Interaction Hamiltonian between an electron and polar surface vibrations in a symmetrical three-layer structure

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The Hamiltonian of the interaction between an electron and surface vibrations for a three-layer symmetrical structure is obtained. In the limiting case of infinite outer-layer thickness, the known results by Mori and Ando are reproduced.

Early achievements in theoretical investigations of the phonon spectrum and the electron-phonon interaction in a separate thin polar layer, both surfaces of which interface with a vacuum, were described in Refs. 1–3. Subsequently, papers appeared concerning the vibrational states and electron-phonon interactions in compositional structures.[4–6] We draw attention to Ref. 4 as an early publication on the problem. In Ref. 4, the theory of polar vibrational excitations and the electron-phonon interaction in multilayer structures composed of an arbitrary number of polar layers was developed, and represents a generalization of the results of Refs. 2 and 3. Based on theory, the states of both the polarons at the polar-polar crystal interface,[7] and the polaronic excitons in thin layers[8,9] were considered, and the theory of Raman scattering in superlattices[10] was developed. All the above problems are discussed in detail in Ref. 11.

The recent papers[12,13] and also Ref. 6 report on the investigation of three-layer symmetrical structure, containing the polar layer (layer $n=2$) of finite thickness sandwiched between two semi-infinite polar layers (layer $n=1$ and 3), the host materials of which are the same. Here we aim to find analytically the normal phonon frequencies and the Hamiltonians of the electron-phonon interaction for such a structure.

We investigate the case of the symmetrical three-layer structure when the electric potentials at external surfaces are zero. Symmetry consideration, both of thicknesses and dielectric permittivities, make it possible to obtain the general solution of the problem in explicit form.

To solve the problem we use the general formula obtained in Ref. 4 for the Fourier transform of the potential resulting from the polarization vibrations in the multilayer structure with an arbitrary number of layers. Here, we reproduce the surface part of the potential,

$$\varepsilon_0 V_n^P(\boldsymbol{\eta},z)=\frac{\pi}{\eta}\sum_{k=I}^{K}\frac{1}{l_k}\left[\frac{\zeta_k}{2\tanh(\zeta_k/2)}\right]^{1/2}\left[\sinh\frac{\zeta_k}{2}\right]^{-1} \times\left\{\left[\sum_{l=1}^{2}\mathcal{K}_{1n,lk}\tilde{P}_{lk}(\boldsymbol{\eta},0)\right]\cosh w_n + \left[\sum_{l=1}^{2}\mathcal{K}_{2n,lk}\tilde{P}_{lk}(\boldsymbol{\eta},0)\right]\sinh w_n\right\} . \quad (1a)$$

In Eq. (1a) lk is the thicknesses of the kth layer of the multilayer structure, $\zeta_k=\eta l_k$; $\mathcal{K}_{jn,lk}$ are the coefficients given in Ref. 5 in their explicit form; $j,l=1,2$ are the numbers of surface polarization modes $\tilde{P}_{lk}(\boldsymbol{\eta},0)$; k is the number of layers, $w_n=\eta[z-(z_n+z_{n-1})/2]$ where z_n is the coordinate of the interface between the nth and $(n-1)$th layer, $\boldsymbol{\eta}$ is the two-dimensional (2D) wave vector perpendicular to the stratification axis $0Z$.

Let us write down the formula for the surface part of the potential in the middle ($n=2$) layer. For the considered three-layer structure, the symmetry properties are of the following form: the thicknesses of the first and third layers are equal to each other $l_1=l_3$, and the dielectric permittivities of the corresponding layers are one and the same, $\varepsilon_1=\varepsilon_3$, $\varepsilon_{10}=\varepsilon_{30}$. Then

$$\varepsilon_0 V_2^P(\boldsymbol{\eta}Z)=[\pi/\eta\sinh(\zeta_2/2)]((1/c_1 l_1)\{\mathcal{K}_{12,21}[1+\tanh^2(\zeta_1/2)]^{1/2}[\tilde{P}'_{21}(\boldsymbol{\eta},0)-\tilde{P}'_{23}(\boldsymbol{\eta},0)]\cosh w_2\} +\mathcal{K}_{22,21}[1+\tanh^2(\zeta_1/2)]^{1/2}[\tilde{P}'_{21}(\boldsymbol{\eta},0)+\tilde{P}'_{23}(\boldsymbol{\eta},0)]\sinh w_2 +(1/c_2 l_2)[\mathcal{K}_{12,12}\tilde{P}_{12}(\boldsymbol{\eta},0)\cosh w_2+\mathcal{K}_{22,21}\tilde{P}'_{22}(\boldsymbol{\eta},0)\sinh w_2]) , \quad (1b)$$

where we have introduced the polarizations $\tilde{P}'$ related to the polarizations $\tilde{P}$ by the definitions

$$[1+\tanh^2(\zeta_1/2)]^{1/2}\tilde{P}'_{21}\equiv\tanh(\zeta_1/2)\tilde{P}_{11}(\boldsymbol{\eta},0)+\tilde{P}_{21}(\boldsymbol{\eta},0) , \quad (2a)$$

$$[1+\tanh^2(\zeta_1/2)]^{1/2}\tilde{P}'_{23}\equiv-\tanh(\zeta_1/2)\tilde{P}_{13}(\boldsymbol{\eta},0)+\tilde{P}_{23}(\boldsymbol{\eta},0) . \quad (2b)$$

From Eq. (1) we can easily see that the chosen symmetry of the structure leads naturally to the hybridization of the polar vibrations of the first and third layers, generating two types of contributions into the potential. First is the symmetrical one with respect to the reflection in the middle plane of the second layer, while the other is antisymmetrical. Henceforth we call the corresponding polarizations antisymmetrical (3a) and symmetrical (3b),

$$\tilde{P}'_{21}(\boldsymbol{\eta},0)+\tilde{P}'_{23}(\eta,0)\equiv\sqrt{2}\tilde{P}_{+}\ , \tag{3a}$$

$$\tilde{P}'_{21}(\boldsymbol{\eta},0)-\tilde{P}'_{23}(\eta,0)\equiv\sqrt{2}\tilde{P}_{-}\ . \tag{3b}$$

For the structure under consideration, we use short notations for the mode indices

$$(n=1,j=2)\rightarrow r=3;\ (n=3,j=2)\rightarrow r=4\ ,$$

$$(n=2,j=1)\rightarrow r=1;\ (n=2;j=2)\rightarrow r=2\ .$$

Now the set [see Eq. (35c) in Ref. 4] of four equations of motion for the normal vibrational modes $\sum_{r'=1}^{4}N_{rr'}W'_{r'}=0$ reduces to two mutually independent sets of equations. This occurs because the symmetry properties of the matrix $N_{rr'}$ are of the following form:

$$N_{14}=N_{41}=-N_{13};\ N_{42}=N_{24}=N_{23};\ N_{43}=N_{34};\ N_{44}=N_{33}\ .$$

Thus, the set of two equations of motion for the antisymmetrical modes is written as

$$\sqrt{2}N_{23}W_2+(N_{33}+N_{34})W_{+}=0\ , \tag{4a}$$

$$N_{22}W_2+\sqrt{2}N_{23}W_{+}=0\ , \tag{4b}$$

and the set for the symmetrical modes is

$$\sqrt{2}N_{13}W_2+(N_{33}-N_{34})W_{-}=0\ , \tag{5a}$$

$$N_{11}W_2+\sqrt{2}N_{13}W_{-}=0\ , \tag{5b}$$

where in Eqs. (4a), (4b), (5a) and (5b), W is the amplitude renormalized by the definition

$$P_n(\boldsymbol{\eta},q_{11})=\omega_n\sqrt{L_xL_yl_n(\varepsilon_{n0}-\varepsilon_n)\varepsilon_0}(2\pi)^{-3}W_n\ . \tag{6}$$

For the coefficients in Eqs. (4a), and (4b), using their definition we find

$$N_{33}+N_{34}=-\coth(\zeta_2/2)/\varepsilon_2^{(1)}\ , \tag{7a}$$

$$N_{23}-N_{24}=\omega^2-\omega_1^2[\varepsilon_2+\varepsilon_{20}\coth\zeta_1\coth(\zeta_2/2)]\ , \tag{7b}$$

$$N_{13}=-\tfrac{1}{2}[1+\tanh^2(\zeta_1/2)]^{1/2}\omega_1\omega_2[(\varepsilon_{10}-\varepsilon_1)(\varepsilon_{20}-\varepsilon_2)/\tanh(\zeta_1/2)\tanh(\zeta_2/2)]^{1/2}/\varepsilon_2^{(1)}\ , \tag{7c}$$

$$N_{23}=\tfrac{1}{2}[1+\tanh^2(\zeta_1/2)]^{1/2}\tanh(\zeta_2/2)\omega_1\omega_2[(\varepsilon_{10}-\varepsilon_1)(\varepsilon_{20}-\varepsilon_2)/\tanh(\zeta_1/2)\tanh(\zeta_2/2)]^{1/2}/\varepsilon_2^{(2)}\ , \tag{7d}$$

$$N_{11}=\omega^2-\omega_2^2(\varepsilon_{20}^{(1)}/\varepsilon_2^{(1)})\ , \tag{7e}$$

$$N_{22}=\omega^2-\omega_2^2(\varepsilon_{20}^{(2)}/\varepsilon_2^{(2)})\ , \tag{7f}$$

where

$$\varepsilon_{20}^{(2)}\equiv\varepsilon_{20}+\varepsilon_1\coth\zeta_1\tanh(\zeta_2/2)\ , \tag{8a}$$

$$\varepsilon_{20}^{(1)}\equiv\varepsilon_{20}+\varepsilon_1\coth\zeta_1\coth(\zeta_2/2)\ . \tag{8b}$$

From the equations of motion, (4a) and (4b) and (5a) and (5b), the dispersion equations for the antisymmetrical normal vibrations are found,

$$N_{22}(N_{33}+N_{34})-2N_{23}^2=0\ , \tag{9a}$$

$$\omega^4-\omega^2\{\omega_2^2\varepsilon_{20}^{(2)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_2/2)]\}/\varepsilon_2^{(2)}+\omega_1^2\omega_2^2[\varepsilon_{20}+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_2/2)]/\varepsilon_2^{(2)}=0 \tag{9b}$$

with the frequencies

$$\Omega_{1,2}^2=[\{\omega_2^2\varepsilon_{20}^{(2)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_2/2)]\}$$
$$\pm(\{\omega_2^2\varepsilon_{20}^{(2)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_2/2)]\}^2-4\omega_1^2\omega_2^2\varepsilon_2^{(2)}[\varepsilon_{20}+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_2/2)])^{1/2}]/2\varepsilon_2^{(2)}\ . \tag{9c}$$

In a similar way, for the symmetrical normal modes we obtain the following equations:

$$N_{11}(N_{33}-N_{34})-2N_{13}^2=0\ , \tag{10a}$$

$$\omega^4-\omega^2\{\omega_2^2\varepsilon_{20}^{(1)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)]\}/\varepsilon_2^{(1)}+\{\omega_1^2\omega_2^2[\varepsilon_{20}+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)]\}/\varepsilon_2^{(1)}=0 \tag{10b}$$

with frequencies

$$\Omega_{3,4}^2=[\{\omega_2^2\varepsilon_{20}^{(1)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)]\}$$
$$\pm(\{\omega_2^2\varepsilon_{20}^{(1)}+\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)]\}^2-4\omega_1^2\omega_2^2\varepsilon_2^{(1)}[\varepsilon_{20}+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)])^{1/2}]/2\varepsilon_2^{(1)}\ . \tag{10c}$$

Owing to the conditions (9a) and (10a) imposed on the coefficients in the equations of motion, (4a) and (4b) and (5a) and (5b), it is possible to use one equation at a time in each set, relating the amplitudes of one and the same frequency inside the second layer $W_{1,2}=W_{12,22}$ and out of it $W_{\pm}\sim\tilde{P}'_{21}(\boldsymbol{\eta},0)\pm\tilde{P}'_{23}(\boldsymbol{\eta},0)$ in order to obtain the normal vibrational amplitudes. By making use of the definition

$$W_2(\Omega_1)=C_1(\Omega_1)Z_1(\Omega_1) \tag{11a}$$

and a corollary from the Eq. (4b)

$$W_+(\Omega_1)=-(N_{22}/\sqrt{2}N_{23})W_2=-F_{21}(\Omega_1)C_1(\Omega_1)Z_1(\Omega_1)\ , \tag{11b}$$

$$F_{21}(\Omega_1)=(N_{22}/\sqrt{2}N_{23})\ , \tag{11c}$$

we find the amplitude of the normal mode $Z_1(\Omega_1)$ with frequency Ω_1. This mode is a characteristic of the whole structure and is normalized by the condition

$$\tfrac{1}{2}(\dot{W}_{1,2}^2+\dot{W}_{\pm}^2)=\tfrac{1}{2}\dot{Z}_s^2;\quad s=1,2,3,4 \tag{12}$$

with the normalizing constant

$$C_1(\Omega_1)=1/\sqrt{1+F_{21}^2(\Omega_1)}\ . \tag{13}$$

From the same set, Eqs. (4a) and (4b), the amplitude of the second antisymmetrical mode $Z_2(\Omega_2)$ is determined analogously as

$$W_+(\Omega_2)=C_2(\Omega_2)Z_2(\Omega_2)\ , \tag{14a}$$

$$W_2(\Omega_2)=-[(N_{33}+N_{34})/\sqrt{2}N_{23}]C_2(\Omega_2)Z_2(\Omega_2) =-F_{12}(\Omega_2)C_2(\Omega_2)Z_2(\Omega_2)\ , \tag{14b}$$

with the normalizing constant

$$C_2(\Omega_2)=1/\sqrt{1+F_{12}^2(\Omega_2)}\ . \tag{14c}$$

For the symmetrical modes with frequencies Ω_3 and Ω_4, the similar procedure gives, respectively,

$$\Omega_3:\ W_1(\Omega_3)=C_3(\Omega_3)Z_3(\Omega_3)\ ;\quad C_3(\Omega_3)=1/\sqrt{1+F_{43}^2}\ ,\qquad W_-(\Omega_3)=-\frac{N_{11}}{\sqrt{2}N_{13}}W_1=-F_{43}(\Omega_3)C_3(\Omega_3)Z_3(\Omega_3)\ , \tag{15}$$

$$\Omega_4:\ W_-(\Omega_4)=C_4(\Omega_4)Z_4(\Omega_4);\quad C_4(\Omega_4)=1/\sqrt{1+F_{34}^2}\ ,\qquad W_1(\Omega_4)=-[(N_{33}-N_{34})/\sqrt{2}N_{13}]W_-(\Omega_4) =-F_{34}(\Omega_4)C_4(\Omega_4)Z_4(\Omega_4)\ . \tag{16}$$

The values Z_1, Z_2, Z_3, and Z_4 are the normal amplitudes.[4]

Next, to deduce the Hamiltonian of the interaction between an electron and the field of polarizational vibrations, we multiply the potential V_n^P by a factor $(-e)$ (where e is the charge of an electron) and then express the polarization vector components in terms of the mode amplitudes Z_1, Z_2, Z_3, Z_4 and represent the latter in the form of second quantization, assuming

$$Z_s(\Omega_s)=[(\hbar/2\Omega_s)]^{1/2}[\hat{b}_s^\dagger(-\boldsymbol{\eta},0)+\hat{b}_s(\boldsymbol{\eta},0)]\ . \tag{17}$$

In a particular case, for an electron placed in the second layer, the above-mentioned procedures are performed by use of Eqs. (1b), (3a), (3b), and (11a)–(16). In order to simplify the transition to the limit $(l\rightarrow\infty)$ described in Ref. 6, we obtain after some transformations the results in the following form:

$$\hat{H}_{2,e\text{-ph}}(\boldsymbol{\rho},z)=-\frac{e\sqrt{\hbar}}{2\sqrt{L_xL_y\varepsilon_0}}\sum_{\boldsymbol{\eta},s=1}^{4}\frac{e^{i\boldsymbol{\eta}\cdot\boldsymbol{\rho}}}{\sqrt{\eta}}(\beta_{1s}+\beta_{2s})^{-1/2}\frac{f_s(z)}{\sqrt{\Omega_s}}\times[b_s^\dagger(-\boldsymbol{\eta})+\hat{b}_s(\boldsymbol{\eta})] \tag{18}$$

with the notations

$$\beta_{1s}\equiv\beta_1(\Omega_s)=\frac{\omega_1^2(\varepsilon_{10}-\varepsilon_1)}{(\Omega_s^2-\omega_1^2)^2}\coth\zeta_1,\{s=1,2\}\ , \tag{19a}$$

$$\beta_{2s}\equiv\beta_2(\Omega_s)=\frac{\omega_2^2(\varepsilon_{20}-\varepsilon_2)}{(\Omega_s^2-\omega_2^2)^2}\begin{Bmatrix}\coth(\zeta_2/2);s=1,2\\ \tanh(\zeta_2/2);s=3,4\end{Bmatrix}, \tag{19b}$$

$$\begin{aligned}&f_1=\sinh w_2/\sinh(\zeta_2/2)\ ,\\ &f_2=-\mathrm{sgn}(\Omega_2^2-\omega_2^2)f_1\ ,\\ &f_3=-\cosh w_2/\cosh(\zeta_2/2)\ ,\\ &f_4=\mathrm{sgn}(\Omega_4^2-\omega_2^2)f_3\ ,\end{aligned} \tag{20a}$$

$$z_1\le z\le z_2\ . \tag{20b}$$

$z_{1,2}$ is the coordinate of the external surface of the first layer. The Hamiltonian for an electron in the outer layer differs from the Hamiltonian (18) only by the form of the coordinate function, which becomes one and the same for all s,

$$f_s(z)=\frac{\tanh(\zeta_1/2)\cosh w_1+\sinh w_1}{2\sinh(\zeta_1/2)}\ ;\quad z_0\le z\le z_1 \tag{21}$$

and has no definite parity.

The Fourier transform of the polarization vector in the second layer has the explicit form

$$\begin{aligned}P_2(\boldsymbol{\eta},z)=&-\omega_2\sqrt{\eta L_xL_y(\varepsilon_{20}-\varepsilon_2)\varepsilon_0}/(2\pi)^2\sqrt{\sinh\zeta_2}\\ &\times\Bigg\{(i\hat{\boldsymbol{\eta}}\sinh w_2+\mathbf{e}_3\cosh w_2)\Bigg[-Z_1(\Omega_1)\Bigg[1+\frac{\alpha_1^{(2)}(\Omega_1)}{\alpha_2^{(2)}(\Omega_1)}\Bigg]^{-1/2}+Z_2(\Omega_2)\Bigg[1+\frac{\alpha_1^{(2)}(\Omega_2)}{\alpha_2^{(2)}(\Omega_2)}\Bigg]^{-1/2}\Bigg]\\ &+(i\hat{\boldsymbol{\eta}}\cosh w_2+\mathbf{e}_3\sinh w_2)\Bigg[Z_3(\Omega_3)\Bigg[1+\frac{\alpha_1^{(1)}(\Omega_3)}{\alpha_2^{(1)}(\Omega_3)}\Bigg]^{-1/2}+Z_4(\Omega_4)\Bigg[1+\frac{\alpha_1^{(1)}(\Omega_4)}{\alpha_2^{(1)}(\Omega_4)}\Bigg]^{-1/2}\Bigg]\Bigg\},\quad\hat{\boldsymbol{\eta}}=\frac{\boldsymbol{\eta}}{\eta},\end{aligned} \tag{22a}$$

where

$$\alpha_1^{(2)}(\Omega)=\Omega^2-\omega_2^2(\varepsilon_{20}^{(2)}/\varepsilon_2^{(2)});\quad \alpha_2^{(2)}=\Omega^2-\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\tanh(\zeta_1/2)]/\varepsilon_2^{(2)}\ , \tag{22b}$$

$$\alpha_1^{(1)}(\Omega)=\Omega^2-\omega_2^2(\varepsilon_{20}^{(1)}/\varepsilon_2^{(1)});\quad \alpha_2^{(1)}=\Omega^2-\omega_1^2[\varepsilon_2+\varepsilon_{10}\coth\zeta_1\coth(\zeta_2/2)]/\varepsilon_2^{(1)}\ . \tag{22c}$$

It is seen from Eqs. (22) that two components $P_{2,z} \equiv P_2^{\parallel}$ and $P_{2,\eta} \equiv P_2^{\perp}$, being due to the normal modes $Z_1(\Omega_1)$ and $Z_2(\Omega_2)$, are antisymmetrical when reflecting in the XOY plane chosen in the middle of the second layer. At the same time, $P_{2,z}$ and $P_2^{\perp}$, which are due to the normal modes $Z_3(\Omega_3)$ and $Z_4(\Omega_4)$, are the symmetrical ones under the above-mentioned reflection.

Notice that the results of Ref. 7 follow immediately from Eq. (18) in the limit $l_1 \to \infty$. The further simplification $\varepsilon_{10} \to \varepsilon_1$ (nonpolar outer layers) and $\varepsilon_1 = 1$ (polar layer in a vacuum) gives the results of Refs. 2 and 3, respectively.

In conclusion, we consider the evolution of the surface vibrational spectra of three-layer symmetrical structure when varying l_1 at the fixed value of l_2. At $\eta \to 0$ from Eqs. (9c) and (10c) for antisymmetrical and symmetrical branches, respectively, in the limit $l_1 \to \infty$, it follows that $\Omega_1 = \max(\omega_{20}, \omega_1)$ and $\Omega_2 = \min(\omega_{20}, \omega_1)$, whereas $\Omega_3 = \max(\omega_{10}, \omega_2)$ and $\Omega_4 = \min(\omega_{10}, \omega_2)$. In the other limit case $l_1 \to 0$, we have $\Omega_1 = \max(\omega_{10}, \omega_2)$ and $\Omega_2 = \Omega_4 = \min(\omega_{10}, \omega_2)$. From the formulas (9c) and (10c) it can be seen that in the limit $\eta \to 0$ only the frequencies of antisymmetrical branches Ω_1 and Ω_2 depend on l_1.

As a numerical example, the structure InAs-GaSb-InAs is considered and the following parameters are used: $\omega_{10} = 30$ meV, $\omega_1 = 27.2$ meV, $\varepsilon_{10} = 14.5$ meV, $\varepsilon_1 = 11.6$ meV, $\omega_{20} = 29.8$ meV, $\omega_2 = 28.6$ meV, $\varepsilon_{20} = 16.1$ meV, and $\varepsilon_2 = 14.4$ meV. For these values, at $l_1 \to \infty$ and $\eta \to 0$, the limit values of Ω_s are arranged in the following order: $\Omega_3 > \Omega_1 > \Omega_4 > \Omega_2$ where it follows unambiguously that $\Omega_1(l_1) \to \Omega_3 = \omega_{10} = \text{const}$ and $\Omega_2(l_1) \to \Omega_4 = \omega_2 = \text{const}$. The results of the calculations of Eqs. (9c) and (10c) are shown in Fig. 1 demonstrating explicitly the evolution of the frequencies.

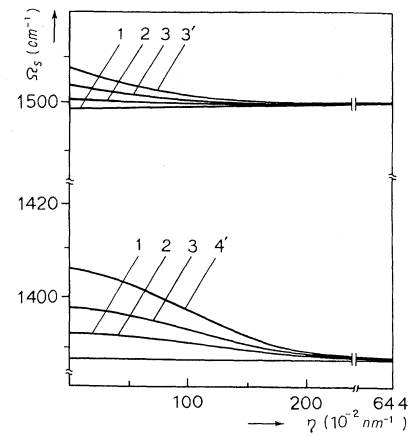

FIG. 1. The dispersion law of the surface modes as the function of the outer-layer thickness of the three-layer structure. $l_2 = 10$ nm and $l_1 = 4.5$ nm; 2.5 nm, and 0.5 nm, respectively; 1, 2, and 3 are the dispersion curves of the nonsymmetrical frequencies Ω_1, Ω_2, depending on l_1; 3′ and 4′ are for the symmetrical frequencies Ω_3 and Ω_4.

We have arrived at the following general conclusions: (1) The hybridization of the vibrations from the outer layers as well as the projective transition from the variables describing the polarization states in the separate layers to these, which are general for all the structure, creates the normal symmetrical and antisymmetrical modes in these layers. (2) At $\eta \to 0$ and for l_1 varying from ∞ to 0, the limit frequencies Ω_1 and Ω_2 of the antisymmetrical vibrations are shifted towards the frequencies of symmetrical vibrations Ω_3 and Ω_4, respectively, coinciding in pairs in the limit $l_1 \to 0$. The concrete scheme of the transformation is determined by the relative magnitudes of the parametric frequencies ω_{10}, ω_1, ω_{20}, and ω_2.

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