Solid-state shot noise

Rolf Landauer
IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598
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Shot noise and thermal-equilibrium noise are often, and incorrectly, viewed as additive and independent noise sources. The two forms of noise can even be identical, as recognized by several pioneering investigators long ago. Shot noise measures the graininess of the conduction process, and in samples with a fixed number of carriers, where the mean free path is short compared to the sample length, it is determined by the passage of a carrier through its mean free path. For a classical nondegenerate conductor we show that shot noise calculated that way *is* the thermal-equilibrium current. For mesoscopic conductors that transmit elastically with a probability small compared to unity, thermal-equilibrium noise is also shown to be approximately the sum of two shot-noise terms, one for the arriving stream at each reservoir. For a semiclassical metallic conductor, at zero-temperature, the noise proportional to current flow is shot noise related to the charge transferred between sample electrodes, when a carrier goes through a mean free path. At higher temperatures, a suggestive physical argument is used to show that noise increases above thermal-equilibrium noise according to the square of the ratio of the energy gained in one mean free path, to the thermal energy.

I. INTRODUCTION

The existence of shot noise, in thermionic diodes, was described by Schottky[1] in 1918. In diodes with a sufficiently large voltage applied to the anode, so that all the independently emitted electrons cross and are not turned back to the cathode by the Coulomb effect of other electrons, the mean-squared noise current in a frequency range of width $\Delta\nu$ is

$$\langle i^2\rangle_{\Delta\nu} = 2eI\Delta\nu. \tag{1.1}$$

e is the electronic charge and I the current. This is the noise that arises from the *graininess* of the current. In a given time interval there may be more electrons emitted, or fewer, than the number determined by the average current I. Equation (1.1) is valid at frequencies low compared to the electron transit time. In the case of the vacuum diode, with a sufficiently large applied voltage, the external circuit is not relevant because any voltage fluctuations which result from the shot noise do not, in turn, influence the current. In general, however, expressions of the form of Eq. (1.1), as found in the noise literature and in this paper, specify the noise current that will flow if the circuit element under consideration is short-circuited in the frequency range under consideration. Or, alternatively, such expressions give the size of the current flowing out of a noise current source, in parallel with the element.

Equation (1.1) can be written in a more revealing, but uncommon, form,[2]

$$\langle i^2\rangle_{\Delta\nu} = 2e^2(dn/dt)\Delta\nu. \tag{1.2}$$

Here, dn/dt represents the rate of electron passage. The current flow consists of a series of randomly spaced (Poisson distribution) δ functions, which contribute incoherently to the current component at a particular frequency. (dn/dt) specifies the rate of these contributions, and $e = \int i\,dt$ measures the size of each electron's contribution to a Fourier integral, at frequencies low compared to the transit time. Thus, in general, we can see that shot noise measures the size of the stochastically independent charge-transfer event. In the vacuum diode, or in tunneling,[3,4] that happens to be the transfer of a complete electron through the sample. We are now ready to anticipate one of our principal conclusions. In a macroscopic resistor the stochastically independent event is the motion of a carrier through a mean free path ℓ. If ℓ is small compared to the sample length L, then the charge transfer between terminating electrodes, due to the motion through a mean free path, becomes of order $(e\ell/L)$. This will be much smaller than the electronic charge e, and we cannot expect shot noise comparable to Eq. (1.1) in magnitude.[5,6] This point has also been emphasized, and treated analytically, in Ref. 7. Much of our ensuing discussion can be considered to be an elaboration of Ref. 7. Another related discussion can be found in Ref. 8, but is much further from the viewpoint expounded subsequently. We have, of course, focused here on samples with a fixed number of carriers, characteristic of metals as a result of charge neutrality. In the case of semiconductors, however, with partially ionized donors, the carrier population will fluctuate. This fluctuation in the resistance will give a noise signal proportional to the current flow,[9] and is independent of the thermal-equilibrium noise. The true additivity of the two noise sources, in this case, may be part of the background for a more widespread informal assumption that shot noise and thermal noise are independent and additive.

Thermal-equilibrium noise[10] is present in the absence of a current. The thermal-equilibrium spectral noise current density of a conductor is

$$\langle i^2\rangle_{\Delta\nu} = 4Gk_B\Theta\Delta\nu. \tag{1.3}$$

G is the conductance, Θ the temperature, and k_B is Boltzmann's constant. Equation (1.3) specifies the noise in a short-circuited resistor, or more generally the equivalent noise current generator, in parallel with the conductor. As stated above, there has been a tendency to think of shot noise and thermal-equilibrium noise as separate and additive. For example, one recent and otherwise perceptive paper tells us: "First there is Johnson noise ... present even at equilibrium. ... The second source, our main concern, is shot noise ... ". One of our purposes is to rebut this notion. Shot noise and thermal-equilibrium noise are special limits of a more general noise formula. Early insights into this came in Refs. 2 and 11, which pointed out that a linear resistor could be built out of two opposing thermionic emitters, with the potential of each cathode controlling the emission of the other. They showed that the noise predicted by Eq. (1.3) was just the sum of the two independent shot-noise contributions of the two opposing streams. There are more modern and similar discussions oriented to solid-state devices, e.g., Ref. 12.

In recent years noise in mesoscopic systems has drawn considerable attention. Three early papers[13,14] were followed by a stream which cannot possibly be listed here, and I cite only a few papers[15] in addition to those invoked elsewhere in this discussion. There are also papers related to resonant tunneling and double barrier tunneling, with and without attention to electron-electron interaction. We cite a sampling of this stream,[16] but it is essentially unrelated to our material. Reference 17 discusses the noise in a single channel conductor with transmission probability T. The result is

$$\begin{aligned}\langle i^2\rangle_{\Delta\nu} = {} & \frac{4e^2}{\pi\hbar}k_B\Theta T\Delta\nu + \frac{2e^2}{\pi\hbar}T(1-T)\Delta\nu eV \\ & +\frac{4e^2}{\pi\hbar}T(1-T)\Delta\nu k_B\Theta \\ & \times\left[\frac{eV/k_B\Theta}{\exp(eV/k_B\Theta)-1}-1\right] . \end{aligned} \tag{1.4}$$

This can be used to illustrate the point made above, concerning a single result with separate shot-noise limits and thermal-equilibrium-noise limits. At high temperature, the first right-hand-side term dominates, leading to the thermal-noise expression. At low temperature, the second term corresponds to a modified shot-noise formula. It is modified through appearance of the factor $(1-T)$, discussed subsequently. The last term in the above equation gives the first-order corrections to the shot-noise and thermal-noise limits:

$$k_B\Theta \gg eV;\ \langle i^2\rangle_{\Delta\nu} = 4k_B\Theta\Delta\nu G + \frac{e^2}{3\pi\hbar}\Delta\nu T(1-T)\frac{(eV)^2}{k_B\Theta}, \tag{1.5a}$$

$$k_B\Theta \ll eV;\ \langle i^2\rangle_{\Delta\nu} = 2eI_{\max}\Delta\nu T(1-T) + \frac{4e^2}{\pi\hbar}k_B\Theta\Delta\nu T^2. \tag{1.5b}$$

The reader needs to be cautioned: Eq. (1.5a) was given incorrectly in the original version of Ref. 17, and corrected in Ref. 18. $I_{\max}$ in Eq. (1.5b) is the maximum current carried in the channel within an energy range of eV, at the Fermi surface, counting all the electrons moving in one direction. This is the current emitted by the higher-energy reservoir, and is the actual current, if $T=1$. For $T \ll 1$, $I_{\max}T$ is the current and $2eI_{\max}T\Delta\nu$ is the shot noise given by Eq. (1.1). For larger values of T we deviate from Eq. (1.1). The Pauli exclusion principle spaces the electrons more uniformly than presumed in Eq. (1.1). For $T=1$, a fully occupied stream, we again reach a noiseless condition, just as for $T=0$. For T very close to 1 it is the current of uncorrelated holes, $I_{\max}(1-T)$, which can be considered to be the source of shot noise.

An alternative form of Eq. (1.5a) will be given here, for future reference, in the limit $T \ll 1$,

$$\langle i^2\rangle_{\Delta\nu} = 4k_B\Theta G\Delta\nu\left[1+\frac{1}{12}\left(\frac{eV}{k_B\Theta}\right)^2\right]. \tag{1.6}$$

Equation (1.6) shows that in the presence of transport, noise increases quadratically, at first. The relevant parameter is the electron energy gain, eV, compared to $k_B\Theta$. At large applied voltages, Eq. (1.5b) applies and noise increases linearly with voltage.

We do not, in this paper, address resonant-tunneling structures explicitly and separately. Furthermore we address only the most fundamental and inevitable noise sources, and ignore those due to time-dependent changes in the sample. Thus, $1/f$ noise and random telegraphic signal noise are outside of our view.

Equation (1.5a) can be given another interesting form to illustrate the transition away from thermal-equilibrium noise. That is,

$$\langle i^2\rangle_{\Delta\nu} = 4k_B\Theta G\Delta\nu + \frac{1}{6}\,\frac{eV}{k_B\Theta}\langle i_s^2\rangle_{\Delta\nu}, \tag{1.7}$$

where $\langle i_s^2\rangle_{\Delta\nu}$ designates the first right-hand-side ($\Theta=0$) term in Eq. (1.5b). All of this discussion emphasizes that the relationship of eV to $k_B\Theta$ determines whether we are in the thermal-noise limit or the modified shot-noise limit.

II. MESOSCOPIC CONDUCTORS

In this section we will supply an adapted version of the arguments of Refs. 2 and 11 to single channel mesoscopic samples whose conductance is determined by elastic transmission, with probability T. The conductance, if the potential difference is measured between wide reservoirs, will be $Te^2/\pi\hbar$. In contrast to Refs. 2 and 11, our analysis is only approximate. In equilibrium the two reservoirs will be at the same electrochemical potential. The fully occupied energy ranges well below the Fermi level, as well as the empty range way above, do not contribute to the noise. That comes from an energy range with width of order $k_B\Theta$, at the Fermi level. Within an energy range $k_B\Theta$ we have a current of order $ek_B\Theta/\pi\hbar$ emerging from each reservoir. A fraction T of that will be transmitted. Let us now specialize to the case $T \ll 1$. The shot noise associated with that, according to Eq. (1.1), will be

$$\langle i^2\rangle_{\Delta\nu} = 2e\left(\frac{ek_B\Theta}{\pi\hbar}T\right)\Delta\nu. \tag{2.1}$$

For two opposing streams, with independent fluctuations, we have

$$\langle i^2\rangle_{\Delta\nu} = (4e^2k_B\Theta T/\pi\hbar)\Delta\nu. \tag{2.2}$$

$Te^2/\pi\hbar$ is the conductance G. Therefore Eq. (2.2) can be written as

$$\langle i^2\rangle_{\Delta\nu} = 4\, k_B\Theta G\, \Delta\nu \tag{2.3}$$

in accordance with Eq. (1.3). Thus, thermal-equilibrium noise is simply the sum of the two separate shot-noise terms for the countervailing streams. The multichannel case will not be discussed in detail. But it becomes a trivial generalization of the above, using the technique described in Ref. 19, invoking a basis reducing the multichannel case to a set of parallel one-dimensional channels. If $T_i \ll 1$, for all of these channels, then both the conductance and noise are simply a sum of separate contributions of the form discussed above, one per channel.

III. CLASSICAL MAXWELL-BOLTZMANN CONDUCTORS

In this section we will show, as in the mesoscopic sample of Sec. II, that thermal-equilibrium noise is the sum of shot-noise contributions. In this case, however, we will consider a Maxwell-Boltzmann carrier distribution. We assume that the carrier scattering probability, per unit of time, is $1/\tau$. As a result of the scattering event, the carrier's velocity is randomized. The *nonfluctuating* carrier density is n. The sample has length L in the direction of current flow, taken to be the z direction, and has a cross-sectional area A. m is the carrier mass. The conductance is $G = (A/L)(ne^2\tau/m)$. A carrier moving with velocity v_z in the z direction, for a time t, will contribute ev_zt/L to the charge transfer between terminating electrodes. In case this is not a well known result to the reader, it can be rationalized on the basis that only if $v_zt = L$ will a complete change be transferred. Or, more authoritatively, it can be shown to be the change of the charge induced on the electrodes at the ends of the sample. References 5 and 20 provide a more detailed analysis. We will now invoke Eq. (1.2), repeated for the reader's convenience:

$$\langle i^2\rangle_{\Delta\nu} = 2q^2(dn/dt)\Delta\nu. \tag{3.1}$$

Here, we have replaced the original e of Eq. (1.2) with q, to emphasize that the charge transferred between electrodes in an elementary stochastic event is not, in general, the carrier's charge. If we take dn/dt to be the total number of terminated carrier flights per second, we do not need to add a contribution from right-moving carriers to one from left-moving carriers. This yields

$$(dn/dt) = A\,Ln/\tau. \tag{3.2}$$

Thus

$$\langle i^2\rangle_{\Delta\nu} = (2q^2ALn/\tau)\Delta\nu, \tag{3.3}$$

or more precisely

$$\langle i^2\rangle_{\Delta\nu} = (2\langle q^2\rangle_{Av}\,ALn/\tau)\Delta\nu. \tag{3.4}$$

Equation (3.4) allows for the fact that q is a function of the flight time t, which is distributed statistically.

We now come to the evaluation of

$$\langle q^2\rangle_{Av} = \langle e^2v_z^2t^2/L^2\rangle_{Av}. \tag{3.5}$$

Consider, first, a particular value of v_z^2. The probability of a flight time between t and $t+dt$ is given by

$$\rho(t)dt = \tau^{-1}e^{-t/\tau}dt \tag{3.6}$$

and this leads to

$$\langle t^2\rangle_{Av} = \int_0^\infty t^2\rho(t)dt = 2\tau^2. \tag{3.7}$$

v_z^2 in Eq. (3.5) is distributed according to the Maxwell-Boltzmann distribution with the weighting $\exp(-mv_z^2/2k_B\Theta)$. This yields

$$\langle v_z^2\rangle_{Av} = (k_B\Theta/m). \tag{3.8}$$

Thus, inserting the results of Eqs. (3.8) and (3.7) into Eq. (3.5) yields

$$\langle q^2\rangle_{Av} = 2\tau^2\,\frac{e^2}{L^2}\,\frac{k_B\Theta}{m}. \tag{3.9}$$

Inserting this value for $\langle q^2\rangle_{Av}$ into Eq. (3.4) gives

$$\langle i^2\rangle_{\Delta\nu} = 4k_B\Theta\,\frac{A}{L}\,\frac{e^2n\tau}{m}, \tag{3.10}$$

which in turn is simply

$$\langle i^2\rangle_{\Delta\nu} = 4k_B\Theta G\Delta\nu. \tag{3.11}$$

Thus the combined effect, arising from separate mean-free-path contributions, yields the expected thermal-equilibrium noise.

IV. METALLIC SHOT NOISE

This section will be devoted to an analysis of a large (not mesoscopic) metallic conductor. Microscopic models for thermal-equilibrium noise in metallic conduction were studied more than half a century ago.[21] Our primary analysis will, instead, be for $\Theta = 0$, where thermal-equilibrium noise is absent, with only some intuitive suggestions regarding the high-temperature case. A semiclassical approach, as in Sec. III, with a relaxation time τ defining the scattering probability, will be invoked.

At $\Theta = 0$, in equilibrium, we have an occupied Fermi sphere with a sharp cutoff at a wave vector of magnitude k_F. There is no noise associated with this distribution. In the presence of an applied field E, the carriers will be accelerated. For simplicity in signs we will consider positive carriers with charge e. The final results will, in any case, depend on the square of the carrier charge. Thus the acceleration gives

$$dk/dt = eE/\hbar. \tag{4.1}$$

Carriers will be carried past the Fermi-sphere surface in the positive k direction. They will be scattered out of their states according to

$$(\partial f/\partial t) = -(f - f_0)/\tau, \tag{4.2}$$

where f_0 is the equilibrium distribution. For a sharp Fermi sphere, at $\Theta = 0$, $f_0 = 0$ for carriers carried past the Fermi sphere's surface and we have

$$\partial f/\partial t = -f/\tau. \tag{4.3}$$

We will now concentrate on the excess carriers taken to the right, past the original equilibrium Fermi surface. After evaluating their contribution to the noise, we double it to account for the similar events taking place on the left half of the sphere. There, of course, we have unoccupied states being brought into the originally occupied sphere. Carriers in the occupied states which have gone past the Fermi surface will be scattered out of these states with a probability $1/\tau$, per unit time. It is the variability in the length of the occupation time, allowed by a scattering rate τ^{-1}, which produces noise. The average time for a state to remain occupied after passing the Fermi surface is τ. During this time τ the state produces a charge transfer between electrodes given by

$$\bar{q} = ev_z\tau/L, \tag{4.4}$$

where v_z is the z component of velocity at the Fermi surface. We will ignore the small fractional change in v_z as the carrier continues to be accelerated before scattering. Carriers which are scattered in a time less than τ will give a charge transport less than that of Eq. (4.4), and conversely for carriers which remain unscattered for a longer time. The mean-squared variation in the transferred charge determines the noise. We once again resort to Eq. (1.2),

$$\langle i^2\rangle_{\Delta\nu} = 2\langle(q-\bar{q})^2\rangle_{Av}(dn/dt)\Delta\nu. \tag{4.5}$$

Note that in Eqs. (1.1) and (1.2) it was the lack of correlation in pulse timing which eliminated the cross effect, in Fourier component evaluation, between different pulses. Here, instead, it is the lack of correlation in deviation from the average.

What is the value of (dn/dt) needed in Eq. (4.5)? The density of states in the conductor, in k space, is $2AL\,(2\pi)^{-3}$, where AL is the volume. The electric field produces a rate of motion, in k space, given by $eE/\hbar$. The Fermi sphere has a projected area, perpendicular to the direction of field and current which is πk_F^2, where k_F is the wave-vector magnitude of the Fermi sphere. Thus the rate at which carriers cross the right half of the Fermi surface is

$$2AL(2\pi)^{-3}(eE/\hbar)\pi k_F^2 = (2\pi)^{-2}ALk_F^2(eE/\hbar). \tag{4.6}$$

If we want to invoke the total rate of Fermi-surface crossing events, including both hemispheres, we must take twice that given in Eq. (4.6), and we will use that hereafter.

Now we come to the evaluation of

$$\langle(q-\bar{q})^2\rangle_{Av} = \frac{e^2v_z^2}{L^2}\;\overline{(t-\tau)^2}. \tag{4.7}$$

t is the elapsed time between Fermi-surface crossing and the scattering event. v_z is the z component of the surface Fermi velocity appropriate to a particular portion of the Fermi surface, and we will take its variation into account later. For the moment we will focus on the stochastic variation related to the final right-hand-side factor in Eq. (4.7).

The probability that the scattering event takes place between t and $t+dt$ is

$$\rho(t)dt = (\tau^{-1})e^{-t/\tau}dt \tag{4.8}$$

and $\bar{t} = \langle t\rangle_{Av} = \tau$, whereas $\langle t^2\rangle_{Av} = 2\tau^2$. Thus

$$\langle(t-\tau)^2\rangle_{Av} = \tau^2 \tag{4.9}$$

and

$$\langle(q-\bar{q})^2\rangle_{Av} = e^2\,v_z^2\,\tau^2/L^2. \tag{4.10}$$

The effective value of v_z^2 in Eq. (4.10) is reached by averaging v_z^2 over the Fermi surface, projected onto the same equatorial plane at $v_z = 0$, invoked in the calculation leading to Eq. (4.6). This yields an averaged value for v_z^2 of $\frac{1}{2}v_F^2$, where v_F is the magnitude of the Fermi-surface velocity. Substituting this value of v_z^2, as well as Eq. (4.10), and twice the value for dn/dt given in Eq. (4.6), into Eq. (4.5), yields

$$\langle i^2\rangle_{\Delta\nu} = \frac{1}{(2\pi)^2\hbar}\,\frac{A}{L}\,e^3\,v_F^2\,k_F^2\,\tau^2\Delta\nu. \tag{4.11}$$

The density of the states per unit volume is

$$n = k_F^3/3\pi^2. \tag{4.12}$$

Utilizing this in Eq. (4.11) and also setting $v_F = \hbar k_F/m$ gives

$$\langle i^2\rangle_{\Delta\nu} = \frac{3}{4}\,\frac{A}{L}\,\frac{e^3En}{m}\,\tau^2\,v_F\Delta\nu. \tag{4.13}$$

Using $\sigma = ne^2\tau/m$ turns this into

$$\langle i^2\rangle_{\Delta\nu} = \tfrac{3}{4}GeE\tau v_F\Delta\nu. \tag{4.14}$$

$E = V/L$, where V is the applied voltage. Also $\ell = v_F\tau$, where ℓ is the mean free path. Therefore Eq. (4.14) becomes

$$\langle i^2\rangle_{\Delta\nu} = \tfrac{3}{4}GV\frac{\ell}{L}e\Delta\nu. \tag{4.15}$$

The current $I = GV$ and this turns Eq. (4.15) into

$$\langle i^2\rangle_{\Delta\nu} = \tfrac{3}{8}\;(2eI\Delta\nu)\;\frac{\ell}{L}. \tag{4.16}$$

Thus, shot noise is reduced below $2eI\Delta\nu$ by $\frac{3}{8}\ell/L$. The exact numerical coefficient $\frac{3}{8}$ in Eq. (4.16) is not very significant. It is, on the one hand, sensitive to the details of our assumptions. Furthermore, Eq. (4.16) can be given a different appearance if we replace ℓ by the z-directed component of the mean free path, averaged in several possible ways. The reduction factor ℓ/L in Eq. (4.16) is in accordance with the suggestion made in Sec. I and the earlier results of Ref. 7. Our result in Eq. (4.16) is not analytic in I, with a discontinuous derivative at $I = 0$. As we shall see below, this behavior is characteristic of the strict $\Theta = 0$ limit, and disappears once we take $\Theta > 0$.

We now turn to the case where there is a nonvanishing temperature and inquire about the onset of the departure from equilibrium noise, as current flow is set up. Noise, as is evident from much of the literature cited in this paper, occurs when the carrier occupation probability is intermediate between 0 and 1, typically reaching a maximum[19] when $f = \frac{1}{2}$. Thus the number of states in this partially occupied range can be taken as a crude measure of relative noise.

In the presence of an applied field E, Eq. (4.2) is replaced, in the steady state, by

$$\frac{\partial f}{\partial t} = -\frac{\partial f}{\partial k_z}\frac{eE}{\hbar} - \frac{f - f_0}{\tau} = 0. \tag{4.17}$$

We can expand the solution f in power of E, regarded as a perturbation,

$$f = \sum_i fi, \tag{4.18}$$

with f_0 taken to be the thermal-equilibrium distribution. Inserting Eq. (4.18) into Eq. (4.17) yields

$$f_{i+1} = -\tau(\partial fi/\partial k_z)\left(\frac{eE}{\hbar}\right). \tag{4.19}$$

Thus, to second order in E, we have

$$f = f_0 - \left(\frac{eE\tau}{\hbar}\right) f_0' + \left(\frac{eE\tau}{\hbar}\right)^2 f_0''. \tag{4.20}$$

The f_0' term represents a simple displacement of the distribution and does not change the width of the transition region where f changes from 1 to 0. It is the final right-hand-side term in Eq. (4.20) which can change the width. If the final right-hand-side term of Eq. (4.20) had a coefficient $\frac{1}{2}$, then Eq. (4.20) would represent the initial terms in a Taylor expansion, for a shifted distribution. It is only the additional term $\frac{1}{2}(eE\tau/\hbar)^2 f_0''$, which needs to be examined as a source of broadening. Thus, we are inquiring about the changed width of

$$f = f_0 + \frac{1}{2}\left(\frac{eE\tau}{\hbar}\right)^2 f_0'', \tag{4.21}$$

due to its final right-hand-side term. The size of the region near $f = \frac{1}{2}$ is inversely proportional to the magnitude of the derivative, $\partial f/\partial k_z$, at the region in $\mathbf{k}$ space where $f = \frac{1}{2}$. Equation (4.21) yields

$$\partial f/\partial k_z = \partial f_0/\partial k_z + \frac{1}{2}\left(\frac{eE\tau}{\hbar}\right)^2 f_0'''. \tag{4.22}$$

We now use

$$\partial/\partial k_z = (\partial U/\partial k_z)(\partial/\partial U) = \hbar v_z(\partial/\partial U), \tag{4.23}$$

with U the energy. We also take into account, in the higher derivatives, that the derivatives of v_z are small compared to those of f. We then find

$$\frac{\partial f}{\partial k_z} = \frac{\partial f_0}{\partial U}\hbar v_z + \frac{1}{2}\left(\frac{eE\tau}{\hbar}\right)^2 \frac{\partial^3 F_0}{\partial U^3}\hbar^3 v_z^3. \tag{4.24}$$

Now at the Fermi level (FL), where $f_0 = \frac{1}{2}$, we have

$$\frac{\partial f_0}{\partial U} = -\frac{1}{4k_B\Theta}, \quad \frac{\partial^3 f_0}{\partial U^3} = \frac{1}{8(k_B\Theta)^3}. \tag{4.25}$$

Thus, Eq. (4.24) becomes

$$\left.\frac{\partial f}{\partial k_z}\right|_{\mathrm{FL}} = -\frac{1}{4k_B\Theta}\hbar v_z + \frac{1}{16}\frac{1}{(k_B\Theta)}{}^3\left(\frac{eE\tau}{\hbar}\right)^2 \hbar^3 v_z^3\,. \tag{4.26}$$

FL designates a derivative at the Fermi level. The magnitude of the inverse of this derivative is enlarged, due to its final right-hand-side term, by a factor η, with

$$\begin{aligned}\eta &= (\partial f/\partial k_z)_{\mathrm{FL},E=0} \;/(\partial f/\partial k_z)_{\mathrm{FL}} \\ &= 1 + \frac{1}{4}\left(\frac{eE\tau}{k_B\Theta}\right)^2 v_z^2.\end{aligned} \tag{4.27}$$

$eE\tau v_z$ is the energy gain of an electron, accelerated for a time τ, and we will designate this as δU. Thus Eq. (4.28) reaches the form

$$\eta = 1 + \frac{1}{4}\left(\frac{\delta U}{k\Theta}\right)^2. \tag{4.28}$$

In terms of noise

$$\langle i^2\rangle_{\Delta\nu} = 4Gk\Theta\Delta\nu\left[1 + \frac{1}{4}\left(\frac{\delta U}{k_B\Theta}\right)^2\right]. \tag{4.29}$$

Thus, we go above thermal-equilibrium noise quadratically with $(\delta U/k_B\Theta)$. The exact coefficient $\frac{1}{4}$ in Eq. (4.29) is once again a consequence of our simplistic approach, and should not be taken very seriously. Note the similarity of Eq. (4.29) to Eq. (1.6). There the final right-hand-side factor invoked eV, the energy gained in crossing the sample. Here it is δU, the energy gained in a mean free path. But both of these can be given one common definition: The energy gained between phase destructive events.

A more general note of caution can also be supplied. Clearly noise depends on the range of states in which f changes from 1 to 0. This is essentially a *hot-electron* problem. How much energy do electrons pick up? A highly idealized model, such as Eq. (4.17), or the one used in Ref. 7, where inelastic effects are provided by attachment of additional reservoirs to the conductor, cannot do real justice to the kinetics of electron energy loss via lattice vibrations. Thus, these methods lead, at best, to suggestive answers.

V. OVERVIEW AND SUMMARY

Our discussion was based entirely on an independent electron picture. Coulomb interactions were ignored. Coulomb interactions keep nearby electrons apart and regularly spaced and are, presumably, an additional source of noise reduction, above and beyond that provided by the Pauli principle. This was discussed in Ref. 17. The effect can be expected to be strongest in the case where nearby electrons stay together. That occurs in single-channel mesoscopic conductors and in vacuum tubes. Space-charge smoothing in vacuum tubes was analyzed many decades ago.[17] In Sec. III, devoted to the classical Maxwell-Boltzmann conductor, we obtained *exact* agreement with the usual thermal-equilibrium noise formula. We did this, from a shot-noise viewpoint, without any concern for Coulomb interaction. Why, then, was exact agreement obtained? There is an obvious guess: Consistent approximations were used; the expression for conductance invoked in the calculation of thermal-equilibrium noise is also the result of an independent electron picture.

The methodology of Sec. IV, treating the ordinary metallic conductor, poses another question. We assign the carrier to a particular free-electron state and, at a certain time, view it as exiting from that state. If we were measuring the momentum of each electron separately, that would be the obviously correct approach. But that is not what we are doing — we are measuring the total noise current. Our approach is, of course, closely akin to the second quantization methods and the Keldysh approach used in a number of the more formal mesoscopic noise papers, where little attempt is made to relate the choice of theory to the experimentally measured quantity. (For an exception, see Ref. 22.) Our approach is also akin to that used in the wave-packet approach of Refs. 13, 17, and 19 for mesoscopic conductors.

A wave packet incident on the sample is occupied or empty, and is then taken to be transmitted or reflected. But there, the occupation number for the wave packet, arriving at its destination, taken to be 0 or 1, corresponds more clearly to the electron-counting process which is what a noise measurement essentially does. In the case of the macroscopic conductor, noise measurements do not, equally obviously, seem to check on the arrival of the carrier at a certain position in the sample. But that may be an excessively cautious reservation. After all, a noise measurement responds to *all* changes in current. And a transition of a carrier, from one plane-wave state to another, is exactly that. Thus, I believe that the method of Sec. III is sound, but also admit that a more formal supporting argument would be welcome.

In the case of a classical Maxwell-Boltzmann conductor we showed that thermal-equilibrium noise is simply the shot noise due to electrons going through a mean free path. In the case of the mesoscopic sample, attached to reservoirs of Fermi carriers, unmodified shot noise due to uncorrelated carrier transmission occurs only at small transmission probabilities. In that case we showed, approximately, that thermal-equilibrium noise was just the sum of two shot-noise contributions, arriving at each reservoir. A more accurate theory than given in Sec. II would, essentially, have brought us back to the full treatment given in Ref. 13. Finally, we discussed the macroscopic metallic semiclassical conductor at zero temperature. There it was shown that the noise consisted of shot noise, proportional to current flow, but with the elementary stochastic charge-transfer event resulting from the motion of an electron through a mean free path, rather than through the whole sample. A suggestive physical argument was used to discuss this case at higher temperatures. That led to the conclusion that the parameter which determines the deviations from thermal-equilibrium noise was $\delta U/k_B\Theta$, the ratio of the energy gain in a mean free path to the thermal energy.

Note added in proof. The preceding discussion stressed the role of $eV/k_\beta\Theta$ as a boundary marker between thermal equilibrium noise and excess noise proportional to current. This was already recognized in Ref. 23.

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