Conductivity of coupled quantum wells with nonsymmetric scattering

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The Green function of an electron in tunnel-coupled quantum wells with different momentum relaxation times cannot be diagonalized when $\Delta \cong T$ is comparable to the collisional broadening of states (Δ is the level splitting and T is the tunnel matrix element). In this case, a quasiparticle description of the energy spectrum is invalid and the conductivity σ is determined by the formulas of quantum transport theory [the dependence $\sigma(\Delta)$ is obtained for the short-range scattering case]. The frequency dispersion of conductivity that may be realized in the low-frequency region is also discussed.

Tunnel-coupling electronic states in double quantum wells (DQW's) have been a topic of active research interest in recent years. Direct measurements of electron tunneling relaxation in the structures that use time-resolved luminescence have been carried out by various authors (see Ref. 1). Infrared transitions in DQW structures have also been investigated.[2] Experimental and theoretical studies of the longitudinal "resistance resonance" caused by electron tunneling between the left (l) and right (r) QW with different mobilities have been made by Palevski *et al.*[3] and by Vinter and Tardella.[4] These calculations and discussions of the experimental data used an electron-energy-spectrum description which included the two-dimensional (2D) kinetic energy and transverse energy that are similar to a two-level-system energy.

However, such a point of view appears to be unacceptable for a description of electronic states in DQW structures with nonsymmetric scattering when the level-splitting and tunnel-mixing energies (Δ and T) are comparable to the difference of collisional broadenings, $\hbar(1/\tau_l - 1/\tau_r)$; here τ_l and τ_r are the relaxation times, respectively, of the l and r QW. Here we report that diagonalization of the one-particle matrix Green function for tunnel-coupled electronic states in the above-mentioned relation between parameters cannot be realized. Thus, a quasiparticle description of the energy spectrum is invalid and the conductivity σ should be determined instead by formulas of quantum transport theory. In the interpretation[3] of increased resistance arising in DQW structures with nonsymmetric scattering when $\Delta = 0$ (resistance resonance), classical formulas are used. Such an interpretation is used in both the nonresonanant case (for large Δ or for $T = 0$, the conductivity in the l and r QW's is given) and the resonant case [for $\Delta = 0$ and with σ given by the usual expression that contains the total 2D concentration n and the common relaxation frequency $(1/\tau_l + 1/\tau_r)$] cases. The consideration which follows demonstrates that, for the intermediate region of parameter variation, σ is a function of T, Δ and the nonsymmetric-collisional-broadening energy $\hbar(1/\tau_l - 1/\tau_r)$. Thus, our consideration demonstrates the necessity of a *quantum-mechanical* description of *macroscopic* transport phenomena in DQW structures with nonsymmetric scattering.

The anomalous frequency dispersion of the DQW-structure conductivity is also under consideration.[5] In the nonsymmetric-scattering case, the additional frequency dispersion of the conductivity arises near the tunneling transition frequency $\hbar/T$ that is located below the Drude dispersion region if $T < \hbar(1/\tau_1 + 1/\tau_r)$.

Electron states in tunnel-coupled DQW structures are formed with the l- and r-QW orbital basis[6] and these orbitals overlap and determine a tunnel matrix element T. In such a representation, the 2×2 matrix Hamiltonian is

$$h = \varepsilon_p + \tfrac{1}{2}\Delta\sigma_z + T\sigma_x \ , \tag{1}$$

where $\varepsilon_p = p^2/2m$ is the kinetic energy of the longitudinal motion (the effective masses m in the l and r QW are equal), σ_x and σ_z are the usual 2×2 Pauli matrices. A simple model of the random potential energies in l and r QW's, $U_{l,r}(\mathbf{x})$ (here we did not consider a variation of the tunnel-barrier transmission) is used for the consideration of scattering processes. Such a case may be realized when the scattering processes are produced by imperfections of the outward heterojunctions of the DQW structure or by nonsymmetric doping of l and r wide-gap spatial domains (this case was realized in Ref. 3). It is described by a 2×2 matrix for the potential energy,

$$U_l(\mathbf{x})P_+ + U_r(\mathbf{x})P_- \quad \text{with } P_\pm = (1 \pm \sigma_z)/2 \ , \tag{2}$$

where $P_\pm$ are projection operators.

For the sake of simplicity in further calculations, we will consider only the case of scattering on statistically independent random potentials $U_{l,r}$ with a zero correlation radius. In this case, the one-particle Green-function diagrammatric expansion contains only P_+ and P_- proportional correlation functions of the random fields. In the Born approximation, the retarded Green function is given by the 2×2 matrix,

$$G_r(E - \varepsilon_p) = [\tilde{h} - E - \tfrac{1}{2}i\hbar(P_+/\tau_l + P_-/\tau_r)]^{-1} \ . \tag{3}$$

Here $\tau_{l,r}$ does not depend on P for short-range scattering processes; the Hamiltonian $\tilde{h}$ differs from (1) because a small Δ renormalization arises (for point-defect scattering, this additional term is logarithmically divergent and a small-distance cutoff has to be introduced[7]). We take the renormalization level splitting Δ to be a definite variable

because it is controlled by the transverse voltage on the DQW structure. It allows us to use Hamiltonian (1) in Green function (3). Expression (3) is an exact equality in the case of completely nonsymmetric scattering (as $\tau_r \to \infty$; we consider the more general case of $\tau_r > \tau_l$) when the self-energy function is P_+ proportional. In the general case, the self-energy function includes σ_x- and σ_y-proportional contributions which arise because of higher-order terms in the diagrammatic expansion (or random contribution in tunnel matrix element). All of the above-mentioned factors will introduce further complications in the conductivity diagrammatic expansion but will not change the results qualitatively.

The denominator of (3) includes the 2×2 real matrix from (1) and a σ_z-proportional imaginary contribution due to nonsymmetric scattering. This expression does not commute with its Hermitian conjugate, and thus the Green function cannot be diagonalized by a unitary transformation.[8] Hence, introduction of the customary description of the energy spectrum of weak-damping quasiparticles in a DQW structure with nonsymmetric scattering is impossible (such an approach is valid only for large Δ). If Δ, T and $\hbar(1/\tau_l - 1/\tau_r)$ are comparable, a quantum-mechanical description of the transport phenomena will be needed even though they are all smaller than the Fermi energy.

The Kubo formula gives a static conductivity per unit area of

$$\sigma = \pi\hbar \left[\frac{e}{m}\right]^2 \sum_{\mathbf{p},\mathbf{p}'} (\mathbf{p}\mathbf{p}')\, \mathrm{tr}\langle G_E(\mathbf{p},\mathbf{p}') G_E(\mathbf{p},\mathbf{p}')\rangle \Big|_{E=\varepsilon_F} \quad (4)$$

which contains an average product of the causal Green functions (tr is the trace of this matrix product) and the Bethe-Salpeter equation for this expression should be considered. For a strongly degenerate electron gas, the Fermi energy ε_F is related to the 2D concentration n by $n = 2\rho_{2D}\varepsilon_F$. This is valid, provided that the level splitting is small and that the Green-function-determined corrections of the 2D density of states ρ_{2D} for small E are negligible (inequalities $\varepsilon_F \gg \Delta, T, \hbar/\tau_{l,r}$ hold well under experimental conditions[3]). Further calculations (4) in the case under consideration generalize the usual method[9] by taking the matrix structures of the Green functions and the correlators of scattering potentials (2) into account. The departure relaxation time coincides with that of transport for the short-range scattering model and (4) may be expressed through Green functions (3) in the ladder approximation,

$$\sigma = \frac{e^2\varepsilon_F}{(2\pi)^2\hbar} \int d\xi\, \mathrm{tr}[G_r(\xi)G_a(\xi) + G_a(\xi)G_r(\xi)] \,. \quad (5)$$

Here we neglect the contributions of the order of $\hbar/\varepsilon_F\tau_{i,r}$ when it is permissible to neglect additional terms in $G_rG_r + G_aG_a$ and to extend the integration over the entire $(-\infty, \infty)$ interval.

After carrying out the matrix summation in (5) using (3) and the integration in (5) using a residual calculation, σ is expressed by the simple formula

$$\sigma = \sigma_0 \left[1 + F_\mu\left[\frac{\Delta\tau}{\hbar}, \frac{2T\tau}{\hbar}\right]\right], \quad \sigma_0 = \frac{e^2 n}{m}\tau \,,$$
$$F_\mu(\delta, t) = \frac{\mu^2(1+\delta^2)}{(1-\mu^2)(1+\delta^2)+t^2} \,, \quad (6)$$

where δ and t are, respectively, the dimensionless level splitting and tunnel matrix element, τ is the average relaxation time, and μ determines the degree of the scattering nonsymmetry,

$$\delta = \frac{\Delta\tau}{\hbar} \,, \quad t = \frac{2T\tau}{\hbar} \,,$$
$$\frac{1}{\tau} = \frac{1}{2}\left[\frac{1}{\tau_l} + \frac{1}{\tau_r}\right], \quad \mu = (\tau_r - \tau_l)/(\tau_r + \tau_l) \,. \quad (7)$$

The case $\mu = 0$ describes symmetric scattering for conductivity σ_0 and the case $|\mu| = 1$ describes completely nonsymmetric scattering. For uncoupled QW's ($t = 0$), we obtain from (6) and (7) the sum of conductivities of each separate QW, $\sigma = e^2 n(\tau_l + \tau_r)/2m$ (two conductors connected in parallel). For the case of strongly tunnel-coupled QW's ($\tau \gg 1$), the function F_μ in (6) is small compared with unity; thus, the conductivity depends on the total concentration n and the average relaxation time (7) which assumes the scattering probabilities summation in the l and r QW has been carried out (i.e., Mathiessen's rule is valid). Such expressions (which do not contain $\hbar$) were used for the interpretation of the resistance resonance in Ref. 3. For intermediate values of the parameters $\sigma(\Delta)$, the quantum character is determined by the relationship between the energies Δ, T and $\hbar/2\tau$ (for $\mu \neq 0$) and this dependence is shown in Fig. 1. The experimen-

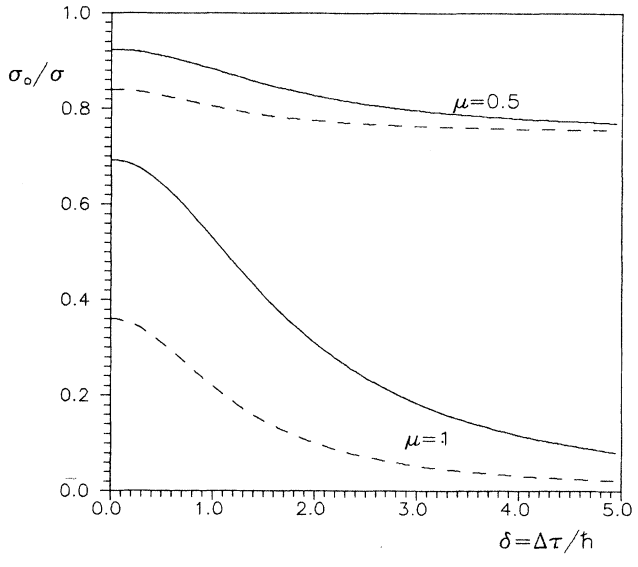

FIG. 1. The shape of the "resistance resonance" peak [the one that depends on the ratio $\sigma_0/\sigma = (1+F_\mu)^{-1}$] for $\mu = 0.5$ and 1 under $t = 0.75$ (dashed lines) and $t = 1.5$ (solid lines). Two classical limits exist: (i) for uncoupled QW's, when $T = 0$ and $\sigma_0/\sigma = 1-\mu^2$ (thus the δ dependence is absent); and (ii) for the symmetric case, when $\mu = 0$ and $\sigma_0/\sigma = 1$.

tal data[3] correspond to the small μ case ($\mu \cong 0.13$ and $t \cong 1.6$) and they are satisfactorily found to be consistent with formula (6). It should be noted also that the resonance amplitude is comparable to the nonresonance resistance value in the case of large nonsymmetry and $t \cong 1$.

Now we consider the dispersion of the real part of the conductivity in the spectral region $\omega \ll \varepsilon_F/\hbar$. The expression for $\mathrm{Re}\sigma(\omega)$ differs from (5) because an energy displacement of the Green-function argument should be carried out and we obtain

$$\mathrm{tr}[G_r(\xi+\hbar\omega)G_a(\xi)+G_a(\xi+\hbar\omega)G_r(\xi)] \ . \tag{8}$$

The trace is taken and the integration over ξ is carried out in analogy to the static case and $\mathrm{Re}\sigma(\omega)$, under the resonance condition (when $\Delta=0$, the general expression is more unwieldy), is then given by

$$\mathrm{Re}\sigma(\omega)=\frac{\sigma_0}{1+(\omega\tau)^2}\left[1+\Phi_\mu\left[\omega\tau,\frac{2T\tau}{\hbar}\right]\right] ,$$
$$\Phi_\mu(\Omega,t)=\frac{\mu^2(1+t^2-\mu^2-3\Omega^2)}{(1+t^2-\mu^2+\Omega^2)^2-4\Omega^2(t^2-\mu^2)} \ . \tag{9}$$

In the strong tunneling approximation ($t \gg 1$), the contribution of Φ_μ is small and (9) describes a Drude dispersion with mean relaxation time τ. In tunnel-uncoupled QW's ($t=0$), this relation transforms into a sum of two Drude contributions with respective characteristic relaxation times τ_l, τ_r and concentrations $n/2$. In the case of weak tunneling and nonsymmetric scattering ($t \ll 1$ and $\mu \cong 1$), a substantial dispersion of Φ_μ near $T/\hbar$ frequencies occurs while the Drude dispersion is not yet dominant. Such a dispersion in the classical region of frequencies is demonstrated in Fig. 2.

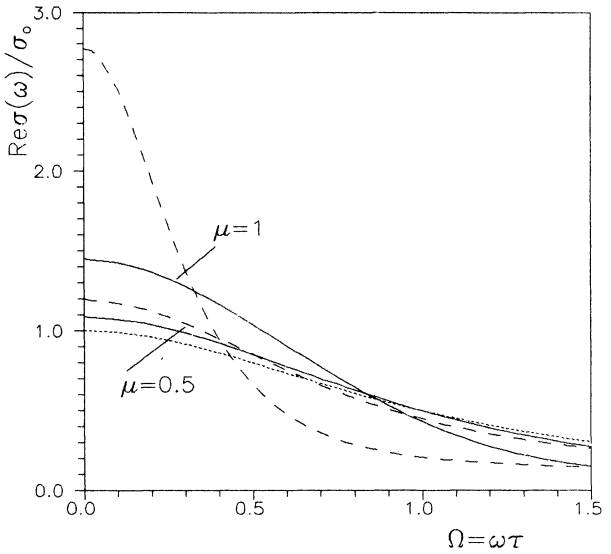

FIG. 2. The dimensionless frequency dispersion of the conductivity, $\mathrm{Re}\sigma(\omega)/\sigma_0=(1+\Phi_\mu)/(1+\Omega^2)$, for $\mu=0.5$ and 1 under $t=0.75$ (dashed lines), $t=1.5$ (solid lines). The dotted curve describes the classical Drude dispersion in the symmetric-scattering case (which is realized for $\mu=0$ or $t=0$).

In conclusion, this Brief Report demonstrates the appearance of macroscopic quantum transport phenomena in DQW structures with nonsymmetric scattering related to the inapplicability of the quasiparticle description in this case. Quantum formulas describing the shape of the resistance resonance and the low-frequency conductivity dispersion are obtained. These results are caused by the mixing of the nonsymmetric part of the dissipation and tunneling under condition of weak coupling between the longitudinal and transversal degrees of freedom in DQW structures.

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