Impurity-screening effects on electronic states of the two-dimensional system under a quantizing magnetic field

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Single-particle energy levels and wave functions for a two-dimensional electron system, confined to an annular film (the Corbino geometry) in the presence of a perpendicular magnetic field are exactly determined in the symmetric gauge. For the ideal system, free of impurities, the problem is shown to be exactly solved. The behavior of the electronic states in some models of impurity potentials is examined. The influence of the impurity screening on the eigenstates for the case of semiconductor inversion layers is analyzed. It is shown that the screening effects are quite important because screening will be reduced in the presence of magnetic fields. Landau levels are determined as a function of several parameters, namely, the strength of interaction, the screening length, and the distance of the electron from impurity. Wave functions, radial densities, and matrix elements are calculated.

I. INTRODUCTION

One of the most interesting aspects of the two-dimensional electron gas (2DEG) confined to semiconductor interfaces is its behavior in the presence of a strong perpendicular magnetic field. The Landau quantization causes the electrons to move in cyclotron orbits parallel to the surface and leads, in an imperfect medium, to the remarkable quantized Hall effect (QHE).[1,2] The electrical transport measurements have usually been carried out on a rectangular device (Hall bar) and a circular sample (Corbino disk). Despite the fact that the Hall geometry is appropriate for measurements of both the longitudinal and Hall resistivities, the Corbino disk has been used to measure the longitudinal conductivity. Very recently, a new method has been established to measure the Hall conductance with a Corbino disk.[3] Instead of the expected plateaus of the QHE, the Hall conductivity shows oscillations as a function of the magnetic field, which are attributed to inhomogeneities of the electron gas. This is experimental evidence of the differences between the geometries of the Hall bar and Corbino disk. On the other hand, theoretical investigations of the integer QHE have been carried out both in the Hall-bar geometry and in the Laughlin-ribbon geometry,[4] in which the Landau gauge becomes appropriate to describe the essentials of the phenomenon. Even though, by general principles, the energy spectrum of the electron gas in the presence of a quantizing magnetic field must be the same, the solution in the symmetric gauge yields a different form for the wave functions. These functions exhibit the physical characteristics of those corresponding to the circular classical orbits instead of the solution given by the Landau method.[5]

The effect of impurities on the QHE has been studied exhaustively since its discovery.[2,6] The accuracy of the QHE in dirty systems such as metal-oxide-semiconductor (MOS) and GaAs-$Ga_xAl_{1-x}As$ heterostructures constituted a challenge for specialists in the search for the correct explanation of the effect. Prange[7] has solved the problem of a single δ-function impurity potential and shown that there is one bound state per Landau level and all the remaining states are extended. The nature of the electronic states of the system in the presence of a general random-impurity potential and under an applied magnetic field has been clarified by a number of authors.[8–12] It can be shown that there must exist at least one extended state at the center of each Landau level and that the others are localized.[6,11] The existence of a mobility gap is fundamental to the gauge arguments of Laughlin[4] and Halperin.[6] The effect of repulsive and attractive scattering potentials on the quantum Hall plateaus has been investigated by Haug *et al.*[13]

We examine the behavior of the electronic states in the presence of impurity potentials and consider smooth-potential models that represent approximately real systems. This is well understood in the case of zero magnetic field. The theory of bound states of a charged impurity, for instance, has been developed from the seminal paper of Stern and Howard[14] concerning the electronic properties of semiconductor inversion layers.[15] They also considered the screening of the carriers but neglected in their calculations the finite thickness of the electron layer. Martin and Wallis[16] eliminated this restriction but neglected the effect of screening. However, Hipólito and Campos[17] include both effects and found that screening effects do indeed lower the electron binding energies. More sophisticated calculations of bound states, which treat screening in a self-consistent manner, were developed by Vinter[18] and Takada[19] (see Ref. 15 for a detailed account). The screening is important because it weakens the potential in the long-wavelength limit.

Stern and Howard[14] found a necessary condition on the strength of the impurity potential in order to have bound states. However, in the presence of a perpendicular magnetic field, all states are localized due to the Landau quantization—even for repulsive impurity potentials—thus leading to new features in the problem. It should be emphasized that in real systems, there are ionized acceptors in the depletion layer of the p-type semiconductors which are negatively charged and thus constitute repulsive centers. On the other hand, there are ionized donors that form attractive centers in the insulator or at the semiconductor-insulator interface. Furthermore, the detailed effects of screening in the magnetic-field case have not been worked out. The inclusion of the finite-thickness correction effectively softens the short-range divergence of the bare Coulomb interaction and will be complementary to the screening smoothing of the potential.

In this paper we investigate, within a single-particle picture, the eigenfunctions and eigenvalues of the electron in a two-dimensional annulus. First, in Sec. II, we consider the ideal system, free of impurities, and the problem is shown to be exactly solved in the symmetric gauge. We compare our exact results with those in the linear-oscillator approximation obtained by an expansion of the effective one-dimensional potential around its minimum.[6] We also discuss the effect of the sample edges on the energy levels. This solution, without taking into account these edge effects, was obtained 40 years ago by Dingle.[20] In Sec. III we examine the behavior of the eigenstates with some model potentials. We start by introducing the simple phenomenological impurity potential $V_0\ln(r)$, with V_0 representing the strength of the attractive or repulsive interaction. This potential arises from the solution of the two-dimensional Poisson equation for a uniform density of space charge and reproduces the potential from charged impurities at semiconductor interfaces in special limits.[14] After this we consider a more realistic potential model consisting of an unscreened point charge at some distance from the interface in a semiconductor inversion layer, such as that found in MOS field-effect transistor (MOSFET) devices. It is shown how the potential lifts the degeneracy of the Landau levels. In Sec. IV, the influence of the impurity screening on the Landau states is analyzed for the case of typical SiO_2-Si interfaces. We point out that the study presented here has obvious limitations. A complete theory of screening of an electron moving in a screened field of an isolated impurity and subjected to an external magnetic field must involve a self-consistent determination of the effective potential and the corresponding electronic-density modulation using Schrödinger's and Poisson's equations. This is a rather trivial matter in the absence of a magnetic field but a difficult task when a magnetic field is present. We use the screened potential given in Ref. 14 which was determined self-consistently, although for the case of zero magnetic field. Then we investigate how the spectrum and the corresponding eigenfunctions of this screened impurity potential depend on the magnetic field and the screening parameter. We assume that the dielectric constant κ is the same in the semiconductor and the insulator so that the image charge effects are neglected.

II. IDEAL ANNULAR FILM

Let us consider an idealized model in which noninteracting electrons are confined to a disc free of impurities shown in Fig. 1. In a magnetic field $\mathbf{B}=B_0\hat{\mathbf{z}}=\nabla\times\mathbf{A}$ the Hamiltonian of an electron of charge $-e$ is given by

$$H=\frac{1}{2m^*}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2+V_{\rm ext}(\mathbf{r}) , \tag{1}$$

where $V_{\rm ext}$ is some external potential including the confining potential. In the symmetric gauge where the vector potential $A_\theta=B_0r/2$, the Schrödinger equation can be written as

$$-\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]\Psi+\frac{e\hbar B_0}{2im^*c}\frac{\partial\Psi}{\partial\theta}+\frac{B_0e^2}{8m^*c^2}r^2\Psi+V_{\rm ext}\Psi=E\Psi . \tag{2}$$

We can separate variables in the usual way and seek solutions in the form

$$\Psi=e^{im\theta}R(r) , \tag{3}$$

where m is an integer. The resulting radial equation is

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}-\left[\frac{m^2}{r^2}+m+\frac{r^2}{4}+V_{\rm ext}(r)-2E\right]R(r)=0 , \tag{4}$$

where r and E are expressed, respectively, in units of $a_m=(\hbar/m^*\omega)^{1/2}$ and $\hbar\omega$ with $\omega=eB_0/m^*c$. In the ideal case, $V_{\rm ext}(r)=0$, we set

$$R(\rho)=\rho^{m/2}e^{-\rho/2}F(\rho) , \tag{5}$$

where $\rho=r^2/2$, and we find that the function F satisfies the confluent hypergeometric equation.[21] The general solution is then

$$\Psi(r,\theta)=e^{im\theta}e^{-\rho/2}[C\rho^m{}_1F_1(m-E+\tfrac{1}{2},m+1;\rho)+D\rho^{-m}{}_1F_1(-E+\tfrac{1}{2},-m+1;\rho)] , \tag{6}$$

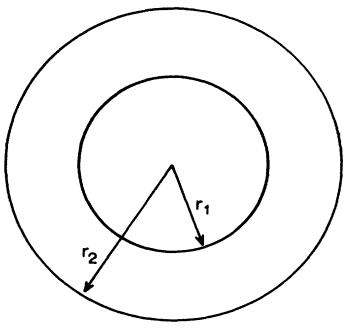

FIG. 1. Annular film of the Corbino geometry.

where ${}_1F_1(a,b;x)$ is the confluent hypergeometric function and C and D are constants to be determined from the boundary conditions. Obviously, the nonedge-sample situation can be recovered from our solution by requiring $D=0$ because of the divergent behavior of the second solution. In this case, the solutions are the associated Laguerre polynomials $L_{n+m}^{m}(\rho)$ and the discrete and degenerate energy levels are given by

$$E=(n+m+\tfrac{1}{2}) , \tag{7}$$

where n is an integer. The wave functions form rings around the origin and the states are thus localized. We note that in the Landau gauge these states are extended and form strips along a line in the plane.

Using $R(r)=r^{-1/2}\chi(r)$ in Eq. (4), we get the required radial differential equation

$$\chi''(r)+\left[2E-m-\frac{r^2}{4}-\frac{(m^2-\frac{1}{4})}{r^2}-V_{\text{ext}}(r)\right]\chi(r)=0 , \tag{8}$$

which, in the absence of external potentials, looks like a one-dimensional Schrödinger equation for the harmonic potential with a centrifugal term that remains even for $m=0$, in contrast with the usual $l(l+1)$ term in the corresponding hydrogen equation. By expanding the total potential of Eq. (8), with $V_{\text{ext}}(r)=0$, in the neighborhood of its minimum, given approximately by $r_m^2=2ma_m^2$, the solutions of the radial equation are then the eigenstates of a one-dimensional harmonic oscillator centered at a radius given by r_m. Furthermore, the eigenvalues are those from the harmonic oscillator $(n+\frac{1}{2})$.[6] Even though the linear-oscillator approximation gives exactly the correct energy levels, the wave functions are only approximate.

If we take $V_{\text{ext}}(r)$ as the confining potential given by infinite walls at $r=r_1$ and $r=r_2$, the boundary conditions on Ψ at the edges of the sample lead to a system of two linear homogeneous algebraic equations for the coefficients C and D. The energy eigenvalues are calculated with the condition that the associated determinant must be zero, i.e.,

$$\begin{vmatrix} F^-(r_1) & F^+(r_1) \\ F^-(r_2) & F^+(r_2) \end{vmatrix}=0 , \tag{9}$$

where

$$F^+(r)=r^m\,{}_1F_1(m-E+\tfrac{1}{2},m+\tfrac{1}{2};r^2/2)$$

and

$$F^-(r)=r^{-m}\,{}_1F_1(-E+\tfrac{1}{2},-m+\tfrac{1}{2};r^2/2) .$$

Thus, the confining potential drastically changes the situation as compared with the unconfined system. The main result concerning the energy spectrum has already been extensively discussed by several authors.[6,9,22,23] For an exact solution, the basic problem in the numerical calculations in the accurate determination of the confluent hypergeometric functions. However, if we use the harmonic-oscillator approximation described above, which, as is well known, gives the correct energy levels, then imposition of the boundary conditions leads to the same quantitative results for the energy spectrum as are found using the Landau gauge.[22] The overall requirement that the wave function vanish at the edges will lift the previous degeneracy of the energy levels. Then the spectrum will depend essentially on r_m for $r_2-a_m<r_m<r_2$. This same behavior occurs for the inner-edge region. The eigenvalue E_{nm} approaches the value given by Eq. (7) for $(r_2-r_m)\gg a_m$ and increases monotonically as r_m increases. As a result, a quasicontinuous band is formed at the edges. The physical properties of the edge states for the electron gas in the Corbino disk have been discussed by Halperin.[6]

III. ANNULAR FILM WITH IMPURITY POTENTIALS

In the presence of an impurity potential, the degenerate bulk energy levels are split, whereas the energy spectrum at the edges are pratically unaffected. Neglecting edge effects now, we consider the potential given by

$$V_{\text{ext}}=-2\lambda e\ln\left[\frac{r}{L}\right] , \tag{10}$$

which comes from the solution of the two-dimensional Poisson equation and is similar to the potential from an infinite string of charges with density λ. L is a characteristic length scale which is taken here as equal to a_m. The general properties of the two-dimensional Schrödinger equation for this potential in the absence of a magnetic field were studied by Atabek, Deutsch, and Lavaud.[24] The spectrum was determined as a function of the coupling parameter $-2\lambda e$ and found to be purely discrete, while the wave functions behave like those of the harmonic oscillator. In the presence of a magnetic field, one can also study the influence of a repulsive potential because there are only bound states induced by Landau quantization.

The radial differential equation was solved numerically by fourth-order Runge-Kutta and Numerov methods. In order to determine the accuracy of our numerical procedure, we first solved the Schrödinger equation with the analytically solvable potential $\alpha^2/r^2+\beta^2r^2$, where α and β are arbitrary parameters. The agreement between the analytical and numerical results were quite good and we are confident that for potentials that diverge more slowly than the centrifugal potential, which appears in the radial equation, these methods work very well.

In Fig. 2, radial densities $r|R_{nm}(r)|^2$ for the ground state and first excited states are plotted for three values of the dimensionless coupling parameter $\Gamma=-2\lambda e/\hbar\omega$. Their shapes are similar to the harmonic-oscillator wave functions, and the effect of the repulsive or attractive potential is indicated by the localization length and the height of their maxima. The corresponding Landau levels are shown in Fig. 3 as a function of the coupling parameter Γ. We observe that the degeneracy of the energy levels given by Eq. (7) is lifted by the inclusion of the impurity potential. An important feature is that the energies of all the states are decreased as Γ becomes more

negative with concavity downward. The asymmetry of the degeneracy lifting of the energies, with respect to Γ, is clearly displayed in Fig. 3 and is a consequence of the fact that the centrifugal potential is always repulsive.

The behavior of the ground state and excited states can also be observed from the data of Table I, where the lowest energy levels and corresponding matrix elements are given for some values of Γ. It was also confirmed that the localization of the wave functions of the excited states increases toward large distances with increasing value of m. This is nothing but a byproduct of the relative weakness of the logarithmic potential as compared with the centrifugal potential near the origin. The effect of the potential can be clearly observed by comparing our numerical results from Table I with the exact expression obtained analytically for the matrix elements of r^2, between identical quantum numbers, for $\Gamma=0$. In this situation, we get

$$\langle nm|r^2|nm\rangle = 2(2n+m+1) ,$$

which is a generalization of the case $n=0$ found in Ref. 25. This result, which is an indication of the relation between interelectron spacing and angular momentum, is substantially altered by the inclusion of an attractive or repulsive impurity potential, as shown in Table I.

We compare our results with those from the harmonic-oscillator approximation discussed in Sec. II, where an expansion of the effective potential around its minimum point is performed. In this approximation, the Landau energy levels are given by

$$E=(n+\tfrac{1}{2})\frac{\Omega}{\omega}+\frac{1}{2}\left[\frac{m^2}{x_0^2}+\frac{x_0^2}{4}+m+\Gamma\ln(x_0)\right] , \qquad (11)$$

where Ω is a renormalized frequency due to the impurity

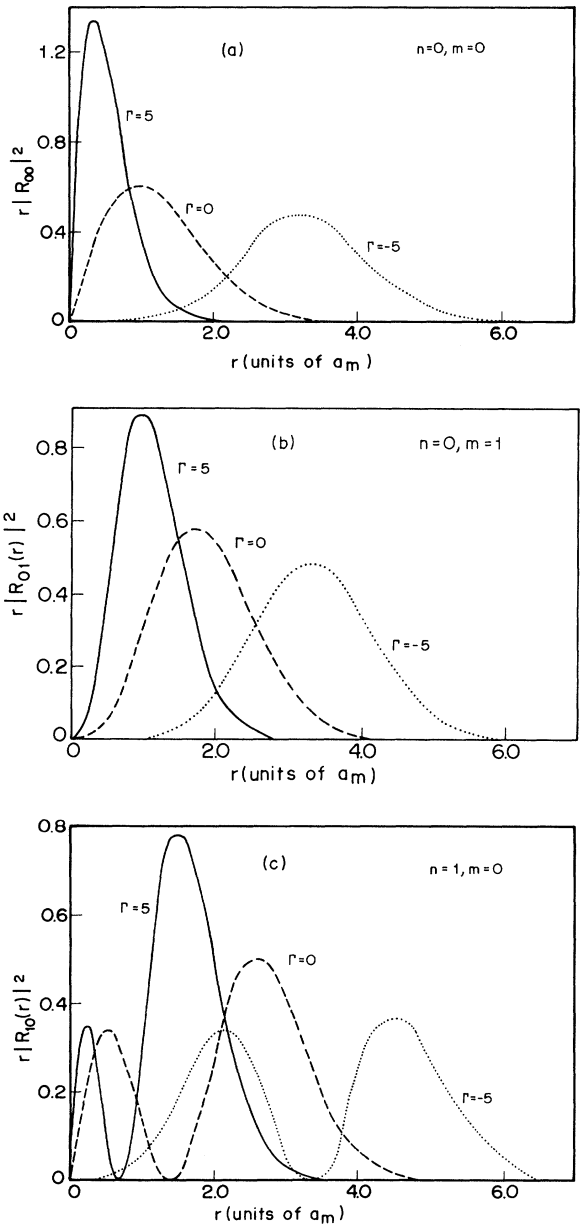

FIG. 2. Radial densities for the ground state and first excited states for the potential model of Eq. (10), for three values of the coupling strength Γ.

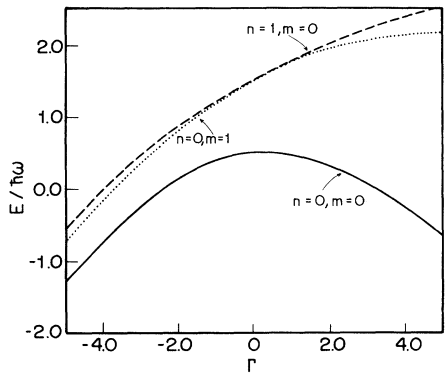

FIG. 3. Energies of the ground state and first excited states as a function of the coupling strength Γ for the potential model of Eq. (10).

TABLE I. Energies and matrix elements of the lowest Landau levels for the logarithmic interaction model potential [Eq. (10)] for some values of the coupling constant Γ. The energy is given in units of $\hbar\omega$ and the distance in units of a_m.

n	m	Γ	E	$\langle r\rangle$	$\langle r^2\rangle$
0	0	5	−0.6424	0.5528	0.4115
0	0	0	0.5000	1.2533	2.0000
0	0	−5	−1.2834	3.2621	11.3325
0	1	5	2.1478	1.1122	1.4467
0	1	0	1.5000	1.8800	4.0000
0	1	−5	−0.7249	3.3423	11.8419
1	0	5	2.5098	1.5038	2.6708
1	0	0	1.5000	2.1933	6.0000
1	0	−5	−0.5634	3.4937	14.2354

potential expressed as

$$\Omega=\omega\left[\frac{3m^2}{x_0^4}+\frac{1}{4}-\frac{\Gamma}{2x_0^2}\right]^{1/2},$$

with $x_0^2=-\Gamma+\sqrt{\Gamma^2+4m^2}$. For $\Gamma=0$, we recover the results of the ideal case. In Fig. 4, we show the Landau levels as a function of the coupling parameter for the first excited state ($n=0$ and $m=1$) for the exact (solid line) and approximated (broken line) solutions. Note that in the regions where the potential is repulsive ($\Gamma<0$), the results from both calculations cannot be distinguished in the scale of the figure.

We also calculate the eigenstates and eigenvalues for a single unscreened impurity of charge Ze located at a distance d from the interface of the insulator-semiconductor inversion layer. In this case, the external potential is given by

$$V_{\text{ext}}(r)=\frac{-Ze^2}{\kappa\sqrt{r^2+d^2}}. \tag{12}$$

In the absence of the magnetic field and for an attractive impurity in the plane of the 2DEG the solution is easily obtained and the eigenvalues are an infinite set of $(2n-1)$-fold degenerate levels

$$E_n=-(e^2/2\kappa a_B)/(n-\tfrac{1}{2})^2,$$

with $a_B=\hbar^2\kappa/m^*e^2$. With the magnetic field, the spectrum is determined as in the preceding case. We assume that if the magnetic field is large enough, the spectrum of the states of the system will still be characterized by a Landau-level structure. However, each Landau level will be broadened by the effect of the random-impurity potential. One considers here mainly the strong magnetic-field limit where only the first Landau level is occupied and virtual inter-Landau-level transitions are allowed. This is the necessary condition for describing true localization in the presence of an electric field. Figure 5 shows the results for the ground-state energy as a function of both the distance d and the renormalized interaction energy $Z=(Ze^2/\kappa a_m)/(\hbar\omega/2)$. The results for the Landau energy levels and matrix elements for the average position $\langle r\rangle$ and $\langle r^2\rangle$ are displayed in Table II. At large distances the energy values tend towards the free-system results and at $d=0$, the behavior of the Landau levels is quite similar to the logarithmic potential. Furthermore, the overall physical description of the electronic states is the same as in the preceding case and no qualitative differences are found.

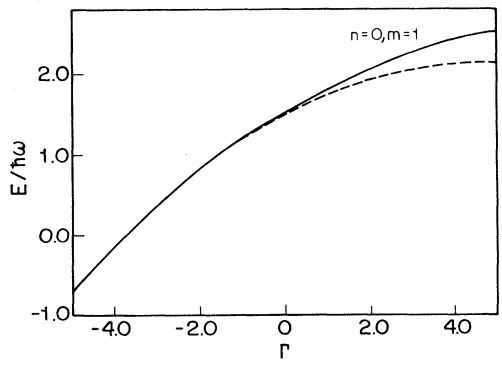

FIG. 4. Comparison between the energy of the Landau level ($n=0, m=1$) as a function of the coupling constant Γ for the logarithmic potential [Eq. (10)], obtained through the exact solution (solid line) and by using the harmonic-oscillator approximation (broken line). Note that for $\Gamma<0$, the results are the same.

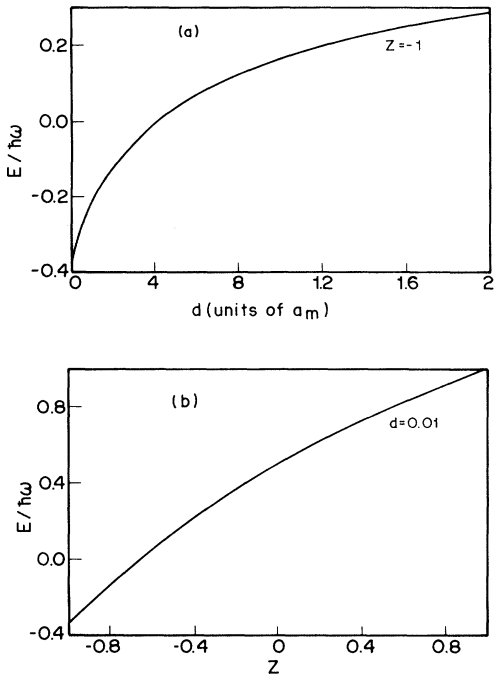

FIG. 5. (a) Landau ground-state energy for the unscreened impurity model potential [Eq. (12)], with $Z=-1$, as a function of the distance d from the interface. (b) Landau ground-state energy for the same model potential as a function of Z for $d=0.01a_m$.

TABLE II. Energies and matrix elements of the lowest Landau levels for the unscreened impurity model potential [Eq. (12)] for a number of values of the parameters Z and d. The energy is expressed in units of $\hbar\omega$ and the distance in units of a_m.

n	m	Z	d	E	$\langle r\rangle$	$\langle r^2\rangle$
0	0	1.0	0.0	1.000	1.637	3.182
0	0	1.0	5.0	0.596	1.257	2.013
0	0	−1.0	0.0	−0.360	0.796	0.889
0	0	−1.0	5.0	0.404	1.249	1.987
0	0	2.0	0.0	1.360	1.904	4.150
0	0	−2.0	0.0	−1.952	0.490	0.371
0	1	2.0	0.0	2.701	2.175	5.236
1	0	2.0	0.0	2.267	2.517	7.811

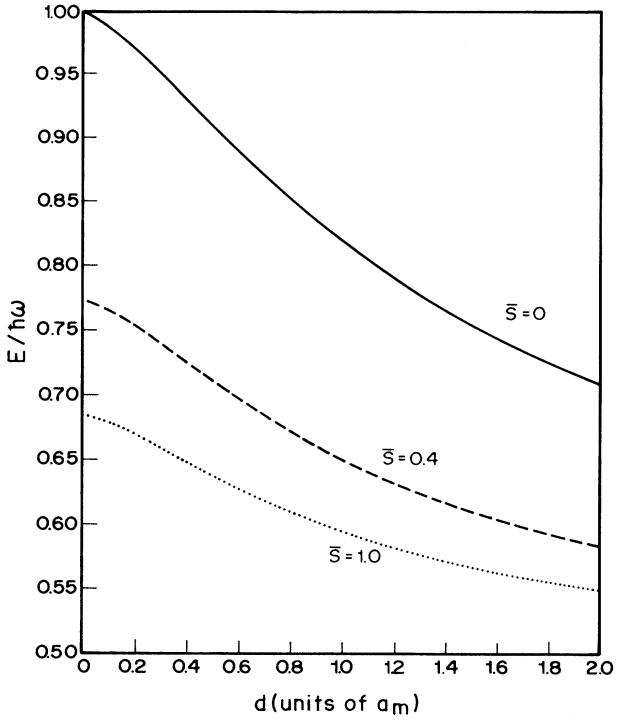

FIG. 6. Calculated energies for the screened impurity model potential [Eq. (13)] as a function of the impurity displacement with screening constant $\bar{s}=0$, 0.4, and 1.0.

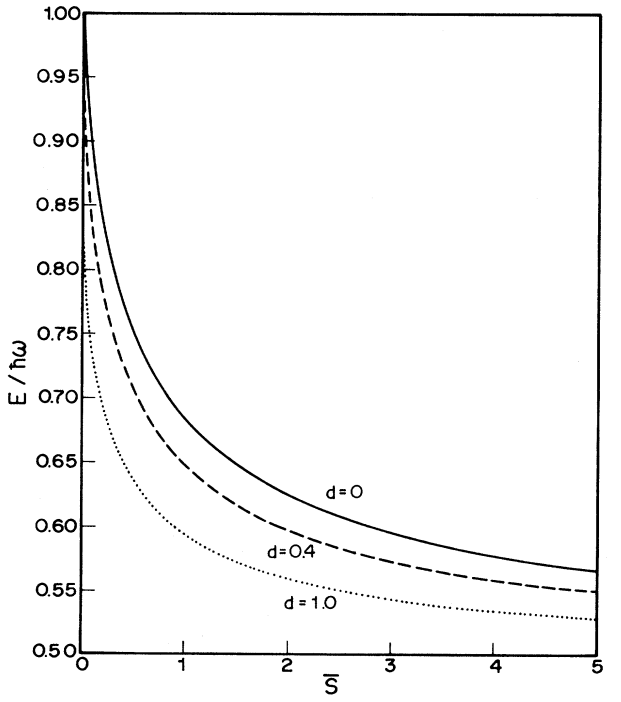

FIG. 7. Energy as a function of screening constant for three distances, calculated from the potential model describing the screened impurity [Eq. (13)].

IV. EFFECTS OF SCREENING

We now discuss the important effect of screening of the impurity charge on the Landau levels. As mentioned in the Introduction, this is not a theory of screening in 2D systems in the presence of the magnetic field. We want to investigate the change of the spectrum when a phenomenological screened potential is considered in Eq. (8). For this we consider, as before, an impurity with a free charge Ze located at a distance d from the surface, which is now screened by the electronic distribution of the 2D electron gas. The effective Coulomb potential was determined by Stern and Howard for the case of zero magnetic field.[14] Their calculation consists of solving Poisson's equation with a mean induced potential due to the electronic-density modulation $g(z)$ in the direction perpendicular to the inversion layer. In the quantum limit, when only the lowest electric subband is occupied, the electronic density was approximated by a variational expression. We restrict ourselves to the strictly two-dimensional situation where $g(z)=\delta(z)$ and the effects of magnetic subbands in the z direction are neglected. The effective screened potential of Ref. 14 is given by

$$V_{\text{ext}}(r)=\frac{-Ze^2}{\kappa}\int_0^\infty \frac{k}{(k+s)}J_0(kr)e^{-kd}dk\ , \tag{13}$$

or, in the more convenient form for numerical calculation,

$$V_{\text{ext}}(r)=\frac{-Ze^2}{\kappa}\left[\frac{1}{\sqrt{(r^2+d^2)}}-se^{sd}\int_d^\infty \frac{e^{-sz}}{\sqrt{r^2+z^2}}dz\right]. \tag{14}$$

Here $s=2n_v m^* e^2/\hbar^2\kappa$ is the density-independent screening constant at zero temperature and n_v the valley degen-

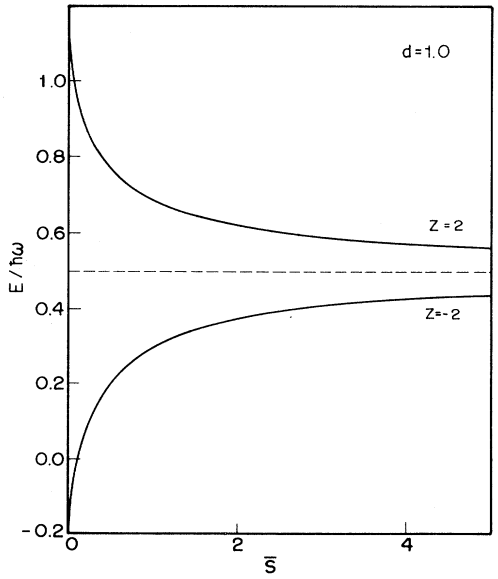

FIG. 8. Energies of the ground state as a function of screening constant using the potential model of Eq. (13) for both attractive and repulsive potentials.

TABLE III. Energies and matrix elements of the lowest Landau level for the screening impurity model potential [Eq. (13)] for a number of parameters Z, $\bar{s}$, and d. The energy is given in units of $\hbar\omega$ and the distance in units of a_m.

Z	d	$\bar{s}$	E	$\langle r \rangle$	$\langle r^2 \rangle$
2.0	0.0	0.0	1.360	0.904	4.145
2.0	0.0	1.0	0.806	1.627	3.121
2.0	0.0	2.0	0.709	1.530	2.802
2.0	1.0	1.0	0.686	1.370	2.353
1.0	0.0	1.0	0.686	1.475	2.640
1.0	0.0	0.0	1.000	1.637	3.182
−1.0	0.0	1.0	0.157	0.938	1.211
−1.0	0.0	2.0	0.212	1.017	1.402
−1.0	1.0	2.0	0.438	1.216	1.892
−1.0	2.0	2.0	0.470	1.243	1.968

eracy of the semiconductor. The dimensionless screening constant $\bar{s}=sa_m$ ranges from 1 to 10 in a large class of semiconductors at $B=10$ T. In the absence of screening, Eq. (14) reduces to Eq. (12).

Without loss of generality we concentrate on the effects of impurity screening in the lowest Landau level. In Fig. 6, we plot the energy for an attractive charge ($Z=1$) as a function of the impurity displacement d for a number of values of the screening constant $\bar{s}$. In particular, as $\bar{s}=0$, we recover the results for the unscreened potential as discussed in Sec. III. For small distances, we find that the energies are lowered by 25–35 % for $\bar{s}$ between 0 and 1, showing that screening effects must be important. As we increase the distance the energy goes down and at very large distances all the curves approach the free-impurity value of $0.5\hbar\omega$. Figure 7 shows how the energies vary with the screening constant for several values of the distance from the interface. For large values of the screening constant, the impurity potential is completely screened out and the free value of the energy is obtained. We see that the energy decreases with increasing distance from the impurity center and screening constant. These results are complementary to those of Stern and Howard,[14] and Hipólito and Campos,[17] where the binding energies are evaluated in the zero-magnetic-field case. In Fig. 8, we present a comparison of the lowest Landau-level energy for both attractive and repulsive potentials and the impurity located at a distance equal to the magnetic length from the interface. We observe that the energy degeneracy is lifted by both potentials. In Table III, we present the energies of the lowest Landau level and corresponding matrix elements for different values of $\bar{s}$, d, and Z. As expected, we find that as long as d and/or $\bar{s}$ go to infinity we recover the ideal conditions. Obviously, this also occurs if Z goes to zero. The screening effects are observed through the localization length related to $\langle r^2 \rangle$ and the height of the maxima of the wave functions associated with the parameters. These effects are quite similar to those arising from the variation of the position of the impurity relative to the 2DEG. For repulsive potentials the electron orbit radius decreases as $\bar{s}$ increases

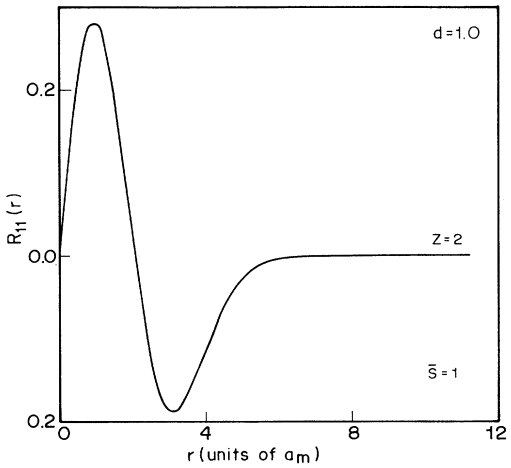

FIG. 9. Electron wave function for the excited state ($n=m=1$) with eigenvalue $E=2.588$ for the potential model of Eq. (13).

due to the screening of the repulsive core. In Fig. 9, we show a typical wave function for the excited state $n=m=1$ with eigenvalue $E=2.588$, $\langle r \rangle=1.116$, $\langle r^2 \rangle=2.132$, and parameters given by $Z=2$, $d=\bar{s}=1$.

V. CONCLUSIONS

In this paper we have calculated single-particle states of the 2DEG in the presence of a quantizing magnetic field. We have used the symmetric gauge and the role of edge effects in the Corbino disk has been worked out. Our exact results were compared with those arising from the harmonic-oscillator approximation. We have discussed the influence of charge impurities on the electronic states. The study was restricted to isolated attractive or repulsive Coulomb centers, i.e., the case of very small impurity concentrations, since the general impurity random potential has already been widely considered in the literature. Without loss of generality, we restricted ourselves to the 2DEG at the insulator-semiconductor interface built in MOSFET's. We showed that screening effects are important and lower the energies of the Landau levels. Our discussion complements, by the presence of a magnetic field, the previous studies of Stern and Howard and Hipólito and Campos about the bound states of electrons in inversion layers.

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