Transport properties and fluctuations in type-II superconductors near H_{c2}

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We study the flux-flow Hall effect and thermomagnetic transport near the upper critical field H_{c2} in extreme type-II superconductors starting from a suitable generalization of the time-dependent Ginzburg-Landau equations. We explicitly incorporate the effects of backflow into the calculations of the local electric field and current, which leads to a current that is properly divergenceless. The Hall conductivity calculated from this current agrees with other mean-field calculations that assume a uniform applied electric field (the Schmid-Caroli-Maki solution), thereby vindicating these simplified treatments. We then use these results to calculate the transverse thermomagnetic effects (the Ettingshausen and Nernst effects). The effects of thermal fluctuations and nonlocal elasticity of the flux lattice are incorporated using a method recently developed by Vecris and Pelcovits [G. Vecris and R. A. Pelcovits, Phys. Rev. B **44**, 2767 (1991)]. We find that the elastic fluctuations of the vortex lattice suppress the conductivities below their mean-field values. Our results, taken together with those of Vecris and Pelcovits, provide a rather complete description of the transport properties of the flux lattice state near H_{c2}, at least within the framework of time-dependent Ginzburg-Landau theory.

I. INTRODUCTION

The thermodynamic and transport properties of the mixed state of type-II superconductors continue to attract the interest of theorists and experimentalists alike, due in large measure to the unusual transport properties of the high-temperature superconductors. High transition temperatures, short coherence lengths, and large anisotropies conspire to produce enhanced thermal fluctuations in these materials, which can significantly modify the mean-field phase diagram; we refer the reader to Ref. 1 for a detailed discussion of these effects. These fluctuations are also apparent in the transport properties, as they lead to a broadened resistive transition in the flux-flow regime near H_{c2} (when pinning is unimportant), and to thermally assisted flux flow at lower temperatures (but away from the putative vortex-glass transition[1]). Indeed, if we had a detailed theory of the transport properties in the presence of fluctuations we could in principle use this to infer properties of the equilibrium phases. So far, most of the theoretical work on transport properties has focused on understanding the behavior of the longitudinal conductivity of the flux lattice. However, it is really the Hall effect which represents the greatest challenge to our understanding of the dynamics of the vortex lattice in superconductors, as evidenced by the experimental observation that the Hall conductivity changes sign upon entering the mixed state in the high-T_c superconductors,[2-7] a feature that is at odds with the classic theories of vortex motion in superconductors.[8-10] Motivated by these observations, in this paper we reexamine the theory of the Hall effect in the mixed state near H_{c2} using a variant of the standard time-dependent Ginzburg-Landau (TDGL) theory. By incorporating the effects of thermal fluctuations, the nonlocal elasticity of the flux lattice, and backflow, we have consolidated the results of several previous authors into a rather complete theory of the Hall effect near H_{c2} (at least within the TDGL framework). As a byproduct, we also study transverse thermomagnetic effects, such as the Ettingshausen effect and the Nernst effect.

As this paper is in a sense a consolidation of the results of several different authors, it is appropriate to first briefly review the history of the subject. Schmid[11] derived a set of TDGL equations from the microscopic Gorkov equations. From these equations he was able to calculate the flux-flow conductivity both near H_{c2} (by solving the linearized equations) and near H_{c1} (for a single vortex). The behavior near H_{c2} was obtained by assuming that the applied electric field **E** was constant in space; the flux lattice is then effectively "boosted" by a velocity $\mathbf{v}=\mathbf{E}\times\mathbf{B}/B^2$, with **B** the induction field. Similar methods were also used by Caroli and Maki[12] to study both the dirty and clean limits. We will henceforth refer to this solution as the Schmid-Caroli-Maki solution. Unfortunately, the local current (which includes the normal current plus the supercurrent) obtained using this method is not divergence free, as was pointed out by Thompson and Hu.[13] To obtain a current with zero divergence, it is necessary to incorporate backflow currents; however, the backflow has zero spatial average, so that the spatially averaged conductivity calculated using this method agrees with the Schmid-Caroli-Maki result. These calculations have recently been taken one step further by Vecris and Pelcovits,[14] who studied the effect of the elastic fluctuations of the flux lattice on the conductivity. Starting from the TDGL equations, these authors calculated the local current with backflow, and incorporated the elastic fluctuations by using a dynamic generalization of the formalism developed by Brandt[15] for the static flux lattice.

In all of these cases, the TDGL equations that were employed had a purely real order-parameter relaxation time, and therefore exhibited a type of "particle-hole" symmetry, which leads to a Hall conductivity that is identically zero.[16–18] To obtain a nonzero Hall conductivity one needs to generalize the TDGL equations by allowing the relaxation time to be complex. The imaginary part of the relaxation time might result from either considerations of Galilean invariance,[18,19] or from microscopic considerations, such as Fermi-surface curvature.[20] Maki[19] and Ebisawa[21] have used TDGL equations with a complex relaxation time to calculate the Hall conductivity using the Schmid-Caroli-Maki method (i.e., without backflow). These equations have also been used to study the fluctuation Hall effect for temperatures $T > T_{c2}$.[16,20] More recently, one of us (A.T.D.) has used the generalized TDGL equations to study the dynamics of a single vortex (i.e., for fields close to H_{c1}).[18]

In this paper we calculate the transport properties for the mixed state of type-II superconductors starting from a set of TDGL equations that have a complex relaxation time. The results of this paper, therefore, complement the single vortex results obtained in Ref. 18. The paper is organized as follows. In Sec. II we calculate the longitudinal and Hall conductivities in mean-field theory by explicitly including backflow, thereby extending the work of Thompson and Hu and Vecris and Pelcovits. One of the important results of this section is that the backflow current, while important in ensuring that the total current has zero divergence, does not contribute to the spatially averaged Hall conductivity (which is of experimental relevance). Hence, the Hall conductivity that we obtain agrees with the result that would be obtained using the Schmid-Caroli-Maki method. We also briefly discuss the relevance of our results to the issue of the sign change of the Hall conductivity in the mixed state of the high-T_c superconductors. In Sec. III we calculate the Ettingshausen and Nernst effects in the presence of backflow. Our derivation, which utilizes a recently discovered "virial theorem" for the equilibrium Ginzburg-Landau equations, is exact within mean-field theory. The effects of elastic fluctuations of the flux lattice are considered in Sec. IV, which follows the work of Vecris and Pelcovits. We find that the amplitude fluctuations in the flux lattice phase suppress the flux-flow conductivities below their mean-field values. Nonlocal effects are extremely important in setting the scale for these fluctuations.

II. MEAN-FIELD THEORY

The TDGL equations consist of an equation of motion for the order parameter ψ,

$$(\gamma_1+i\gamma_2)(\partial_t+i\Phi)\psi=\left[\frac{\nabla}{\kappa}-i\,\mathbf{A}\right]^2\psi+\psi-|\psi|^2\psi\ , \qquad (2.1)$$

along with Ampère's law,

$$\nabla\times\mathbf{h}=\mathbf{J}\ , \qquad (2.2)$$

where $\mathbf{h}=\nabla\times\mathbf{A}$ is the local magnetic induction field. For the current we adopt a two fluid model, so that $\mathbf{J}=\mathbf{J}_n+\mathbf{J}_s$. The normal current is

$$\mathbf{J}_n=\underline{\sigma}^{(n)}\mathbf{E}\ , \qquad (2.3)$$

where the electric field is expressed in terms of the potentials as

$$\mathbf{E}=-\frac{1}{\kappa}\nabla\Phi-\partial_t\,\mathbf{A}\ . \qquad (2.4)$$

In the normal current we include both the longitudinal and the transverse response of the normal carriers; the underbar that appears in Eq. (2.3) denotes a tensor with $\underline{\sigma}^{(n)}$ the normal-state conductivity tensor,

$$\underline{\sigma}^{(n)}=\begin{bmatrix}\sigma_{xx}^{(n)} & \sigma_{xy}^{(n)}\\ -\sigma_{xy}^{(n)} & \sigma_{xx}^{(n)}\end{bmatrix}\ . \qquad (2.5)$$

The signs used in $\sigma_{xy}^{(n)}$ are appropriate for positive carriers. The supercurrent is given by

$$\mathbf{J}_s=\frac{1}{2\kappa i}(\psi^*\nabla\psi-\psi\nabla\psi^*)-|\psi|^2\,\mathbf{A}\ . \qquad (2.6)$$

These equations are written in dimensionless variables such that lengths are scaled by the magnetic penetration depth λ, time is scaled by $\hbar/2m\xi^2$ with ξ the coherence length, magnetic fields are scaled by $\sqrt{2}H_c$ with H_c the thermodynamic critical field; $\kappa=\lambda/\xi$ is the usual Ginzburg-Landau parameter. As an aid to the reader, important results will be explicitly expressed in both dimensionless and conventional units. The quantities γ_1 and γ_2 are the real and imaginary parts of the dimensionless order-parameter relaxation time. The scalar potential is denoted by Φ; the difference between the scalar potential and the electrochemical potential will be ignored here (see Refs. 11 and 14 for a more extended discussion). Since in equilibrium we will assume that we have local charge neutrality, out of equilibrium any excess charge density must be $O(v)$, with $\mathbf{v}$ the velocity of the vortex lattice; therefore the time variation of the charge density is $O(v^2)$, and will be neglected in the spirit of the linear response calculation of this paper. As a result, the total current must be divergenceless; i.e., $\nabla\cdot(\mathbf{J}_n+\mathbf{J}_s)=0$.

Before attempting to solve the TDGL equations, it is useful to first simplify them somewhat. To do this, we write the order parameter in terms of an amplitude and a phase,

$$\psi(\mathbf{r},t)=f(\mathbf{r},t)\exp[i\varphi(\mathbf{r},t)]\ .$$

In terms of the gauge-invariant quantities $\mathbf{Q}\equiv\mathbf{A}-\nabla\varphi/\kappa$ and $P\equiv\Phi+\partial_t\varphi$, the magnetic and electric fields are

$$\mathbf{h}=\nabla\times\mathbf{Q}\ , \qquad (2.7)$$

$$\mathbf{E}=-\frac{1}{\kappa}\nabla P-\partial_t\mathbf{Q}\ , \qquad (2.8)$$

and the supercurrent is

$$\mathbf{J}_s=-f^2\mathbf{Q}\ . \qquad (2.9)$$

The real part of Eq. (2.1) is

$$\gamma_1\partial_t f-\gamma_2 Pf=\frac{1}{\kappa^2}\nabla^2 f-Q^2 f+f-f^3\ , \qquad (2.10)$$

while the imaginary part is

$$\gamma_2\partial_t f+\gamma_1 Pf+\frac{1}{\kappa}f\nabla\cdot\mathbf{Q}+\frac{2}{\kappa}\mathbf{Q}\cdot\nabla f=0\ , \qquad (2.11)$$

and Eqs. (2.2)–(2.6) become

$$\nabla\times\nabla\times\mathbf{Q}=\underline{\sigma}^{(n)}\left[-\frac{1}{\kappa}\nabla P-\partial_t\mathbf{Q}\right]-f^2\mathbf{Q}\ . \qquad (2.12)$$

An explicit equation for P may be obtained as follows. First, multiply Eq. (2.11) by f; the gradient terms can be combined as $\nabla\cdot\mathbf{J}_s$; then use the fact that $\nabla\cdot\mathbf{J}_s=-\nabla\cdot\mathbf{J}_n$. We finally obtain

$$\frac{1}{\kappa}\nabla\cdot\left[\underline{\sigma}^{(n)}\left[-\frac{1}{\kappa}\nabla P-\partial_t\mathbf{Q}\right]\right]+\gamma_1 f^2P+\gamma_2 f\partial_t f=0\ . \qquad (2.13)$$

We begin by calculating the local electric field for the moving flux lattice. First, we assume that the lattice translates uniformly, so that f, $\mathbf{Q}$, and P are only functions of $\mathbf{r}-\mathbf{v}t$. Therefore, we replace all time derivatives in Eqs. (2.10), (2.12), and (2.13) by $-\mathbf{v}\cdot\nabla$. Second, as we are concerned with linear response in this paper, we keep only terms of order the flux lattice velocity $\mathbf{v}$. In this spirit, we expand all quantities in powers of the velocity, with the order of expansion denoted by a superscript: $f=f^{(0)}+f^{(1)}$, $\mathbf{Q}=\mathbf{Q}^{(0)}+\mathbf{Q}^{(1)}$, where $f^{(1)}$ and $\mathbf{Q}^{(1)}$ are $O(v)$. Note that P is $O(v)$, since the electric field vanishes in equilibrium. The $O(1)$ equations are simply the equilibrium Ginzburg-Landau equations. The electric field can therefore be written as

$$\mathbf{E}=-\mathbf{v}\times\mathbf{h}^{(0)}-\nabla\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]\ . \qquad (2.14)$$

Upon averaging over the volume V of the sample, we find for the spatially averaged electric field

$$\langle\mathbf{E}\rangle\equiv\frac{1}{V}\int d^3r\,\mathbf{E}(\mathbf{r})=-\mathbf{v}\times\mathbf{B}\ , \qquad (2.15)$$

since the average of the gradient term in Eq. (2.14) can be converted to a surface term that vanishes at the boundaries; $\mathbf{B}=\langle\mathbf{h}^{(0)}\rangle$ is the (equilibrium) macroscopic induction field. Although the gradient term in Eq. (2.14) does not contribute to the spatially averaged electric field, it does contribute to the local electric field. We therefore need to calculate $P/\kappa-\mathbf{v}\cdot\mathbf{Q}^{(0)}$; an equation for this quantity follows from Eq. (2.13):

$$\nabla\cdot\left[\underline{\sigma}^{(n)}\nabla\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]\right]-\kappa^2\gamma_1\omega^{(0)}\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]=-\nabla\cdot\left[\underline{\sigma}^{(n)}(\mathbf{v}\times\mathbf{h}^{(0)})+\frac{\gamma_2}{2}\omega^{(0)}\mathbf{v}\right]+\kappa^2\gamma_1\omega^{(0)}\mathbf{v}\cdot\mathbf{Q}^{(0)}\ , \qquad (2.16)$$

where for simplicity we have introduced $\omega^{(0)}=(f^{(0)})^2$. The last term on the right-hand side of Eq. (2.16) can be further simplified by noting that from the equilibrium equations we have

$$\mathbf{v}\cdot(\omega^{(0)}\mathbf{Q}^{(0)})=\nabla\cdot(\mathbf{v}\times\mathbf{h}^{(0)})\ .$$

Simplifying the derivatives on the left-hand side, we finally arrive at

$$\sigma_{xx}^{(n)}\nabla^2\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]-\kappa^2\gamma_1\omega^{(0)}\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]=-\nabla\cdot\mathbf{j}\ , \qquad (2.17)$$

where we have defined

$$\mathbf{j}(\mathbf{r})\equiv\underline{\sigma}^{(n)}[\mathbf{v}\times\mathbf{h}^{(0)}(\mathbf{r})]-\kappa^2\gamma_1\mathbf{v}\times\mathbf{h}^{(0)}(\mathbf{r})+\frac{\gamma_2\kappa}{2}\omega^{(0)}(\mathbf{r})\mathbf{v}\ . \qquad (2.18)$$

Next, define the local deviations from the average equilibrium values of the magnetic induction field and the square of the order parameter as

$$\delta\mathbf{h}(\mathbf{r})\equiv\mathbf{h}^{(0)}(\mathbf{r})-\mathbf{B}\ , \qquad (2.19)$$

$$\delta\omega\equiv\omega^{(0)}-\langle\omega^{(0)}\rangle\ , \qquad (2.20)$$

so that $\langle\delta\mathbf{h}(\mathbf{r})\rangle=\langle\delta\omega\rangle=0$. When these expressions are substituted into Eq. (2.18), there will be a constant piece that can be discarded as it will not contribute to Eq. (2.17). Noting that for the equilibrium state $\delta\mathbf{h}=-(\delta\omega/2\kappa)\hat{\mathbf{z}}$, which is correct to $O(\delta\omega^2)$,[22] we then see that it is possible to write Eq. (2.18) in the following form:

$$\mathbf{j}(\mathbf{r})=\underline{\sigma}^{(n)}[\mathbf{v}\times\delta\mathbf{h}(\mathbf{r})]-\frac{\gamma_1\kappa}{2}\delta\omega(\mathbf{r})(\hat{\mathbf{z}}\times\mathbf{v})+\frac{\gamma_2\kappa}{2}\delta\omega(\mathbf{r})\mathbf{v}\ . \qquad (2.21)$$

Eqs. (2.14), (2.17), and (2.21) will together determine the local electric field, and therefore the local normal current.

The solution to Eq. (2.17) is

$$\frac{1}{\kappa}P(\mathbf{r})-\mathbf{v}\cdot\mathbf{Q}^{(0)}(\mathbf{r})=-\int d^3r'G(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{j}(\mathbf{r}')$$
$$=\int d^3r'\mathbf{j}(\mathbf{r}')\cdot\nabla'G(\mathbf{r},\mathbf{r}')\ , \qquad (2.22)$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function that satisfies

$$[\sigma_{xx}^{(n)}\nabla^2-\kappa^2\gamma_1\omega^{(0)}(\mathbf{r})]G(\mathbf{r},\mathbf{r}')=\delta^{(3)}(\mathbf{r}-\mathbf{r}')\ , \qquad (2.23)$$

and where in the second line of Eq. (2.22) we have integrated by parts and neglected a surface contribution. With this solution it is possible to calculate the normal current. First, take a gradient of Eq. (2.23) and multiply by the normal-state conductivity tensor; after using several vector identities, we find

$$\underline{\sigma}^{(n)}\nabla\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}_0\right]=\int d^3r'\,\mathbf{j}(\mathbf{r}')\sigma_{xx}^{(n)}\nabla\cdot\nabla'G(\mathbf{r},\mathbf{r}')-\nabla\times\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla'G(\mathbf{r},\mathbf{r}')] \ . \quad (2.24)$$

The flux lattice state is not translationally invariant, so that $G(\mathbf{r},\mathbf{r}')$ is not a function of the coordinate difference alone. However, sufficiently close to H_{c2} the order parameter amplitude is small, and we can replace $\omega^{(0)}(\mathbf{r})$ in Eq. (2.23) by its spatial average $\langle\omega^{(0)}\rangle$, which is correct to $O(\delta\omega)$. Within this approximation, the Green's function $G(\mathbf{r},\mathbf{r}')=G(\mathbf{r}-\mathbf{r}')$; then

$$\nabla\cdot\nabla'G(\mathbf{r},\mathbf{r}')=-\nabla^2G(\mathbf{r}-\mathbf{r}') \ .$$

Combining this result with Eq. (2.23) for the Green's function, we find that Eq. (2.24) becomes

$$\underline{\sigma}^{(n)}\nabla\left[\frac{1}{\kappa}P-\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]=-\mathbf{j}(\mathbf{r})-\kappa^2\gamma_1\langle\omega^{(0)}\rangle\int d^3r'\,G(\mathbf{r}-\mathbf{r}')\mathbf{j}(\mathbf{r}')-\nabla\times\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla'G(\mathbf{r}-\mathbf{r}')] \ . \quad (2.25)$$

The second term on the right-hand side of Eq. (2.25) will generally be quite small near the transition, as $\mathbf{j}$ itself is $O(\delta h)$; this is then multiplied by $\langle\omega^{(0)}\rangle$, rendering the second term doubly small near the transition. We will therefore drop this term in what follows. The normal current is obtained by combining Eq. (2.25) with the definition of the normal current, Eq. (2.3), along with the expression for the electric field, Eq. (2.14); the final result is

$$\mathbf{J}_n(\mathbf{r})=-\underline{\sigma}^{(n)}[\mathbf{v}\times\mathbf{B}]-\frac{\gamma_1\kappa}{2}\delta\omega(\mathbf{r})(\hat{\mathbf{z}}\times\mathbf{v})+\frac{\gamma_2\kappa}{2}\delta\omega(\mathbf{r})\mathbf{v}+\nabla\times\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla'G(\mathbf{r}-\mathbf{r}')] \ . \quad (2.26)$$

As a check on our result, we note that the spatial average of the last three terms on the right-hand side of Eq. (2.26) is zero, so that

$$\langle\mathbf{J}_n\rangle=-\underline{\sigma}^{(n)}(\mathbf{v}\times\mathbf{B})=\underline{\sigma}^{(n)}\langle\mathbf{E}\rangle \ ,$$

as required. It is also straightforward to show that when $\sigma_{xy}^{(n)}=\gamma_2=0$, Eq. (2.26) reduces to the analogous results derived by Thompson and Hu[13] and Vecris and Pelcovits[14] in two and three dimensions, respectively.

We next calculate the linearized supercurrent for the moving flux lattice. This calculation is most conveniently carried out in the symmetric gauge, rather than the Landau gauge used by Thompson and Hu.[13] Following Vecris and Pelcovits,[14] we start by using a postulated solution for the order parameter of a uniformly translating flux lattice,

$$\psi_l(\mathbf{r}(t))=\exp\left[-\frac{B\kappa}{4}r^2(t)\right]g[x(t)+iy(t)] \ , \quad (2.27)$$

where the subscript l indicates a linearized solution, $\mathbf{r}(t)=\mathbf{r}-\mathbf{v}t$ is the coordinate in the moving frame, and $g[x(t)+iy(t)]$ is an analytic function (to be specified later). In the presence of an electric field there are corrections to κ of $O(v^2)$, which we drop in the spirit of our linear response calculation.[14] Substituting Eq. (2.27) into the first TDGL equation, Eq. (2.1), and dropping terms proportional to $\nabla g/g$, we find the required symmetric gauge potential near H_{c2},

$$\mathbf{A}=\frac{B}{2}\hat{\mathbf{z}}\times\mathbf{r}-\frac{\gamma_1\kappa}{2}\hat{\mathbf{z}}\times\mathbf{v}+\frac{\gamma_2\kappa}{2}\mathbf{v} \ . \quad (2.28)$$

We use an analytic function, $g[x(t)+iy(t)]$,[14,15] appropriate for a translating flux-line lattice with flux lines located at $\mathbf{r}_\nu$ and parallel to the z axis,

$$g[x(t)+iy(t)]=\prod_{\nu=1}^{N}\{x(t)-x_\nu+i[y(t)-y_\nu]\} \ . \quad (2.29)$$

This form of the order parameter has zeros at the instantaneous vortex positions $\mathbf{r}_\nu$. We therefore have for the square of the amplitude of the order parameter,

$$\omega_l(\mathbf{r}(t))=\exp\left[-\frac{\kappa B}{2}r^2(t)\right]\prod_{\nu=1}^{N}|\mathbf{r}(t)-\mathbf{r}_\nu|^2 \ , \quad (2.30)$$

and for the phase of the order parameter

$$\varphi(\mathbf{r}(t))=\sum_{\nu=1}^{N}\tan^{-1}\left[\frac{y(t)-y_\nu}{x(t)-x_\nu}\right] \ . \quad (2.31)$$

For the gauge-invariant vector potential we then have

$$\begin{aligned}\mathbf{Q}&=\mathbf{A}-\frac{1}{\kappa}\nabla\varphi\\&=\frac{B}{2}\hat{\mathbf{z}}\times\mathbf{r}-\frac{\gamma_1\kappa}{2}\hat{\mathbf{z}}\times\mathbf{v}+\frac{\gamma_2\kappa}{2}\mathbf{v}-\frac{1}{\kappa}\sum_{\nu=1}^{N}\frac{\hat{\mathbf{z}}\times[\mathbf{r}(t)-\mathbf{r}_\nu]}{|\mathbf{r}(t)-\mathbf{r}_\nu|^2}\\&=-\frac{\hat{\mathbf{z}}\times\nabla\omega_l}{2\kappa\omega_l}-\frac{\gamma_1\kappa}{2}\hat{\mathbf{z}}\times\mathbf{v}+\frac{\gamma_2\kappa}{2}\mathbf{v} \ . \end{aligned}\quad (2.32)$$

The linearized supercurrent $\mathbf{J}_s=-\omega\mathbf{Q}$ is, therefore,

$$\mathbf{J}_s=\mathbf{J}_s^{(0)}+\frac{\gamma_1\kappa}{2}\omega_l(\hat{\mathbf{z}}\times\mathbf{v})-\frac{\gamma_2\kappa}{2}\omega_l\mathbf{v} \ , \quad (2.33)$$

where $\mathbf{J}_s^{(0)}=\hat{\mathbf{z}}\times\nabla\omega_l/2\kappa$ is the uniformly translating equilibrium supercurrent.

The total current $\mathbf{J}=\mathbf{J}_n+\mathbf{J}_s$ is obtained by adding our expression for the normal current, Eq. (2.26), to our expression for the supercurrent, Eq. (2.33), which we obtained above. Using $\mathbf{v}=\langle\mathbf{E}\rangle\times\mathbf{B}/B^2$, we find

$$\mathbf{J}(\mathbf{r})=\underline{\sigma}\langle\mathbf{E}\rangle+\mathbf{J}_s^{(0)}+\nabla\times\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla'G(\mathbf{r}-\mathbf{r}')] \ , \quad (2.34)$$

where the conductivity tensor is

$$\sigma=\begin{vmatrix} \sigma_{xx}^{(n)}+\dfrac{\gamma_1\kappa\langle\omega^{(0)}\rangle}{2B} & \sigma_{xy}^{(n)}-\dfrac{\gamma_2\kappa\langle\omega^{(0)}\rangle}{2B} \\ -\sigma_{xy}^{(n)}+\dfrac{\gamma_2\kappa\langle\omega^{(0)}\rangle}{2B} & \sigma_{xx}^{(n)}+\dfrac{\gamma_1\kappa\langle\omega^{(0)}\rangle}{2B} \end{vmatrix} . \tag{2.35}$$

The spatial averages of the last two terms on the right-hand side of Eq. (2.34) are zero, so that $\langle \mathbf{J}\rangle=\underline{\sigma}\,\langle \mathbf{E}\rangle$; we also have $\nabla\cdot\mathbf{J}=0$, since $\nabla\cdot\mathbf{J}_s^{(0)}=0$. We have therefore found a current that is properly divergenceless, as required. The various terms on the right-hand side of Eq. (2.34) also have simple interpretations—the first term is a uniform transport current, the second is the uniformly translating equilibrium supercurrent, and the last term is the backflow current. This form of the local current was first obtained by Thompson and Hu[13] for the case $\sigma_{xy}^{(n)}=\gamma_2=0$; our result is a generalization to the situation in which there is particle-hole asymmetry.

In mean-field theory the Abrikosov value for $\langle\omega^{(0)}\rangle$ is (see Ref. 22, for instance)

$$\langle\omega^{(0)}\rangle=\frac{m}{2\pi\hbar e^*}\frac{H_{c2}-B}{(2\kappa^2-1)\beta_A+1} , \tag{2.36}$$

where $\beta_A=\langle(\omega^{(0)})^2\rangle/(\langle\omega^{(0)}\rangle)^2$, which is 1.16 in mean-field theory for a triangular flux lattice.[22] This leads to the following expressions for the conductivities in mean-field theory, in conventional units:

$$\sigma_{xx}=\sigma_{xx}^{(n)}+\frac{\gamma_1 m}{2\pi\hbar}\frac{1}{(2\kappa^2-1)\beta_A+1}\frac{H_{c2}-B}{B} \tag{2.37}$$

and

$$\sigma_{xy}=\sigma_{xy}^{(n)}-\frac{\gamma_2 m}{2\pi\hbar}\frac{1}{(2\kappa^2-1)\beta_A+1}\frac{H_{c2}-B}{B} . \tag{2.38}$$

Notice that the conductivities have contributions from both the normal carriers and from the vortex motion. The real part of the order-parameter relaxation time, γ_1, is always positive, so that this contribution is additive for the longitudinal conductivity. However, the sign of γ_2 is most likely determined by microscopic considerations;[20] if $\gamma_2>0$, then it is possible for the Hall conductivity to change sign in the mixed state. Further microscopic calculations are needed to determine if this is the source of the sign change that has been observed in the high-T_c superconductors.

It is also possible to calculate the corrections to the local magnetic field for a moving flux lattice. We can do this by expressing the local current as a curl:

$$\begin{aligned}\nabla\times\mathbf{h}(\mathbf{r})&=\mathbf{J}(\mathbf{r})\\ &=-\nabla\times[\hat{\mathbf{z}}(\mathbf{J}_t\times\hat{\mathbf{z}})\cdot\mathbf{r}]-\nabla\times[\hat{\mathbf{z}}\omega^{(0)}(\mathbf{r})/2\kappa]\\ &\quad+\nabla\times\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla' G(\mathbf{r}-\mathbf{r}')] ,\end{aligned} \tag{2.39}$$

where $\mathbf{J}_t=\underline{\sigma}\langle\mathbf{E}\rangle$ is the uniform transport current. Integrating, we obtain $[\mathbf{h}(\mathbf{r})=h(\mathbf{r})\hat{\mathbf{z}}]$:

$$\begin{aligned}h(\mathbf{r})&=h^{(0)}(\mathbf{r})-(\mathbf{J}_t\times\hat{\mathbf{z}})\cdot\mathbf{r}\\ &\quad+\int d^3r'\,\mathbf{j}(\mathbf{r}')\times[\underline{\sigma}^{(n)}\nabla' G(\mathbf{r}-\mathbf{r}')] ,\end{aligned} \tag{2.40}$$

where $h^{(0)}=B-\delta\omega/2\kappa$ is the equilibrium local magnetic field. This is a generalization to the particle-hole asymmetric case of the result of Vecris and Pelcovits [see Eq. (2.23) of Ref. 14]. As noted by these authors, the second term in Eq. (2.40), which grows linearly with distance within the sample, is typical of magnetostatics problems in the presence of a uniform current density.

To summarize our results so far, we have explicitly calculated the total current for a moving flux lattice starting from the generalized TDGL equations. This current has zero divergence; however, the conductivities calculated from this current are identical to those that would be obtained by using the Schmid-Caroli-Maki solution.

III. THERMAL TRANSPORT

The moving flux lattice not only produces dissipation but also transports energy, in a direction parallel to its velocity. In order to calculate the transported energy in the mixed state, we start from the expression for the energy current, which is due to Schmid:[11]

$$\mathbf{J}^h=2\mathbf{E}\times\mathbf{h}-2\mathbf{E}\times\mathbf{B}+2\left[\left[\frac{\nabla}{\kappa}-i\,\mathbf{A}\right]\psi(\partial_t-i\Phi)\psi^*+\text{c.c.}\right] , \tag{3.1}$$

where $\mathbf{J}^h$ is in units of $(H_c^2/4\pi)(\hbar/2m)(\kappa^2/\lambda)$ (i.e., units of energy per unit volume times velocity). The second term, which was not considered by Schmid, is necessary in order to subtract out the contribution from the uniform background field $\mathbf{B}=\langle\mathbf{h}\rangle$.[23] This expression may be greatly simplified by using a sequence of transformations that were used in Ref. 18; these are reproduced here for completeness. First, in terms of the potentials P and $\mathbf{Q}$, along with the order-parameter amplitude f, to $O(v)$ Eq. (3.1) becomes

$$\begin{aligned}\mathbf{J}^h&=2\left[-\frac{1}{\kappa}\nabla P+\mathbf{v}\cdot\nabla\mathbf{Q}^{(0)}\right]\times(\nabla\times\mathbf{Q}^{(0)})-2\mathbf{E}\times\mathbf{B}\\ &\quad+\frac{2}{\kappa}\left[-\frac{1}{\kappa}(\mathbf{v}\cdot\nabla f^{(0)})(\nabla f^{(0)})+P\mathbf{Q}^{(0)}(f^{(0)})^2\right] .\end{aligned} \tag{3.2}$$

Using

$$\nabla\times\nabla\times\mathbf{Q}^{(0)}+(f^{(0)})^2\mathbf{Q}^{(0)}=0 ,$$

the first and last terms in Eq. (3.2) may be combined:

$$(\nabla P)\times(\nabla\times\mathbf{Q}^{(0)})+P\nabla\times\nabla\times\mathbf{Q}^{(0)}=\nabla\times(P\nabla\times\mathbf{Q}^{(0)}) , \tag{3.3}$$

where a vector identity has been used. The second term on the left-hand side of Eq. (3.2) may be written as

$$(\mathbf{v}\cdot\nabla\mathbf{Q}^{(0)})\times(\nabla\times\mathbf{Q}^{(0)})=\nabla\times[(\mathbf{v}\cdot\mathbf{Q}^{(0)})\nabla\times\mathbf{Q}^{(0)}]+\mathbf{v}(\nabla\times\mathbf{Q}^{(0)})^2-(\mathbf{v}\cdot\mathbf{Q}^{(0)})\nabla\times\nabla\times\mathbf{Q}^{(0)} , \tag{3.4}$$

where we have again used several vector identities. Combining Eqs. (3.2)–(3.4), we have

$$\mathbf{J}^h=2\nabla\times\left[\left[-\frac{1}{\kappa}P+\mathbf{v}\cdot\mathbf{Q}^{(0)}\right]\mathbf{h}^{(0)}\right]-2\mathbf{E}\times\mathbf{B}+2\left[\frac{1}{\kappa^2}(\mathbf{v}\cdot\nabla f^{(0)})(\nabla f^{(0)})+(f^{(0)})^2(\mathbf{v}\cdot\mathbf{Q}^{(0)})\mathbf{Q}^{(0)}+\mathbf{v}(h^{(0)})^2\right]. \quad (3.5)$$

Therefore, the backflow terms only appear in the first two terms on the right-hand side of Eq. (3.5). However, when we calculate the spatially averaged heat current, the first term on the right-hand side of Eq. (3.5) can be converted into a surface term, which vanishes; the second term yields $-2\langle\mathbf{E}\rangle\times\mathbf{B}=-2B^2\mathbf{v}$; therefore the backflow corrections do not enter into the calculation of the spatially averaged energy current. After performing the spatial average, we find that $\langle\mathbf{J}^h\rangle=nU_\phi\mathbf{v}$, where $n=\kappa B/2\pi$ is the vortex density ($n=B/\phi_0$ in conventional units), and where U_ϕ is the transport energy per vortex; we have

$$nU_\phi=\frac{1}{V}\int d^3r\left[\frac{1}{\kappa^2}(\nabla f^{(0)})^2+(f^{(0)})^2(Q^{(0)})^2+2(h^{(0)})^2\right]-2B^2\ , \quad (3.6)$$

(a factor of $\frac{1}{2}$ arises from an angular average in the integral). The first two terms are half of the kinetic energy of the superfluid, while the third term is the magnetic field energy. Recently, Doria *et al.* have proved a virial theorem for the equilibrium Ginzburg-Landau equations,[24] which shows that the integral that appears in Eq. (3.6) is precisely equal to $2\mathbf{H}\cdot\mathbf{B}$. Therefore, we find that

$$nU_\phi=-2(\mathbf{B}-\mathbf{H})\cdot\mathbf{B}=-8\pi\mathbf{M}\cdot\mathbf{B}\ , \quad (3.7)$$

with $\mathbf{M}=(\mathbf{B}-\mathbf{H})/4\pi$ the spatially averaged equilibrium magnetization of the sample. Therefore, the transport energy is

$$U_\phi=-\frac{4\pi}{\kappa}4\pi M=-\phi_0 M\ , \quad (3.8)$$

where the second line is in conventional units. Sufficiently close to H_{c2}, we can substitute for M the Abrikosov value for the magnetization[22] to obtain the mean-field transport energy

$$U_\phi=\frac{4\pi}{\kappa}\frac{\kappa-H}{\beta_A(2\kappa-1)}=\frac{\phi_0}{4\pi}\frac{H_{c2}-H}{\beta_A(2\kappa^2-1)}\ , \quad (3.9)$$

where the second line is again in conventional units. Using the linearized microscopic theory near H_{c2}, Maki[25] obtained the result $U_\phi=-\phi_0 ML_D(t)$, where t is the reduced temperature; $L_D(t)\approx 1$ in the dirty limit near H_{c2}, so that our results agree in this limit. Note that our result is much more general, as the derivation did not invoke the assumption of linearity of the order parameter, but only the assumption of linear response in the flux lattice velocity. Therefore, our result holds for the entire mixed state (but only within the TDGL framework and in mean-field theory), and not just near H_{c2}.

The thermomagnetic transport coefficients for a superconductor in the mixed state are discussed in Ref. 18, to which we refer the reader for details. The Nernst coefficient is defined as $\nu=E_y/H(\partial T/\partial x)$, under the conditions of $J_x=J_y=\partial T/\partial y=0$. Introducing the transport coefficient α_{xy} through $\langle\mathbf{J}^h\rangle=\alpha_{xy}\langle\mathbf{E}\rangle\times\hat{\mathbf{z}}$, then it is possible to write the Nernst coefficient as $\nu\approx(1/TH)(\alpha_{xy}/\sigma_{xx})$, where σ_{xx} is the full conductivity (including both the normal-state and flux-flow contributions); but from the above discussion we see that $\alpha_{xy}=U_\phi/\phi_0$. We therefore find for the Nernst coefficient,

$$\nu=\frac{1}{\phi_0 TH}\frac{U_\phi}{\sigma_{xx}}\ . \quad (3.10)$$

The Ettingshausen coefficient is defined as $\mathcal{E}=(\partial T/\partial y)/HJ_x$ under the conditions $J_y^h=J_y=\partial T/\partial x=0$. Using the Onsager relations, it is possible to show that $\mathcal{E}=T\nu/\kappa_{xx}$, where κ_{xx} is the thermal conductivity.

IV. FLUCTUATIONS

Having calculated the transport properties in the mean-field regime, we now turn to the study of the effects of thermal fluctuations of the flux lattice on the transport properties. To do this, one first assumes that the flux lines are located at $\mathbf{r}_\nu(z)=\mathbf{R}_\nu+\mathbf{s}_\nu(z)$, where $\{\mathbf{R}_\nu\}$ are the positions of the flux lines in mean-field theory (which form a triangular lattice), and $\{\mathbf{s}_\nu(z)\}$ are the deviations from the mean-field positions. Expanding about the mean-field solution, and then taking the continuum limit by replacing $\mathbf{s}_\nu(z)$ by $\mathbf{s}(\mathbf{r})$, the free energy becomes $F=F_0+F_{\rm el}$, where F_0 is the free energy of the mean-field Abrikosov state and $F_{\rm el}$ is the elastic free energy given by

$$F_{\rm el}=\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}s_i(-\mathbf{k})\{[c_{11}(\mathbf{k})-c_{66}(\mathbf{k})]k_ik_j+\delta_{ij}[c_{66}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2]\}s_j(\mathbf{k})\ , \quad (4.1)$$

where c_{11}, c_{44}, and c_{66} are the uniaxial compression modulus, tilt modulus, and shear modulus, respectively. The derivation of the nonlocal elastic moduli from Ginzburg-Landau theory was first carried out by Brandt;[15] his results have recently been generalized to the case of anisotropic superconductors by Houghton, Pelcovits, and Sudbø.[26] The current must now be averaged with respect to an ensemble specified by the elastic free energy; as shown by Vecris and Pelcovits,[14] this is equivalent to replacing the spatial average of the square of the mean-field order parameter, $\langle \omega(\mathbf{r}) \rangle$, which appears in the expression for the mean-field conductivities, Eq. (2.35), by the spatial and ensemble average of the square of the order parameter, $\langle \langle \omega(\mathbf{r}) \rangle_{\text{th}} \rangle$, where $\langle \cdots \rangle_{\text{th}}$ is the ensemble average (we will drop the superscript on ω for simplicity). Therefore, in the presence of thermal fluctuations the longitudinal conductivity (first obtained by Vecris and Pelcovits) becomes, in conventional units,

$$\sigma_{xx} = \sigma_{xx}^{(n)} + \gamma_1 e^* \langle \langle \omega(\mathbf{r}) \rangle_{\text{th}} \rangle / B \ , \tag{4.2}$$

and the Hall conductivity (our new result) becomes

$$\sigma_{xy} = \sigma_{xy}^{(n)} - \gamma_2 e^* \langle \langle \omega(\mathbf{r}) \rangle_{\text{th}} \rangle / B \ . \tag{4.3}$$

The quantity $\langle \langle \omega(\mathbf{r}) \rangle_{\text{th}} \rangle$ has been calculated in the large-κ limit by Maki and Thompson[27] and by Ikeda and co-workers,[28,29] with the result

$$\langle \langle \omega \rangle_{\text{th}} \rangle = \langle \omega_0 \rangle \left[1 - 0.49 \left[\frac{2\pi \epsilon_G m_z}{m} \right]^{1/2} \times \frac{1}{(1-t)^{1/2}} \frac{b}{(1-b)^{3/2}} \right] , \tag{4.4}$$

where $\langle \omega_0 \rangle$ is the value of the mean-square order parameter in mean-field theory, $b = B / H_{c2}(T)$ is the reduced magnetic field, $t = T / T_c(0)$ is the reduced critical temperature, m_z and m are the effective masses along the z axis and in the x-y plane, respectively, and ϵ_G is the Ginzburg parameter given by

$$\epsilon_G = \frac{16\pi^3 \kappa^4 (k_B T_c)^2}{\phi_0^3 H_{c2}(0)} \ . \tag{4.5}$$

There are two points worth noting. First, the fluctuations in the flux lattice state *reduce* $\langle \langle \omega \rangle_{\text{th}} \rangle$ below its mean-field value, in contrast to the normal state. Second, since the high-T_c superconductors typically have $(\epsilon_G m_z / m)^{1/2}$ of $O(1)$ or larger,[1,26] these fluctuations are relatively large in these materials as compared to the low temperature superconductors. These fluctuations lead to a reduction of the conductivity and a significant rounding of the resistive transition in a magnetic field.

The calculations of Maki and Thompson[27] and of Ikeda and co-workers[28,29] made use of the fact that the spatially averaged mean-square order parameter is equal to the derivative of the free energy with respect to the coefficient of the quadratic term in the Ginzburg-Landau Hamiltonian. Here we will repeat the calculation using a rather different method, which is to thermally average the order parameter directly (this approach was motivated by a suggestion in the work of Vecris and Pelcovits);[14] we hope that some readers will find this alternative method of calculation instructive. We start with a generalized form for the square of the order parameter in the mixed state of anisotropic superconductors, which was suggested by Brandt[15] for isotropic superconductors (the anisotropic generalization is implicit in the work of Houghton, Pelcovits, and Sudbø).[26] The order parameter is

$$\omega(\mathbf{r}) = N(B) \exp \left\{ -4\pi \sum_{\nu} \int dz' \int \frac{dk_z}{2\pi} \int_{\text{BZ}} \frac{d^2 k_\perp}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot[\mathbf{r} - \mathbf{r}_\nu(z')]}}{k_\perp^2 + \gamma^2 k_z^2 + k_\psi^2} \right\} , \tag{4.6}$$

where $N(B)$ is a magnetic-field-dependent normalization constant, $\gamma^2 = m / m_z$ is the mass anisotropy, with m the effective mass in the plane and m_z the effective mass along the z axis, $\mathbf{k}_\perp = (k_x, k_y)$, BZ denotes an integration over the first Brillouin zone, and $k_\psi^2 = 2(1-b)/\xi_{ab}^2(T)$ in conventional units, with ξ_{ab} the in-plane coherence length. As argued by Brandt, this form of the square of the order parameter has the proper second-order zeros at the vortex positions $\{\mathbf{r}_\nu\}$, and reduces to the correct forms for both large and small inductions. Expanding the exponent to first order in $\mathbf{s}_\nu$, and taking the continuum limit, we obtain

$$\omega(\mathbf{r}) = \omega_0(\mathbf{r}) \exp \left[i 4\pi n \int \frac{dk_z}{2\pi} \int_{\text{BZ}} \frac{d^2 k_\perp}{(2\pi)^2} \frac{\mathbf{k}\cdot\mathbf{s}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}}}{k_\perp^2 + \gamma^2 k_z^2 + k_\psi^2} \right] , \tag{4.7}$$

where $\omega_0(\mathbf{r})$ is the square of the mean-field order parameter with zeros at $\{\mathbf{R}_\nu\}$, and n is the vortex density; the $k_\perp$ integration is now over a circular Brillouin zone of radius

$$k_{\text{BZ}} = (4\pi n)^{1/2} = (2b)^{1/2} / \xi_{ab}(T)$$

(this choice preserves the volume of the Brillouin zone). Performing the thermal average, we obtain

$$\langle \omega(\mathbf{r}) \rangle_{\text{th}} = \omega_0(\mathbf{r}) e^{-W} , \tag{4.8}$$

where W is a suppression factor, given by

$$W = \tfrac{1}{2} (4\pi n)^2 \int \frac{dk_z}{2\pi} \int_{\text{BZ}} \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{(k_\perp^2 + \gamma^2 k_z^2 + k_\psi^2)^2} k_i k_j \langle s_i(\mathbf{k}) s_j(-\mathbf{k}) \rangle_{\text{th}} \ . \tag{4.9}$$

The fluctuation propagator is given by

$$\langle s_i(\mathbf{k}) s_j(-\mathbf{k}) \rangle_{\text{th}} = k_B T [P_{ij}^T G_T(\mathbf{k}) + P_{ij}^L G_L(\mathbf{k})] \ , \tag{4.10}$$

where $P^T_{ij}=\delta_{ij}-k_ik_j/k_\perp^2$ and $P^L_{ij}=k_ik_j/k_\perp^2$ are the transverse and longitudinal projection operators, respectively, and where the transverse and longitudinal propagators are given by

$$G_T(\mathbf{k})=\frac{1}{c_{66}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2}\ ,\quad G_L(\mathbf{k})=\frac{1}{c_{11}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2}\ . \tag{4.11}$$

Carrying out the implicit summation in Eq. (4.9), we obtain

$$W=\frac{1}{2}k_BT(4\pi n)^2\int\frac{dk_z}{2\pi}\int_{\rm BZ}\frac{d^2k_\perp}{(2\pi)^2}\frac{k_\perp^2}{(k_\perp^2+\gamma^2k_z^2+k_\psi^2)^2}\frac{1}{c_{11}(\mathbf{k})k_\perp^2+c_{44}(\mathbf{k})k_z^2}\ . \tag{4.12}$$

The spatial average is trivial; our final result is

$$\langle\langle\omega(\mathbf{r})\rangle_{\rm th}\rangle=\langle\omega_0(\mathbf{r})\rangle e^{-W}\ , \tag{4.13}$$

with $\langle\omega_0(\mathbf{r})\rangle$ the spatial average of the square of the mean-field order parameter given in Eq. (2.36).

In order to calculate the suppression factor W we first rescale the momenta by the Brillouin-zone radius by introducing a dimensionless variable $\mathbf{q}=\mathbf{k}/k_{\rm BZ}$. Then we have

$$W=\frac{16\pi k_BT}{\phi_0^2k_{\rm BZ}}\frac{B^2}{4\pi}\int_0^\infty dq_z\int_0^1 dq_\perp\frac{q_\perp^3}{[q_\perp^2+\gamma^2q_z^2+m_\xi^2]^2}\frac{1}{c_{11}(\mathbf{q})q_\perp^2+c_{44}(\mathbf{q})q_z^2}\ , \tag{4.14}$$

where $m_\xi^2=(k_\psi/k_{\rm BZ})^2=(1-b)/b$. The nonlocal, anisotropic elastic coefficients are given by[26,32]

$$c_{11}(\mathbf{q})=\frac{B^2}{4\pi}m_\lambda^2\left[\frac{q^2+\gamma^2m_\lambda^2}{(q^2+m_\lambda^2)(q_\perp^2+\gamma^2q_z^2+\gamma^2m_\lambda^2)}-\frac{1}{q_\perp^2+\gamma^2q_z^2+m_\xi^2}\right]\ , \tag{4.15}$$

$$c_{44}(\mathbf{q})=\frac{B^2}{4\pi}m_\lambda^2\gamma^2\left[1+\frac{1}{q_\perp^2+\gamma^2q_z^2+\gamma^2m_\lambda^2}\right]\ , \tag{4.16}$$

with $m_\lambda^2=(1-b)/2\beta_A\kappa^2b$. The integral is rather formidable, and we have not succeeded in evaluating it in a general form. However, progress is possible if we consider the limit $m_\lambda^2\ll 1$, i.e., $1/(2\kappa^2)\ll b/(1-b)$, which is easily satisfied for most of the mixed state in the high-T_c superconductors. In this limit it is possible to take to nonlocal limit of the elastic coefficients, i.e., take $m_\lambda^2\to 0$ in the expressions for c_{11} and c_{44}, Eqs. (4.15) and (4.16). In this limit the anisotropy factor γ only enters when multiplied by q_z, so it may be scaled out of the integral. The integral on q_z can be performed analytically, although the result is quite complicated. The remaining $q_\perp$ integral can then be performed in the limits $m_\xi^2\gg 1$ or $m_\xi^2\ll 1$. We will consider these cases in turn.

(i) $m_\xi^2\gg 1$; i.e., $1/(2\kappa^2)\ll b/(1-b)\ll 1$. In this limit, which applies to most of the mixed state, we obtain

$$W=\frac{\sqrt{2}}{8}\beta_A\left[\frac{2\pi\epsilon_G m_z}{m}\right]^{1/2}\frac{1}{(1-t)^{1/2}}\frac{b^{5/2}}{(1-b)^3}\ , \tag{4.17}$$

where we have used $\xi=\xi_{ab}(0)(1-t)^{-1/2}$.

(ii) $m_\xi^2\ll 1$; i.e., $1/(2\kappa^2)\ll 1\ll b/(1-b)$. This is the appropriate limit for fields near the mean-field upper critical field H_{c2}. We obtain for the suppression factor

$$W=\frac{(2-\sqrt{2})}{2}\beta_A\left[\frac{2\pi\epsilon_G m_z}{m}\right]^{1/2}\frac{1}{(1-t)^{1/2}}\frac{b}{(1-b)^{3/2}}. \tag{4.18}$$

Again, we note the appearance of the Ginzburg parameter ϵ_G. We would also like to point out that W exhibits an interesting scaling behavior in this limit; i.e.,

$$W(B,T)=\mathcal{F}\left\{\left[\frac{\Lambda_T\gamma(1-t)^{1/2}}{\kappa^2\xi_{ab}b}\right]^{2/3}(1-b)\right\}\ , \tag{4.19}$$

where $\Lambda_T=\phi_0^2/16\pi^2k_BT$ is a thermal length,[1] and where the scaling function $\mathcal{F}(x)=0.34x^{-3/2}$. Therefore the conductivities near H_{c2} exhibit a scaling behavior identical to the scaling behavior that is inherent in transport calculations which use the lowest Landau-level Hartree approximation.[28,16] This scaling behavior has recently been observed in measurements on the high-T_c superconductors.[30] Finally, note that our result in this limit is quite close to the result of Maki and Thompson and of Ikeda and co-workers, Eq. (4.4). This can be seen by noting that in order for the harmonic approximation to be valid, the suppression factor W must be small; expanding the exponential in Eq. (4.13) for small W, and using our result for W in Eq. (4.18), we obtain

$$\langle\langle\omega\rangle_{\rm th}\rangle=\langle\omega_0\rangle\left[1-0.34\left[\frac{2\pi\epsilon_G m_z}{m}\right]^{1/2}\times\frac{1}{(1-t)^{1/2}}\frac{b}{(1-b)^{3/2}}\right]\ . \tag{4.20}$$

We see that aside from the numerical factor, our result is identical to Eq. (4.4).

The importance of nonlocal effects in setting the scale for thermal fluctuations that may melt the flux lattice was pointed out by Brandt.[31] This can also be observed in the amplitude fluctuations. In the isotropic limit ($\gamma=1$) the suppression factor in the local limit is easily calculated; we find

$$W_{\text{local}} = \frac{\sqrt{2}}{12}\frac{\xi_{ab}}{\Lambda_T}\frac{b^{3/2}}{(1-b)^2}\ , \tag{4.21}$$

which lacks the important factor of κ^2, which appears in the nonlocal expression.

There are several features of our result for the thermally averaged order parameter that are noteworthy. First, note that there are no divergences in the amplitude fluctuations that we consider here. The amplitude fluctuations, while suppressing the conductivity below the mean-field value, do not drive the flux-flow contribution to the conductivity to zero. This is in contrast to the phase fluctuations, which diverge with the system size.[33,34,32] This divergence is often taken as an indication of the absence of off-diagonal long-range order in the flux-lattice state[34,32] although this interpretation is still subject to some controversy.[33] Second, the amplitude fluctuations are longitudinal, unlike the phase fluctuations, which are transverse. As a result, our expression for W does not involve the shear modulus c_{66}. It would then appear that the conductivities are relatively insensitive to a vortex lattice melting transition,[1,26,35] at which the shear modulus would be abruptly driven to zero in crossing the liquid-solid phase boundary.[36] However, this observation may be significantly modified once we account for vortex pinning.[1,37,38]

V. CONCLUSIONS

To summarize, we have calculated the transport coefficients in the mixed state using a generalized TDGL theory. Our calculations have explicitly incorporated "backflow" effects, yielding a current that is properly divergenceless. However, the results which we obtain are wholly equivalent to the Schmid-Caroli-Maki solution of the TDGL equations, since the backflow current has zero spatial average. Therefore, at least within the framework of TDGL theory, the backflow currents associated with vortex motion have little bearing on the question of the sign change of the Hall conductivity. We also calculated the thermomagnetic transport properties in mean-field theory, and found that under quite general circumstances the transport energy is proportional to the equilibrium magnetization. Finally, we find that elastic fluctuations of the vortex lattice tend to suppress the conductivities below their mean-field values.

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