Nonlinear helicon-wave propagation in a layered medium

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In this paper, we have developed a theory that describes the propagation of nonlinear helicon waves in a layered structure. The reductive perturbation method is used to derive the nonlinear-evolution equation. We have shown that this equation has a one-soliton solution and this solution has been derived. Periodic-boundary conditions have been used and expressions relating different quantities in different layers have been derived, thus indicating how a nonlinear-dispersion relation for a layered medium may be obtained.

I. INTRODUCTION

Helicons are transverse, circularly polarized electromagnetic waves propagating in a conducting medium along the direction of an externally applied magnetic field. Their existence has been established for more than a quarter of a century now, and finds applications in many areas of plasma physics as well as condensed-matter physics. Recently, interest in the study of helicon waves has been revived because of the manifest importance of the exotic properties they exhibit while propagating in superlattices and in layered media. In the present analysis, we have tried to highlight the nonlinear aspect of the propagation of helicons in a superlattice semiconductor plasma.

The linear theory was well explained when Baynham and Boardman[1] studied the propagation of helicon waves in a Kronig-Penny-type[2] periodic structure. This work was initiated about twenty years ago and forms a watershed for later development in this area. They found that in the absence of scattering, the dispersion equation breaks up into a band structure consisting of allowed and forbidden propagation regions in a manner similar to an electron band structure. When scattering is included, the band edge blurs, resulting in a second propagation solution. If the scattering is further enhanced, the two propagating circularly polarized solutions collapse into a single polarized wave. For mathematical simplicity, Baynham and Boardman[1,2] chose to limit themselves to the Kronig-Penny-type sandwich structures only.

Later, the Kronig-Penny model in the δ-function version was used by Tselis, Gonzales De La Cruz, and Quinn[3] in explaining their theory of helicon-wave propagation based on the linear-response function. They did not find band-structure-type effects. Kushwaha[4] included the imaginary part for the frequency in the dispersion relation and treated the problem incorporating collisional effects as well. Again, no band structure as depicted in Ref. 1 was found, nor did the second propagation solution become apparent.

Currently, the electrodynamics of semiconductor superlattices is the subject of many theoretical and experimental papers,[5] since these properties provide the basis for developing appropriate devices. (A conspicuous illustration of this fact is provided by semiconductor superstructures.) In an ordinary semiconductor, the energy of the conduction electron is a parabolic function of the quasimomentum. This is because of the fact that the allowed bandwidth exceeds the thermal energy of the electron. However, invoking Bloch's theorem,[6] Bass and Tetervov[7] reiterated that the spectrum of the electron in a periodic field is inherently nonparabolic. The inherent periodicity of the energy as a function of the quasimomentum leads to refined nonlinearity effects, which form the basis for the development of many semiconductor devices: oscillators, amplifiers, mixers, frequency multipliers, dividers detectors, etc. Further, in Ref. 7, these authors, in their detailed review, have particularly studied high-frequency phenomena in superlattices. They have shown that the theory explaining this phenomenon predicts a large number of oscillatory, nonlinear, and resonance behavior of the high-frequency waves within the superlattice.

More recently, Achar[8] has reexamined the helicon wave propagation in a periodic structure to see under what prevalent conditions "band-structure" effects appear. In place of the Kronig-Penny model the author has used a sinusoidal structure as an alternative model with the advantage that the sinusoidal modulation is generic to the complex periodic modulation, since it represents a single Fourier component of a general periodic structure. The constraint the author has applied is a local approximation, which is valid when the fields vary slowly over distances of the order of the mean free path and during the time between collisions. This approximation is met for the parameters of the model used in Ref. 1. The author has shown that the wave propagation is governed by Mathieu's equation. Further numerical calculations do not exhibit strong band-gap effects in the dispersion.

In Ref. 8 the same author has asserted the fact that the propagation of helicon waves is rendered possible by the occurrence of the Hall effect. One can consider the helicon waves to be the dynamical manifestation of the Hall effect, which accounts for their use in contactless measurements of the Hall effect. In a later publication, Achar and Ferguson[9] have concluded that in a three-dimensional Kronig-Penny model system the plateaus due to helicon damping become distorted initially when the barrier is made higher and narrower. In Ref. 10, a plasma theory of high-T_c superconductivity has been proposed. This is based upon the fact that all oxide superconductors have layered superlattice structures, and on the idea that the pairing of carriers in a single plasma results from the attraction arising from the exchange of virtually excited plasmons.

In the present paper, we have studied the propagation of helicons in a periodically layered structure. We have established a nonlinear-evolution equation for the propagation of helicons, using a fluid description for the electrons and a Kronig-Penny model for the periodic structure. The equation that we have obtained is similar to the standard Korteweg–de Vries (KdV) and modified nonlinear Schrödinger (NLS) equations, for which we have obtained a one-soliton solution. Finally, we have introduced periodic-boundary conditions and indicated how a nonlinear dispersion relation may be obtained for a layered structure.

II. MATHEMATICAL FORMULATION

In order to study the propagation of helicons in a layered medium of a semiconductor superstructure, we solve the fundamental equations using a modified form of the reductive perturbation technique. Confining ourselves to a magnetic field directed along the z direction, which, in our case, is the direction of the normal to the layered medium, the basic equations governing the transmission of circularly polarized helicons can be written for each layer as follows:[11]

$$\frac{\partial}{\partial t}n+\frac{\partial}{\partial z}(nv_z)=0 , \tag{1}$$

$$\frac{\partial}{\partial t}v_\pm+v_z\frac{\partial}{\partial z}v_\pm=-\frac{e}{m}E_\pm\mp i\frac{e}{m}[v_zB_\pm-\omega_c v_\pm] , \tag{2}$$

$$\frac{\partial}{\partial t}v_z+v_z\frac{\partial}{\partial z}v_z=-\frac{e}{m}[v_xB_y-v_yB_x] , \tag{3}$$

$$\frac{\partial}{\partial z}E_\pm=\pm i\frac{\partial}{\partial t}B_\pm , \tag{4}$$

$$\frac{\partial^2}{\partial z^2}E_\pm=\mu_0\left[\frac{\partial^2}{\partial t^2}D_\pm+\frac{\partial}{\partial t}j_\pm\right] , \tag{5}$$

$$D_\pm=\varepsilon' E_\pm , \tag{6}$$

$$j_\pm=-nev_\pm . \tag{7}$$

In the above equations, n, v_z, $v_\pm$, $E_\pm$, $B_\pm$, and $j_\pm$ denote the number density of the electrons, the parallel electron velocity, the perpendicular electron velocity of the waves, the electric and magnetic fields, and the current density, respectively. The remaining quantities μ_0, ε', and ω_c are the magnetic susceptibility, the lattice dielectric constant, and the electron cyclotron frequency. Since helicon waves are circularly polarized, the fluctuating quantities have all been expressed in the form $a_\pm=a_x\pm ia_y$, where the signatures relate to the right- and left-hand polarizations. Clearly, the parameters n and v_z do not contribute to the linear-dispersion relation for the helicons, since the fluctuations in the leading order terms would thereby be of order ε^2. Modifying the reductive perturbation method[12] to suit our system of equations, we perturb the variables as follows:

$$a_\pm=\varepsilon a_{\pm 1}+\varepsilon^3 a_{\pm z}+\cdots , \tag{8a}$$

$$n=n_0+\varepsilon^2 n_1+\cdots , \tag{8b}$$

$$v_z=\varepsilon^2 v_{z1}+\cdots . \tag{8c}$$

Further, we introduce at this stage the stretched variables

$$\xi=z-\lambda t , \quad \tau=\varepsilon^2 t . \tag{9}$$

Consequently, the operators become

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial \xi}+\frac{\partial}{\partial \eta} , \quad \frac{\partial}{\partial t}=-\lambda\frac{\partial}{\partial \xi}+\varepsilon^2\frac{\partial}{\partial \tau} ,$$

where η is the independent variable upon which the variation of the amplitude depends and is also in the $\hat{z}$ direction.

In Eq. (9) λ is a velocity parameter to be determined later. We note that the ordering we have used is the same as in Ref. 12 for obtaining the modified KdV (MKdV) equation for Alfven waves. Substituting the ordering scheme given by Eqs. (8a)–(8c) and (9) and collecting terms in different orders of ε, we obtain the following differential equation in the lowest order of ε:

$$\left[\lambda\frac{\partial}{\partial \xi}\pm i\omega_c\right]\left[\frac{\partial^2}{\partial \eta^2}+2\frac{\partial^2}{\partial \eta\partial \xi}-\left[\frac{\lambda^2}{c^2}-1\right]\frac{\partial^2}{\partial \xi^2}\right]v_{\pm 1}-\lambda\frac{\omega_p^2}{c^2}\frac{\partial}{\partial \xi}v_{\pm 1}=0 . \tag{10}$$

Varying $v_{\pm 1}$ as $\exp(ik_\xi \xi)$ we obtain the dispersion relation for helicons[1,13] as

$$\left[\frac{\partial^2}{\partial \eta^2}+2ik_\xi\frac{\partial}{\partial \eta}+k_\xi^2\left[\frac{\lambda^2}{c^2}-1\right]-\frac{k_\xi\omega_p^2\lambda}{(\lambda k_\xi\pm\omega_c)c^2}\right]v_{\pm 1}=0 . \tag{11}$$

In the above expressions, $\lambda=\omega/k_\xi$ is the phase velocity of the helicon wave. Following Ref. 1 we can obtain a dispersion relation for helicon waves in a layered medium. This is done with the introduction of appropriate boundary conditions and the periodicity of the lattice structure via the Bloch wave number.[1]

Finally, we would like to note here that both the coordinates ξ and η are in the $\hat{z}$ direction, the difference being that the former pertains to the oscillations within each layer and the latter to oscillations across the layers. We also note here that in Eq. (11) the various parameters have different values in different layers.

III. NONLINEAR-EVOLUTION EQUATION

In this section we derive the nonlinear-evolution equation that governs the propagation of helicon waves in a layered medium. This is done by collecting higher-order terms using the reductive perturbation method introduced in the preceding section.

To order ε^2, we get an expression relating the parallel velocity fluctuations to the perpendicular velocity fluctuation as

$$\frac{\partial}{\partial\xi}v_{z1}=\frac{1}{2\lambda}\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]|v|^2\ , \tag{12}$$

where $|v|^2=v_{\pm1}v^*_{+1}$.

Finally, in order ε^3, we obtain the equation for the evolution of the helicon wave. Following Ref. 14, this equation has the form

$$\mathcal{D}v_{\pm1}=0\ , \tag{13}$$

where

$$\begin{aligned}\mathcal{D}=&\frac{\partial}{\partial\tau}\pm2i\lambda\frac{\omega_c}{\omega_p^2}\frac{\partial^2}{\partial\tau\partial\xi}-\frac{1}{2\lambda}\frac{\partial}{\partial\xi}|v|^2-\frac{1}{2\lambda}\frac{\partial}{\partial\xi}\left[\left[\frac{\partial^2}{\partial\eta^2}+\frac{\partial^2}{\partial\eta\partial\xi}\right]\int\int|v|^2d\xi'd\xi''\right]+\frac{c^2}{\omega_p^2}\left[3\frac{\lambda^2}{c^2}-1\right]\frac{\partial^3}{\partial\tau\partial\xi^2}\\&\mp i\frac{\omega_c}{2\lambda^2}\frac{c^2}{\omega_p^2}\left[\frac{\lambda^2}{c^2}-1\right]\frac{\partial^2}{\partial\xi^2}\left[|v|^2+\frac{\partial}{\partial\eta}\int|v|^2d\xi'\right]\\&-\frac{c^2}{\omega_p^2}\left[\frac{\partial^2}{\partial\eta^2}+2\frac{\partial^3}{\partial\xi\partial\eta^2}\right]\left[\frac{\partial}{\partial\tau}+\frac{1}{2\lambda}\left[|v|^2+\frac{\partial}{\partial\eta}\int|v|^2d\xi'\right]\left[\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\xi}\right]\right]\\&+\frac{1}{2\lambda}\frac{c^2}{\omega_p^2}\left[\frac{\lambda^2}{c^2}-1\right]\frac{\partial^2}{\partial\xi^2}\left[|v|^2+\frac{\partial}{\partial\eta}\int|v|^2d\xi'\right]+\frac{1}{2\lambda^2}\frac{c^2}{\omega_p^2}\left[\frac{\partial^2}{\partial\eta^2}+2\frac{\partial^2}{\partial\xi\partial\eta}\right]\left[|v|^2+\frac{\partial}{\partial\eta}\int|v|^2d\xi'\right]\\&\qquad\times\left[\frac{\partial}{\partial\eta}\int\left[\lambda\frac{\partial}{\partial\xi}\pm i\omega_c\right]d\xi'+\lambda\frac{\partial}{\partial\xi}\pm i\omega_c\right]\\&-\frac{1}{2\lambda^2}\frac{c^2}{\omega_p^2}\left[\frac{\lambda^2}{c^2}-1\right]\frac{\partial^2}{\partial\xi^2}\left[|v|^2+\frac{\partial}{\partial\eta}\int|v|^2d\xi'\right]\frac{\partial}{\partial\eta}\int\left[\lambda\frac{\partial}{\partial\xi}\pm i\omega_c\right]d\xi'\ .\end{aligned}\tag{14}$$

In order to obtain an exact solution of Eq. (14) we substitute in the above expression the following transformation:[15]

$$\alpha=K_\xi\xi+K_\eta\eta-\omega^*\tau\ . \tag{15}$$

This step is taken in order to incorporate both time and space variation into a single variable and allows us to integrate once with respect to α. After performing this step we can separate the resulting expression into its imaginary and real parts by substituting

$$v_{\pm1}=A(\alpha)\exp[i\varphi(\alpha)]\ .$$

For the imaginary part we obtain the following expression:

$$\gamma_1\frac{d}{d\alpha}A+\gamma_2\frac{d}{d\alpha}\varphi\frac{d}{d\alpha}A+\gamma_3A\frac{d^2}{d\alpha^2}\varphi+\gamma_4A^2\frac{d}{d\alpha}A=0\ , \tag{16}$$

where

$$\begin{aligned}&\gamma_1=\mp2\lambda\omega_c\omega^*K_\xi/\omega_p^2\ ,\\&\gamma_2=-2c^2\omega^*(K_\xi^2-2K_\xi K_\eta-K_\eta^2)/\omega_p^2\ ,\\&\gamma_3=\gamma_2/2\ ,\\&\gamma_4=\mp(3\omega_cc^2/2\lambda^2\omega_p^2)(K_\xi+K_\eta)^2\\&\qquad\times\left[\frac{\lambda^2}{c^2}-1-K_\eta(K_\eta+2K_\xi)/K_\xi^2\right]\ .\end{aligned}\tag{17}$$

Integrating Eq. (15) once, we get

$$\frac{d}{d\alpha}\phi=Q/(\gamma_3A^2)-\gamma_1/(2\gamma_3)-\gamma_4A^2/(4\gamma_3)\ , \tag{18}$$

where Q is an integration constant and we note that for $Q=0$ we expect a solitary wave solution and for $Q\neq0$ a periodic wave solution.[16] Furthermore, we shall consider the case where $Q=0$ and look for soliton-type solutions only.

For the real part of Eq. (14) we get the following expression:

$$\Gamma_1A+\Gamma_2A\frac{d}{d\alpha}\varphi+\Gamma_3A^3+\Gamma_4\frac{d^2}{d\alpha^2}A+\Gamma_5A\left[\frac{d}{d\alpha}\varphi\right]^2+\Gamma_0A^3\frac{d}{d\alpha}\varphi=0\ , \tag{19}$$

where

$$\begin{aligned}&\Gamma_1=-\omega^*\ ,\\&\Gamma_2=\pm2\lambda\omega_cK_\xi/\omega_p^2\ ,\\&\Gamma_3=-[K_\xi^2-K_\eta(K_\eta+K_\xi)]/(2\lambda K_\xi)\ ,\\&\Gamma_4=-c^2\omega^*[K_\xi^2-K_\eta(K_\eta+K_\xi)]/\omega_p^2\ ,\\&\Gamma_5=-\Gamma_4\ ,\\&\Gamma_6=\pm\omega_cc^2(K_\eta+K_\xi)\\&\qquad\times\left[K_\xi\left[\frac{\lambda^2}{c^2}-1\right]-K_\eta(K_\eta+K_\xi)\right.\\&\qquad\qquad\left.\times(K_\eta+2K_\xi)/(2K_\xi^2)\right]\Big/(\lambda\omega_p)^2.\end{aligned}\tag{20}$$

Substituting Eq. (18) into Eq. (19) for the case $Q=0$, we obtain the following differential equation:

$$\frac{d^2A}{d\alpha^2}+\beta_1 A+\beta_2 A^3+\beta_3 A^5=0 , \quad (21)$$

where

$$\beta_1=[\Gamma_1-\gamma_1\Gamma_2/(2\gamma_3)+\Gamma_5\gamma_1^2/(4\gamma_3^2)]/\Gamma_4 ,$$
$$\beta_2=[-\Gamma_2\gamma_4/(4\gamma_3)+\Gamma_3+\Gamma_5\gamma_1\gamma_4/(4\gamma_3)^2]/\Gamma_4 , \quad (22)$$
$$\beta_3=\gamma_4[\Gamma_5\gamma_4/(4\gamma_3)-\Gamma_6]/(4\gamma_3\Gamma_4) .$$

The solution to Eq. (20) is of the form[11]

$$A=A_0[\lambda_0+\cosh\alpha]^{-1/2} . \quad (23)$$

Substituting Eq. (23) into Eq. (21) and collecting terms independent of $\cosh\alpha$, those proportional to $\cosh\alpha$, and those proportional to $\cosh^2\alpha$ and setting them separately equal to 0, we get

$$\beta_1=-\tfrac{1}{4} ,$$
$$\lambda_0=\pm 3(16\beta_1/\beta_2^2-3)^{-1/2} , \quad (24)$$
$$A_0^2=\pm 12\beta_1(16\beta_1/\beta_2^2-3)^{-1/2}/\beta_2 .$$

We can now substitute expression (23) into Eq. (19) and obtain an expression for φ that reads as follows:

$$\varphi=-\left\{\int[\gamma_1+\gamma_4 A^2]d\alpha'\right\}\Big/(2\gamma_3) . \quad (25)$$

Thus, the one-soliton solution of Eq. (14) is given by Eqs. (23) and (25). Equation (23) defines the variation of the amplitude of the nonlinear wave and the parameter φ appearing in Eq. (25) describes the variation of the envelope of the nonlinear wave. It should be noted here that expression (25) is integrable[17] but gives different results for different combinations of the signs of the coefficients entering under the integral.

We can now obtain an expression relating the nonlinear wave numbers and frequency, i.e., K_ξ, K_η, and ω^*, respectively. This is done by equating the expressions for β_1 given by the set of expressions (22) and (24). Thus, we obtain

$$\omega_{pi}^2+2\lambda_i^2\omega_{ci}K_{\xi i}^2/\{\omega_i^* c_i^2[K_{\xi i}^2-K_{\eta i}(2K_{\xi i}+K_{\xi i})]\}-\lambda_i^2\omega_{ci}^2K_{\xi i}^2/c_i^2=-c_i^2[K_{\xi i}^2-K_{\eta i}(2K_{\xi i}+K_{\eta i})]/4 . \quad (26)$$

Here, the subscript i has been introduced so that differentiation between the different layers can be made. Thus, expression (26) relates the wave numbers and frequency within each layer and this expression can be viewed as the nonlinear analog to the linear-dispersion relation, which relates k_ξ, k_η, and ω to one another.

IV. PERIODIC-BOUNDARY CONDITIONS

In this section we shall introduce boundary conditions similar to those used in Ref. 1 and discuss how a nonlinear-dispersion relation can be obtained for a layered medium. The solution to the nonlinear-evolution equation [Eq. (14)] for each layer taken separately can be rewritten with the help of the subscript i and this is given by the following:

$$v_{\pm 1i}=A_{0i}[\lambda_{0i}+\cosh(K_{\xi i}\xi+K_{\eta i}\eta-\omega_i^*\tau)]^{-1/2}$$
$$\times\exp[i\varphi_i(\xi,\eta,\tau)] , \quad (27a)$$

where

$$\varphi_i=-\left\{\int[\gamma_{1i}+\gamma_{4i}A_i^2]d\alpha_i'\right\}\Big/(2\gamma_{3i}) . \quad (27b)$$

For the sake of simplicity, we assume that our layered structure consists of two layers which are repeated periodically. The thickness of the first layer is d_1 and that of the second layer is d_2, and the subscripts $i=1,2$ in Eqs. (27) and subsequently refers to the layer under consideration. This scenario is illustrated in Fig. 1 and is similar to that used in Ref. 1. Following Ref. 1, the periodic-boundary conditions are

$$v_{\pm 1}(d_1)=v_{\pm 2}(d_1) ,$$
$$\left.\frac{dv_{\pm 1}}{d\eta}\right|_{\eta=d_1}=\left.\frac{dv_{\pm 2}}{d\eta}\right|_{\eta=d_1} ,$$
$$v_{\pm 1}(0)=\exp(i\mathcal{K}d)v_{\pm 2}(d) , \quad (28)$$
$$\left.\frac{dv_{\pm 1}}{d\eta}\right|_{\eta=0}=\exp(i\mathcal{K}d)\left.\frac{dv_{\pm 2}}{d\eta}\right|_{\eta=d} .$$

Substituting the periodic-boundary conditions given by expressions (28) into the expression (26) and separating into real and imaginary parts, we get the following results:

$$A_{01}p_{1d_1}^{-1/2}=A_{02}p_{2d_1}^{-1/2} , \quad (29)$$

$$\varphi_{1d_1}=\varphi_{2d_1} , \quad (30)$$

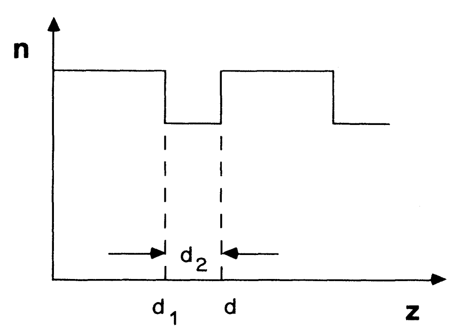

FIG. 1. Periodic modulation of the carrier density.

$$A_{02}^2 Q_{1d_1} = A_{01}^2 Q_{2d_1} , \tag{31}$$

$$\frac{d\varphi_{1d_1}}{d\eta} = \frac{d\varphi_{2d_1}}{d\eta} , \tag{32}$$

$$\varphi_{10} = \varphi_{2d} + \mathcal{K} d , \tag{33}$$

$$A_{01} p_{10}^{-1/2} = A_{02} p_{24}^{-1/2} , \tag{34}$$

$$A_{02}^2 Q_{10} = A_{01}^2 Q_{2d} , \tag{35}$$

$$\frac{d\varphi_{10}}{d\eta} = \frac{d\varphi_{2d}}{d\eta} , \tag{36}$$

where

$$P_{1d_1} = \lambda_{01} + \cosh(K_{\xi 1}\xi - \omega_1^* \tau)\cosh K_{\eta 1} d_1 + \sinh(K_{\xi 1}\xi - \omega_1^* \tau)\sinh K_{\eta 1} d_1 , \tag{37}$$

$$P_{2d_1} = \lambda_{02} + \cosh(K_{\xi 2}\xi - \omega_2^* \tau)\cosh K_{\eta 2} d_1 + \sinh(K_{\xi 2}\xi - \omega_2^* \tau)\sinh K_{\eta 2} d_1 , \tag{38}$$

$$\varphi_{1d_1} = -\frac{1}{2\gamma_{3,1}} \int [\gamma_{1,1} + \gamma_{4,1} A_{0,1}^2 p_{1\sigma}^{-1}] d\alpha' |_{\sigma = d_1} , \tag{39}$$

$$\varphi_{2d_1} = -\frac{1}{2\gamma_{3,2}} \int [\gamma_{1,2} + \gamma_{4,2} A_{02}^2 p_{2\sigma}^{-1}] d\alpha' |_{\sigma = d_1} , \tag{40}$$

$$Q_{1d_1} = K_{\eta 1}\cosh(K_{\xi 1}\xi - \omega_1^* \tau)\sinh K_{\eta 1} d_1 + K_{\eta 1}\sinh(K_{\xi 1}\xi - \omega_1^* \tau)\cosh K_{\eta 1} d_1 , \tag{41}$$

$$Q_{2d_1} = K_{\eta 2}\cosh(K_{\xi 2}\xi - \omega_2^* \tau)\sinh K_{\eta 2} d_1 + K_{\eta 2}\sinh(K_{\xi 2}\xi - \omega_2^* \tau)\cosh K_{\eta 2} d_1 , \tag{42}$$

$$\varphi_{10} = -\frac{1}{2\gamma_{3,1}} \int [\gamma_{1,1} + \gamma_{4,1} A_{01}^2 p_{1\sigma}^{-1}] d\alpha' |_{\sigma = 0} , \tag{43}$$

$$\varphi_{2d} = -\frac{1}{2\gamma_{3,2}} \int [\gamma_{1,2} + \gamma_{4,2} A_{02}^2 p_{2\sigma}^{-1}] d\alpha' |_{\sigma = d} , \tag{44}$$

$$P_{10} = \lambda_{01} + \cosh(K_{\xi 1}\xi - \omega_1^* \tau) , \tag{45}$$

$$P_{2d} = \lambda_{02} + \cosh(K_{\xi 2}\xi - \omega_2^* \tau)\cosh K_{\eta 2} d + \sinh(K_{\xi 2}\xi - \omega_2^* \tau)\sinh K_{\eta 2} d , \tag{46}$$

$$Q_{10} = K_{\eta 1}\cosh(K_{\xi 1}\xi - \omega_1^* \tau) , \tag{47}$$

$$Q_{2d} = K_{\eta 2}\cosh(K_{\xi 2}\xi - \omega_2^* \tau)\sinh K_{\eta 2} d + K_{\eta 2}\sinh(K_{\xi 2}\xi - \omega_2^* \tau)\cosh K_{\eta 2} d . \tag{48}$$

In the above set of expressions, i.e., (29)–(48), the subscripts for P, Q, and φ have been used in the following way: the first subscript refers to the layer that is being considered (see Fig. 1) and the second refers to the point within the layer at which any one of the quantities P, Q, or φ is being evaluated. We further note that for the coefficients γ and A_0 the second subscript relates to the layer under consideration.

V. CONCLUSIONS

In the present paper we had set out to describe nonlinear helicon-wave propagation in a layered superstructure. We have been successful in achieving this end in that we have been able to derive a nonlinear-evolution equation [Eq. (14)]. We have also been able to derive its one-soliton solution [given by (23) and (25)]. We have also derived a nonlinear-dispersion relation for helicon-wave propagation in each layer separately. This is given by expression (26). In Sec. IV we have introduced standard periodic-boundary conditions for a layered medium and have derived expressions relating different coefficients across the layers.

At this point we would like to note that a nonlinear-dispersion relation can be obtained for a periodic layered structure that would be analogous to the linear-dispersion relation derived by Baynham and Boardman [Eq. (3.6) of Ref. 1] by using expressions (29)–(32) and (34)–(36) in the relationship given by (33). However, an explicit expression for the nonlinear-dispersion relation for a layered structure has not been given. This is because Eq. (25) has not been explicitly integrated because, as has been pointed out above, the integration yields different results for different combinations of the coefficients entering this expression. We have kept our treatment of the problem very general in that we have not tried to apply our results to a specific layered structure. This has not been done, since we have considered an idealized situation in which collisions in the medium have been neglected. Inclusion of collisions would lead to a fast damping of the helicon wave and in order to overcome the effect of damping, an external electric field has to be taken into account. We hope to report on the inclusion of these effects soon.

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