Quantum collective creep: Effects of anisotropy, layering, and finite temperature

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We determine the creep rates for classical and quantum motion in uniaxially anisotropic and layered superconductors within the framework of weak collective-pinning theory. In particular, we concentrate on the low- and intermediate-magnetic-field regime where single-vortex collective pinning is relevant. For a field aligned with the crystal c axis, we find that both the classical and the quantum creep rates are enhanced as compared with the isotropic results due to the increased elasticity of the vortices. For anisotropic superconductors the creep rates do not depend on the angle ϑ between the magnetic field and the crystal ab plane and are also independent of the direction of motion. Identical results are obtained for layered superconductors within the large-angle region $\vartheta > \epsilon$, where $\epsilon^2 = m/M < 1$ denotes the mass anisotropy ratio. A more complex behavior is obtained in the small-angle region $|\vartheta| < \epsilon$, where the structure of the vortex cannot be approximated by a simple rectilinear object. Here the creep rates depend both on the angle ϑ and on the direction of motion. We discuss the finite-temperature corrections to the quantum motion and determine the crossover temperature to the classical thermally activated behavior.

I. INTRODUCTION

The magnetic properties of type-II superconductors exposed to fields larger than the lower critical field H_{c_1} are determined by the static and dynamic properties of the vortices. Material defects acting as pinning centers trap the vortices in a critical state[1] which is characterized by a nonuniform vortex density. From the thermodynamic point of view, the critical state is only metastable and thus decays by thermally activated vortex motion in order to achieve a more uniform vortex density. This phenomenon is known as creep and the classical theory by Anderson[2] predicts relaxation rates (for the diamagnetic moment or for the persistent current) which vanish linearly with decreasing temperature. In the oxide superconductors the observed magnetic relaxation rate is particularly large (giant creep[3]), indicating weak pinning of the vortices. In addition, recent experiments have shown that the low-temperature relaxation rate does not extrapolate to zero,[4–6] suggesting a decay of the critical state by quantum tunneling. Similar effects have been observed in a chevrel phase,[7] in heavy fermions,[8] and in organic superconductors.[9] Thus, the question arises what is the fundamental parameter governing the quantum motion of the vortices and distinguishing the oxide, organic, heavy-fermion, and the chevrel phase superconductors from "more conventional" superconductors which do not show quantum effects.

Recently, the tunneling rates for single vortices and for vortex bundles have been determined within the framework of weak collective pinning theory.[10] The tunneling rate is determined by the (effective) Euclidean action $S_E^{(\mathrm{eff})}$ of the process.[11] For the most important case of single-vortex tunneling in the limit of strong dissipation, the dimensionless effective Euclidean action becomes $S_E^{\mathrm{eff}}/\hbar \simeq (\hbar/e^2)(\xi/\rho_n)\sqrt{j_0/j_c}$. The fundamental dimensionless parameter is given by the resistance ratio $(\rho_n/\xi)/(\hbar/e^2)$, where ρ_n is the normal-state resistivity and ξ is the coherence length of the superconductor. Furthermore, j_0 and j_c denote the depairing and the critical current densities. Superconductors characterized by a large normal-state resistivity and a small coherence length, such that $\rho_n/\xi \gtrsim 1\mathrm{k}\Omega$, are good candidates for the observation of quantum creep. The exotic superconductors listed above belong to this class of materials.

The theory as developed in Ref. 10 applies to isotropic materials, whereas the oxides as well as the organic superconductors are characterized by large anisotropies. In this paper we extend the theory of quantum collective creep to uniaxially anisotropic and layered materials. Thereby we restrict ourselves to the most important case of single-vortex pinning which describes well the situation for small and moderate magnetic fields. In general, the creep rate may depend on the angle ϑ between the magnetic field and the superconducting layers and on the direction of the motion (motion parallel to the planes denoted "in-plane", versus motion within the plane containing the vortex and the crystal c axis denoted "out-of-plane"). Our main results are the following: In an anisotropic material and for a field directed along the c axis, the elasticity of the vortices is enhanced with respect to the isotropic material, leading to a *reduction* of the

TABLE I. Angular dependencies of the phenomenological parameters for anisotropic and layered superconductors and for in-plane (||) as well as out-of-plane (⊥) motions. Parameters for in-plane motion of Josephson vortices are denoted by J. All angular dependencies are expressed in terms of ϵ_ϑ, with $\epsilon_\vartheta^2=\epsilon^2\cos^2\vartheta+\sin^2\vartheta$. The other parameters are $\varepsilon_0=(\Phi_0/4\pi\lambda)^2$, $c_{66}=B\Phi_0/(8\pi\lambda)^2$, $M_v=(2/\pi^2)mK_F$, and $\eta=\Phi_0^2/2\pi c^2\xi^2\rho_n$. $f_{\rm pin}$ is the individual mean pinning force acting on a vortex aligned parallel with the crystal c axis.

Motion	Elasticity	Shear	Distance	Pinning	Mass	Friction
$\parallel$	$\frac{\varepsilon_0\epsilon^2}{\epsilon_\vartheta}$	$\epsilon_\vartheta^3 c_{66}$	ξ	$f_{\rm pin}$	$\epsilon_\vartheta M_v$	$\epsilon_\vartheta\eta$
$\perp$	$\frac{\varepsilon_0\epsilon^2}{\epsilon_\vartheta^3}$	$\frac{c_{66}}{\epsilon_\vartheta}$	$\epsilon_\vartheta\xi$	$\frac{f_{\rm pin}}{\epsilon_\vartheta}$	$\frac{M_v}{\epsilon_\vartheta}$	$\frac{\eta}{\epsilon_\vartheta}$
J	$\varepsilon_0\epsilon$	ϵ^3c_{66}	Λ	$f_{\rm pin}\left[\frac{\xi}{\Lambda}\right]^3$	ϵM_v	$\epsilon\left[\frac{\xi}{\Lambda}\right]^2\eta$

(effective) Euclidean action $S_E^{(\text{eff}),c}$ and thus to an enhanced magnetic relaxation rate $d\ln M/d\ln t\simeq-\hbar/S_E^{(\text{eff}),c}$. For the case of strong dissipation, the coherence length ξ in the effective Euclidean action $S_E^{\text{eff},c}$ has to be replaced by the coherence length ξ_c along the c axis. For anisotropic materials we find $S_E^{(\text{eff})}=S_E^{(\text{eff}),c}$ independent of the direction of the field and of the direction of motion. For layered materials the results agree with those obtained for the anisotropic case within the large-angle region $\vartheta>\epsilon$, where $\epsilon^2=m/M<1$ is the mass anisotropy ratio. For small angles, $|\vartheta|<\epsilon$, a strong dependence of the action upon the angle ϑ is found. The case where the magnetic field is precisely aligned with the ab plane, $\vartheta=0$, has been discussed extensively by Ivlev, Ovchinnikov, and Thompson.[12] All results of the present paper are summarized in Tables I and II.

The second important aspect discussed in this paper is the finite-temperature enhancement of quantum creep and the crossover to the classical, i.e., thermally activated, regime. It turns out that, once the length of the tunneling segment of the vortex has been determined, the dynamic component of the tunneling process corresponds to the tunneling of a pointlike object in a renormalized potential. Therefore, the results obtained for the finite-temperature enhancement of macroscopic quantum tunneling of a pointlike object[13,14] can easily be generalized to the string problem. The basic quantity determining the temperature dependence of quantum creep is the dissipation. For the most important case of ohmic dissipation, a T^2 behavior is obtained for the finite-temperature correction $\Delta S_E^{\text{eff}}(T)=S_E^{\text{eff}}(T)-S_E^{\text{eff}}(0)$ to the effective action. If the coupling to the environment has a low-frequency cutoff, the importance of dissipation is reduced and the temperature dependence of the action $S_E^{\text{eff}}(T)$ is exponentially small.

The tunneling rate is determined by the saddle-point solution of the (effective) Euclidean action $S_E^{(\text{eff})}$ of the tunneling process,[15–17,10] whereas the classical thermally activated process is described by the saddle-point solution of the free energy. The quantum motion is thus an $(n+1)$-dimensional generalization of the classical n-dimensional process, where the role of the additional dimension is played by the time coordinate. Thus, in order to describe quantum motion we first have to study classical creep in anisotropic and layered materials. In Sec. II, we start with a discussion of classical creep in anisotropic and layered materials. Here, we refer to an "anisotropic superconductor" as a material where an anisotropic *continuous* Ginzburg-Landau or London free-energy functional gives an accurate description of the physics, whereas we use the term "layered superconductor" to refer to a material which has to be described by the *discrete* (orthogonal to the layers) Lawrence-Doniach[18] model. In Sec. III, we determine the quantum-mechanical tunneling rate using the classical results for the calculation of the (effective) Euclidean action $S_E^{(\text{eff})}$ of the tunneling process. Section IV contains a discussion of finite-temperature effects and we summarize and conclude our work in Sec. V.

II. CLASSICAL CREEP IN ANISOTROPIC AND LAYERED MATERIALS

A. Anisotropic superconductors

Let us consider an anisotropic superconductor characterized by its mass anisotropy ratio $\epsilon^2=m/M<1$. We choose a coordinate system where the z axis is aligned parallel to the crystal c axis. A magnetic field $\mathbf{H}$ enclosing an angle ϑ_H with the ab plane (an angle $\theta_H=\pi/2-\vartheta_H$ with the c axis) is applied to the sample. To fix ideas we assume $\mathbf{H}$ to lie in the yz plane of our coordinate system. The magnitude of $\mathbf{H}$ is chosen to be much larger than the lower critical field $H_{c_1}(\theta_H)$. The correct angular dependence of $H_{c_1}(\theta_H)$ still seems to be a rather controversial issue. A particularly simple expression can be obtained within the London approximation[19]

$$H_{c_1}(\theta_H)=\frac{\Phi_0}{4\pi\lambda^2}\frac{\epsilon}{\epsilon_{\theta_H}}\ln\left[\frac{\lambda}{\xi}\right],$$

where λ denotes the London penetration depth in the ab plane, ξ denotes the planar coherence length, $\Phi_0=hc/2e$ is the flux quantum, and

$$\epsilon_\vartheta^2=\epsilon^2(\vartheta)=\epsilon^2\cos\vartheta+\sin^2\vartheta.$$

An analysis of based on the Ginzburg-Landau theory

TABLE II. Angular dependencies of the collective-pinning length L_c, the collective-pinning energy U_c, the hopping length L_h, the tunneling time t_c, and the (effective) Euclidean action $S_E^{(\text{eff})}$ for anisotropic and layered superconductors and for in-plane ($||$) as well as out-of-plane ($\perp$) motions. CMP denotes the result for the cubic model potential. Angular dependencies are expressed in terms of ϵ_ϑ, with $\epsilon_\vartheta^2=\epsilon^2\cos^2\vartheta+\sin^2\vartheta$. The other parameters are $L_c^c\simeq(\varepsilon_0^2\xi^2\epsilon^4/Wa^2)^{1/3}$, $U_c^c\simeq\varepsilon_0\epsilon^2\xi^2/L_c^c$, $t_c^M\simeq(M_v/\varepsilon_0)^{1/2}L_c^c/\epsilon$, $S_E^c/\hbar\simeq\xi^2\epsilon k_F K_F(l/\xi_0)^{1/2}$, $t_c^\eta\simeq(\eta/\varepsilon_0)(L_c^c/\epsilon)^2$, and $S_E^{\text{eff},c}/\hbar\simeq(\hbar/e^2)(\epsilon\xi/\rho_n)(j_0/j_c)^{1/2}$.

Motion	L_c	U_c	L_h/L_c	t_c ($\eta=0$)	S_E	t_c ($\eta>0$)	S_E^{eff}	ϑ
Anisotropic								
$\|\|$ or $\perp$	$\frac{L_c^c}{\epsilon_\vartheta}$	U_c^c	1	t_c^M	S_E^c	t_c^η	$S_E^{\text{eff},c}$	$0<\vartheta<\frac{\pi}{2}$
CMP	$\frac{L_c^c}{\epsilon_\vartheta}$	$\left[\frac{\pi}{4}\right]^2 U_c^c$	1		$\left[\frac{2\sqrt{2}}{5}\right]S_E^c$		$\left[\frac{\pi^2}{18}\right]S_E^{\text{eff},c}$	$0<\vartheta<\frac{\pi}{2}$
Layered								
$\|\|$	$\frac{L_c^c}{\epsilon}\left[\frac{\Lambda}{\xi}\right]^2$	U_c^c		$t_c^M\left[\frac{\Lambda}{\xi}\right]^2$	$S_E^c\left[\frac{\Lambda}{\xi}\right]^2$	$t_c^\eta\left[\frac{\Lambda}{\xi}\right]^2$	$S_E^{\text{eff},c}\left[\frac{\Lambda}{\xi}\right]^2$	$\|\vartheta\|<\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}\left[\frac{\xi}{\Lambda}\right]^3$
	$\frac{d}{\|\vartheta\|}$	$U_c^c\left[\frac{d}{L_c^c}\right]^{1/2}$	1	$t_c^M\left[\frac{d\epsilon^2}{L_c^c\vartheta^2}\right]^{1/4}$	$S_E^c\left[\frac{d^3\epsilon^2}{L_c^{c3}\vartheta^2}\right]^{1/4}$	$t_c^\eta\left[\frac{\xi}{\Lambda}\right]^3\left[\frac{d\epsilon^2}{L_c^c\vartheta^2}\right]^{1/2}$	$S_E^{\text{eff},c}\left[\frac{\xi}{\Lambda}\right]^2\frac{d\epsilon}{L_c^c\|\vartheta\|}$	$\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}\left[\frac{\xi}{\Lambda}\right]^3<\vartheta<\epsilon\left[\frac{\xi}{\Lambda}\right]^2$
	$\frac{d}{\|\vartheta\|}$	$U_c^c\left[\frac{d}{L_c^c}\right]^{1/2}$	1	$t_c^M\left[\frac{d\epsilon^2}{L_c^c\vartheta^2}\right]^{1/4}$	$S_E^c\left[\frac{d^3\epsilon^2}{L_c^{c3}\vartheta^2}\right]^{1/4}$	$t_c^\eta\left[\frac{d}{L_c^c}\right]^{1/2}$	$S_E^{\text{eff},c}\frac{d}{L_c^c}$	$\epsilon\left[\frac{\xi}{\Lambda}\right]^2<\vartheta<\epsilon\left[\frac{d}{L_c^c}\right]^{3/2}$
	$\frac{d}{\|\vartheta\|}$	$U_c^c\left[\frac{d}{L_c^c}\right]^{1/2}$	$\left[\frac{L_c^{c3}\vartheta^2}{d^3\epsilon^2}\right]^{1/4}$	$t_c^M\left[\frac{d\epsilon^2}{L_c^c\vartheta^2}\right]^{1/4}$	S_E^c	$t_c^\eta\left[\frac{d}{L_c^c}\right]^{1/2}$	$S_E^{\text{eff},c}\left[\frac{d\vartheta^2}{L_c^c\epsilon^2}\right]^{1/4}$	$\epsilon\left[\frac{d}{L_c^c}\right]^{3/2}<\vartheta<\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}$
	$\frac{L_c^c}{\epsilon}\frac{\|\vartheta\|}{\epsilon}$	$U_c^c\frac{\|\vartheta\|}{\epsilon}$	$\frac{\epsilon}{\|\vartheta\|}$	t_c^M	S_E^c	$t_c^\eta\left[\frac{\|\vartheta\|}{\epsilon}\right]$	$S_E^{\text{eff},c}\frac{\|\vartheta\|}{\epsilon}$	$\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}<\vartheta<\epsilon$
Same as anisotropic superconductor								$\epsilon<\vartheta$
$\perp$	$\frac{d}{\|\vartheta\|}$	$U_c^c\left[\frac{d}{L_c^c}\right]^{1/2}$	1	$t_c^M\left[\frac{d}{L_c^c}\right]^{1/4}$	$S_E^c\left[\frac{d}{L_c^c}\right]^{3/4}$	$t_c^\eta\left[\frac{d}{L_c^c}\right]^{1/2}$	$S_E^{\text{eff},c}\frac{d}{L_c^c}$	$\|\vartheta\|<\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}$
	$\frac{L_c^c}{\epsilon}\frac{\|\vartheta\|}{\epsilon}$	$U_c^c\frac{\|\vartheta\|}{\epsilon}$	1	$t_c^M\left[\frac{\|\vartheta\|}{\epsilon}\right]^{1/2}$	$S_E^c\left[\frac{\|\vartheta\|}{\epsilon}\right]^{3/2}$	$t_c^\eta\frac{\|\vartheta\|}{\epsilon}$	$S_E^{\text{eff},c}\left[\frac{\vartheta}{\epsilon}\right]^2$	$\epsilon\left[\frac{d}{L_c^c}\right]^{1/2}<\vartheta<\epsilon$
Same as anisotropic superconductor								$\epsilon<\vartheta$

provides the additional angular dependence in the logarithm (plus additional nonlogarithmic corrections), which can lead to substantial modifications of the above simple result.[20,21] Our analysis below will not rely on the detailed value of H_{c_1}, and the above result is merely quoted for an order-of-magnitude estimate for the regime of applicability of our results.

In anisotropic superconductors the direction of the external magnetic field ϑ_H in general deviates from the direction ϑ of the vortices. Here, ϑ is again measured with respect to the *ab* plane, see Fig. 1. For an Abrikosov lattice in an equilibrium state this deviation is given by[19,22,23]

$$\sin(\vartheta_H-\vartheta)=\frac{H_{c_1}^c}{H}\frac{\sin\vartheta\cos\vartheta}{2\epsilon_\vartheta}(1-\epsilon^2)\frac{\ln[H_{c_2}(\vartheta)/B]}{\ln(\lambda/\xi)}.$$

Here, $H_{c_1}^c$ is the lower critical field along the c axis and $H_{c_2}(\vartheta)=\Phi_0/2\pi\xi^2\epsilon_\vartheta$ is the upper critical field along ϑ. An additional complication occurs in strongly layered superconductors, where the vortices are locked to the *ab* plane[23] below the critical angle ϑ_l which is given by the relation $\vartheta_l \simeq H_{c_1}^c/H\ln(\lambda/\xi)$. For large enough fields,

$$H \gg \frac{1}{\epsilon_\vartheta}H_{c_1}^c, \tag{1}$$

the relative difference between the external angle ϑ_H and the internal angle ϑ as well as the locking angle ϑ_l become small and we can neglect this complication in the following analysis, where we express all quantities by the internal field variable ϑ. For small field values the situation is more difficult: Our main focus in this paper is on pinning, thus the vortices go into a metastable critical state rather than to the stable equilibrium configuration. The internal angle ϑ then depends on the condition under which the critical state is formed: If the field is switched off after field cooling (magnetic remanence), the vortices are pinned initially in a direction parallel to the former external field. This initial angle is changed due to flux flow and creep as the metastable critical state evolves in time, as has been shown, e.g., by Tuominen *et al.*,[24] who observe an alignment of the remanent diamagnetic moment with the c axis after the external field is switched off. On the other hand, in a zero-field-cooled situation, vortices entering the sample will be directed along the external magnetic-field direction at the surface, however, it is unclear along which direction the vortices will point within the interior as this is the result of flux flow and creep during the creation time of the critical state itself. In this paper we will concentrate on a specific part of this problem, which is pinning and creep in anisotropic and layered material due to classical and quantum motion. We therefore express all quantities by the (local) internal field angle ϑ and leave the problem of relating internal and external angles under nonequilibrium conditions for future studies.

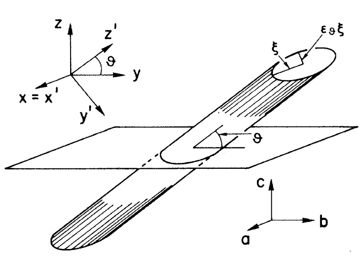

FIG. 1. Coordinate systems used to describe vortex motion in anisotropic and layered superconductors. The vortices are directed along z'. In-plane motion of a vortex is along $x=x'$, whereas out-of-plane motion is along y'. In an anisotropic superconductor the Abrikosov vortex has core dimensions ξ and $\epsilon_\vartheta\xi$ along the x and y' axes, respectively.

Vortices entering the sample will minimize their energies with respect to the weak random pinning potential $U_{\rm pin}$. For not too large fields, the interaction between the vortices is small compared with the interaction of a single vortex with the pinning centers, such that we can study the single-vortex free energy

$$\mathcal{F}[\mathbf{u}]=\int dz'\left[\frac{\varepsilon_l^{\parallel}(\vartheta)}{2}(\partial_{z'}u_x)^2+\frac{\varepsilon_l^{\perp}(\vartheta)}{2}(\partial_{z'}u_{y'})^2 + U_{\rm pin}(z',\mathbf{u})\right]. \tag{2}$$

Here we have introduced the rotated coordinate system with z' pointing along the external field $\mathbf{H}$ and a common x axis, $x=x'$, see Fig. 1. The elasticities $\varepsilon_l^{\parallel}(\vartheta)$ and $\varepsilon_l^{\perp}(\vartheta)$ for in-plane and out-of-plane motions can be obtained in the following way: The line energy of a vortex segment of length L enclosing an angle ϑ with the *ab* plane is[20,19] $E(L,\vartheta)=\varepsilon_0\ln(\lambda/\xi\epsilon_\vartheta)\varepsilon_\vartheta L$, with

$$\varepsilon_0=\left[\frac{\Phi_0}{4\pi\lambda}\right]^2.$$

The tilt modulus is determined by the increase in energy due to a transverse fluctuation of the vortex position: symmetrically deforming the vortex on a distance $2L$ by an amplitude $L\delta\phi$, the energy increase of one segment with length L is

$$\delta E=\frac{\partial E}{\partial L}\delta L+\frac{\partial E}{\partial\vartheta}\delta\vartheta+\frac{1}{2}\frac{\partial^2E}{\partial\vartheta^2}(\delta\vartheta)^2. \tag{3}$$

For an out-of-plane tilt by an angle $\delta\phi=\delta\vartheta$ the length of the segment L is increased by $\delta L\simeq L(\delta\vartheta)^2/2$. The out-of-plane elasticity then is defined by the relation

$$\tfrac{1}{2}\varepsilon_l^{\perp}(\vartheta)L(\delta\vartheta)^2=\frac{\partial E}{\partial L}\delta L+\frac{1}{2}\frac{\partial^2E}{\partial\vartheta^2}(\delta\vartheta)^2. \tag{4}$$

Here the linear term in (3) has dropped out as the total deformation involves two segments of length L with opposite angular corrections $\pm\delta\vartheta$. After a few algebraic manipulations we obtain the result[25,46] $\varepsilon_l^{\perp}(\vartheta)\simeq\varepsilon_0\epsilon^2/\epsilon_\vartheta^3$. For an in-plane tilt by an angle $\delta\phi$, the length of the segment L is increased by $\delta L\simeq L(\delta\phi)^2/2$, whereas the change in the angle ϑ is given by the equation

$$\tan(\vartheta+\delta\vartheta)=L\sin\vartheta\sqrt{(L\cos\vartheta)^2+(L\delta\phi)^2},$$

resulting in $\delta\vartheta = -(\tan\vartheta)(\delta\phi)^2/2$. The in-plane elasticity is defined by

$$\tfrac{1}{2}\varepsilon_l^{\parallel}(\vartheta)L(\delta\phi)^2 = \frac{\partial E}{\partial L}\delta L + \frac{\partial E}{\partial \vartheta}\delta\vartheta, \tag{5}$$

where we have dropped the term quadratic in $\delta\vartheta$ as the angular correction $\delta\vartheta$ is already of second order in $\delta\phi$. The final result for the in-plane elasticity is $\varepsilon_l^{\parallel}(\vartheta) \simeq \varepsilon_0\epsilon^2/\epsilon_\vartheta$. All angular dependences for the material parameters are summarized in Table I.

Within weak collective pinning theory[26] vortex segments of length L_c are pinned independently. Each segment L_c is subject to the competition between the elastic tilt energy and the pinning energy, such that the individual pinning forces add up only randomly within the collective-pinning volume $V_c \propto L_c$. On the other hand, the net pinning forces of the segments add up linearly, resulting in a finite pinning force density. The collective-pinning length L_c is determined by minimization of the free energy (2). In the following we will study a pinning potential with a minimal characteristic length $r_p \simeq \xi$ given by the spatial extent of the vortex core. Such a model is appropriate if the size of the defect is smaller than the coherence length ξ. Depending on the mode of relaxation, we will have to consider the coherence length ξ in the ab plane $[\mathbf{u}=(u_x,0,0)]$ or the coherence length $\epsilon_\vartheta\xi$ along $y'[\mathbf{u}=(0,u_{y'},0)]$, see Fig. 1. In the high-T_c superconductors weak collective pinning by pointlike oxygen defects[27] is believed to be the main source of pinning and our model should be applicable.

The solution minimizing the free energy (2) can be obtained using dimensional estimates (in these estimates we will express all the quantities through their natural parameters and will drop the numericals which usually combine to a factor of the order of unity). Within the collective-pinning volume $V_c \simeq \epsilon_\vartheta\xi^2 L_c$, the elastic energy competes with the pinning energy and the equality between the two energies determines the length L_c. We first study the relaxation mode with $\mathbf{u}$ in the ab plane: For this case the relevant length scale is ξ, the elastic energy is $\varepsilon_l^{\parallel}(\vartheta)\xi^2/L_c^{\parallel}$, and the pinning energy is $U_c^{\parallel} = (f_{\rm pin}^2 n_d L_c^{\parallel}\xi^2\epsilon_\vartheta)^{1/2}\xi$. Here n_d denotes the defect density and $f_{\rm pin}$ is the individual mean pinning force. Solving for the collective-pinning length $L_c^{\parallel}$ we obtain

$$L_c^{\parallel} \simeq \left[\frac{\varepsilon_0^2\xi^2\epsilon^4}{Wa^2\epsilon_\vartheta^3}\right]^{1/3} \simeq \frac{L_c^c}{\epsilon_\vartheta}. \tag{6}$$

Here we have introduced the mean-squared random force density $W = f_{\rm pin}^2 n_d(\xi/a)^2$, with $a = \sqrt{\Phi_0/B}$ the mean vortex separation. L_c^c is the collective-pinning length for a field applied parallel to the c axis of the crystal. Note that L_c^c is reduced by a factor $\epsilon^{4/3}$ with respect to the isotropic result.[28] The result (6) has to be compared with the relaxation mode within the yz plane, $\mathbf{u}=(0,u_{y'},0)$. For this situation the relevant length scale is given by $\epsilon_\vartheta\xi$, the elastic energy becomes $\varepsilon_l^{\perp}(\vartheta)(\epsilon_\vartheta\xi)^2/L_c^{\perp}$, and the pinning energy takes the form

$$U_c^{\perp} = [(f_{\rm pin}/\epsilon_\vartheta)^2 n_d L_c^{\perp}\xi^2\epsilon_\vartheta]^{1/2}\epsilon_\vartheta\xi.$$

For this mode the individual pinning force is enhanced by a factor of $1/\epsilon_\vartheta$ due to the reduced length scale involved in the pinning. The collective pinning length becomes

$$L_c^{\perp} \simeq \frac{L_c^c}{\epsilon_\vartheta},$$

which is identical to the result for in-plane motion. The vortex relaxes to the pinning potential by choosing the mode characterized by the smaller collective-pinning length. Here, $L_c^{\parallel} \simeq L_c^{\perp}$ and thus the relaxation of the vortex involves both in-plane and out-of-plane motions. The collective-pinning energy is $U_c^{\parallel} \simeq U_c^{\perp} \simeq U_c^c$ with

$$U_c^c \simeq (Wa^2L_c^c)^{1/2}\xi \simeq \varepsilon_0\epsilon^2\xi^2/L_c^c$$

and hence is independent of the angle ϑ between the field and the ab plane and of the direction of relaxation.

The pinning potential enters the expression for the collective-pinning length (6) via the mean pinning force density W. Since W is not directly accessible by experiment, we relate the collective-pinning length L_c^c for a field $\mathbf{H}\| z$ to the corresponding critical current density j_c in the ab plane: The critical current density is determined by the equality between the driving Lorentz force $j_c\Phi_0 L_c^c/c$ and the pinning force U_c^c/ξ. Using the definition of the depairing current density $j_0 = c\Phi_0/12\sqrt{3}\pi\lambda^2\xi$, we obtain

$$L_c^c \simeq \epsilon\xi\left[\frac{j_0}{j_c}\right]^{1/2}, \tag{7}$$

expressing L_c^c by experimentally accessible quantities.

Finally, we have to determine the range of validity of our results. As the external field is increased, the vortex separation decreases and the interaction between neighboring vortices becomes increasingly important. The range of applicability of the single-vortex pinning results is given by the condition that the elastic energy due to tilt of an individual vortex be larger than the interaction energy with the nearest neighbor. Consider first the case of in-plane relaxation: An estimate for the interaction energy is given by the elastic shear energy $a^2\bar{L}_c^{\parallel}c_{66}^e(\vartheta)(\xi/a\sqrt{\epsilon_\vartheta})^2$ within the volume $a^2\bar{L}_c^{\parallel}$. The (easy) shear modulus[29] is given by $c_{66}^e(\vartheta) = \epsilon_\vartheta^3 c_{66}$, with $c_{66} = B\Phi_0/(8\pi\lambda)^2$ the shear modulus for the field aligned with the crystal c axis (note that λ is the London penetration depth in the ab plane and not an effective mean penetration depth). In addition, we have used the smallest intervortex distance[29] $a\sqrt{\epsilon_\vartheta}$ in our expression for the shear energy. Equating the shear energy with the elastic energy $\varepsilon_l^{\parallel}(\vartheta)\xi^2/\bar{L}_c^{\parallel}$ and solving for the maximal single-vortex pinning length $\bar{L}_c^{\parallel}$, we obtain the condition

$$L_c^{\parallel} < \bar{L}_c^{\parallel} \simeq \frac{\epsilon}{\epsilon_\vartheta}\frac{a}{\sqrt{\epsilon_\vartheta}}. \tag{8}$$

Next, we study the case of out-of-plane relaxation, where the elastic shear energy is given by $a^2\bar{L}_c^{\perp}c_{66}^h(\vartheta)[\xi\epsilon_\vartheta/(a/\sqrt{\epsilon_\vartheta})]^2$. The (hard) shear modulus[29] is given by $c_{66}^h(\vartheta) = c_{66}/\epsilon_\vartheta$ and we have used the larger intervortex distance $a/\sqrt{\epsilon_\vartheta}$ in the determination of the shear energy. The maximal collective pinning length involving a single vortex then is $\bar{L}_c^{\perp} \simeq \epsilon a / \epsilon_\vartheta^{3/2}$, which coincides with the result (8) for in-plane relaxation. Using the results (6)–(8), we find that our theory is applicable for fields

$$B \lesssim \alpha \frac{j_c}{j_0} H_{c_2}(\vartheta), \tag{9}$$

with α a numerical of the order of 10. Using a typical value for the critical current ratio $j_c/j_0 \simeq 10^{-2}$ for the oxide superconductors we expect that the single-vortex pinning regime extends up to fields of the order of 10 T. Additional complications due to the difference in the external and the internal field angles can be avoided [see Eq. (1)] if we choose the induction to lie within the interval

$$H_{c_1}^c \ll \epsilon_\vartheta B \ll \alpha \frac{j_c}{j_0} H_{c_2}^c .$$

B. Layered material

We refer to a "layered superconductor" as a material which has to be described in terms of a Lawrence-Doniach[18] model. In layered superconductors the vortices are not simple rectilinear objects but are rather viewed as chains of two-dimensional (2D) pancake vortices connected by interplanar Josephson-type vortices,[30–32] see Fig. 2. A pancake vortex can be viewed as a finite segment of height d, the interlayer spacing, of an Abrikosov vortex directed along the c axis. In particular, as is the case for the Abrikosov vortex, we can associate two length scales with a pancake vortex, the coherence length ξ, which describes the extent of the core region where the order parameter of the superconducting layer goes to zero, and the planar London penetration depth λ, which describes the magnetic extent of the pancake vortex. In order to understand the nature of a Josephson vortex consider first a uniaxially anisotropic superconductor with field directed orthogonal to the c axis. The number of relevant scales then doubles and the core of the vortex is described by the scales ξ (perpendicular to c) and $\epsilon\xi$ (parallel to c), whereas the magnetic extent of the vortex is given by λ/ϵ and λ. As the anisotropy increases we go over to the layered superconductor and the Abrikosov vortex turns into a Josephson vortex. Regarding the magnetic extent of the Josephson vortex, nothing changes as compared with the Abrikosov vortex. On the other hand, the Josephson vortex has no core in the usual sense, as the superconducting order parameter is strongly suppressed in between the layers. Therefore, the Josephson vortex is only defined by its current flow pattern.[33] For scales smaller than the magnetic extent of the vortex, the current flow pattern can be found from the solution of a coupled set of nonlinear differential equations involving the two length scales d (the interlayer spacing) and $\Lambda = d/\epsilon$. The phase and hence the current pattern in a Josephson vortex then change rapidly on the scales d (perpendicular to the layers) and Λ (along the layers). Whereas for distances $z < d$, $x < \Lambda$, the layered structure is relevant (due to the rapid change in phase), an accurate description of the current pattern can be obtained at large distances, $z \gg d$ and $x \gg \Lambda$, on the basis of a continuum approximation. Also, due to the rapid change in phase, the planar order parameter is slightly suppressed on the scales d and Λ (see below). Therefore, we call this inner region of the Josephson vortex, where the phase changes rapidly, the "phase core" of the Josephson vortex.

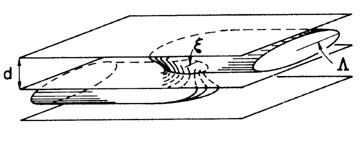

FIG. 2. Mixed Josephson-pancake vortex in a strongly layered superconductor. The (phase) core dimensions of the Josephson vortex are d along the z axis and $\Lambda = d/\epsilon$ along the planes. The pancake vortex has a core with vanishing order parameter with a size ξ (drawing not to scale).

In a next step, let us discuss the pinning properties of pancake and Josephson vortices. The pancake vortices are pinned against motion within the ab plane, the characteristic length scale for the pinning being the coherence length ξ. Let us denote the individual pinning force of one small defect acting on a pancake vortex by f_{pin}. This pinning force is identical to the one acting on an Abrikosov vortex pointing along the c axis. On the other hand, the Josephson vortices are intrinsically pinned against motion parallel to the c axis, however, the pinning with respect to motion along the ab plane is very weak. In our model we assume the intrinsic pinning force to be infinite. The pinning force against motion within the plane is suppressed compared with the pinning force f_{pin} affecting a pancake vortex. The origin of this reduction in the individual pinning force is twofold: First of all, the relevant extent of the Josephson vortex Λ is much larger than the corresponding length ξ in a pancake vortex, hence the pinning force, being a derivative of the pinning energy with respect to distance, is reduced by a factor ξ/Λ. Second, the relative suppression of the order parameter within the layers adjacent to the Josephson vortex is only small, in fact, only by a fraction $(\xi/\Lambda)^2$, whereas the relative suppression of the order parameter in the core of a pancake vortex is of order unity. Hence, the possible energy gain due to the presence of a defect within the superconducting layer is reduced by a factor $(\xi/\Lambda)^2$ for a Josephson vortex as compared with a pancake vortex. Together, we then obtain a suppression of the pinning force acting on a Josephson vortex by a factor $(\xi/\Lambda)^3$ as compared to the pinning force acting on a pancake vortex. A more quantitative analysis[34] of this effect involves the following steps: The force $\mathbf{f}(\mathbf{r})$ acting on a vortex due to an individual pin a distance $\mathbf{r}$ away is given by[35,26]

$$\mathbf{f}(\mathbf{r}) = N(\varepsilon_F) \int d^3r' g_1(\mathbf{r}') \nabla |\Delta_v(\mathbf{r}+\mathbf{r}')|^2 + N(\varepsilon_F) \int d^3r' g_2(\mathbf{r}') \xi^2 \nabla |\nabla \Delta_v(\mathbf{r}+\mathbf{r}')|^2, \tag{10}$$

where $N(\varepsilon_F)$ is the density of states at the Fermi level and $\Delta_v(\mathbf{r})$ is the gap function in the presence of a vortex. The first term describes pinning due to disorder in the attractive electron-electron interaction g_1 (variations in the transition temperature T_c), whereas the second term is due to randomness in the coherence length of the superconductor, as caused, e.g., by disorder in the mean free path of the electrons. For a small defect with size $r_0 < \xi$, the variation in Δ_v is small on the scale of the variation in the coupling g and we obtain

$$\mathbf{f}(\mathbf{r}) \simeq N(\varepsilon_F)\left[\nabla|\Delta_v(\mathbf{r})|^2 \int d^3r' g_1(\mathbf{r}') + \xi^2 \nabla|\nabla\Delta_v(\mathbf{r})|^2 \int d^2r' g_2(\mathbf{r}')\right]. \quad (11)$$

The first term describes pinning due to a variation in the modulus of Δ_v. For a Josephson vortex the modulus $|\Delta|$ in the superconducting layers is suppressed only indirectly via the coupling to the phase φ of the gap function. We calculate this suppression of the order parameter perturbatively. The order parameter $\Delta = |\Delta|\exp(i\varphi)$ (Δ is normalized to unity asymptotically) of an isolated layer is given by the solution of the planar Ginzburg-Landau equation

$$\xi^2 e^{-i\varphi}\nabla^2(|\Delta|e^{i\varphi}) + |\Delta| - |\Delta|^3 = 0.$$

With a Josephson vortex present directed along the y axis, the phase $\varphi(x)$ changes rapidly on a scale $x \lesssim \Lambda$, such that, to leading order,

$$|\Delta_v| \simeq 1 - \tfrac{1}{2}\xi^2(\partial_x\varphi)^2.$$

The maximal suppression of the order parameter then is found near the center of the Josephson vortex and can be estimated to be

$$\delta|\Delta_v| \simeq \frac{1}{2}\left[\frac{\xi}{\Lambda}\right]^2.$$

The corresponding result for a pancake vortex, of course, gives unity. Hence we see that comparing the possible gain in condensation energy due to the presence of a small defect within the superconducting layer, the Josephson vortex is pinned less effectively with a suppression factor given by $(\xi/\Lambda)^2$. Moreover, the relevant scale for the pinning of a Josephson vortex is given by the dimension Λ of the phase core, whereas it is ξ for the case of a pancake vortex. In calculating the pinning force, we then obtain an additional reduction factor ξ/Λ, such that the first term in Eq. (11) finally produces a pinning force $(\xi/\Lambda)^3 f_{\text{pin}}$, which is reduced by a factor $(\xi/\Lambda)^3$ as compared with the pinning force f_{pin} exerted by the same defect on the pancake vortex. A more precise numerical calculation[34] gives a suppression factor $(\xi/0.66\Lambda)^3$. On the other hand, pinning by disorder affecting the coherence length (more precisely, the mean free path) is mainly due to the variation in the phase φ of the gap function, as described by the second term above. Using $|\nabla\Delta_v|^2 \simeq |\Delta_v|^2/\Lambda^2$, we obtain again a reduced pinning force $f^J_{\text{pin}} \simeq (\xi/\Lambda)^3 f_{\text{pin}}$, more precisely,[34] the reduction factor for the case of pinning by disorder in the mean free path is $(\xi/0.71\Lambda)^3$. Thus, the reduction factor $\simeq(\xi/\Lambda)^3$ is essentially independent of the type of pinning, whether produced by spatial fluctuations in the transition temperature or by variations in the coherence length.[36]

Before discussing collective pinning in layered superconductors we have to determine the elasticities $\varepsilon_l^{\parallel}(\vartheta)$ and $\varepsilon_l^{\perp}(\vartheta)$ of the pancake-chain vortex. The line energy of a vortex segment of length L enclosing an angle ϑ with the superconducting layers is obtained by summing up the energies of the individual pancake vortices along the z direction and their interactions[31] along the y axis. Note that the interaction between the individual pancake vortices consists of two parts, a magnetic one and a second due to the Josephson coupling between the layers. Here we concentrate on the second contribution, which is the dominant one under conditions typical for the oxide superconductors. For large angles $\vartheta > \epsilon$ the separation between neighboring pancake vortices is less than Λ and the line energy becomes

$$E(L,\vartheta > \epsilon) = \varepsilon_0 L|\sin\vartheta|[1 + (\epsilon^2/2)\cot^2\vartheta],$$

the first term describing the individual pancake energies and the second term being due to the interaction produced by the Josephson coupling between the layers.[31] For small angles $|\vartheta| < \epsilon$, the interaction energy grows only linearly with distance and the line energy becomes

$$E(L,|\vartheta| < \epsilon) = \varepsilon_0 L|\sin\vartheta|[1 + \epsilon|\cot\vartheta| + (1/2\epsilon)|\tan\vartheta|].$$

The third term is a higher-order correction[37] to the interaction energy between two pancake vortices, which has to be taken into account in order to obtain a nonzero result for the out-of-plane elasticity $\varepsilon_l^{\perp}(\vartheta)$. Note that, in the present case, the two pancake vortices are parallel instead of antiparallel as is the case for the situation discussed in Ref. 37, hence, the interaction given by Eq. (19) of Ref. 37 is repulsive in our case. Using again the definitions (4) and (5) for $\varepsilon_l^{\perp}(\vartheta)$ and for $\varepsilon_l^{\parallel}(\vartheta)$ above, as well as the expressions for the change in length δL and angle $\delta\vartheta$, we obtain the results $\varepsilon_l^{\perp}(\vartheta > \epsilon) = \varepsilon_0\epsilon^2/|\sin^3\vartheta|$ and $\varepsilon_l^{\perp}(|\vartheta| < \epsilon) = \varepsilon_0/\epsilon\cos^3\vartheta$ for the out-of-plane tilt modulus and $\varepsilon_l^{\parallel}(\vartheta > \epsilon) = \epsilon_0\epsilon^2/|\sin\vartheta|$ and $\varepsilon_l^{\parallel}(|\vartheta| < \epsilon) = \varepsilon_0\epsilon\cos\vartheta$ for the in-plane tilt modulus. Note that the out-of-plane tilt modulus is infinite for the case where the field is applied parallel to the layers ($\vartheta = 0$) due to our assumption of an infinite intrinsic pinning force. However, at finite angles $|\vartheta| > 0$, a finite density of kinks (pancakes) occurs which are mobile and hence the out-of-plane tilt modulus becomes finite. Using $\epsilon_\vartheta^2 = \epsilon^2\cos^2\vartheta + \sin^2\vartheta$ we find that the elasticities for the anisotropic and for the layered superconductor are roughly identical.

Let us now study the pinning properties of a single vortex in a layered superconductor. We start with the simplest case of pure Josephson pinning which becomes relevant in the limit $\vartheta \to 0$. Keeping in mind that the relevant length scale is given by the dimension of the phase core Λ, we obtain for the elastic energy of a segment of length L^J of a Josephson vortex deformed by $u_x \simeq \Lambda$ the expression $\varepsilon_0\epsilon\Lambda^2/L_c^J$, whereas the (collective) pinning energy acting on this segment is given by $U_c^J = [(f_{\rm pin}^J)^2 n_d L^J \Lambda d]^{1/2}\Lambda$. Here we have used the $\vartheta \to 0$ limit of the in-plane elasticity $\varepsilon_l^{\parallel}(\vartheta)$. Equating the elastic and the pinning energy, we obtain the collective-pinning length L_c^J for the Josephson vortices

$$L_c^J \simeq \left[\frac{\varepsilon_0^2 \epsilon^2 \Lambda^7}{W a^2 \xi^4 d}\right]^{1/3} \simeq \frac{L_c^c}{\epsilon}\left[\frac{\Lambda}{\xi}\right]^2, \tag{12}$$

where the final expression has been obtained by using $\Lambda = d/\epsilon$. The result shows that the collective-pinning length for the Josephson vortices is considerably enhanced over the result L_c^c/ϵ for Abrikosov vortices in an anisotropic material. The collective pinning energy U_c^J remains again unchanged, $U_c^J \simeq U_c^c$.

Next we consider vortices tilted by an angle ϑ with respect to the layers. For angles $\vartheta > \vartheta_J \simeq \epsilon(d/L_c^c)^{1/2}(\xi/\Lambda)^3$ pinning is dominated by the 2D pancake vortices, whereas for angles $|\vartheta| < \vartheta_J$, pinning due to Josephson vortices is more important. The crossover angle ϑ_J can be determined by a comparison of the critical current densities as produced by the pinning of the pancake vortices and of the Josephson vortices, respectively, see below. Again we have to study the in-plane and the out-of-plane relaxation modes and compare the two collective pinning lengths. We start with in-plane relaxation: the elastic energy is again $\varepsilon_l^{\parallel}(\vartheta)\xi^2/L_c^{\parallel}$ but the pinning energy is modified, $U_c^{\parallel} = (f_{\rm pin}^2 n_d \xi^2 L_c^{\parallel}|\sin\vartheta|)^{1/2}\xi$, since only the vertical component of the vortex (pancakes) is pinned. As a result we obtain the pinning length

$$L_c^{\parallel} \simeq \left[\frac{\varepsilon_0^2 \xi^2 \epsilon^4}{W a^2 \epsilon_\vartheta^2 |\sin\vartheta|}\right]^{1/3} \simeq \frac{L_c^c}{\epsilon_\vartheta}\left[\frac{\epsilon_\vartheta}{|\sin\vartheta|}\right]^{1/3}. \tag{13}$$

This has to be compared with the collective-pinning length for out-of-plane relaxation,

$$L_c^{\perp} \simeq \left[\frac{\varepsilon_0^2 \xi^2 \epsilon^4 |\sin^3\vartheta|}{W a^2 \epsilon_\vartheta^6}\right]^{1/3} \simeq \frac{L_c^c}{\epsilon_\vartheta}\frac{|\sin\vartheta|}{\epsilon_\vartheta}, \tag{14}$$

which is obtained by equating the elastic energy $\varepsilon_l^{\perp}(\vartheta)(\xi\sin\vartheta)^2/L_c^{\perp}$ and the pinning energy $(f_{\rm pin}^2 n_d \xi^2 L_c^{\perp}|\sin\vartheta|)^{1/2}\xi$. Note here that the vortex relaxes by a distance ξ along the layer such that the relevant orthogonal projection along y' is $\xi|\sin\vartheta|$. Comparing (13) and (14) we find that the ratio $L_c^{\perp}/L_c^{\parallel} = (|\sin\vartheta|/\epsilon_\vartheta)^{4/3}$ drops below unity as $\vartheta \simeq \epsilon$, such that for small angles $|\vartheta| < \epsilon$ out-of-plane relaxation dominates. For larger angles $\vartheta > \epsilon$, $L_c^{\parallel} \simeq L_c^{\perp}$, and we obtain no preferred direction of relaxation as for the case of the anisotropic material discussed above. Furthermore, the collective-pinning length for the layered superconductor becomes equal to the corresponding quantity for the anisotropic material within the large-angle region $\vartheta > \epsilon$. The collective-pinning energy for the out-of-plane relaxation relevant for small angles $|\vartheta| < \epsilon$ takes the form $U_c^c|\vartheta|/\epsilon$, whereas for large angles the pinning energy $U_c \simeq U_c^c$ becomes again independent of the angle and identical with the anisotropic result.

For the case of small angles ($\vartheta \to 0$) $L_c^{\perp} \simeq L_c^c|\vartheta|/\epsilon^2$ becomes shorter than the distance between neighboring pancake vortices $d/|\vartheta|$ and we enter a regime where each single pancake vortex is pinned independently. The collective-pinning energy in this regime becomes $U_c^c(d/L_c^c)^{1/2}$. The condition for the realization of single pancake pinning is $L_c^{\perp} \le d/|\vartheta|$ or, using Eq. (14), $|\vartheta| < \epsilon(d/L_c^c)^{1/2}$.

It remains to determine the crossover from Josephson pinning to single pancake pinning. This crossover takes place when the Josephson pinning starts to dominate over single pancake pinning and an appropriate condition is given by comparing the two critical current densities j_c^J and j_c^p. For the case of Josephson pinning the Lorentz force $j_c^J \Phi_0 L_c^J/c$ acts on the length L_c^J and the pinning force is given by U_c^c/Λ, resulting in a critical current density $j_c^J = \epsilon(\xi/\Lambda)^3 j_c$. Again, we use as our reference value j_c the critical current density for the case of a magnetic field directed along the crystal c axis and a current flowing in the ab plane. On the other hand, single pancake pinning produces a critical current density $j_c^p = |\vartheta|(L_c^c/d)^{1/2} j_c$ which decreases linearly with the angle ϑ: whereas the Lorentz force $j_c^p \Phi_0 d/|\vartheta|$ acts on an increasingly longer vortex segment $d/|\vartheta|$, the single pancake pinning force remains unchanged, $U_c^p/\xi = U_c^c(d/L_c^c)^{1/2}/\xi$. The crossover between the two pinning regimes is given by the equality $j_c^J \simeq j_c^p$. Thus, we obtain that, for angles bigger than $\vartheta \simeq \epsilon(d/L_c^c)^{1/2}(\xi/\Lambda)^3$, the Josephson part of the vortex is free for current densities $j \simeq j_c^p$ and pinning is due to the 2D pancake vortices.

Summarizing, for the case of layered superconductors we find four different pinning regimes: Josephson pinning for angles $|\vartheta| < \epsilon(d/L_c^c)^{1/2}(\xi/\Lambda)^3$, single pancake pinning for

$$\epsilon(d/L_c^c)^{1/2}(\xi/\Lambda)^3 < \vartheta < \epsilon(d/L_c^c)^{1/2},$$

multipancake collective pinning with out-of-plane relaxation for $\epsilon(d/L_c^c)^{1/2} < \vartheta < \epsilon$, and again multipancake pinning but with mixed in-plane and out-of-plane relaxations in the remaining regime $\epsilon < \vartheta$. Note that for angles $\vartheta > \epsilon$ the results for the anisotropic superconductor and for the layered superconductor agree with each other—any different behavior between anisotropic and layered materials is restricted to small angles $|\vartheta| < \epsilon$. The results for the collective-pinning lengths L_c and for the collective-pinning energies U_c are summarized in Table II.

Regarding the regime of validity, the results for the layered materials remains unchanged with respect to the anisotropic case for the regime $\vartheta > \epsilon$. For small angles $|\vartheta| < \epsilon$, the relaxation is out of plane and we have to compare the elastic energy $\varepsilon_l^{\perp}(\vartheta)(\xi\sin\vartheta)^2/\bar{L}_c^{\perp}$ with the shear energy $a^2\bar{L}_c^{\perp} c_{66}^h(\vartheta)[\xi\sin\vartheta/(a/\sqrt{\epsilon_\vartheta})]^2$, leading to a maximal single-vortex pinning length $\bar{L}_c^{\perp} \simeq a/\sqrt{\epsilon}$ and using (6) and (7) we obtain again the condition $B \lesssim (j_0/j_c)H_{c_2}(\vartheta)$ as for the anisotropic situation. However, there is a second constraint limiting the validity of our approach, which is given by the assumption of weak collective pinning. For the layered materials discussed above, the interplanar distance d defines a second length scale besides the collective-pinning length L_c. A vortex aligned parallel to the crystal c axis can be considered as a chain of pancake vortices. If the collective-pinning length L_c^c drops below the distance d between neighboring pancakes, we enter the regime of strong pinning where each pancake vortex is pinned individually. This provides a second constraint to our theory: $L_c^c > d$.

C. Classical creep

With the determination of the single-vortex configuration minimizing our free energy (2), we have performed the main important steps in our effort to solve the problem of classical creep for current densities near to the critical current density. Minimizing the free energy, we have obtained the energy scale for the pinning, the collective-pinning energy. Equating this energy to the product of the Lorentz force and the relevant length scale for the pinning potential we immediately obtain the critical current densities. However, the creep rate is determined by the saddle-point solution of the free-energy functional (2). In general, the elasticity involved in the relaxation of the vortex to the pins may differ from the elasticity involved in the hop, leading to an increase in the length L_h of the hopping segment with respect to the collective-pinning length L_c.

Let us first consider the case of an anisotropic superconductor. Here the elastic energy density is identical for the two cases of in-plane and out-of-plane relaxation. Therefore, the elastic energy density involved in the hop always agrees with the one involved in the relaxation process. Consequently, the minimum and the saddle-point solution for the free energy agree with each other, $L_h \simeq L_c$, and the typical activation energy for creep is U_c^c, independent of the angle ϑ or the direction of the hop.

Second, let us turn to layered superconductors. For large angles $\vartheta > \epsilon$ the results for the layered and the anisotropic materials coincide and the above formulas for the activation energy can be applied. Let us then consider small angles with $|\vartheta| < \epsilon$: The relevant relaxation mode involves out-of-plane motion. For a current flow along the x direction producing out-of-plane motion, the length of the hopping segment remains unchanged, $L_h^\perp = L_c^\perp$, and the activation energy for creep is given by $U_c^c|\vartheta|/\epsilon$ for $\epsilon(d/L_c^c)^{1/2} < \vartheta < \epsilon$ and $U_c^c(d/L_c^c)^{1/2}$ for the single pancake pinning regime $|\vartheta| < \epsilon(d/L_c^c)^{1/2}$. On the other hand, a current flow along y' produces in-plane motion and we have to determine the length $L_h^\parallel$ of the hopping segment. Equating the in-plane elastic energy density during the hop, $\varepsilon_l^\parallel(\vartheta)(\xi/L_h^\parallel)^2$, with the out-of-plane elastic energy density of the relaxed vortex, $\varepsilon_l^\perp(\vartheta)(\xi\vartheta/L_c^\perp)^2$, we find for the hopping length

$$L_h^\parallel \simeq \frac{\epsilon}{|\vartheta|} L_c^\perp \simeq \frac{L_c^c}{\epsilon} \geq L_c^\perp . \tag{15}$$

Thus, for angles $\epsilon(d/L_c^c)^{1/2} < \vartheta < \epsilon$ the activation energy for an in-plane hop is $U_c^c(|\vartheta|/\epsilon)(L_h^\parallel/L_c^\perp) \simeq U_c^c$. For angles $|\vartheta| < \epsilon(d/L_c^c)^{1/2}$ we enter the single pancake pinning regime but, as long as $L_h^\parallel > d/|\vartheta|$, creep still proceeds by simultaneous hopping of many pancake vortices. The length $L_h^\parallel$ is then determined by equating the elastic energy density involved in the hop to the mean pinning energy density $U_c^c\sqrt{d/L_c^c}|\vartheta|/d$, resulting in $L_h^\parallel/L_c^\perp \simeq \sqrt{|\vartheta|/\epsilon}(L_c^c/d)^{3/4}$ with $L_c^\perp \simeq d/|\vartheta|$. The activation energy for creep is given by

$$U_c^c\sqrt{d/L_c^c}(L_h^\parallel/L_c^\perp) \simeq U_c^c(L_c^c/d)^{1/4}\sqrt{|\vartheta|/\epsilon}$$

and decreases until we reach the single pancake hopping regime at small angles $|\vartheta| < \epsilon(d/L_c^c)^{3/2}$, where the hopping length $L_h^\parallel$ becomes larger than the separation $d/|\vartheta|$ between two pancakes and the activation energy saturates to $U_c^c\sqrt{d/L_c^c}$. Finally, for very small angles $|\vartheta| < \epsilon(d/L_c^c)^{1/2}(\xi/\Lambda)^3$, in-plane creep is determined by the Josephson vortices and the activation energy jumps back to U_c^c. This completes our discussion of classical creep near j_c. Note that as the driving current density j decays far below j_c, these results have to be modified by taking the dependence of the collective pinning energy on the current density j into account.[28]

III. QUANTUM MOTION IN ANISOTROPIC AND LAYERED MATERIALS

A. Zero dissipation limit

Let us now turn to quantum motion. The Lagrangian generating the classical equation of motion for the vortex is given by

$$\mathcal{L}[\mathbf{u}] = \int dz' \left[\frac{M^\parallel(\vartheta)}{2}(\partial_t u_x)^2 + \frac{M^\perp(\vartheta)}{2}(\partial_t u_{y'})^2 \right] - \mathcal{F}[\mathbf{u}] . \tag{16}$$

The vortex masses $M^\parallel(\vartheta)$ and $M^\perp(\vartheta)$ have to be determined by calculating the kinetic energy of a moving vortex or by studying the inertial response of a vortex to an external force. For the isotropic case, corresponding calculations have been carried out by Suhl[38] and by Kupriyanov and Likharev,[39] using different kinds of time-dependent Ginzburg-Landau theories but providing consistent results. It is simple to generalize Suhl's analysis to the anisotropic case: The electronic part of the mass is determined by the time-derivative term l_t in the Lagrangian density which has the form

$$l_t \propto \int dx\, dy' |\partial_t \Psi_v(\mathbf{r}' - \mathbf{v}'t)|^2 , \tag{17}$$

where Ψ_v denotes the order parameter in the presence of an Abrikosov vortex pointing along z' and positioned at the origin of the (rotated) coordinate system. $\mathbf{v}'$ is the velocity of the vortex in the rotated reference frame. Using $\partial_t = v_x\partial_x + v_{y'}\partial_{y'}$ and the approximations $|\partial_x\Psi_v| \simeq |\Psi_\infty|/\xi$ and $|\partial_{y'}\Psi_v| \simeq |\Psi_\infty|/\epsilon_\vartheta\xi$ within the core region, we obtain the results

$$M^\parallel(\vartheta) = \epsilon_\vartheta M_v$$

and (18)

$$M^\perp(\vartheta) = \frac{M_v}{\epsilon_\vartheta}$$

for the angular dependences of the vortex masses. Here Ψ_∞ denotes the asymptotic value of the order parameter far away from the vortex core and M_v is the vortex mass for the case of a magnetic field applied parallel to the crystal c axis. In Ref. 10 we have presented a simple estimate for the vortex mass in isotropic superconductors which reproduces the results of Suhl and Kupriyanov and Likharev. This estimate can be generalized to anisotropic materials and we first consider the case where the magnetic field is applied parallel to the c axis. The basic idea is that the electronic contribution to the vortex mass is due to the local change in dispersion within the vortex core. The number of electrons experiencing this change is $\pi\xi^2 N(\varepsilon_F)\delta\varepsilon$. $\delta\varepsilon \simeq \hbar v_F/\pi\xi$ is the change in energy due to the confinement to the vortex core and $N(\varepsilon_F)=K_F m/\hbar^2\pi^2$ is the density of states at the Fermi level for the anisotropic material, m denoting the planar mass and K_F and v_F are the Fermi wave vector along the c axis and the Fermi velocity in the plane, respectively. The effective mass of these electrons will be modified by an amount of the order of $m\delta\varepsilon/\varepsilon_F$ and we obtain the mass M_v of the vortex, $M_v=(2/\pi^3)mK_F$. For the isotropic case this result agrees with Suhl's expression. Finally, the angular dependences of the masses can be understood by noting the following two points: (i) The vortex core size $\epsilon_\vartheta\xi^2$ depends on the angle ϑ such that the vortex mass is reduced by a factor ϵ_ϑ for motion along x. (ii) In addition, for motion along y' the effective mass of the electrons is increased by a factor ϵ_ϑ^{-2}, leading to an increase of the vortex mass by a factor ϵ_ϑ^{-1} for this case, in agreement with Eq. (18).

Besides the electronic contribution, a second term $M_{\rm em}$ of electromagnetic origin[38] contributes to the vortex mass. Typically, the electronic contribution discussed above is the dominant part, but for the case of a Josephson vortex in a layered superconductor $M_{\rm em}$ can become large. The latter is given by the relation

$$\frac{M_{\rm em}}{2}v^2=\int dx\,dz\frac{E^2}{8\pi}\simeq\int dx\frac{C}{2}\frac{\hbar^2}{4e^2}\dot{\varphi}^2, \tag{19}$$

where $C=\varepsilon_d/4\pi d$ is the capacity per unit area between two layers (ε_d is the dielectric constant) and φ is the phase difference between the two neighboring layers enclosing the phase core of the Josephson vortex. Using the soliton solution $\varphi\simeq 4\arctan\{\exp[(x-vt)/\Lambda]\}$ for the phase core of the Josephson vortex, we obtain

$$M_{\rm em}\simeq\frac{\epsilon\varepsilon_d}{2\pi d^2}\frac{\hbar^2}{e^2}. \tag{20}$$

Depending on the value of the dielectric constant ε_d, the electromagnetic mass of the Josephson vortex can become large. Note, that the electronic mass of the Josephson vortex remains unchanged with respect to the result for the Abrikosov vortex in an anisotropic medium: The different core size $d\Lambda$ is compensated by a reduced suppression of the order parameter $[\delta|\Delta_v|^2\simeq(\xi/\Lambda)^2|\Delta|^2]$ in the Josephson vortex. Using the planar effective Bohr radius $a_B^p=\hbar^2/me^2$, the ratio $M_{\rm em}/\epsilon M_v$ takes the simple form $M_{\rm em}/\epsilon M_v=(\pi^2/4)(a_B^p/d)\varepsilon_d/dK_F$. For the oxide superconductors we use the estimates $\varepsilon_d\simeq 20$, $a_B^p\lesssim 0.5$ Å, $d\simeq 10$ Å, and $K_F\simeq\pi/d$, resulting in a mass ratio of the order of unity. In the following we use the electronic mass ϵM_v for the Josephson vortices (see also Table I) but remark that the dominant mass contribution has to be determined for each material.

Let us now turn to the tunneling process. In generalizing the WKB approximation to the many-dimensional case, Banks, Bender, and Wu[40] pointed out that the tunneling particle chooses to follow a path of minimal resistance. Coleman[16] then showed that the tunneling path obeys the Lagrangian equations for a particle moving in an inverted potential (or in imaginary time) and performing a bounce trajectory (instanton solution). The bounce or instanton solution corresponds to a saddle-point solution of the Euclidean action. This reformulation of the tunneling process allows straightforward generalization to the case where the tunneling object is not a pointlike object.[16] Here the tunneling object is a 1D string and the tunneling rate is determined by the saddle-point solution of the Euclidean action of the vortex

$$S_E=\int dt\left\{\int dz'\left[\frac{M^{\parallel}(\vartheta)}{2}(\partial_t u_x)^2+\frac{M^{\perp}(\vartheta)}{2}(\partial_t u_{y'})^2\right]+\mathcal{F}[\mathbf{u}]\right\}, \tag{21}$$

$$\frac{d\ln M}{d\ln t}\simeq-\frac{\hbar}{S_E|_{\rm bounce}}. \tag{22}$$

Thus, the quantum problem corresponds to the $(n+1)$-dimensional generalization of the n-dimensional classical problem. For the string $n=1$, whereas $n=3$ for the problem of moving vortex bundles.[10] The additional dimension becoming relevant for the quantum motion is time: Quantum mechanically, energy conservation can be violated in a virtual process. However, the amplitude of the process decays exponentially with the size of the time interval during which the energy conservation is violated.

Whereas we had to determine the saddle-point solution of the free energy in order to find the classical creep rate, we now have to determine the saddle-point solution of the Euclidean action. We use again the method of dimensional estimates in order to find the relevant dimensions of the bounce. The geometric dimension of the bounce has already been determined: The tunneling segment has a length L_h and we have to use the appropriate hopping length for each case as discussed above. The estimate for the characteristic tunneling time t_c is obtained by equating the kinetic and elastic energy densities in (21). Note that by construction, the elastic energy density involved in the hop is equal to the elastic energy density involved in the relaxation to the pinning potential.

Consider first the anisotropic superconductors and motion in the plane, i.e., the driving current flows along y'. For the bounce solution the kinetic-energy density $M^{\parallel}(\vartheta)(\xi/t_c^{\parallel})^2$ is equal to the elastic energy density $\varepsilon_l^{\parallel}(\vartheta)(\xi/L_c^{\parallel})^2$. Inserting the result (6) for $L_c^{\parallel}$, we obtain for the tunneling time $t_c^{\parallel}\simeq(M_v/\varepsilon_0)^{1/2}L_c^c/\epsilon=t_c^M$, where t_c^M is the tunneling time for a vortex aligned with the crystal c axis. Finally, the Euclidean action of the bounce is (S_E^c is the action for a vertex parallel to the c axis)

$$\frac{S_E^{\parallel}}{\hbar} \simeq \frac{t_c^{\parallel}}{\hbar} U_c^c \simeq \epsilon\xi^2 \left[\frac{\varepsilon_0 M_v}{\hbar^2}\right]^{1/2} \simeq \xi^2 \epsilon K_F k_F \left[\frac{l}{\xi_0}\right]^{1/2} = \frac{S_E^c}{\hbar}. \quad (23)$$

Here we have used standard formulas to relate the London penetration depth λ to the density of electrons $n = k_F^2 K_F / 3\pi^2$ in the system. For a clean superconductor, the mean free path l has to be substituted by the coherence length ξ_0. For a free-electron-like parabolic dispersion, the above formula can be further simplified by using $\epsilon K_F = k_F$. Aligning the driving current with the x axis, the vortex motion involves the out-of-plane mass and elasticity. The tunneling time is again obtained by equating the kinetic and the elastic energy densities,

$$M^{\perp}(\vartheta)(\epsilon_\vartheta \xi / t_c^{\perp})^2 \simeq \varepsilon_l^{\perp}(\vartheta)(\epsilon_\vartheta \xi / L_c^{\perp})^2,$$

resulting in a tunneling time $t_c^{\perp} \simeq t_c^M$ which is again independent of the angle ϑ and agrees with the tunneling time for in-plane motion. Thus, we obtain the simple result that the Euclidean action for the bounce is independent of the angle ϑ and of the direction of motion for an anisotropic superconductor, $S_E^{\perp} \simeq S_E^c$.

Let us turn to the layered superconductors now and consider first the simpler case of out-of-plane motion (current along the x axis) at small angles $|\vartheta| < \epsilon$. The tunneling process involves motion of pancake vortices of mass dM_v by a distance ξ within the ab plane. For the multipancake pinning regime, the number of pancake vortices hopping simultaneously is $L_c^{\perp}|\vartheta|/d$ resulting in a kinetic-energy density $|\vartheta| M_v (\xi / t_c^{\perp})^2$. Equating this expression with the elastic energy density $\varepsilon_l^{\perp}(\vartheta)(\xi\vartheta / L_c^{\perp})^2$ we obtain the tunneling time $t_c^{\perp} \simeq t_c^M (|\vartheta|/\epsilon)^{1/2}$. Multiplying by the pinning energy $U_c^c |\vartheta|/\epsilon$ the result for the Euclidean action is

$$S_E^{\perp} \simeq S_E^c \left[\frac{|\vartheta|}{\epsilon}\right]^{3/2}, \quad \epsilon\left[\frac{d}{L_c^c}\right]^{1/2} < \vartheta < \epsilon. \quad (24)$$

For the single pancake pinning regime at small angles $|\vartheta| < \epsilon(d/L_c^c)^{1/2}$, we have to equate the kinetic energy $dM_v(\xi/t_c^{\perp})^2$ to the pinning energy $U_c^c(d/L_c^c)^{1/2}$. The tunneling time is $t_c^{\perp} \simeq t_c^M (d/L_c^c)^{1/4}$ and the action takes the form

$$S_E^{\perp} \simeq S_E^c \left[\frac{d}{L_c^c}\right]^{3/4}, \quad |\vartheta| < \epsilon\left[\frac{d}{L_c^c}\right]^{1/2}. \quad (25)$$

If the current flows along the y' axis, the in-plane hop involves the motion of the pancake vortices *and* the Josephson vortices. The latter are not pinned at the current densities considered here but contribute to the mass of the moving object. Comparing the pancake vortex mass dM_v with the mass of the Josephson vortex $\epsilon M_v d/|\vartheta|$, we find that for small angles $|\vartheta| < \epsilon$ the Josephson vortex mass is dominant. Thus, the kinetic-energy density involved in the hop is modified and reads $\epsilon M_v (\xi / t_c^{\parallel})^2$. Equating the kinetic-energy density to the elastic energy density involved in the relaxation $\varepsilon_l^{\perp}(\vartheta)(\xi\vartheta/L_c^{\perp})^2$, results in an enhanced tunneling time $t_c^{\parallel} \simeq t_c^M$. In addition, inplane hops proceed via larger vortex segments $L_h^{\parallel} \simeq L_c^c/\epsilon$, leading to an additional enhancement of the action by a factor $L_h^{\parallel}/L_c^{\perp}$. The result for the multipancake creep regime then is

$$S_E^{\parallel} \simeq t_c^M U_c^c \frac{|\vartheta|}{\epsilon} \frac{\epsilon}{|\vartheta|} \simeq S_E^c, \quad \epsilon\left[\frac{d}{L_c^c}\right]^{1/2} < \vartheta < \epsilon. \quad (26)$$

For angles $|\vartheta| < \epsilon(d/L_c^c)^{1/2}$ we enter the regime of single pancake pinning where the elementary pinning energy is $U_c^{\perp} \simeq U_c^c (d/L_c^c)^{1/2}$ and equating this to the kinetic energy $\epsilon M_v (d/|\vartheta|)(\xi/t_c^{\parallel})^2$ the tunneling time becomes $t_c^{\perp} \simeq t_c^M (d/L_c^c)^{1/4} \sqrt{\epsilon/|\vartheta|}$. The enhancement factor takes the form $L_h^{\parallel}/L_c^{\perp} \simeq \sqrt{|\vartheta|/\epsilon}(L_c^c/d)^{3/4}$ resulting in an action

$$S_E^{\parallel} \simeq S_E^c, \quad \epsilon\left[\frac{d}{L_c^c}\right]^{3/2} < \vartheta < \epsilon\left[\frac{d}{L_c^c}\right]^{1/2}. \quad (27)$$

Below the angle $\epsilon(d/L_c^c)^{3/2}$ the hopping length $L_h^{\parallel}$ drops below the mean pancake vortex separation $d/|\vartheta|$ and we enter the regime of single pancake hops where the action increases with decreasing angle ϑ,

$$S_E^{\parallel} \simeq S_E^c \left[\frac{d}{L_c^c}\right]^{3/4} \left[\frac{\epsilon}{|\vartheta|}\right]^{1/2},$$
$$\epsilon\left[\frac{d}{L_c^c}\right]^{1/2} \left[\frac{\xi}{\Lambda}\right]^3 < \vartheta < \epsilon\left[\frac{d}{L_c^c}\right]^{3/2}. \quad (28)$$

Finally, at very small angles, we enter a regime where tunneling is dominated by the Josephson vortices. Comparing the kinetic-energy density $\epsilon M_v (\Lambda / t_c^J)^2$ with the elastic energy density $\varepsilon_0 \epsilon (\Lambda / L_c^J)^2$, we obtain the tunneling time $t_c^J \simeq t_c^M (\Lambda/\xi)^2$, which is considerably enhanced due to the increased dimensions Λ and L_c^J of the tunneling object. Correspondingly the action is very large and the tunneling rate for the Josephson vortices is small,

$$S_E^J \simeq S_E^c \left[\frac{\Lambda}{\xi}\right]^2, \quad |\vartheta| < \epsilon\left[\frac{d}{L_c^c}\right]^{1/2} \left[\frac{\xi}{\Lambda}\right]^3. \quad (29)$$

B. Finite dissipation

The above results apply to the limit of vanishing dissipation. Usually, macroscopic quantum tunneling is an inherently dissipative process as the macroscopic variable is coupled to environmental degrees of freedom. By coupling the macroscopic variable to a bath of harmonic oscillators, Caldeira and Leggett[17] have been able to take the effect of dissipation into account in an elegant way. Integrating out the harmonic-oscillator degrees of freedom they were able to derive an *effective* Euclidean action S_E^{eff} for the macroscopic variable coupled dissipatively to the environment. Generalizing their result for the SQUID to the case of a tunneling vortex, the environment can be accounted for by adding a term

$$\int dt\,dt'dz' \left\{ \frac{\eta^{\parallel}(\vartheta)}{4\pi} \left[\frac{u_x(z',t)-u_x(z',t')}{t-t'} \right]^2 + \frac{\eta^{\perp}(\vartheta)}{4\pi} \left[\frac{u_{y'}(z',t)-u_{y'}(z',t')}{t-t'} \right]^2 \right\} \tag{30}$$

to the Euclidean action (21). Here we have assumed that the dissipation is ohmic and we will comment in some detail on this assumption in Sec. IV. The viscous drag coefficients $\eta^{\parallel}(\vartheta)$ and $\eta^{\perp}(\vartheta)$ depend on the direction of the applied field and on the direction of motion. Again the two effects of vortex core size $\epsilon_\vartheta \xi^2$ and of the electronic mass m/ϵ_ϑ^2 compete with each other, resulting in

$$\eta^{\parallel}(\vartheta)=\epsilon_\vartheta \eta$$

and (31)

$$\eta^{\perp}(\vartheta)=\frac{\eta}{\epsilon_\vartheta}.$$

Here $\eta=\Phi_0^2/2\pi c^2\xi^2\rho_n$ is the viscous drag coefficient for a vortex aligned with the c axis of the crystal. As we redirect the field along ϑ, the vortex core size ξ^2 changes to $\epsilon_\vartheta \xi^2$, producing a correction factor ϵ_ϑ^{-1}. For the case of out-of-plane motion, the electric field generated by the moving vortex points along x, such that we have to use the in-plane resistivity ρ_n. On the other hand, the in-plane motion of the vortex produces an electric field along y' and we have to use the corresponding resistivity $\rho_n/\epsilon_\vartheta^2$ in our expression for η. Finally, for the case of layered superconductors the viscous drag coefficient is

$$\eta^J=\epsilon \left[\frac{\xi}{\Lambda} \right]^2 \eta, \tag{32}$$

as can be easily seen by substituting $d\Lambda$ for ξ^2 and ρ_n/ϵ^2 for ρ_n in the formula for η. The result (32) agrees with the result found by Clem and Coffey.[33]

The correction (30) is nonlocal in time and in order to treat this term we transform the effective action to Fourier space,

$$S_E^{\text{eff}}=\int \frac{d\omega}{2\pi}\frac{dq}{2\pi} \left\{ \frac{1}{2} \left[\left[M^{\parallel}(\vartheta)+\frac{\eta^{\parallel}(\vartheta)}{|\omega|} \right] \omega^2+\varepsilon_l^{\parallel}(\vartheta)q^2 \right] |u_x(q,\omega)|^2 + \frac{1}{2} \left[\left[M^{\perp}(\vartheta)+\frac{\eta^{\perp}(\vartheta)}{|\omega|} \right] \omega^2+\varepsilon_l^{\perp}(\vartheta)q^2 \right] |u_{y'}(q,\omega)|^2 + U_{\text{pin}}(q,\mathbf{u}) \right\}. \tag{33}$$

The inclusion of dissipation leads to renormalized dispersive masses $M_{\text{eff}}^{\parallel}=M^{\parallel}(1+\eta^{\parallel}/M^{\parallel}|\omega|)$ and $M_{\text{eff}}^{\perp}=M^{\perp}(1+\eta^{\perp}/M^{\perp}|\omega|)$. The tunneling times $t_c^{\parallel}$ and $t_c^{\perp}$ for in-plane and out-of-plane motions are obtained by equating the kinetic- and the elastic energy densities,

$$M_{\text{eff}}^*(\vartheta,\omega_c^*)(\xi^*\omega_c^*)^2 \simeq \varepsilon_l^*(\vartheta)(\xi^* q_c)^2,$$

$q_c=2\pi\epsilon_\vartheta/L_c^c$, and solving for $\omega_c^*=2\pi/t_c^*$. Here the superscript $*$ stands for $\parallel$ or $\perp$ and we have substituted the result (6) for the collective-pinning length into the expression for the wave vector $q_c=q_c^*=2\pi/L_c^*$. The mass enhancement factor $1+\eta^*/M^*|\omega_c^*|$ becomes equal to $1+2/[(1+\nu^*)^{1/2}-1]$, with $\nu^*=4M^*\varepsilon_l^*(q_c/\eta^*)^2$. Inserting all angular dependencies one finds $\nu^{\parallel}=\nu^{\perp}=\nu$ with $\nu=16\pi^2 M_v \varepsilon_0 \epsilon^2/(L_c^c)^2\eta^2$. In the following we wish to concentrate on the limit of large damping, where ν is small. It seems that this limit is applicable for the description of the oxide superconductors. In order to estimate ν we use standard expressions to relate the London penetration depth λ to the electronic density $n=k_F^2 K_F/3\pi^2$ and obtain

$$\nu=(32/3\pi)^4(\xi^2\epsilon K_F k_F)^2(l/\xi_0)(\hbar/S_E^{\text{eff},c})^2.$$

Here we have already anticipated the result (34) below. Oxide superconductors are characterized by a small electron density $n\simeq 5\times 10^{21}$ cm^{-3}, a large anisotropy $\epsilon\sim 10^{-1}$, and a small coherence length $\xi\simeq 20$ Å ($\rightarrow$clean limit). Approximating K_F by the reciprocal-lattice vector, $K_F\simeq \pi/d$ with an interlayer distance $d\simeq 10$ Å, we obtain $(\xi^2\epsilon K_F k_F)^2\simeq 10^2$. For a typical relaxation rate $\hbar/S_F^{\text{eff},c}\simeq 1\%$ we find $\nu \ll 1$. The mass enhancement factors $\eta^*/M^*|\omega_c^*|\simeq 4/\nu$ then are large, corresponding to the limit of strong dissipation.

Note that a similar expression for a dispersive mass is obtained by starting from a purely dissipative dynamics involving a dispersive friction coefficient $\eta(\omega)$: Expanding $\eta(\omega)$ for small frequencies $|\omega| \ll \Delta/\hbar$, we obtain a term of the form $[\eta(0)/2](1+\hbar|\omega|/\Delta)|\omega|u^2$ in the Euclidean Lagrangian. Using standard expressions for the friction η and for the gap parameter Δ, the second term reduces to a kinetic-energy term with a mass $M_v \simeq (1/2\pi)mK_F$,[41] in rough agreement with our expression above. Using the result below for the tunneling time t_c^η as well as Eq. (7), we obtain $\hbar\omega_c/\Delta \simeq j_c/j_0$, such that under the condition of weak pinning, $j_c/j_0 \ll 1$, the dissipative term in the Lagrangian is always dominant.

In the limit of large damping, the determination of the tunneling time simplifies considerably and we obtain the results $t_c^{\parallel}\simeq t_c^{\perp}\simeq t_c^{\eta}=(\eta/\varepsilon_0)(L_c^c/\epsilon)^2$. For anisotropic superconductors the final expression for the effective action is

$$\frac{S_E^{\parallel}}{\hbar}\simeq\frac{S_E^{\perp}}{\hbar}\simeq\frac{t_c^{\eta}}{\hbar}U_c^c \simeq \frac{\eta\xi^2 L_c^c}{\hbar} \simeq \frac{\hbar}{e^2}\frac{e\xi}{\rho_n}\left[\frac{j_0}{j_c}\right]^{1/2} = \frac{S_E^{\text{eff}},c}{\hbar}, \tag{34}$$

independent of the angle ϑ and the direction of motion.

Finally, we have to derive the expressions for the effective Euclidean action for a layered superconductor where we have to concentrate on the small-angle region $|\vartheta| < \epsilon$. Out-of-plane motion again splits into a multipancake pinning regime, where $t_c^\perp \simeq t_c^\eta |\vartheta|/\epsilon$, and a single pancake pinning regime with $t_c^\perp \simeq t_c^\eta (d/L_c^c)^{1/2}$. The corresponding expressions for the effective action are

$$S_E^\perp \simeq t_c^\perp U_c^c \frac{|\vartheta|}{\epsilon} \simeq S_E^{\text{eff},c} \left[\frac{\vartheta}{\epsilon} \right]^2 , \quad \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} < \vartheta < \epsilon, \tag{35}$$

$$S_E^\perp \simeq t_c^\perp U_c^c \left[\frac{d}{L_c^c} \right]^{1/2} \simeq S_E^{\text{eff},c} \frac{d}{L_c^c}, \quad |\vartheta| < \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} . \tag{36}$$

The case of in-plane motion is again complicated by the enhancement length of the hopping segment producing a shift of the single pancake tunneling regime towards smaller angles. The results are

$$S_E^\parallel \simeq S_E^{\text{eff},c} \frac{|\vartheta|}{\epsilon}, \quad \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} < \vartheta < \epsilon, \tag{37}$$

$$S_E^\parallel \simeq S_E^{\text{eff},c} \left[\frac{d}{L_c^c} \right]^{1/4} \left[\frac{|\vartheta|}{\epsilon} \right]^{1/2} , \quad \epsilon \left[\frac{d}{L_c^c} \right]^{3/2} < \vartheta < \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} . \tag{38}$$

At small angles the friction produced by the moving Josephson vortex, $\epsilon\eta(\xi/\Lambda)^2 d/|\vartheta|$, competes with the friction due to the moving pancake vortex, ηd, and for angles $|\vartheta| < \epsilon(\xi/\Lambda)^2$ the single pancake hops are damped by the Josephson vortex dragged along. Thus, the single pancake hop regime splits into two parts with actions

$$S_E^\parallel \simeq S_E^{\text{eff},c} \frac{d}{L_c^c}, \quad \epsilon \left[\frac{\xi}{\Lambda} \right]^2 < \vartheta < \epsilon \left[\frac{d}{L_c^c} \right]^{3/2} , \tag{39}$$

and

$$S_E^\parallel \simeq S_E^{\text{eff},c} \frac{d}{L_c^c} \left[\frac{\xi}{\Lambda} \right]^2 \frac{\epsilon}{|\vartheta|}, \quad \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} \left[\frac{\xi}{\Lambda} \right]^3 < \vartheta < \epsilon \left[\frac{\xi}{\Lambda} \right]^2 . \tag{40}$$

The regime of Josephson vortex pinning at very small angles is characterized by a large action and a correspondingly small tunneling rate,

$$S_E^J \simeq S_E^{\text{eff},c} \left[\frac{\Lambda}{\xi} \right]^2 , \quad |\vartheta| < \epsilon \left[\frac{d}{L_c^c} \right]^{1/2} \left[\frac{\xi}{\Lambda} \right]^3 . \tag{41}$$

This completes our analysis of quantum collective creep in anisotropic and layered superconductors for the two limits of vanishing and strong damping. All results are summarized in Table II.

IV. THERMAL ENHANCEMENT OF TUNNELING

The instanton approach to the tunneling problem allows one to formulate the temperature dependence of the tunneling process in a straightforward way:[13,14] For finite temperatures $T > 0$, the limits of the imaginary-time integrals in the effective Euclidean action [see Eqs. (21) and (30)] have to be cut off at $\pm\beta/2$, where $\beta = \hbar/k_B T$. The bounce trajectory is deformed to a periodic orbit with a period given by the inverse temperature β. This deformation mainly affects the motion near the metastable potential minimum. At zero temperature the bounce trajectory is a homoclinic orbit where the metastable potential minimum (local maximum of the inverted potential) defines the homoclinic point. At finite temperatures the bounce trajectory is periodic within a finite interval and thus avoids the homoclinic point. For the case of zero dissipation, this corresponds to an orbit characterized by a finite excitation energy. Thus, the effect of finite temperature is to cut off the infinitely slow motion near the homoclinic point and thereby reduce the (effective) Euclidean action. The determination of the finite-temperature corrections to the action involves the precise knowledge of the bounce solution near the homoclinic point—a quantity which is not accessible by our simple dimensional estimates.

However, for the case where the tunneling object is a pointlike object, Grabert, Weiss, and Hänggi[14] have shown that the finite-temperature corrections to the bounce trajectory are mainly determined by the dissipation mechanism alone: For the zero damping limit, the corrections are exponentially small, $\propto \exp(-\hbar\omega_0/2k_B T)$. Here ω_0 denotes the frequency of small oscillations of the collective coordinate around the metastable minimum. For nonzero damping, the finite-temperature corrections to the bounce trajectory show power-law behavior, $\propto T^{2(n+1)}$, with an exponent $n \geq 0$ given by the first nonvanishing term in the Taylor expansion of the frequency-dependent damping coefficient $\eta(\omega)$, $\partial_\omega^{2n}\eta(\omega)|_0 \neq 0$. The most pronounced temperature dependence is a T^2 law[13] ($n=0$) which is realized for the case of ohmic dissipation with $\eta(\omega=0) > 0$. Note that if the coupling to the environment has a low-frequency cutoff $\delta/\hbar$, such that $\partial_\omega^{2n}\eta(\omega)|_0 = 0$ for all n, the finite temperature ($k_B T < \delta$) corrections to the bounce trajectory are again exponentially small. Furthermore, the finite-temperature corrections to the effective Euclidean action show the same dependence on the low-frequency behavior of the damping coefficient $\eta(\omega)$ and do not depend on the details of the potential, the only relevant quantity being the tunneling time t_t. In the zero dissipation limit

$$\Delta S_E(T)=S_E(T)-S_E(0)\simeq -S_E(0)\exp(-\hbar/k_B T t_t),$$
$$t_t\simeq 1/\omega_0\ ,$$

whereas for strong (ohmic) damping

$$\Delta S_E^{\rm eff}(T)\simeq -S_E^{\rm eff}(0)(k_B T t_t/\hbar)^2,\quad t_t\simeq \eta/M\omega_0^2\ .$$

These results can be applied to the problem of quantum motion of vortices where the tunneling object is a string. In fact, in the above derivation of the saddle-point solution to the Euclidean action we first determined the length L_c of the optimal segment. In a second step, this segment was treated as a pointlike object in the determination of the collective tunneling time t_c. The reduction of the string problem to a particle problem can be done explicitly for a cubic model potential describing the tunneling of a vortex segment of length L_c under a barrier of (maximal) height U_0. For an initial configuration

$$u(z',t=-\infty)=(-u_0/2)\sin kz',$$

$k=\pi/L_c$, and an eigenfrequency ω_0 for the small oscillations around the metastable configuration, the impurity potential takes the form $[\gamma(z')=\sin kz']$

$$U_{\rm pin}(u,z')=\frac{27}{4}U_0\left[\frac{\gamma(z')^2}{8}+\frac{\gamma(z')}{4}\left[\frac{u}{u_0}\right]-\left[\frac{1}{2}+\alpha^2\right]\left[\frac{u}{u_0}\right]^2-\frac{1}{\gamma(z')}\left[\frac{u}{u_0}\right]^3\right], \tag{42}$$

with $\alpha^2=\varepsilon_l k^2/M_v\omega_0^2$. The effective potential $U_{\rm eff}=U_{\rm pin}+(\varepsilon_l/2)(\partial_{z'}u)^2$ then has a (metastable) minimum with zero energy along the line $u(z')=(-u_0/2)\sin kz'$ and a symmetric zero energy line at $u(z')=(u_0/2)\sin kz'$, see Fig. 3. Note that the condition $(1/M)\partial^2U_{\rm eff}/\partial u^2|_{u(z',\infty)}=\omega_0^2$ has a solution $\omega_0^2=\frac{27}{2}U_0/Mu_0^2$, such that the expression for the effective

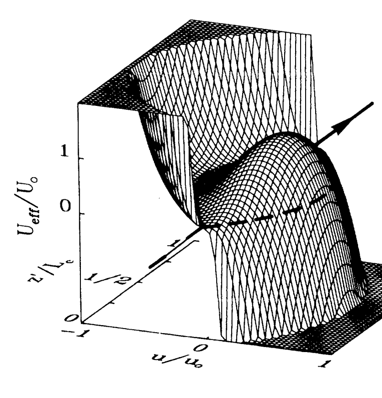

FIG. 3. Effective potential pinning a vortex in a metastable state. The units are given by L_c (scale along the vortex), by u_0 (scale transverse to the vortex), and by U_0 (effective pinning potential). Also shown is the final state of the vortex after the tunneling process for the case of vanishing dissipation.

potential takes the form of $U_{\rm pin}$ but with $\alpha=0$. The saddle-point solution of the free energy has the form $u_s(z')=(u_0/6)\sin kz'$ and produces a collective-pinning energy $U_c=L_cU_0/2$. For weak collective pinning, $u_0=\xi$, and the collective-pinning energy U_c is equal to the elastic energy of the relaxed string $(\pi/4)^2\varepsilon_l\xi^2/L_c$. Using these conditions we obtain $U_0=(\pi^2/8)\varepsilon_l(\xi/L_c)^2$, $\alpha^2=2(\frac{2}{3})^3$, and $\omega_0=ck/\alpha$ with $c^2=\varepsilon_l/M_v$. Generalizing the results to anisotropic superconductors we have to choose the appropriate length scale $u_0^{\parallel}=\xi$ and $u_0^{\perp}=\epsilon_\vartheta\xi$ for in-plane and out-of-plane motions as well as the corresponding elasticities. The collective-pinning energy becomes

$$U_c=(\pi/4)^2\varepsilon_0\epsilon^2\xi^2/L_c^c=(\pi/4)^2\varepsilon_0\epsilon\xi\sqrt{j_c/j_0},$$

independent of the angle ϑ and of the direction of motion, in agreement with our results above.

Using an ansatz $u(z',t)=u_0q(t)\sin(kz')$ for the bounce solution, the geometric component of the problem can be integrated out and we obtain an effective Euclidean action for the coordinate $q(t)$,

$$S_E^{\rm eff}=\frac{L_cu_0^2}{2}\int dt\left\{\frac{M_v}{2}\{\dot q^2+\omega_0^2[(q+\tfrac{1}{2})^2-(q+\tfrac{1}{2})^3]\}+\frac{1}{2}\int dt'\alpha(t-t')[q(t)-q(t')]^2\right\}. \tag{43}$$

Up to a trivial shift in the coordinate system, the above effective Euclidean action is identical to the one solved by Caldeira and Leggett[17] for the SQUID problem. The kernel $\alpha(t)$ is determined by the spectral density $J(\omega)$ of the environment,[17]

$$\alpha(t)=\int_0^\infty(d\omega/2\pi)J(\omega)\exp(-\omega|t|),$$

and reduces to the simple from $\alpha(t)=\eta/2\pi t^2$ for the important case of ohmic dissipation. Inserting the bounce solutions found by Caldeira and Leggett for the two limits of zero and strong damping, we obtain for the (effective) Euclidean actions

$$S_E=(\sqrt{3}/5)\epsilon\xi^2\sqrt{\varepsilon_0M_v}=(2\sqrt{2}/5)\hbar(\xi^2\epsilon K_Fk_F)\sqrt{l/\xi_0}$$

and

$$S_E^{\rm eff}=(\pi/9)L_c^c\eta\xi^2$$
$$=(\pi^2/18)(\hbar/e)^2(\epsilon\xi/\rho_n)(j_0/j_c)^{1/2}\ ,$$

independent of the angle ϑ and of the direction of motion, in agreement with our previous results. Note that for the above cubic model potential we obtain numerical factors of order unity, $2\sqrt{2}/5\simeq 0.5$ and $\pi^2/18\simeq 0.5$, in the (effective) Euclidean actions, in very good agreement with our dimensional estimates. Thus, the tunneling problem for the vortex has been reduced to the tunneling of a pointlike object and we can apply the results of the analysis of Grabert, Weiss, and Hänggi.[14]

For the two limits of zero dissipation and of strong damping, the finite-temperature corrections

$$\Delta S_E^{(\mathrm{eff})}(T)=S_E^{(\mathrm{eff})}(T)-S_E^{(\mathrm{eff})}(0)$$

to the Euclidean action take the forms

$$\Delta S_E(T)\simeq -S_E(T=0)e^{-\hbar/k_B T t_c} \tag{44}$$

and

$$\Delta S_E^{\mathrm{eff}}(T)\simeq -S_E^{\mathrm{eff}}(T=0)\left[\frac{k_B T t_c}{\hbar}\right]^2, \tag{45}$$

where we have to insert the appropriate tunneling time t_c as given in Table II.

The crossover temperature T_{qc} from the quantum to the classical regime of motion is determined by the condition $\Delta S_E^{(\mathrm{eff})}/S_E^{(\mathrm{eff})}\simeq 1$ and thus depends on the tunneling time t_c,

$$k_B T_{qc}\simeq\frac{\hbar}{t_c}. \tag{46}$$

Note that, for weak collective pinning, the zero-dissipation tunneling time t_c^M is determined by the elasticity of the string, $\omega_c=ck$ with $c=\sqrt{\varepsilon_0\epsilon^2/M_v\epsilon_\vartheta^2}$ and $k\simeq\epsilon_\vartheta/L_c^c$, and not by the curvature of the bare pinning potential. The time scale for tunneling in the strong damping limit is enhanced over the zero-dissipation tunneling time

$$t_c^\eta=(\eta/M_v)(t_c^M)^2>(\eta/M_v\omega_c^\eta)t_c^M>t_c^M,$$

and thus the crossover temperature T_{qc} becomes small for the large damping limit. Using $S_E^{(\mathrm{eff})}(T=0)\simeq t_c U_c$, we obtain the relation

$$\frac{S_E^{(\mathrm{eff})}(T=0)}{\hbar}\simeq\frac{U_c}{k_B T_{qc}} \tag{47}$$

between the zero-temperature (effective) Euclidean action $S_E^{(\mathrm{eff})}(T=0)$, the classical activation energy U_c, and the crossover temperature T_{qc}.

Finally, we make a few remarks concerning the energy dissipation produced by a moving vortex. Following Larkin and Ovchinnikov,[42,43] the Bardeen-Stephen formula for the viscous drag coefficient η can be used also at low temperatures where quantum motion is relevant. However, in this calculation[43] the discreteness of the quasiparticle spectrum[44] within the vortex core has been neglected. If the dissipation is due to the quasiparticle current flowing across the vortex core, the spectral density $J(\omega)$ of the environment will have a low-frequency cutoff at the lowest quasiparticle bound-state energy which is of the order of Δ^2/ε_F, with Δ denoting the superconducting gap energy. This is similar to the problem of macroscopic quantum tunneling in a SQUID as described by Eckern, Schön, and Ambegaokar[45] who assume that the dissipation is due to quasiparticle tunneling across the oxide layer in the Josephson junction. Thus, in a very clean superconductor we have to expect an exponential low-temperature behavior for the action in the regime $k_B T<\Delta^2/\varepsilon_F$. On the other hand, in a dirty superconductor the quasiparticle levels will be broadened by finite lifetime effects. If the level broadening $\hbar/\tau$ is larger than the quasiparticle gap Δ^2/ε_F, the low-frequency cutoff in the spectral density $J(\omega)$ vanishes and the Bardeen-Stephen formula can be used. Here τ denotes the quasiparticle lifetime. The condition for the applicability of ohmic dissipation, $l<\xi_0(\varepsilon_F/\Delta)$, with $l=v_F\tau=$mean free path, is less stringent than the condition $l<\xi_0$ defining a dirty superconductor. Thus, we may expect that the model of ohmic dissipation is applicable for the oxide superconductors which rather belong to the class of clean superconductors due to their short coherent length.

V. SUMMARY AND CONCLUSION

In this work we have determined the rates for classical and quantum creep of vortices in anisotropic and layered superconductors. Both creep rates are determined by the saddle-point solutions to some specific functional. The classical motion is characterized by an optimal geometric configuration of the hopping segment and the rate is determined by the saddle point of the free energy. Quantum motion additionally involves an optimal dynamics of the tunneling object and therefore the rate is determined by the saddle-point solution of the Euclidean action. Thus, the quantum problem can be viewed as the $(n+1)$-dimensional generalization of the n-dimensional classical problem. In a first step we have solved the classical problem in order to find the optimal configuration for the moving vortex. Once the optimal length for the hopping segment is known, we immediately obtain the activation barrier against classical creep. In a second step we then have determined the optimal tunneling time for the bounce, and using the result for the classical activation energy, we have obtained the action determining quantum creep.

Classical as well as quantum motion of vortices is a very complex problem which is beyond direct microscopic description. The problem is even too complicated to be described on the level of Ginzburg-Landau. In our approach we have based the description on the Lagrangian for the macroscopic variable, which is the displacement field $\mathbf{u}(\mathbf{r})$ for the vortex position. This Lagrangian contains the mass M_v, the elasticity ε_l, and the viscous friction coefficient η as phenomenological parameters, which have been obtained by going back to a Ginzburg-Landau-type description. This is also the place where the anisotropic properties of the material enter into the model.

The determination of the saddle-point solutions for the free energy and for the (effective) Euclidean action has been done by using dimensional estimates. In anisotropic superconductors the mass, the density, the pinning force, and the friction depend on the angle ϑ which the vortex encloses with the *ab* plane and on the direction of motion. Nevertheless, it turns out that for classical as well as for quantum motion these dependencies cancel each other such that the final results for the activation energy U_c and for the (effective) Euclidean action $S_E^{(\mathrm{eff})}$ are independent of the angle ϑ as well as the direction of motion. For layered superconductors identical results have been obtained within the large-angle region $\vartheta>\epsilon$. For smaller angles $|\vartheta|<\epsilon$, the deviation of the vortex structure from a simple rectilinear object becomes important. Pinning is affecting mainly the component parallel to the c axis, the pancake vortices. After a transition region, classical as well as quantum motion proceeds in terms of single pancake vortex hops characterized by a reduced activation energy $U_c^c(d/L_c^c)^{1/2}$ and a reduced effective Euclidean action $S_E^{\text{eff},c}d/L_c^c$. For the case of vanishing dissipation, the Euclidean action is reduced for the case of out-of-plane motion, $S_E^c(d/L_c^c)^{3/4}$, however, for in-plane motion the action increases as the pancake vortex has to carry the large mass of the Josephson vortex along. The transition region depends on the direction of motion and the results within this region in addition also depend on the angle ϑ, such that the crossover from anisotropic behavior at large angles to the single pancake pinning regime at small angles is smooth. Finally, for very small angles where the distance between neighboring pancake vortices becomes large, in-plane motion is determined by the pinning of Josephson vortices. Whereas the classical activation barrier remains unchanged for this regime, $U_c^J \simeq U_c^c$, the action is considerably enhanced due to the large tunneling time t_c^J for the Josephson vortex.

The independence of the classical and quantum creep rates upon the field direction for anisotropic superconductors suggests that magnetic relaxation rates for polycrystalline and for single-crystal material should be the same, in rough agreement with the results of Refs. 4 and 6. Also, it would be very interesting to investigate quantum creep in the strongly layered Bi-based high-temperature superconductors where our theory predicts large relaxation rates due to the large anisotropy. The determination of the critical current density j_c in the planes can be used to check the validity of the weak collective-pinning approach: With $d \simeq 15$ Å, $\xi \simeq 30$ Å, $\epsilon \simeq \frac{1}{50}$, and $j_0 \simeq 10^8$ A cm^{-2}, we obtain the condition $j_c \lesssim 10^5$ A cm^{-2} $(d < L_c^c)$.

Regarding the finite-temperature corrections to the Euclidean action we have found a T^2 dependence for the most important case of strong ohmic dissipation. This T^2 law applies to a very limited regime at low temperatures below the crossover temperature $T_{qc} \simeq U_c \hbar / S_E^{\text{eff}} k_B$, with U_c the high-temperature activation energy and S_E^{eff} the zero-temperature effective Euclidean action determining classical and quantum creep, respectively. It would be very interesting to determine the exact behavior of the low-temperature corrections to the action, as such measurements can provide information on the relevant dissipation mechanism. Within the restricted temperature regime $T < T_{qc}$, the action $S_E^{(\text{eff})}(T)$ changes only by a factor of the order of unity and therefore the relevant quantity is

$$\Delta S_E^{(\text{eff})}(T) = S_E^{(\text{eff})}(T) - S_E^{(\text{eff})}(0),$$

asking for very accurate measurements. Also note that with increasing dissipation the effective Euclidean action $S_E^{(\text{eff})}(0)$ becomes larger and hence the regime $T < T_{qc}$ for quantum creep becomes smaller.

Let us close with a final remark regarding the concept of weak collective pinning. The only unknown parameter entering the theory is the parameter W in the force-force correlator. All other relevant parameters (e.g., the elasticity, the vortex mass, the viscous friction coefficient) can be determined by independent measurements (e.g., λ, ξ, ϵ, ρ_n). Thus, with a single adjustable parameter we can determine the critical current densities, the activation energies for classical creep, and now also the Euclidean action for quantum motion, as well as their temperature and field dependencies. The concept of weak collective pinning seems to be able to produce correct order-of-magnitude estimates for all these quantities which is emphasizing the consistency of the theory.

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[1]C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962).
[2]P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962).
[3]Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).
[4]A. C. Mota, A. Pollini, P. Visani, K. A. Müller, and J. G. Bednorz, Phys. Scr. **37**, 823 (1988).
[5]R. Griessen, J. G. Lensink, T. A. M. Schröder, and B. Dam, Cryogenics **30**, 563 (1990).
[6]L. Fruchter, A. P. Malozemoff, I. A. Campbell, J. Sanchez, M. Konczykowski, R. Griessen, and F. Holtzberg, Phys. Rev. B **43**, 8709 (1991).
[7]A. V. Mitin, Zh. Eksp. Teor. Fiz. **93**, 590 (1987) [Sov. Phys. JETP **66**, 335 (1987)].
[8]A. C. Mota, P. Visani, and A. Pollini, Phys. Rev. B **37**, 9830 (1988); A. C. Mota, A. Pollini, P. Visani, G. Juri, and J. J. M. Franse, *Progress in High Temperature Superconductivity*, Proceedings of ICTPS 90, edited by R. Nicolsky (World Scientific, Singapore, 1990), Vol. 25.
[9]A. C. Mota, A. Pollini, G. Juri, P. Visani, and B. Hilti, Physica A **168**, 298 (1990); and further results on [bis(ethylenedithio tetrahthiafulvalene)] $Cu(SCN)_2$ at low temperatures, A. C. Mota (private communication).
[10]G. Blatter, V. B. Geshkenbein, and V. M. Vinokur, Phys. Rev. Lett. **66**, 3297 (1991).
[11]We consistently use S_E to denote the Euclidean action in the absence of any dissipation and S_E^{eff} for the effective Euclidean action including a term due to finite dissipation.
[12]B. I. Ivlev, Yu. N. Ovchinnikov, and R. S. Thompson, Phys. Rev. B **44**, 7023 (1991).
[13]A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 322 (1983) [JETP Lett. **37**, 382 (1983)].
[14]H. Grabert, U. Weiss, and P. Hänggi, Phys. Rev. Lett. **52**, 2193 (1984); H. Grabert and U. Weiss, Z. Phys. B **56**, 171 (1984); Phys. Rev. Lett. **53**, 1787 (1984).
[15]J. S. Langer, Ann. Phys. **41**, 108 (1967).
[16]S. Coleman, Phys. Rev. D **15**, 2929 (1977).
[17]A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. **149**, 374 (1983).

[18]W. E. Lawrence and S. Doniach, in *Proceedings of the 12th International Conference on Low Temperature Physics,* Kyoto, edited by E. Kanda (Keigaku, Tokyo, 1971), p. 361.

[19]A. V. Balatskii, L. I. Burlachkov, and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **90**, 1478 (1986) [Sov. Phys. JETP **63**, 866 (1986)].

[20]R. A. Klemm and J. R. Clem, Phys. Rev. B **21**, 1868 (1980).

[21]Recently, Klemm [R. A. Klemm, Phys. Rev. B **41**, 117 (1990)] found an additional correction to the results of Refs. 20 and 19 due to the existence of a finite magnetic-field transverse to the vortex direction.

[22]V. G. Kogan, Phys. Rev. B **38**, 7049 (1988).

[23]L. N. Bulaevskii, Phys. Rev. B **44**, 910 (1991).

[24]M. Tuominen, A. M. Goldman, Y. C. Chang, and P. Z. Jiang, Phys. Rev. B **42**, 8740 (1990).

[25]The single-vortex line tension is logarithmically dispersive (Ref. 46), however, since pinning involves only short-wavelength distortions with $k \simeq 1/L_c$, we neglect here this logarithmic correction factor, which is of order unity. Also note that the new result found by Klemm (Ref. 21) for the vortex line energy does not effect the short-wavelength line tension which is determined by the redistribution of currents near the core due to the deformation, whereas the magnetic field remains essentially unchanged.

[26]A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[27]M. Tinkham, Helv. Phys. Acta **61**, 443 (1988).

[28]M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).

[29]V. G. Kogan and L. J. Campbell, Phys. Rev. Lett. **62**, 1552 (1989).

[30]S. Doniach, in *The Phenomenology of Flux Motion in High Temperature Superconductors,* Proceedings of the Los Alamos Symposium 1989, edited by K. S. Bedell, D. Coffey, D. E. Meltzer, D. Pines, and J. R. Schrieffer (Addison-Wesley, Reading, MA, 1990), p. 406.

[31]M. V. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Physica C **167**, 177 (1990).

[32]J. R. Clem, Phys. Rev. B **43**, 7837 (1991).

[33]J. R. Clem and M. W. Coffey, Phys. Rev. B **42**, 6209 (1990).

[34]D. Perruchoud, Diploma thesis, ETH-Zürich, Switzerland, 1991.

[35]A. I. Larkin, Zh. Eksp. Teor. Fiz. **58**, 1466 (1970) [Sov. Phys. JETP **31**, 784 (1970)].

[36]The above discussion applies to disorder within the superconducting planes. Disorder in the interlayer coupling (which we do not consider here) may produce additional pinning which cannot be directly related to $f_{\rm pin}$.

[37]S. Chakravarty, B. I. Ivlev, and Yu. N. Ovchinnikov, Phys. Rev. B **42**, 2143 (1990).

[38]H. Suhl, Phys. Rev. Lett. **14**, 226 (1965).

[39]M. Yu. Kupriyanov and K. K. Likharev, Zh. Eksp. Teor. Fiz. **68**, 1506 (1975) [Sov. Phys. JETP **41**, 755 (1975)].

[40]T. Banks, C. Bender, and T. T. Wu, Phys. Rev. D **8**, 3366 (1973).

[41]L. I. Glazman and N. Ya. Fogel, Fiz. Nauk. Temp. **10**, 95 (1984) [Sov. J. Low Temp. Phys. **10**, 51 (1984)].

[42]A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity,* edited by D. N. Langenberg and A. I. Larkin (Elsevier Science, New York, 1986), p. 509.

[43]L. P. Gor'kov and N. B. Kopnin, Zh. Eksp. Teor. Fiz. **65**, 396 (1973) [Sov. Phys. JETP **38**, 195 (1974)].

[44]C. Caroli, P. G. De Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).

[45]U. Eckern, G. Schön, and V. Ambegaokar, Phys. Rev. B **30**, 6419 (1984).

[46]A. Sudbø and E. H. Brandt, Phys. Rev. B **43**, 10482 (1991).