RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 24 15 JUNE 1993-II

Magnetic-flux-induced conductance steps in microwires

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(Received 28 January 1993)

We predict a new type of steplike fluctuations in the magnetoconductance of three-dimensional ballistic microwires. These mesoscopic fluctuations, which appear on a scale corresponding to a small fraction of the quantum unit of magnetic flux $\Phi_0 = hc/e$, are a novel manifestation of the Aharonov-Bohm effect. The sharp conductance steps are caused by the shift of electronic levels through the Fermi level in a magnetic field. The flux-induced steps should be observable in, e.g., submicrometer Bi whiskers at temperatures of order 0.1 K. Sample-characteristic fluctuations are predicted to appear in fields as low as a few gauss.

In this paper we present a theory for the magnetoconductance of a three-dimensional (3D) microwire suspended between macroscopic leads. For a ballistic wire we predict sample-characteristic fluctuations in the form of sharp steps in the trace of the conductance as a function of a weak magnetic field. These mesoscopic conductance fluctuations appear on a new scale corresponding to a longitudinal magnetic flux through the wire of only a small fraction of the quantum unit $\Phi_0 = hc/e$. The sharp conductance steps are caused by the energy shifts of electronic levels in a magnetic field; the new scale is set by the flux change needed to shift energy levels a distance of the order of the average spacing between quantized *transverse* energy levels in the 3D wire, $E_F/(k_F a)^2$ (E_F, k_F is the Fermi energy and wave vector; a is the radius of the wire). These shifts result in charge carrying single-particle modes in the wire being "switched" on or off as energy levels move across the Fermi level and get (de)populated. The small magnetic flux scale is characteristic for a wire—a solid cylinder—and does not appear in the ballistic hollow cylinders first studied.[1] This is because the average spacing between transverse modes is larger by a factor $k_F a$ in such a geometry, which makes the corresponding magnetic scale coincide with the scale Φ_0 of the usual Aharonov-Bohm (AB) oscillations.

The concept of a quantized conductance that changes in a steplike manner with the number of current-carrying modes is familiar from studies of quantum ballistic transport in constrained 2D electron gas systems (for a review see, e.g., Ref. 2). In such systems a current can be forced to flow between two large 2D reservoirs through a microconstriction defined by the electrostatic field of a split gate. For a smooth enough constriction adiabatic charge transport occurs in an integer number of effectively one-dimensional modes that each contribute a quantum unit, $G_0 = 2e^2/h$, to the total conductance.[3] Without a magnetic field the number of active modes and hence the conductance is determined by the minimum width of the constriction, which can be conveniently controlled by a gate voltage.

The effect of a magnetic field on the conductance of a microconstriction is also known.[2] Because of the 2D character of the transport process, a magnetic field has a substantial effect only if it is so strong that the radius of the cyclotron orbit, $l_c = \hbar k_F c/eB$, is of the order of the constriction width or smaller. For typical GaAs-based heterostructures this criterion corresponds to large fields of several tesla.

The situation changes qualitatively if we consider electron transport in 3D ballistic wires. In this case the cross section of the wire is 2D and (specular) surface scattering produces closed orbits in the transverse directions without the help of a strong magnetic field, i.e., even if $l_c \gg a$. The transport properties of a number of systems are sensitive to a weak magnetic field (for a review see, e.g., Refs. 4 and 5 and references therein); the magnetoconductance or magnetic moment (persistent current) of doubly connected systems like cylindrical metal films or contacts, metal rings, and even a singly connected system[6] like a solid microwire[7,8] may have an oscillatory component with a period corresponding to a quantum flux unit Φ_0. These *smooth* conductance variations with field are associated with the AB effect and depend on the changing amount of magnetic flux enclosed by typical electron trajectories. The conductance steps predicted by our theory may be considered to be a *discrete* manifestation of the AB effect.

The geometry of the transport problem considered here is shown in Fig. 1. The ballistic microwire is modeled as a cylindrical channel attached to two large reservoirs. A bias voltage between the reservoirs drives a current through the thin cylinder. The channel length L

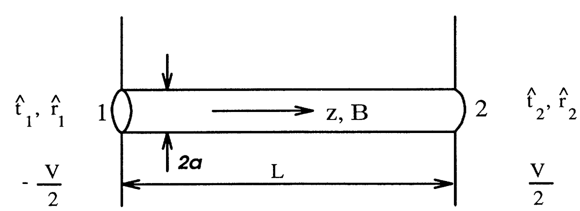

FIG. 1. A ballistic microwire modeled as cylindrical channel between two large electron reservoirs. The *probabilites* for scattering at contact 1 (2) between modes in the reservoir and the channel (transmission) or between modes in the reservoir (reflection) are given by the elements of the matrices $\hat{t}_{1(2)}$ and $\hat{r}_{1(2)}$.

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is assumed to be, on the one hand, small compared to the elastic and inelastic mean free paths of the electrons (corresponding to the ballistic limit), and, on the other hand, large compared to the normal metal coherence length $\hbar v_F/k_B T$. The latter criterion ensures that the interference between different single-particle wave functions in the cylinder are unimportant. Hence it is sufficient to describe quantum transport through the microwire in terms of probabilities rather than probability amplitudes for electron transmission and reflection at the entrance and/or exit of the cylindrical channel. There is no need for the passage between the cylinder and the reservoirs to be adiabatic. On the contrary, we predict sharp conductance steps, albeit of smaller magnitude, even if scattering is strong in the contact regions labeled 1 and 2 in Fig. 1.

Current is a conserved quantity in our problem and is most conveniently evaluated in the microwire. We parametrize the spatially quantized electron states in the wire by $\gamma=(m,n,p)$ where m and n are discrete transverse quantum numbers and the continuous variable p is the longitudinal momentum. The corresponding electron states in the reservoirs are denoted by $\mathbf{k}=(\mathbf{k}_\perp,k_z)$. It follows that the current through a cross section of the cylinder can be expressed as

$$I=(2e/hm^*)\sum_{m,n}\int_{-\infty}^{\infty}dp\,pf(E_\gamma)\ , \tag{1}$$

where m^* is the effective mass of the electron. To proceed it is convenient to write the distribution function $f(E_\gamma)$ for electrons inside the channel as a sum of the two functions $f^>(E_\gamma)$ for right-moving electrons ($p>0$) and $f^<(E_\gamma)$ for those moving to the left ($p<0$). These functions can then be expressed in terms of the equilibrium distribution functions in the two reservoirs and the probability for electrons to enter the microwire. One finds that

$$f^{\gtrless}(E_\gamma)=\sum_{\mathbf{k}_\perp,E_k=E_\gamma}\hat{T}_1^{\gtrless}(\mathbf{k},\gamma)f_0[(E_\gamma+eV/2-\mu)/T]+\sum_{\mathbf{k}_\perp,E_k=E_\gamma}\hat{T}_2^{\gtrless}(\mathbf{k},\gamma)f_0[(E_\gamma-eV/2-\mu)/T]\ , \tag{2}$$

where T is the temperature, μ the chemical potential, $\hat{T}^>_{1(2)}(\mathbf{k},\gamma)$ and $\hat{T}^<_{1(2)}(\mathbf{k},\gamma)$ are the probabilities for an electron in state $\mathbf{k}$ in the left (right) reservoir to enter the microwire and end up in a state γ with momentum $p>0$ and $p<0$, respectively.

The probabilities $\hat{T}^>_{1,2}$ and $\hat{T}^<_{1,2}$ discussed above are related to the probabilities for single elastic scattering events at the contacts between wire and reservoirs, either for transmission from one mode i in the reservoir to another mode j in the wire $\hat{t}_{1,2}(i,j)$, or for reflection back into the wire, $\hat{r}_{1,2}(i,j)$. It is reasonable to assume that the scattering of electrons in the contact regions is strong, which means that there is a large probability of mode mixing. This is enhanced if the transmission probabilities are low so that each electron suffers multiple reflections before it escapes from the wire. In this situation there is an equal probability for an electron that tunnels into the cylindrical channel from any of the M modes of the reservoir to eventually end up in any of the N cylinder modes. With the approximation that the mode mixing is completely random in *each* single scattering event, the problem becomes effectively one dimensional and the probability matrices for single tunneling simplify to read

$$\hat{t}_{1,2}=t_{1,2}(1/M)\hat{I},\ \ \hat{r}_{1,2}=r_{1,2}(1/N)\hat{J}\ . \tag{3}$$

Here $t_{1,2}$ and $r_{1,2}$ are c numbers while $\hat{I}$ and $\hat{J}$ are matrices of order $M\times N$ and $N\times N$, respectively, with all elements equal to unity. The current through the microwire is determined by the difference $T_1^>-T_1^<=(\hat{t}_1+\hat{r}_1\hat{r}_2\hat{t}_1+\cdots)-(\hat{r}_2\hat{t}_1+\hat{r}_2\hat{r}_1\hat{r}_2\hat{t}_1+\cdots)$, which is trivial to evaluate because of the simplification (3). Summing over the M reservoir modes one finds

$$\sum_{\mathbf{k}_\perp}(\hat{T}_1^>-\hat{T}_1^<)=t_2t_1/(1-r_1r_2)\ . \tag{4}$$

Finally, by appealing to the Landauer formula, we can relate the coefficients $t_{1,2}(=1-r_{1,2})$ to the contact resistances $R_{1,2}=R_0/(t_{1,2}N)$, $R_0=h/2e^2$ which allows us to express the linear conductance as[9]

$$G=(2e^2/h)T_0\int d\epsilon(-\partial f_0/\partial\epsilon)\sum_{m,n}\Theta(\epsilon-E_{m,n})\ , \tag{5}$$

where $T_0=(R_0/N)/(R_1+R_2-R_0/N)$. In Eq. (5) $E_{m,n}$ is the transverse part of the electron energy in the microconstriction, and we have used a sum rule,

$$\sum_{\mathbf{k}_\perp}(\hat{T}_1^{\gtrless}+\hat{T}_2^{\gtrless})=1\ , \tag{6}$$

obtained from the definition (2), which simply expresses the obvious fact that an electron occupying the state γ in the microwire has to come from either the left or the right reservoir. The presence of the term R_0/N in the denominator of the expression for T_0 distinguishes our result for the ballistic microwire from the usual result for a diffusive system for which the contact resistances simply add.

The geometry discussed here, with an abrupt transition region between microwire and leads causing strong elastic scattering, differs significantly from that of an adiabatic contact whose dimensions change slowly on the scale of the Fermi wavelength (some properties of 3D adiabatic contacts are discussed in Ref. 10). An adiabatic contact causes no scattering and the probabilty matrices $\hat{t}_{1,2}$ are simply unit matrices. In spite of this difference, the expression for the conductance of a microchannel with adiabatic contacts to the reservoirs coincides with Eq. (5) if we set $T_0=1$.

We now proceed to analyze the current through the microwire in the presence of a longitudinal magnetic field using (5). The field is taken to be so weak that the cyclotron radius is much larger than the radius of the wire, $l_c\gg a$ or equivalently $k_Fa\gg\Phi/\Phi_0$ where $\Phi=\pi a^2B$ is the magnetic flux through a cross section of the wire. The transverse energy eigenvalues in a weak magnetic field of an electron in a microwire approximated as a perfectly smooth cylindrical channel are[11]

$$E_{m,n}=\frac{\hbar^2}{2m^*a^2}\left[\gamma_{m,n}^2+2\alpha m+\frac{\alpha^2}{3}\left[1+\frac{2(m^2-1)}{\gamma_{m,n}^2}\right]\right]. \tag{7}$$

In this expression $\alpha=\Phi/\Phi_0$ is the dimensionless magnetic flux, and $\gamma_{m,n}$ is the nth root of the mth Bessel function, i.e., $J_m(\gamma_{m,n})=0$, $n=1,2,\ldots$ and $m=0,\pm1,\pm2,\ldots$. In the semiclassical approximation the electrons whose energy spectrum is given by (7) propagate along complicated trajectories. However, when projected onto a cross section of the cylinder they do not penetrate inside a region bounded by a circle (caustic) whose radius a_0 is connected to the magnetic quantum number m by the expression[7]

$$m=\pm ka_0+\pi a_0^2B/\Phi_0 . \tag{8}$$

It follows from Eq. (8) that the radius of the caustic grows when the absolute value of m increases.

The states corresponding to the largest values of m are localized in a narrow layer near the surface of the cylinder. These edge states are responsible[12] for the AB effect, i.e., they give rise to an oscillatory dependence of the conductivity on magnetic flux with period Φ_0.[7,8] The spectrum of these states is obtained from Eq. (7) by using the approximation $\gamma_{m,n}\approx m+\frac{1}{2}m^{1/3}[3\pi(n-\frac{1}{4})]^{2/3}+\cdots$, which follows from an analysis of an asymptotic form of the Bessel functions.[8] The result is

$$E_{m,n}=\frac{\hbar^2}{2m^*a^2}\{(m+\alpha)^2+m^{4/3}[3\pi(n-\tfrac{1}{4})]^{2/3}\} . \tag{9}$$

We wish to analyze the expression (5) for the conductance using the energy spectrum (9) for small wires (small values of k_Fa) at low temperatures. This is best done numerically. In contrast, at slightly elevated temperatures and larger wires it is convenient to use the Poisson summation formula as in Refs. 11 and 7 to get an analytical expression for the conductance. At temperatures such that $k_BT>E_F/(k_Fa)$, only the leading term in a Fourier series expansion contributes and gives a smooth oscillatory contribution of period Φ_0 to the magnetoconductance.

In order to better understand the numerical results it is useful to consider first the zero magnetic field and zero-temperature limit. It follows then from Eqs. (5) and (7) that a variation of the parameter k_Fa leads to nonequidistant conductance steps. The distribution of the steps is determined by the roots $\gamma_{m,n}$ of the Bessel functions. In zero magnetic field ($\alpha=0$) the height of the conductance steps are either $T_0(2e^2/h)$ or $2T_0(2e^2/h)$. The "double height" steps appear because the states with nonzero magnetic quantum number m are degenerate at $\alpha=0$ (since $\gamma_{m,n}=\gamma_{-m,n}$). An infinitely weak magnetic field removes the degeneracy, see Fig. 2. Note also that the conductance steps occur more frequently for larger k_Fa, when the roots of the Bessel functions are more densely spaced.

Conductance steps can also occur when the magnetic field is varied, obviously a more convenient parameter from an experimental point of view. To see this, let us return to the conductance formula (5). Electronic states in

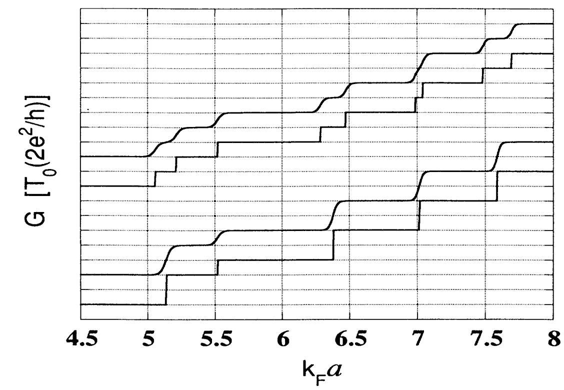

FIG. 2. Conductance of a microwire as a function of the Fermi wave vector k_F (or radius a). Results are given for zero magnetic field ($\alpha=0$, lower two curves) and finite field ($\alpha=0.2$, upper two curves). For each field the conductance is plotted for zero temperature ($T=0$, sharp steps) and finite temperature ($T=0.2\hbar^2/m^*a^2k_B$, rounded steps). The step height is either $T_0(2e^2/h)$ or $2T_0(2e^2/h)$ as explained in text, and the curves are offset for clarity. The value of T_0 is related to scattering at the contacts between the ballistic microwire and the macroscopic leads and is unity for adiabatic contacts.

the channel are characterized by two quantum numbers m and n; for illustration we fix n and determine the maximum value of m as a function of α (field). This is most easily done for small n. The spectrum of these states is given by (9). Since m is large and of the same order as k_Fa, the second term can be omitted. The remainder corresponds to the electronic states in a one-dimensional ring. The allowed values of m in this case are confined to the interval $-k_Fa-\alpha\le m\le k_Fa-\alpha$. It follows that the number of allowed states (m values) varies periodically between two values as a function of α. The period in α is unity and the plateau width (corresponding to an "extra" allowed state) depends on the fractional part of k_Fa. At integer or half-integer values of k_Fa the plateaus degenerate into points. With increasing n the fluctuation pattern gets more complicated; for large values of n the maximum magnetic quantum number m is smaller, corresponding to a smaller flux enclosed by the electronic orbit and thus larger periods in α ($\Delta\alpha>1$). The full result with a fairly complicated structure of conductance steps is a superposition of the magnetic flux dependence of all the states as shown in Fig. 3.

As expected the conductance steps are broadened at finite temperatures, as can be seen in Figs. 2 and 3. The temperature scale for broadening is related to the average energy level spacing, $E_F/(k_Fa)^2\sim\hbar^2/m^*a^2$, of transverse (i.e., 2D) states in the wire, which is not changed by a weak magnetic field. This scale differs substantially from the temperature scale $E_F/(k_Fa)$ associated with the ordinary AB oscillations.

In conclusion we have identified a new type of fluctuations in the magnetoconductance of a 3D ballistic microwire. These fluctuations are reproducible for a given sample and form its "magnetic fingerprint." In order to resolve the steplike conductance fluctuations the surface of the wire has to be nearly perfect with a high degree of

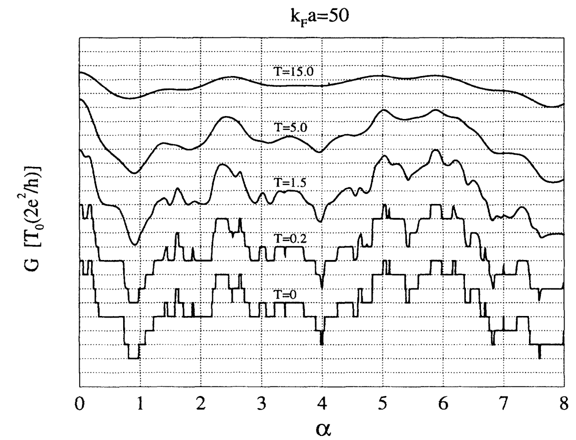

FIG. 3. Conductance as a function of normalized magnetic flux through the microwire. Results are shown for $T=0$, 0.2, 1.5, 5.0, and 15.0 (in units of $\hbar^2/m^*a^2k_B$). At temperatures below unity in these units, sharp fluctuations appear on the scale of only a fraction of a flux quantum ϕ_0; a new scale for mesoscopic phenomena. At higher temperatures only smooth conductance oscillations on the scale of ϕ_0 remain. All curves were calculated for $k_Fa=50$.

specular reflection. In addition a small variation of the radius on a scale smaller than the Fermi wavelength is required. Nonspecular surface scattering will, like temperature, broaden the sharp conductance steps. A simple estimate[13] leads to the criterion that $\sigma<\lambda_F/(k_Fa)$, where σ is a length characterizing the surface roughness. Recently magnetoconductance fluctuations were observed in ballistic gold nanobridges in a transverse magnetic field.[14] The scale of these oscillatory fluctuations were related to the separation of energy levels in the system. Our theory would predict steplike conductance variations on the same scale in a longitudinal field. We propose that suitable ballistic microwires can also be made from bismuth whiskers. The AB effect in the form of smooth magnetoconductance oscillations has already been observed in such systems.[15] The new type of submicrometer-size bismuth microbridges used in Ref. 16 may be even more suitable. The Fermi wavelength in bismuth is a few hundred angstroms, and $E_F/k_B \sim 200$ K. For a submicrometer bismuth wire a surface roughness of at most about 5% of the wire radius would be sufficient to make it possible to observe sharp conductance steps in magnetic fields of about a few gauss and at temperatures about 0.1 K.

This work was supported by the Swedish Royal Academy of Sciences (KVA), the Natural Science Research Council (NFR), and by NUTEK. One of us (E.B.) acknowledges the hospitality of the Department of Applied Physics, CTH/GU.

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