sim\omega^{-1}$ at very high frequencies. We believe some of these results are due to the period effect. After eliminating the period effect by an extrapolation (see Figs. 3 and 4, we find the high-frequency results as follows.

(1) For $R^2<2$, the conductivity has the same high-frequency behavior as in the ordinary periodic chain

$$\mathrm{Re}(\sigma) = m_1; \quad \mathrm{Im}(\sigma) \sim \omega^{-1} \ , \tag{30}$$

with

$$m_1 = \frac{1}{2^N}\sum_n W_n = \frac{R-1}{R-2}\left[\frac{R}{2}\right]^N - \frac{1}{R-2} \ , \tag{31}$$

which is consistent with the analytical expression due to AD.[12]

(2) For $2\le R^2<4$, the high-frequency behavior of $\mathrm{Re}(\sigma)$ preserves ordinary as in the case of $R^2<2$, whereas $\mathrm{Im}(\sigma)$ displays an anomalous power-law behavior, i.e.,

$$\mathrm{Re}(\sigma) = m_1; \quad \mathrm{Im}(\sigma) \sim \omega^{\beta} \ , \tag{32}$$

with

$$\beta = \frac{\ln R - \ln 2}{\ln R} \ . \tag{33}$$

Thus, the crossover for a transition from the ordinary to anomalous power-law decay behavior of $\mathrm{Im}(\sigma)$ is observed at $R=\sqrt{2}$.

(3) For $R^2>4$, both $\mathrm{Re}(\sigma)$ and $\mathrm{Im}(\sigma)$ display the anomalous power-law growth

$$\mathrm{Re}(\sigma) \sim \omega^{\beta}; \quad \mathrm{Im}(\sigma) \sim \omega^{\beta} \ , \tag{34}$$

with β given by Eq. (33). Clearly a transition for the high ω dependence of $\mathrm{Re}(\sigma)$ occurs at $R=2$.

(4) For $R=2$, our numerical results show evidence that $\mathrm{Re}(\sigma)$ and $\mathrm{Im}(\sigma)$ may display logarithmic growth as $\omega\to\infty$.

In fact, the critical value R_c for high ω behavior of $\mathrm{Re}(\sigma)$ and $\mathrm{Im}(\sigma)$ can be obtained by an iterative procedure.[12] According to AD, as $\omega\to\infty$, we have $\mathrm{Re}(\sigma)=m_1$ and $\omega\,\mathrm{Im}(\sigma)=a$ with m_1 given by Eq. (31) and

$$a = \frac{1}{2^N}\sum_n (W_n^2 - W_n W_{n+1}) = \frac{R}{2-R} + \frac{1}{2-R^2} + \frac{2(1-R)}{2-R}\left[\frac{R}{2}\right]^N + \frac{(1-R^2)}{2-R^2}\left[\frac{R^2}{2}\right]^N \ . \tag{35}$$

Obviously, $\mathrm{Re}(\sigma)$ goes to infinity as $N\to\infty$ for $R>2$, while $\omega\,\mathrm{Im}(\sigma)=a$ tends to infinity for $R^2>2$, in agreement with our numerical results for the critical values of

R. Furthermore, from the expression (35), we conjecture that the logarithmic corrections to the power-law decay of $\mathrm{Im}(\sigma)$ at high frequencies are possible in the anomalous regime $2 < R^2 < 4$, i.e., instead of the second relation in Eq. (32), we suggest

$$\mathrm{Im}(\sigma) \sim \omega^{\beta}(\ln\omega)^{\beta'} , \quad (36)$$

with β' dependent on R. The anomalous high-frequency dependences of the conductivity are displayed in Figs. 3 and 4 for $R = 1.8$ and $R = 2.5$, which are typical for $\sqrt{2} < R < 2$ and $R > 2$, respectively.

Finally, there are still several points which should be noticed. The first one is that we do not find any special characteristic of the conductivity in the intermediate range of frequencies. This is rather different from the results of the Fibonacci chain[13] and the Thue-Morse chain,[12] where strong differences in frequency dependence of the conductivity between the periodic chain and the aperiodic ones are manifested in the intermediate range. The second point worth noticing is that our results for the conductivity of the hierarchical system yield a smooth curve for σ against ω and display no self-similarity which was found in the electron and phonon spectra, despite the strong similarity between the MA equations and the tight-binding equations in the electron and phonon problem.[6-9] The explanation lies in the fact that the oscillatory solutions for the I_n are prohibited in the present model in order that there should be a finite current flowing throughout the infinite sample. It is believed that it is these oscillatory solutions that give the complex, self-similar structure of the electron and phonon spectra.[13] The last point comes from the fact that our results for the conductivity show no oscillatory behavior with the length of the chain, which is analogous to the results for the Fibonacci chain.[13] Indeed, since these results are obtained by solving a finite set of MA equations with the periodic boundary conditions, the results so obtained are believed to be similar to those for the infinite chain, i.e., the variation with the length of the sample is not available by the hopping model.[13] This is a genuine shortfall of the present model.

IV. CONCLUSION

By a real-space renormalization-group method, we have calculated, at all frequencies of the driving electric field, the hopping conductivity of a hierarchical lattice. It has been found that the low- and high-frequency dependences of the conductivity are strongly dependent on the hierarchical parameter R. Various types of power-law behavior, with possible logarithmic correction in some particular cases, have been found for the real part and the imaginary part of the conductivity at low and high frequencies. The power-law exponents for both the real part and the imaginary part of the conductivity may undergo dynamical transition at several values of R. This remarkable type of behavior differs considerably from those for the Fibonacci chain and the Thue-Morse chain, at which the low- and high-frequency dependences of the conductivity are independent of the diluted parameter W_A/W_B association with the two building elements in the aperiodic chain.[12,13] Finally, the conductivity of the present model displays no special characteristic in the intermediate range of frequencies, which is also contrary to those for the Fibonacci and the Thue-Morse aperiodic chains.[12,13]

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