Tunneling in double-layered quantum Hall systems

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We study interlayer tunneling in some double-layered quantum Hall states that contain a neutral gapless superfluid mode. A tunneling current less than a certain critical value will not cause any voltage drop between the two layers. A finite voltage V between the layers will induce an ac current with a frequency $\omega=eV/\hbar$. In contrast to tunneling between superconductors, the critical current here is linear rather than quadratic in the tunneling amplitude.

I. INTRODUCTION

Quantum Hall states are known to have long-range coherence. One naturally asks whether these states have some properties similar to those of superconductors, which also have long-range coherence. It would appear that various superconducting effects, such as the Josephson effect, would be impossible for the quantum Hall states because they are incompressible. However, it has been shown that some multilayered fractional quantum Hall (FQH) states[1-3] may contain gapless modes.[4-7] The dynamical properties of the gapless mode were studied in detail in Refs. 4, 5, 6, and 7; in particular a semiquantitative dispersion relation of the low-lying mode in the presence of interlayer tunneling was obtained by MacDonald, Platzman, and Boebinger.[6] In the absence of tunneling, the mode becomes gapless, corresponding to a (neutral) superfluid mode and the system demonstrates many superfluid properties.[7]

In this paper, we will study electron tunneling between layers in these systems. We predict that a small current passing through the barrier between the layers does not cause any voltage drop. This dissipationless current resembles the supercurrent in a superconductor-insulator-superconductor junction. We predict further that when a dc voltage V is applied across the two layers, an alternating tunneling current

$$J=J_0\sin(eVt/\hbar) \tag{1.1}$$

is generated. This results from the operator $c_1^\dagger c_2$ developing a long-range order in these FQH states (even in the absence of the interlayer tunneling) and from the essential angular nature of the order parameter. (Here $c_{1,2}$ are the electron operators in the two layers). Thus, the physics behind this effect is basically the same as that behind the Josephson effect.[8] However, one crucial difference is that the Josephson current between two superconductors is proportional to $\sin(2eVt/\hbar)$. The frequency of the tunneling current in the two-layered Hall system is half as much as the frequency of the Josephson current. Another important difference, as we will see, is that in the Hall system the current depends linearly on the tunneling amplitude, rather than quadratically, as is the case in superconductivity.

The plan of this paper is as follows. In Sec. II, we derive, using the field theory formalism of Ref. 7 the low-energy effective theory that takes into account the angular character of the order parameter. From the effective theory, the result (1.1) may be derived heuristically. We go through this derivation partly to assure ourselves that the effects of a certain massive gauge field (called α_+ below) may be neglected. Next, in Sec. III, we discuss the effects of dissipation and derive the tunneling current in situations relevant to realistic experiments. The experimentally inclined reader may wish to skip directly to Sec. III.

II. LOW-ENERGY EFFECTIVE THEORY

We being by giving an exceedingly brief review of the theoretical framework discussed in Ref. 7 to which the reader may wish to turn for further details. As we have discussed in a series of papers,[9] the long-distance physics of the Hall fluid may be described in terms of several gauge potentials $\alpha_I, I=1,2,\ldots$ interacting via the Lagrangian

$$\mathcal{L}=\frac{1}{4\pi}\alpha_\mu K_{\mu\nu}\epsilon^{\mu\nu\lambda}\partial_\nu\alpha_\lambda+2A_\mu\epsilon^{\mu\nu\lambda}\partial_\nu\alpha_\lambda+\text{Maxwell terms}\ , \tag{2.1}$$

where α_μ and β_μ are gauge potentials. Here K is a matrix in terms of which the long-distance properties of the Hall fluid, such as the filling factor (and conductance), and the charge and statistics of the quasiparticles, are completely determined.

In this paper, we specialize to $\dim\underline{K}=2$ and $\underline{K}=m\binom{11}{11}$ for m odd, corresponding to a $\nu=1/m$ Hall fluid in a two-layered system. The electromagnetic current in layer $I, I=1,2$ is given by

$$J_\mu^{(I)}=\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_{I\lambda}\ , \tag{2.2}$$

with the total current $J_\mu = J_\mu^{(1)} + J_\mu^{(2)}$ coupling to the electromagnetic potential A_μ. Evidently, $\underline{K}$ has eigenvalues $2m$ and 0, associated with α_+ and α_- defined by $a_\pm = \alpha_1 \pm \alpha_2$. The Maxwell terms in (2.1) are given by

$$\frac{1}{g_+^2} f_+^2 + \frac{1}{g_{+-}^2} f_+ f_- + \frac{1}{64\pi^2 g^2} f_-^2 \ , \qquad (2.3)$$

where $f_{\pm\mu\nu} = \partial_\mu \alpha_{\pm\nu} - \partial_\nu \alpha_{\pm\mu}$. Note that if the two layers are identical we expect the Lagrangian under the transformation $\alpha_+ \to \alpha_+$ and $\alpha_- \to -\alpha_-$ and thus in this case $1/g_{+-}^2 = 0$.

Because α_+ has a finite-energy gap, we may effectively set $f_+ = 0$ and the g_{+-} and g_+ terms are irrelevant at low energies. The gapless excitation described by α_- is identified as the neutral superfluid mode. With conventional units the last term in (2.3) would be written as

$$\frac{\kappa}{32\pi^2} \left[-(f_{-,12})^2 + \frac{1}{2v^2} (f_{-,0i})^2 \right] , \qquad (2.3')$$

where κ^{-1} is the capacitance (per unit area) between the two layers, and v is the velocity of the neutral superfluid mode. Note $f_{-,12}/2\pi = (n_1 - n_2)$ is the difference of the electron densities in the two layers. In (2.3) we have chosen units such that $v = 1$. We see that g^2 and κ are related through $g^{-2} = \kappa$, where κ may also be identified as the compressibility of the superfluid.

The Lagrangian (2.1), however, is incomplete: it does not include interlayer electron tunneling. As explained in Ref. 7, this corresponds to nonperturbative instanton effects. In the following we would like to include interlayer electron tunneling, or nonperturbative instanton effects in our effective theory.

When an electron tunnels from one layer to the other, the current $J_{-\mu} \equiv J_\mu^{(1)} - J_\mu^{(2)}$ is no longer conserved. Indeed, we have

$$\int d^2x dt\, \partial^\mu J_{-\mu} = \int d^2x dt \frac{1}{2\pi} \partial_\mu \epsilon_{\mu\nu\lambda} \partial_\nu \alpha_{-\lambda}$$
$$= \pm 2 \ , \qquad (2.4)$$

indicating the presence of "magnetic" monopole or antimonopole in Euclidean spacetime coupling to the gauge potential α_-. Evidently, (2.3) cannot be satisfied by a smooth gauge potential α_0. If we insist on smooth gauge potentials, we have to modify the current $J_{-\mu}$ to be equal to

$$J_-^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu \alpha_{-\lambda} + \frac{\partial^\mu}{\partial^2} \rho \ , \qquad (2.5)$$

where $\rho = \sum_a q_a \delta^{(3)}(x - x_a)$ is the monopole density. Here $q_a = \pm 1$ are the magnetic charges and x_a the locations of the monopoles and antimonopoles.

Long ago Polyakov[10] analyzed the influence of a monopole-antimonopole plasma on $(2+1)$-dimensional electrodynamics. Our problem differs from Polyakov's only in the presence of the Chern-Simon term $\alpha_{+\mu} \epsilon_{\mu\nu\lambda} \partial_\nu \alpha_{+\lambda}$. The partition function of the monopole plasma is given by

$$Z_M = \sum_{N=0}^{\infty} \frac{\zeta^N}{N!} \sum_{\{q_a\}} \prod_a \int dx_a \exp \left[-\sum_{b,c} \frac{q_b q_c}{g^2} \frac{1}{|x_b - x_c|} \right] . \qquad (2.6)$$

The Coulomb interaction between the monopoles (in Euclidean 3-space) generated by α_- reflects the fact that tunneling events are correlated: a tunneling event from layer 1 to layer 2 is more likely followed by a tunneling event from layer 2 to layer 1 occurring close by in spacetime. Here ζ denotes the fugacity for creating a monopole and is determined by short-distance physics. The partition function may be transformed to

$$Z_M = \int \mathcal{D}\theta \exp \left[i \int \tfrac{1}{2} g^2 (\partial_\mu \theta)^2 + \zeta \cos\theta \right] , \qquad (2.7)$$

the well-known sine-Gordon representation of a Coulomb gas.[11] Here $\theta(x)$ is a scalar field introduced to reproduce the Coulomb interaction between the monopoles. The quantization of magnetic charge leads to the angular character of θ.

The path integral of the field theory (2.1) factorizes

$$Z = Z_\alpha Z_M \ , \qquad (2.8)$$

with

$$Z_\alpha = \int \mathcal{D}\alpha_+ e^{iS_+(\alpha_+)} \int \mathcal{D}\alpha_- e^{iS_-(\alpha_+, \alpha_-)} \ , \qquad (2.9)$$

where

$$S_+(\alpha_+) = \frac{1}{g_{++}^2} f_+^2 + A_\mu \epsilon_{\mu\nu\lambda} \partial^\nu \alpha_{+\lambda} + \alpha_{+\mu} \epsilon_{\mu\nu\lambda} \partial_\nu \alpha_{+\lambda} \qquad (2.10)$$

and

$$S_-(\alpha_+, \alpha_-) = \frac{1}{64\pi^2 g^2} f_-^2 + \frac{1}{g_{+-}^2} f_+ f_- \ . \qquad (2.11)$$

In Ref. 7, we showed that the fugacity ζ is related to experimentally measurable quantities, since ζ is essentially the density of monopoles, or equivalently the number of tunneling events in a unit volume of spacetime, and thus may be estimated to be $\Delta_{\rm SAS}/l_B^2$, where l_B is the magnetic length and $\Delta_{\rm SAS}$ is the measured energy gap between the symmetric and antisymmetric states in the two-layered systems. ($\Delta_{\rm SAS}$ is essentially the tunneling amplitude.)

Polyakov showed that the correlation function $\langle J_{-\mu} J_{-\nu} \rangle$ did not have a massless pole when the effects of the monopole plasma were included. In the context of our problem, this means that tunneling opens a gap for the superfluid mode.[7] Note that Polyakov's result (and hence much of our work in Ref. 7) does not depend essentially on the angular character of θ; specifically, it may be derived by approximating $\cos\theta$ in (2.7) by $(1 - \theta^2/2)$. We see that the θ field has a gap $(\zeta g^2)^{1/2} \sim (\kappa \Delta_{\rm SAS})^{1/2}/l_B$. In this paper we discuss the physics associated with the angular character of θ.

Our problem is to calculate the tunneling current, evidently $(\partial/\partial t)\langle J_{-0} \rangle$ by definition, generated by an external potential U. In contrast to Polyakov, we are interested in the one-point function, rather than the two-point function. The effects of the external potential may be included by introducing into the integrand in (2.8) the factor

$$\exp\left[i\int eU_\mu J_-^\mu\right] \tag{2.12}$$

and agreeing to set at the end of the calculation U_i to zero and U_0 to U.

Using (2.5) and integrating by parts we may write

$$U_\mu J_-^\mu=\frac{1}{2\pi}U_\mu\epsilon^{\mu\nu\lambda}\partial_\nu\alpha_{-\lambda}-\partial_\mu U^\mu\frac{1}{\partial^2}\rho \ . \tag{2.13}$$

The effect of the first term is to add to S_- the term $(1/2\pi)eU_\mu\epsilon^{\mu\nu\lambda}\partial_\nu\alpha_\lambda$. The second term in (2.13) introduces into the monopole plasma an external charge density $\partial_\mu U^\mu$. The net effect is to change Z_M in (2.7) to

$$Z_M(U)=\int\mathcal{D}\theta\exp\left[i\int\tfrac{1}{2}g^2(\partial_\mu\theta-2U_\mu)^2+\zeta\cos\theta\right] . \tag{2.14}$$

When $\zeta=0$ the above partition function is invariant under the following gauge transformation $U_\mu\to U_\mu+\partial_\mu\lambda$. This is associated with the conservation of $J_{-\mu}$. The gauge invariance is violated by the interlayer tunneling, which results in a nonzero ζ.

For the case of interest, we have $U_i=0$ and $U_0=U$ a constant. In this case Z_α is independent of U. After integrating out $\alpha_\pm$, we find the total partition function is given by

$$Z=\int\mathcal{D}\theta e^{i\mathcal{L}} \tag{2.15}$$

with the effective Lagrangian:

$$\mathcal{L}=\tfrac{1}{2}g^2[(\partial_t\theta-2U)^2-(\partial_i\theta)^2]+\zeta\cos\theta \ . \tag{2.16}$$

Now we can calculate J_{-0} from $\langle J_{-0}\rangle=(\delta/\delta U)\ln Z(U)$:

$$\langle J_{-0}\rangle=\langle n_1-n_2\rangle=2g^2(\partial_t\langle\theta\rangle-2U) \ . \tag{2.17}$$

The total energy of the system is

$$E=\int d^2x\{\tfrac{1}{2}g^2[(\partial_t\theta)^2+(\partial_i\theta)^2]-2g^2U^2-\zeta\cos\theta\} \ . \tag{2.18}$$

From (2.17) and (2.18) we see that moving an electron from one layer to the other will cost an energy

$$V\equiv\partial_t\theta \ . \tag{2.19}$$

Thus V is just the difference of the chemical potentials between the two layers. Equation (2.19) can be written as $V=(1/2g^2)(n_1-n_2)+2U$, which may be expected, since the chemical potential depends on both the charge imbalance n_1-n_2 and the external potential U.

We now calculate the tunneling current $J_T=(\partial/\partial t)\langle J_{-0}\rangle|_{U_0=U,\ U_i=0}$. From the equation of motion we see that $J_T=g^2\partial_t^2\theta=\zeta\sin\theta$. Then (2.19) implies that $J_T=\zeta\sin(e\hbar^{-1}Vt)$ (if V is independent of time). Notice that, as mentioned, J_T depends linearly on the tunneling amplitude ζ.

Incidentally, omitting the Chern-Simon term in (2.1) we have the so-called dual representation of a superconductor.[12] Thus, the manipulations from (2.4) to (2.19) may be used to derive the Josephson effect in a superconductor-superconductor tunneling junction, with the appropriate replacement $e\to2e$. It is satisfying to see the Josephson effect emerge from Polyakov's formulation of (2+1)-dimensional electrodynamics in the presence of Dirac monopoles. To our knowledge, this derivation has not been given before in the literature.

We would like to mention that the effective Lagrangian (2.16) describe the low-energy dynamics of the system and (2.17) express the physical quantities in terms of the effective field θ. Thus (2.16) and (2.17) provide a complete description of the low-energy properties of the system.

We also like to point out that the effective theory (2.16) is just the XY model in the presence of a magnetic field.[6,7] We may treat the two-layered system as a system of electrons that carry a pseudospin $S=\frac{1}{2}$. The electron with $S_z=\frac{1}{2}$ corresponds to the electron in the first later and $S_z=-\frac{1}{2}$ the second layer. In the pseudospin language, $e^{i\theta}\sim S_x+iS_y$ and $n_1-n_2\sim S_z$, where S_i is the pseudospin operators. The interlayer electron hopping operator $c_1^\dagger c_2+\text{H.c.}\sim S_x\sim\cos\theta$. All the above results can also be derived from the pseudospin picture and the related XY model. The above formalism about the gauge field and monopoles is just the dual representation of the XY model in magnetic field.

III. TUNNELING AND DISSIPATION

The low-energy properties of the system are entirely determined by the effective theory (2.16). In the following we are going to apply the effective theory to some realistic situations and study the tunneling between the layers. In particular we would like to include the effects of dissipation and of the leads. For our problem, we are interested only in the space-independent mode in $\theta(\mathbf{x},t)$. Thus, we let θ to be $\theta(t)$ independent of $\mathbf{x}$ and we integrate the Lagrangian (density) $\mathcal{L}$ over space to find the Lagrangian

$$L\equiv\int d^2x\,\mathcal{L}=\frac{1}{2}M\left[\frac{d\theta}{dt}-2U\right]^2+r\cos\theta \ , \tag{3.1}$$

with $M\equiv g^2A$ and $r\equiv\zeta A$ where A denotes the area of the fluid. This describes a particle of mass M moving on a ring (of radius 1) with a potential $-r\cos\theta$ that tends to keep the particle near $\theta=0$ and with magnetic flux $2\pi U$ threading through the ring. In the thermodynamic limit, both the mass M and the scale of the potential τ go to ∞ like the area A.

The momentum conjugate to θ is

$$P=\frac{\delta L}{\delta\dot\theta}=M(\dot\theta-2U) \tag{3.2}$$

and the corresponding Hamiltonian

$$H=\frac{P^2}{2M}-\frac{2}{M}PU-\tau\cos\theta \ . \tag{3.3}$$

[From (2.17) we see that $N_-/2=P$, where $N_-=N_1-N_2$ is the difference of the numbers of the electrons in the two layers. Thus $N_-/2$ and θ are canonically conjugate to each other.

Heisenberg's equations of motion consist of (3.2) and

$$\dot{P}=-\tau\sin\theta \ . \tag{3.4}$$

The corresponding Schrödinger equation can actually be solved. In fact, however, in the thermodynamic limit, θ becomes a classical variable because θ is basically the position of a very heavy particle trapped in a steep well. It is easy to estimate from (3.3) that the mean-square derivation of θ from its classical value scales like $(\Delta\theta)^2\sim(M\tau)^{1/2}\propto A^{-1}$.

A heuristic derivation of (1.1) may now be given. In the absence of tunneling, states with the same $N_+=N_1+N_2$ but different $N_-=N_1-N_2$ are essentially degenerate. The states $|\theta\rangle\equiv\sum_{N_-}e^{i\theta N_-/2}|N_-\rangle$ all have the same energy. A tunneling Hamiltonian H_T lifts degeneracy and $|\theta\rangle$ has the energy $-\tau\cos\theta$, with τ some constant. Note the factor of 2, stemming from the fact that under the action of H_T, as an electron hops from one layer to the next, N_- changes by ±2. An external voltage U can now be applied by adding the Hamiltonian H_U defined by $H_U|N_-\rangle=eUN_-|N_-\rangle$. In a tunneling event, the energy changes by $2eU$. The tunneling current is defined by

$$\begin{aligned}J_T&=\frac{1}{2}e\frac{d}{dt}\langle\hat{N}_-\rangle=\frac{i}{2}e\langle[H,\hat{N}_-]\rangle\\&=\frac{1}{2}e\frac{\partial}{\partial\theta}\langle H\rangle=J_c\sin\theta \ ,\end{aligned} \tag{3.5}$$

where J_c depends on the constant τ and we have restored $\hbar$. From (2.19) we see that $J_T=J_c\sin(eVt/\hbar)$.

This heuristic derivation is the same as Josephson's original derivation of his effect, but given here in a different context. We have to go through the more elaborate discussion from (2.4) to (2.19) in order to assure ourselves that the presence of another gauge potential α_+ and of Chern-Simon terms, etc., does not affect this result.

We may now incorporate the effects of dissipation phenomenologically into Heisenberg's equations by respecting two general principles, that dissipation terms violate time-reversal invariance and that the angular character of θ must be preserved. We find that (3.2) may not be changed but that two time-reversal violating terms may be added to (3.4) so that

$$\dot{P}=-\tau\sin\theta-\eta P-\xi M\dot{\theta} \ . \tag{3.6}$$

The phenomenological dissipative parameters η and ξ have dimension of 1/time. Later, we will relate these parameters to experimentally measurable resistances. Combining (3.2) and (3.6) we obtain

$$\ddot{\theta}=-\frac{\tau}{M}\sin\theta-(\eta+\xi)\dot{\theta}+\eta U \ . \tag{3.7}$$

In the following we will calculate η and ξ in a realistic model. The low-energy properties of an isolated two-layer system are described by the following equation of motion:

$$\frac{A}{\kappa}\frac{d^2}{dt^2}\theta=A\xi\sin\theta \ , \tag{3.8}$$

where A is the area of the system. The difference of the numbers of the electrons in the two layers is given by [see Eq. (2.17)]

$$N_-=2\frac{A}{\kappa}\frac{d}{dt}\theta \ . \tag{3.9}$$

Thus the left-hand side of (3.8) is the time derivative of the electron number in one layer. (Note $N_+=N_1+N_2$ is always fixed due to the incompressibility.) The right-hand side of (3.8) is the current of the quantum tunneling (or, the phase-coherent tunneling). From the total energy (2.18) and (3.9) we see that $A/\kappa=C$ is just the capacitance between the two layers. Now let us add one lead to each layer, and the system becomes an open system. In this case $d/dt(N_-/2)$ also contains a contribution from the leads, in addition to that from the quantum tunneling. The current injected from the lead is given by $R_{ex}^{-1}(V_{ex}-V)$, where R_{ex} is the contact resistance, V_{ex} the voltage difference between the two leads, and

$$V=\frac{N_-/2}{C}=\kappa N_-/2A=\frac{d}{dt}\theta$$

the difference between the chemical potentials of the electron in the two layers. We should also include incoherent tunneling between the two layers. We assume the incoherent tunneling current to have a linear dependence on V and to be given by $-R_{\text{leak}}^{-1}V$, which also contributes to change of N_-. Combining all the above effects, we reach the following equation of the motion:

$$\frac{A}{\kappa}\frac{d^2}{dt^2}\theta=-A\xi\sin\theta+R_{ex}^{-1}\left[V_{ex}-\frac{d\theta}{dt}\right]-R_{\text{leak}}^{-1}\frac{d\theta}{dt} \ . \tag{3.10}$$

It is gratifying to note that this equation agrees with Eq. (3.7) obtained from general principles, if we identify the phenomenological parameters η and ξ suitably. In experiments we can easily measure the current in the leads $I=R_{ex}^{-1}(V_{ex}-d\theta/dt)$ and the voltage difference between the two layers V (using the four-terminal measurement).

From (3.10) we can obtain I and V directly as functions of V_{ex} and t. In the appendix we sketch the analysis. For small V_{ex} (3.10) has a static solution $d\theta/dt=0$ with a current $I=V_{ex}/R_{ex}$ passing through the barrier. The voltage across the barrier is $V=0$. Such a current is a dissipationless supercurrent. The static solution exists only when the supercurrent is less than a critical value given by $I_c=A\xi\sim eA\Delta_{\text{SAS}}/l_B^2\hbar$. Again, note that the critical current is linear in the tunneling amplitude or Δ_{SAS}. When $V_{ex}/R_{ex}>I_c$ (3.10) has only time-dependent solutions and I and V will acquire some ac components. When $V_{ex}/R_{ex}\gg I_c$ we may treat quantum tunneling term as a perturbation. We find

$$V \approx V_0 + \frac{I_c/(R_{ex}^{-1}+R_{\text{leak}}^{-1})}{\{[V_0 C/(R_{ex}^{-1}+R_{\text{leak}}^{-1})]^2+1/4\}^{1/2}} \sin\left[\frac{eV_0}{\hbar}t\right]$$

$$= V_0 + \frac{2\Delta^2}{\{V_0^2+[(R_{ex}^{-1}+R_{\text{leak}}^{-1})/2C]^2\}^{1/2}} \sin\left[\frac{eV_0}{\hbar}t\right] ,$$

$$I \approx \frac{V_{ex}}{R_{ex}+R_{\text{leak}}} + \frac{I_c}{\sqrt{(R_{ex}CV_0)^2+(1+R_{\text{leak}}^{-1}R_{ex})^2}} \sin\left[\frac{eV_0}{\hbar}t\right] , \tag{3.11}$$

$$V_0 = \frac{R_{\text{leak}}}{R_{ex}+R_{\text{leak}}} V_{ex} ,$$

where $\Delta=\sqrt{\xi\kappa/2}$ is the gap of the neutral superfluid mode due to the interlayer tunneling. (3.11) is valid when the ac component in V is much less than the dc part V_0, that is, when $V_{ex}/R_{ex} \gg I_c$ or $V_0/\Delta \gg 1$. For ideal leads (i.e., $R_{ex}=0$), (3.11) reduces to $V=V_{ex}$ and

$$I = \frac{V_{ex}}{R_{\text{leak}}} + I_c \sin\left[\frac{eV}{\hbar}t\right] .$$

In the above we have assumed that there is no magnetic field parallel to the layers. In practice, it may be difficult to align the sample perpendicular to the magnetic field so that the parallel magnetic field can be ignored. In the presence of a parallel magnetic field, the quantum-tunneling term should be modified to be

$$\xi \cos[\theta(x)+\phi(x)] , \tag{3.12}$$

where $\phi(x)$ is a fixed function that depends on the parallel magnetic field. We may average over ϕ and (3.12) becomes

$$\xi^* \cos\theta, \text{ with } \xi^* = \sqrt{\langle \xi \cos\phi \rangle^2 + \langle \xi \sin\phi \rangle^2} . \tag{3.13}$$

Thus the effect of the parallel magnetic field is to reduce the critical current to $I_c = A\xi^*$, but (3.11) remains valid in the presence of a parallel magnetic field provided that I_c is regarded as the reduced critical current.

In the above discussion we have assumed that the leads are attached to the bulk electrons and are not connected to the edges. In practice the leads are connected to the edges and the transport between the leads also receives contributions from the edge states. To be specific let us connect two leads to the sample. The first lead connects to the first layer on one side of the sample and the second lead to the second layer on the opposite side of the sample. We will ignore the contact resistance. In the above device the chemical potentials of the electrons in each layer will in general depend on position. However, due to the superfluidity, the difference of the chemical potentials in the two layers, V, is a constant in the whole sample, at least in the weak tunneling limit. First let us assume that the voltage difference between the two leads, V_{ex}, is very small and the tunneling current is less than the critical current I_c. In this case the chemical potentials in the two layers are equal, $V=0$. The whole sample behaves like the usual $\nu=1/m$ quantum Hall (QH) state. The two-terminal resistance is determined by edge transport and is given by $R=\nu^{-1}(h/e^2)$. As we increase V_{ex} the tunneling current $I=V_{ex}\nu(e^2/h)$ also increases. When $I>I_c$ or when $V_{ex}>I_c\nu^{-1}(h/e^2)$, the difference of the chemical potential V will no longer be zero. In this case we suddenly lose the dc supercurrent. The two-terminal resistance is expected to suddenly jump to R_{leak}. At nonzero V there is an ac Josephson current. Because V is constant in the whole sample, the frequency of the Josephson current is well defined. In practice, the Josephson current may appear in a form of narrow band noise. From the above discussion, we see that the system with leads attached to the edge is similar to the system discussed in the last few paragraphs with $R_{ex}=\nu^{-1}(h/e^2)$.

IV. DISCUSSIONS

In this paper we studied macroscopic quantum effects in some double-layered QH systems. We find some double-layered systems exhibit phenomena that are very similar to superconductors. The physical phenomena discussed in this paper apply to a much more general class of QH states. Essentially any QH state characterized by the matrix $\underline{K}$ will demonstrate superconducting phenomena if $\det\underline{K}=0$. The tunneling supercurrent and the Josephson effect discussed in this paper are within the reach of present experimental technology. Before ending this paper, we would like to remark that in the above we only discussed the quantum tunneling between the two layers of the *same* QH sample. One can also consider two double-layer samples (labeled by a and b) coupled by weak links. This system is more similar to the usual superconductor-insulator-superconductor junction. We expect a supercurrent of the form j_1-j_2 can pass the weak link. The frequency of the Josephson current will be given by $\omega=e(V_a-V_b)/\hbar$, where V_a and V_b are the differences the chemical potentials in the two layers for the sample a and sample b.

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APPENDIX

We give here a brief analysis of (3.7) or (3.10). Rescaling the time by $t=(1/\sigma)t'$, we obtain

$$\ddot{\theta} = -\dot{\theta} - T\sin\theta + W , \tag{A1}$$

(where the dot denotes differentiation with respect to t') with the rescaled tunneling amplitude

$$T = \frac{\tau}{M}(\eta+\xi)^{-2} \tag{A2}$$

and the driving potential

$$W = U\eta(\eta+\xi)^{-2} . \tag{A3}$$

Note that (for $\eta \sim \xi$) these two dimensionless quantities scale as the inverse of the dissipation.

We first consider the case $W \gg T$ and solve for θ (or more properly $\dot{\theta}$) iteratively. Write $\theta = \theta_0 + \theta_1 + \cdots$ with $\theta_0 = Wt$ and θ_1 the solution of

$$\ddot{\theta}_1 + \dot{\theta}_1 = -T \sin(Wt) \ . \tag{A4}$$

This equation becomes accurate for large time and has the solution

$$\theta_1 = \frac{T}{W}(W^2+1)^{-1/2} \cos(Wt) \tag{A5}$$

up to an unimportant phase shift.

The case $W \ll T$ is essentially trivial, as it is dominated by the friction term. Here the driving term cannot push the system over the hill and can only balance the friction term to give $\sin\theta_0 = W/T$. We then see that θ_1 decays exponentially.

Scaling back to physical quantities we obtain the results given in Sec. III.

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