Low-voltage breakdown of the quantum Hall effect in narrow channels

O. G. Balev
Institute of Semiconductors, Academy of Sciences, 45 Pr. Nauky, Kiev 252650, Ukraine

P. Vasilopoulos
Department of Physics, Concordia University, 1455 de Maisonneuve Boulevard O, Montréal, Québec, Canada H3G 1M8

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Low-voltage breakdown is considered in a quasi-two-dimensional electron gas confined laterally in a narrow channel of width W and subject to a strong perpendicular magnetic field. It is shown that electron-phonon interaction leads to a substantial dissipation due to electron transitions at the edges of the channel and constitutes the main dissipation in the channel if W is not too large. Negative differential conduction is possible when a threshold drift velocity v_D is reached. This leads to an instability of the almost dissipationless regime, i.e., to the breakdown of the quantum Hall effect. Under certain conditions the instability is possible for $v_D < s$ or $v_D \ll s$ (low-voltage breakdown), where s is the speed of sound. The finite thickness of the channel leads, in general, to breakdown velocities smaller than those pertaining to zero thickness. Good agreement is obtained between the theory and the experimental results of Makerov *et al.* (Pis'ma Zh. Eksp. Teor. Phys. **47**, 59 (1988) [JETP Lett. **47**, 71 (1988)]).

I. INTRODUCTION

To date, a generally accepted theory of the breakdown of the quantum Hall effect (QHE) is absent.[1-4] After its experimental discovery[5] in wide channels ($W \approx 1000$ μm) the breakdown has been studied on samples of sufficiently smaller width ($W \approx 1 - 100$ μm).[3,6] Various breakdown characteristics were associated in Ref. 6 with the possibility of persistent current flow along the edges of the channel. This is in line with the theoretical results of Refs. 7 and 8, for wide channels, according to which the current flows along the channel boundaries and the inner regions of the two-dimensional electron gas (2DEG) do not contribute to the current.

Most of the theoretical studies consider the current along the channel as a surface flow and point to the electron-phonon interaction as the origin of the breakdown. In Ref. 9 it was found that when the Fermi level is in the middle between adjacent Landau levels breakdown was possible when the drift velocity $v_x \equiv v_D = E_y/B \geq 10s$, where s is the speed of sound, B the magnetic field, and $E_y = E_H$ the Hall field. This is approximately 10–20 times larger than the observed[5,10,11] value for v_D. In Ref. 2 it was shown that the breakdown of the QHE is possible for $v_D \geq s$ due to the nonheating negative differential conductivity that leads to an instability of the almost dissipationless regime, i.e., $j_y \propto E_y^{-1}$ if $E_y \geq sB$, for a 2DEG interacting with piezoelectric phonons under the assumption $\hbar\omega_c \gg k_BT \gg m^*s^2$, where m^* is the effective mass, $\omega_c = |e|B/m^*$ the cyclotron frequency, and e (< 0) the electron charge. In this case the location of the Fermi level influences the value of the dissipative conductivity σ_{yy} but not its dependence on E_y. Notice that in Refs. 2 and 9 the channels treated were *wide* in the sense, adopted hereafter, that the dissipation, due to electron-phonon interaction, occurred mainly in the channel and the confining potential was neglected; consequently, the role of the channel edges in the dissipation was neglected. Dissipation due to phonon emission is possible for $v_D > s$ when a sufficiently large-scale static potential is taken into account.[12-14] In the last two works the total dissipation, for $T = 0$, is connected with electron transitions at the edges of the channel, i.e., the channel, in our terminology, was *narrow*. In the homogeneous case the results of Ref. 13 were similar to those of Ref. 9.

In what follows we will study the breakdown of the QHE in a *narrow* channel of finite thickness and infinite length in the presence of a strong perpendicular magnetic field B such that $\hbar\omega_c \gg k_BT$. For simplicity we neglect a random static potential as well as the interaction between electrons. The only scattering we consider is electron-phonon interaction in relatively weak applied electric fields (along the channel) when the condition $|E_x/E_H| \ll 1$ is satisfied due to the strong magnetic field. The heating of the 2DEG is neglected. For definiteness we designate a low-voltage regime as that for which $E_H < Bs$, i.e., one for which $v_D < s$. The main result is that electron-phonon interaction leads to dissipation mainly due to electron transitions between edge states and to a possibility of breakdown velocities v_D smaller or much smaller than s. For other conditions the breakdown may occur for $v_D > s$ or $v_D \gg s$; see Sec. II. We further assume that E_H is not large enough to cause interlevel transitions; see Sec. II.

The possibility of low-voltage breakdown is related to finite but not-too-high temperatures. Physically, in a *narrow* channel electron states (and transitions between them) are more pertinent at the edges of the channel than at its interior. Indeed, at the edges of the channel the Landau levels are tilted upwards by the confining poten-

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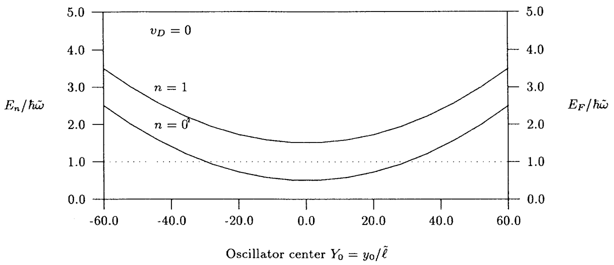

FIG. 1. Schematic energy diagram of the first two Landau levels (solid curves) and the Fermi level (dotted curve) as function of the oscillator center $Y_0 = y_0/\tilde{l}$ for $v_D = 0$.

tial and the Fermi level crosses them. This is illustrated schematically in Figs. 1 and 2, which show the variation across the channel of the Fermi level and the lowest two Landau levels for $v_D = 0$ and $v_D > 0$, respectively. We used a parabolic confining potential of frequency Ω and the eigenvalues given by Eq. (2), see below, with $\omega_c/\Omega = 30$ and $|e|E_H\tilde{l}/\hbar\tilde{\omega} = 0.01$. The dimensionless factor $Y_0 = y_0/\tilde{l}$ gives the position of the oscillator center y_0 in terms of the renormalized, due to the confining potential, magnetic length $\tilde{l}$. As can be seen from Figs. 1 and 2 (and will be detailed later) states within $\tilde{l}$ from the edges are close to the Fermi level and at low temperatures they contribute to dissipation via the electron-phonon interaction much more than the "bulk" states at the interior of the sample since the latter are lying far below the Fermi level. In fact, it has been observed[2] that the contribution of these inner states, lying Δ_{in} below the Fermi level, decreases exponentially with temperature if the condition $\exp(-\Delta_{\text{in}}/k_BT) \ll 1$ is satisfied.

The paper is organized as follows. A criterion for the breakdown is established in Sec. II and the main results are given in Secs. II C, III, and IV. An explanation of the experimental results of Ref. 4 is presented in Secs. II C and IV. The current-voltage characteristics (CVC) for not-too-low and low temperatures are given, respectively, in Secs. III and IV. A low-voltage breakdown related to fluctuations of the confining potential along the z axis is discussed in Sec. V. Conclusions follow in Sec. VI.

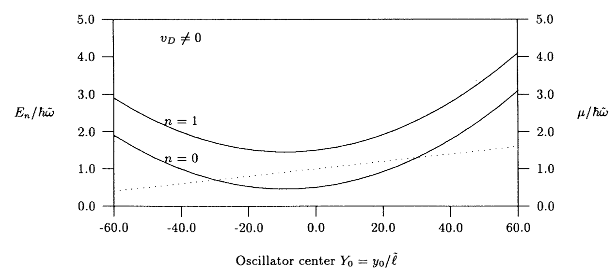

FIG. 2. Solid curves, the same as in Fig. 1, but for $v_D \neq 0$; the dotted curve represents the electrochemical potential $\mu \equiv E_F + E_\alpha - E_\alpha(E_H = 0)$. E_α is given by Eq. (2).

II. CRITERION FOR BREAKDOWN IN NARROW CHANNELS

A. Channel characteristics

We consider a 2DEG confined in a narrow channel in the (x, y) plane of width $L_y = W$, length $L_x = L$, and finite thickness $L_z = d$. For simplicity we take the confining potential along y as parabolic: $V_y = m^*\Omega^2 y^2/2$, where Ω is the confining frequency. However, most of the results hold for the more realistic potential $V_y' = 0$ for $y_l < y < y_r$, $V_y' = m^*\Omega^2(y - y_r)^2/2$ for $y > y_r > 0$, and $V_y' = m^*\Omega^2(y - y_l)^2/2$ for $y < y_l < 0$; this will be indicated below. For the confinement in the z direction we consider a parabolic well of frequency ω_z or the standard triangular well. When an electric field E_x is applied along the channel and a strong magnetic field B along the z axis in the Landau gauge for the vector potential $\mathbf{A} = (-By, 0, 0)$, the one-electron Hamiltonian h^0 is given by

$$h^0 = [(p_x + eBy)^2 + p_y^2]/2m^* + V_y - eE_H y - eE_x x + h_z, \tag{1}$$

where $\mathbf{p}$ is the momentum operator and $h_z = p_z^2/2m^* + V_z$ the z part of h^0 with V_z the confining potential. The third term on the right-hand side represents the Hamiltonian due to the Hall field $E_H \equiv E_y$ as explained in Ref. 15. We assume, in line with most experimental situations, that the 2DEG occupies only the lowest subband and denotes the corresponding eigenvalue and eigenfunction of h_z by E_{z0} and $X_0(z)$, respectively. For strong magnetic fields such that $|E_x/E_H| \ll 1$ we consider the term $-eE_x x$ as perturbation and notice that $\nabla \times \mathbf{E} = 0$ entails that, since $\mathbf{E}$ does not depend on x, E_x is independent of y and z. Without this term the eigenvalues and eigenfunctions corresponding to Eq. (1) are given by

$$\begin{aligned} E_\alpha \equiv E_{n,k_x} = {} & \hbar\tilde{\omega}(n + \tfrac{1}{2}) + \frac{\hbar^2 k_x^2}{2\tilde{m}} \\ & - \frac{eE_H}{\tilde{\omega}^2 m^*}\left(\hbar k_x \omega_c + \frac{eE_H}{2}\right) + E_{z0} \end{aligned} \tag{2}$$

and

$$|\alpha\rangle \equiv e^{ik_x x}\Psi_n(y - y_0)X_0(z)/\sqrt{L}, \tag{3}$$

respectively. Here $\tilde{\omega} = (\omega_c^2 + \Omega^2)^{1/2}, \tilde{m} = m^*\tilde{\omega}^2/\Omega^2, y_0 = (eE_H + \hbar\omega_c k_x)/m^*\tilde{\omega}^2$, and $\Psi_n(y)$ is a harmonic oscillator function. As can be seen from Eq. (2) for $E_H = 0$ the main difference from the corresponding result for $V_y = 0$ is that the k_x degeneracy of the energy levels is lifted by the confining potential ($\Omega > 0$) and the electrons appear heavier since $\tilde{m} > m^*$. For the calculations that will follow we need the following matrix elements:

$$\begin{aligned} &|M_{\alpha\alpha'}(\mathbf{q})|^2 \\ &\quad = |\langle\alpha|e^{i\mathbf{q}\cdot\mathbf{r}}|\alpha'\rangle|^2 \\ &\quad = (n!/n'!)u^{n'-n}e^{-u}[L_n^{n'-n}(u)]^2 F(q_z)\delta_{q_x,k_x-k_x'}, \end{aligned} \tag{4}$$

where $u = [(\omega_c^2/\tilde{\omega}^2)q_x^2 + q_y^2]\tilde{l}^2/2, \tilde{l} = (\hbar/m^*\tilde{\omega})^{1/2}$ is the renormalized magnetic length, and $L_n^{n'}(u)$ is a Laguerre polynomial. For $d = 0$ we have $F(q_z) = 1$. For d finite and $V_z = m^*\omega_z^2 z^2/2$ we have, with $l_z^2 = \hbar/m^*\omega_z \ll \tilde{l}^2$, $F(q_z) = \exp(-q_z^2 l_z^2/2)$ if $\omega_z \gg \tilde{\omega}$. For typical values $\bar{q}_z^2 \ll l_z^{-2}$ the result for $F(q_z)$ is almost equal to that obtained from the variational wave function $X_0(z) = z(b_0^3/2)^{1/2}\exp(-b_0 z/2)$ if $l_z^{-2} = b_0^2/6$, i.e., $F(q_z) = [1 + q_z^2/b_0^2]^{-3}$; in this case the average thickness is $3/b_0$.

B. Current density and criterion for the breakdown

We assume that the main scattering mechanism is the electron-phonon interaction. Using Eq. (1) the relevant Hamiltonian is given by

$$\begin{aligned} H = &\sum_\alpha E_\alpha a_\alpha^\dagger a_\alpha + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \\ &-eE_x \sum_{\alpha\alpha'} \langle\alpha|x|\alpha'\rangle a_\alpha^\dagger a_{\alpha'} \\ &+ \sum_{\mathbf{q},\alpha,\alpha'} \theta(t-\delta t)[M_{\alpha\alpha'}(\mathbf{q})C_{\mathbf{q}} b_{\mathbf{q}} \\ &+M_{\alpha\alpha'}(-\mathbf{q})C_{\mathbf{q}}^* b_{\mathbf{q}}^\dagger] a_\alpha^\dagger a_{\alpha'}. \end{aligned} \tag{5}$$

Here $a^\dagger, a$ and $b_{\mathbf{q}}^\dagger, b_{\mathbf{q}}$ are the creation and annihilation operators for electrons and phonons (of wave vector $\mathbf{q}$), respectively, $C_{\mathbf{q}}$ measures the strength of the electron-phonon interaction, and $\omega_{\mathbf{q}}$ is the phonon frequency. As indicated by the θ function the electron-phonon interaction (fourth term on the right) is absent for $t < \delta t$ (if it is switched on adiabatically[15] at $t = -\infty$ we obtain equivalent results for stationary responses).

Extending the procedure of Ref. 2 for wide channels we evaluate the average current density by substituting the solutions of the equations of motion for the operators $a(t), a^\dagger(t)$, which involves the Hamiltonian H, in the standard expression $j_\mu(t) = (e/LW)\sum_{\alpha\alpha'}\langle\langle a_\alpha^\dagger(t)a_{\alpha'}(t)\rangle\rangle\langle\alpha|v_\mu|\alpha'\rangle$, $\mu = x, y$, and considering the term proportional to $-eE_x$ as perturbation. Because the magnetic field is strong, only the main terms in the expansion of the electron-phonon interaction constant are taken into account.[2] We then obtain for large t the following expressions for the components of the current density averaged over a statistical ensemble and the dimensions of the channel:

$$j_y = \sigma_{yy}(E_H)E_H + \sigma_{yx}^0 E_x = 0, \tag{6}$$

$$j_x = \sigma_{xx}(E_H)E_x + \sigma_{xy}^0 E_H \approx \sigma_{xy}^0 E_H, \tag{7}$$

since $\sigma_{xx}(E_H) \ll \sigma_{xy}^0$ and $E_x/E_H \ll 1$ for strong magnetic fields. Here $\sigma_{yx}^0 = -\sigma_{xy}^0 \propto e^2/2\pi\hbar$ was obtained in the absence of electron-phonon interaction, indicated by the superscript 0. The first term on the right-hand side of Eq. (6), labeled j_d to remind that it expresses the dissipation, is given by

$$\begin{aligned} j_d &= \sigma_{yy}(E_H)E_H \\ &= \frac{2\pi|e|\omega_c}{LWm^*\tilde{\omega}^2} \sum_{\mathbf{q},\alpha,\alpha'} q_x|C_{\mathbf{q}}|^2|M_{\alpha\alpha'}(\mathbf{q})|^2 \\ &\qquad \times[f_{\alpha 0}(1-f_{\alpha' 0}) + n_{\mathbf{q}}(f_{\alpha 0} - f_{\alpha' 0})] \\ &\qquad \times\delta(E_\alpha - E_{\alpha'} - \hbar\omega_{\mathbf{q}}); \end{aligned} \tag{8}$$

here $f_{\alpha 0} = 1/\{1+\exp[(E_{\alpha 0} - E_F)/k_B T]\}$ is the Fermi-Dirac function, $E_{\alpha 0} = E_\alpha(E_H = 0)$, E_F is the Fermi level, and $n_{\mathbf{q}}$ is the equilibrium distribution function for phonons. We emphasize that Eqs. (6)–(8) were obtained using perturbation theory and that the energy spectrum, Eq. (2), is not degenerate with respect to k_x. It can be shown that prior to switching on the electron-phonon interaction, at $t = 0$, we have, from Eqs. (6) and (7), $\sigma_{yy} = 0$, $E_x = 0$; then Eq. (7) is exact. For further details concerning a stationary response after the interaction is turned on see Ref. 2. From Eqs. (6) and (8) we can express E_H as function of E_x, $E_H = E_H(E_x)$. Then from Eq. (7) the CVC, $j_x = j_x(E_x)$, and the condition for negative differential conduction (NDC), $\partial j_x/\partial E_x < 0$, are written as

$$j_x = \sigma_{xy}^0 E_H(E_x) \tag{9}$$

and

$$\left.\frac{\partial j_d}{\partial E_H}\right|_{E_H(E_x)} < 0, \tag{10}$$

respectively. Criterion (10), which follows from $\partial j_x/\partial E_x < 0$ and Eqs. (6)–(8), is a condition for the breakdown of the QHE. The more general condition $\partial j_d/\partial E_H \neq 0$, which describes breakdown as well, will be discussed in Sec. III. To proceed further we must specify the kind of phonons. For the very low temperatures pertinent to the QHE we consider only the standard acoustical (DA) and piezoelectrical (PA) phonons for which $\omega_{\mathbf{q}} = sq$, and $C_{\mathbf{q}}^2 = (c'/L_x L_y L_z)q^{\pm 1}$ where c' is a constant. Moreover, we will assume $\Omega \ll |\omega_c|$, i.e., that the confining potential affects the eigenfunctions $|\alpha\rangle$ very little, but it substantially changes the eigenvalues E_α. This condition is usually fulfilled if the magnetic field is not too weak.[16] We further assume that E_H is not strong enough to cause interlevel transitions. As shown in Ref. 9 for interlevel transitions $\sigma_{yy} \propto \exp[-\hbar\omega_c/2m^*(v_D - s)^2]$. Then assuming $v_D \gg s$ we obtain $\exp(-\hbar\omega_c/2m^*v_D^2) < \delta y/W \ll 1$ as the condition for neglecting the interlevel contribution to j_d from all occupied states in comparison with the intralevel contribution of the edge regions. For $\nu_D < s$ the condition for neglecting the interlevel contribution to j_d is $\exp(-\hbar\omega_c/k_B T) \ll \delta y/W \ll 1$. The characteristic extent δy of the edge states is estimated below. From this last condition it follows that $|eE_H|\tilde{l} < \hbar\omega_c/\sqrt{2}$, i.e., $|E_H|$ is not too large. Notice that even for a purely parabolic potential V_y we usually have $\delta y/W \ll 1$ due to $\Omega \ll \omega_c$ which is, e.g., the case when the Fermi level is in the middle between the lowest two Landau levels.

C. Dissipation: Main results

From Eq. (8) it can be shown that the intra-Landau level transitions give the main contribution to the current j_d. For definiteness and in order to make contact with the experimental results of Ref. 4 we assume that only the lowest Landau level ($n = 0$) is occupied. Then $L_0(u) = 1$ and, as outlined in the Appendix, Eq. (8) can be written as

$$j_d = j^{\rm I} + j^{\rm II} + j^{\rm III}, \tag{11}$$

where, for $E_H > 0, B > 0$,

$$j^{\rm II} \equiv j^{\rm II}(k_e, k_E) = -\frac{|e|\omega_c c'}{4\pi^2 \hbar s m^* W \tilde{\omega}^2} \int_0^\infty dq_x q_x^2 (q_x v)^{m+1} e^{-(q_x^2 \tilde{l}^2/2)[1+\lambda_+(v^2-1)/2]} I_0[q_x^2 \tilde{l}^2 \lambda_-(v^2-1)/4]\theta(v-1) \times [(1-e^{\hbar q_x s v_0/k_B T})^{-1} - (1-e^{\hbar q_x s v/k_B T})^{-1}], \tag{12}$$

and

$$j^{\rm I} = -j^{\rm II}(-k_e, k_E), \quad j^{\rm III} = -j^{\rm II}(k_e, -k_E). \tag{13}$$

Here $m = -1$ (1) for PA (DA) phonons, $I_0(x)$ is the modified Bessel function, $k_e = (\tilde{\omega}/\hbar\Omega)[2m^*(E_{F0} - \hbar\tilde{\omega}/2)]^{1/2}$, $E_{F0} = E_F - E_{z0}$, $k_E = |e|\omega_c E_H/\hbar\Omega^2$, $\lambda_\pm = 1 \pm l_z^2/\tilde{l}^2$, $\theta(x)$ is the theta function, and $v \equiv v(k_e, k_E) = \hbar(k_e + k_E)/\tilde{m}s$, with $v_0 = v(k_e, 0)$, is a dimensionless number which, for $\Omega \to 0$, reduces to the ratio of the drift velocity to that of sound, $v \to E_H/sB$. From Eqs. (12) and (13) it follows that $j^{\rm I} j^{\rm III} = 0$ because the θ functions cannot be finite simultaneously ($j^{\rm I} \propto \theta[v(-k_e, k_E) - 1]$ and $j^{\rm III} \propto \theta[v(k_e, -k_E) - 1]$). Notice that, as the θ function shows, $j^{\rm II}$ is connected with electron-phonon interaction *at the right edge of the channel* $y_{\rm RE}$ since $y_{\rm RE} = (eE_H + \hbar\omega_c k_e)/m^*\tilde{\omega}^2 \approx \hbar\omega_c k_e/m^*\tilde{\omega}^2 > 0$ (this corresponds to $k_x = k_e$; see Appendix), whereas $j^{\rm I}$ and $j^{\rm III}$ express dissipation *at the left edge of the channel* $y_{\rm LE}$, $y_{\rm LE} = (eE_H - \hbar\omega_c k_e)/m^*\tilde{\omega}^2 \approx -\hbar\omega_c k_e/m^*\tilde{\omega}^2 \approx -y_{\rm RE}$ (this corresponds to $k_x = -k_e$). We use $|e|E_H \ll \hbar\omega_c k_e$, i.e., $(m^* v_x^2/\hbar\tilde{\omega})^{1/2} \ll \tilde{\omega}/\Omega$ follows from the above-stated conditions. For our case in Eqs. (6) and (7) we have $\sigma_{yx}^0 = e^2/2\pi\hbar$ and $W = y_{\rm RE} - y_{\rm LE} = 2\hbar\omega_c k_e/m^*\tilde{\omega}^2$.

Because the main contributions to j_d involve transitions between electron states near the edges of the channel a more general and realistic confining potential can be considered. For instance, $\bar{V}_y = m^*\Omega_+^2(y - y_+)^2/2$ for $y \geq y_+ > 0$, $\bar{V}_y = m^*\Omega_-^2(y - y_-)^2/2$ for $y \leq +y_- < 0$, and $\bar{V}_y = 0$ for $y_- \leq y \leq y_+$, where y_+ and y_- are almost equal to the coordinates of the right and left channel edges, $y_{\rm RE}$ and $y_{\rm LE}$, respectively, if $\max\{y_{\rm RE} - y_+, y_- - y_{\rm LE}\} \ll y_+ - y_- \approx y_{\rm RE} - y_{\rm LE} = W$ and $\Omega_+ \neq \Omega_-$. The connection with the previous potential V_y' is that $y_+ - y_- = y_r - y_l \approx W = y_{\rm RE} - y_{\rm LE}$. Then in Eq. (12) and in the expressions for $\tilde{\omega}, \tilde{l}, k_E$, and k_e, one must substitute Ω_+ for Ω and in Eq. (13) Ω_- for Ω. The characteristic extent of the edge states δy, contributing to j_d in the low-voltage regime, is given by virtue of the conditions $k_e \gg \tilde{l}^{-1}, \hbar\tilde{\omega} \gg k_B T$, stated above, as $\delta y \approx (\hbar\omega_c/m^*\tilde{\omega}^2) \times \max\{\min\{1/\tilde{l}, k_B T/\hbar s\}, k_B T\tilde{m}/\hbar^2 k_e\} \ll y_{\rm RE} - y_+(y_- - y_{\rm LE})$. For clarity we repeat that by *narrow* channel we understand a channel in which the current j_d flows mainly due to electron-phonon processes along its edges and by *wide* channel one in which the current j_d flows mainly due to electron-phonon interaction in the inner regions. We notice in this respect, in line with Ref. 6, that the same channel can be characterized as *wide* for some magnetic field values and as *narrow* for others.

For the more general potential $\bar{V}_y$ we have, using Eq. (11) for constant B, $j_d/E_H = \sigma_{yy}(E_H) \neq \sigma_{yy}(-E_H)$. Then the form of the CVC $j_x = j_x(E_x)$ changes when E_H changes sign since from Eqs. (6) and (7) it follows that $j_x(E_H) = -j_x(-E_H)$ whereas $|E_x(E_H)| \neq |E_x(-E_H)|$. If E_H remains constant and B changes sign it can be shown, using Eq. (11), that $\sigma_{yy}(E_H, B) = \sigma_{yy}(E_H, -B)$. We then have $j_x(E_H, B) = -j_x(E_H, -B)$ and $E_x(E_H, B) = -E_x(E_H, -B)$, i.e., the form of the CVC, for E_H constant, does not change if the signs of B and j_x change simultaneously. These characteristics were observed in Ref. 4 and allow us to assume that the corresponding channel was narrow in the sense defined above and that the confining potential had different form at the two edges. Indeed, the reported results are essentially connected with $\Omega_- \neq \Omega_+$. Support for this conclusion comes from the fact that although the channel was geometrically rather wide, $W \approx 200$ μm, B was in the middle of the Hall plateau for ρ_{xy} or in the middle of the ρ_{xx} minimum and the corresponding values of B were not much different. In this case the contribution to j_d of the inner regions of the channel is exponentially small. Evidently, the purely parabolic model V_y is not realistic for such widths. We point out that from Eqs. (6) and (7) it follows that $j_d E_H = j_x E_x$, i.e., after determining j_d it is not difficult to obtain dissipation in the channel. Because of the relationship $E_x = j_d(E_H)/\sigma_{xy}^0 = \sigma_{yy}(E_H)E_H/\sigma_{xy}^0$, the construction of the CVC $E_x = E_x(j_x)$ is equivalent, in relative units, to the construction of the dependence $j_d = j_d(E_H)$.

III. CVC AND BREAKDOWN FOR NOT-TOO-LOW TEMPERATURES

For not-too-low temperatures, such that $\hbar s/\tilde{l} \ll k_B T$ and $[\exp(\hbar q_x s v/k_B T) - 1]^{-1} \approx k_B T/\hbar q_x s v$, we obtain, from Eqs. (11)–(13), for the PA interaction

$$j_d = j_{\rm PA}^{\rm I} + j_{\rm PA}^{\rm II} + j_{\rm PA}^{\rm III}, \tag{14}$$

where

$$j_{\rm PA}^{\rm II} \equiv j_{\rm PA}^{\rm II}(k_e, k_E) = \left(\frac{|e|\omega_c k_B T c'}{4\pi^2\hbar^3 s^2 \tilde{\omega} W}\right)\left(\frac{v - v_0}{v_0 v^2}\right) \times \left[1 + \frac{l_z^2}{\tilde{l}^2}(v^2 - 1)\right]^{-1/2} \theta(v - 1) \tag{15}$$

and

$$j_{\rm PA}^{\rm I} = -j_{\rm PA}^{\rm II}(-k_e, k_E), j_{\rm PA}^{\rm III} = -j_{\rm PA}^{\rm II}(k_e, -k_E). \tag{16}$$

If $v(k_e, k_E) > 1, |v(-k_e, k_E)| < 1$, then $j_{\rm PA}^{\rm I} = j_{\rm PA}^{\rm III} = 0$, and from the last three equations we obtain

$$j_d = \frac{m^* \tilde{m} \tilde{\omega} k_B T c'}{4\pi^2 \hbar^4 k_e W} \frac{E_H}{(E_H + E_e)^2} \times \left[1 + \frac{l_z^2}{\tilde{l}^2}\left(\frac{(E_e + E_H)^2}{E_s^2} - 1\right)\right]^{-1/2}. \tag{17}$$

Here $E_e = \hbar\Omega^2 k_e / |e|\omega_c$ is the characteristic electric field defining the influence of the channel boundaries, $E_s = \tilde{\omega}^2 s B/\omega_c^2$, and $v_0 = v(k_e, 0) = E_e/E_s$. For a channel of zero thickness, it follows from Eq. (17) that Eq. (10) is fulfilled for $E_H > E_e$, i.e., the smallest E_H for which NDC is possible is $\bar{E} = E_s/2$ if $E_e = E_s/2$. The corresponding threshold speed is ($n_e = m^*\tilde{\omega}^2/2\pi\hbar\omega_c$ is the electron density)

$$\bar{v}_x = \frac{|e|\bar{E}}{2\pi\hbar n_e} = \frac{s}{2}. \tag{18}$$

That is, electron-phonon interaction at the boundaries of a narrow channel can lead to a threshold speed for the breakdown *smaller* than the speed of sound in sharp contrast with wide samples,[2,9] see also Refs. 12–14 for $T = 0$, where this is possible only for $v_x \geq s$. For a channel of finite thickness l_z we have $\bar{E} = E_e/\lambda_+$ assuming that $v = (E_e + \bar{E})/E_s \to 1$ in which case the typical $\bar{q}_z$ tends to 0. Then the minimum threshold speed is

$$\bar{v}_x = \frac{s\bar{E}}{E_s} = \frac{s}{2 + l_z^2/\tilde{l}^2}. \tag{19}$$

If we take $1/l_z^2 = b_0^2/6$ we obtain $\tilde{l}^2/l_z^2 \approx \hbar b_0^2/6m^*\omega_c \approx 1$ or 1.3, in line with the experimental conditions of Refs. 4 and 5. Thus the finite thickness l_z can make the threshold speed substantially smaller than that pertaining to the zero-thickness 2DEG. Notice that for the parabolic potential V_y we have $v_x = |e|E_H/2\pi\hbar n_e \equiv E_H\omega_c^2/B\tilde{\omega}^2 = v_D\omega_c^2/\tilde{\omega}^2 \approx v_D$ because of the stated condition $\omega_c \gg \Omega$. For the more realistic potentials V_y' and $\bar{V}_y$ the difference between v_x and v_d becomes smaller so we can practically neglect it.

Another case where NDC is permissible for all values of E_H is obtained from Eqs. (14) and (15) for $E_H + E_e > E_s$, i.e., for $v(k_e, k_E) > 1$ and $E_H - E_e > E_s$, i.e., for $v(-k_e, k_E) > 1$. Assuming $l_z = 0$ we have $j_{\rm PA}^{\rm III} = 0$ and

$$j_d = \frac{m^* \tilde{m} \tilde{\omega} k_B T c'}{4\pi^2 \hbar^4 k_e W} E_H \left[\frac{1}{(E_H + E_e)^2} + \frac{1}{(E_H - E_e)^2}\right]. \tag{20}$$

This case corresponds to relatively large E_H and the threshold speed v_D is larger than s. From Eq. (20) for $E_H \gg E_e$ we have $j_d \propto 1/E_H$. If finite thickness is taken into account the corresponding result for $(E_H l_z/E_s \tilde{l})^2 \gg 1$ is $j_d \propto 1/E_H^2$. For other values of the parameters we can again have NDC with $\bar{v}_x \ll s$.

For the DA interaction under the conditions $\hbar s/\tilde{l} \ll k_B T$ from Eqs. (11)–(13) we obtain Eqs. (14)–(16) with the subscript PA replaced by DA. Here

$$j_{\rm DA}^{\rm II} \equiv j_{\rm DA}^{\rm II}(k_e, k_E) = A\left(\frac{v - v_0}{v_0 v^2}\right) \frac{v^2 + 1 + (v^2 - 1)l_z^2/\tilde{l}^2}{[1 + (v^2 - 1)l_z^2/\tilde{l}^2]^{3/2}} \theta(v - 1), \tag{21}$$

where $A = |e|\omega_c c' m^* k_B T/4\pi^2\hbar^4 s^2 W$ and

$$j_{\rm DA}^{\rm I} = -j_{\rm DA}^{\rm II}(-k_e, k_E), \; j_{\rm DA}^{\rm III} = -j_{\rm DA}^{\rm II}(k_e, -k_E). \tag{22}$$

From the last two equations for $l_z = 0, E_H + E_e > E_s$, and $|E_H - E_e| < E_s$ we obtain

$$j_d = A\frac{E_H}{E_e}\left[1 + \frac{E_s^2}{(E_H + E_e)^2}\right]. \tag{23}$$

From Eq. (23) it is evident that the breakdown condition (10) is impossible. Consider now Eqs. (21) and (22) for $E_H > E_e + E_s$. In this case $j_{\rm DA}^{\rm III} = 0$ and for the zero thickness channel we have

$$j_d = A\frac{E_H}{E_e}\left[2 + \frac{E_s^2}{(E_H + E_e)^2} + \frac{E_s^2}{(E_H - E_e)^2}\right]. \tag{24}$$

For $E_H \gg E_e$ it follows from Eq. (24) that $j_d \propto E_H$ and $j_x \propto E_x$, i.e., the CVC is Ohmic and NDC is absent. However, if $E_H = \bar{E}$, NDC is possible if $(\bar{E} + E_e)/E_s = 1 + 2E_e/E_s > 1.84$. In Fig. 3, the CVC is shown for different values of E_e/E_s using Eq. (24).

Consider again Eqs. (21) and (22) for $E_e > E_s, E_H$ and zero channel thickness. In this case for $E_e - E_H > E_s$ NDC is absent. However, upon increasing E_H by $\Delta E_H = \Delta E = \max\{\hbar\Omega^2/|e|\omega_c\tilde{l}, k_B T m^*\tilde{\omega}^2/\hbar|e|\omega_c k_e\}$, in the neighborhood of $\bar{E} = E_e - E_s$, NDC appears again as j_d decreases sharply due to the vanishing of $j_{\rm DA}^{\rm III}$. In par-

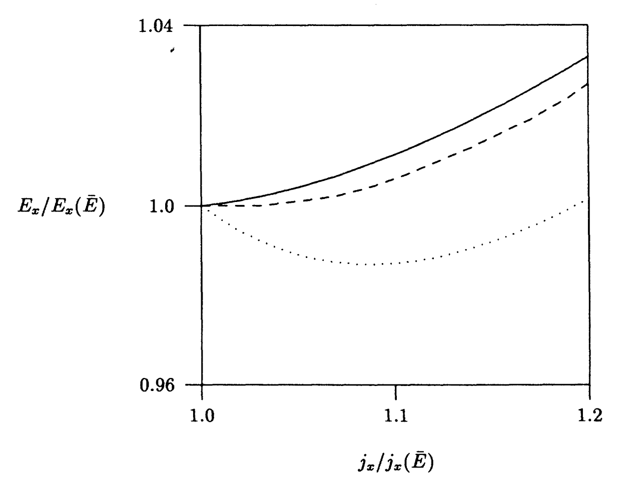

FIG. 3. Current-voltage characteristics for the DA interaction corresponding to Eq. (24) ($E_H \geq E_e + E_s = \bar{E}$); the solid, dashed, and dotted curves correspond to $\eta = E_e/E_s =$ 0.3, 0.5, and 1, respectively.

ticular, for $E_e/E_s - 1 \ll 1, \Delta E \ll \bar{E}$ in the region under consideration of E_H we have $v_x \ll s$, i.e., the breakdown of the QHE here is caused by NDC for relatively small currents and Hall fields.

We notice that from Eqs. (14)–(16), (21), and (22) we have the possibility of dissipation ($j_d \neq 0$), in the absence of NDC, for $E_e/E_s \gtrsim 1$; even if $E_H < E_s$, i.e., if $v_D < s$, it follows that j_d is not exponentially small. Such a weakly dissipative regime would be established for finite E_H if $E_e < E_s$ and immediately for $E_H = 0$ if $E_e \geq E_s$. Below for definiteness we consider the QHE regime as (i) nondissipative if $j_d = 0$ and (ii) weakly dissipative if $j_d \neq 0$. Equation (10) is associated with such a weakly dissipative regime which can be totally unstable if Eq. (10) is fulfilled for all E_H pertinent to this regime.

IV. CVC AND BREAKDOWN AT LOW TEMPERATURES

We now consider temperatures low enough that the inequality $r = (\hbar s v/\tilde{l} k_B T)^2/2 \gg 1$ holds. Then for the PA interaction with $\eta = E_e/E_s < 1$ and $\eta_\pm = (E_H \pm E_e)/E_s$ we assume that $|E_H - E_e| < E_s$ and

$$0 < \eta_+^2 - 1 \ll (r/2)\min\left\{1/\lambda_-; \frac{\lambda_+}{3\lambda_-^2}\right\}, \tag{25}$$

where $\lambda_\pm = 1 \pm l_z^2/\tilde{l}^2$. The last inequality allows us to take $I_0 = 1$ in Eq. (12) and obtain, from Eqs. (11)–(13),

$$\begin{aligned} j_d &= j_{\rm PA}^{\rm II}(E_e, E_H) \\ &= \Gamma\{A_3(\eta x_T) - A_3(\eta_+ x_T) \\ &\quad -3\lambda_+(\eta_+^2 - 1)[A_5(\eta x_T) - A_5(\eta_+ x_T)]\}, \end{aligned} \tag{26}$$

where $\Gamma = |e|\omega_c c'/2\pi^2\hbar^2 s\tilde{l}\tilde{\omega}W$, $x_T = \hbar s/\tilde{l}k_B T$, and $A_n(x) = \exp(x^2/4)D_{-n}(x)$ with $D_{-n}(x)$ being a parabolic cylinder function.[17] We have used the approximation $n_{q_x} \approx \exp(-\hbar q_x s\eta/k_B T)$ justified under the considered conditions. From Eqs. (10) and (26) the condition for breakdown, for $\min E_H = \bar{E} = E_s - E_e$, reads

$$x_T A_4(x_T) - 2\lambda_+[A_5(\eta x_T) - A_5(x_T)] < 0, \tag{27}$$

and $\eta^2 x_T^2/2 > 1$; here $\eta_+ = 1$ and $\bar{q}_z \to 0$. Using the values of B, s, and T from Ref. 4 we have $x_T = 2.5$, $\lambda_+ \approx 2$, and $\lambda_- \approx 0$. We have fulfillment of Eq. (27), i.e., NDC for $\eta < 0.65$; this corresponds to the smallest threshold speed $\bar{v}_x = s\bar{E}/E_s \approx 0.35s$ that is close to the observed value. The condition (27) leads to $\bar{v}_x < s$ for larger values of x_T if η is sufficiently small. For instance, if $\eta^2 x_T^2 \gg 15$ and $\eta^5 \ll 1$ we obtain the breakdown condition for $\eta_+ \geq 1$ in the form

$$2\lambda_+\eta_+^5 > x_T^2\eta^5. \tag{28}$$

If in Eq. (28) we take $\eta_+ = 1$ and denote the maximum η by $\eta_{\rm max}$, we find that the lowest Hall voltage for breakdown corresponds to $\bar{v}_x = (1 - \eta_{\rm max})s$. This value of $\bar{v}_x$ cannot be lowered for $\eta_+ > 1$ as inspection of Eq. (28) shows. A weakly dissipative stable regime is established, in the sense defined above, if $\eta > \eta_{\rm max}$ and the corresponding drift velocities are in the region $s(1-\eta) < v_x < s\eta(1-\eta_{\rm max})/\eta_{\rm max}$. Figure 4 shows the CVC for different η using the values $x_T \approx 2.5, \lambda_+ \approx 2$, and $\lambda_- \approx 0$ as determined from the experimental parameters of Ref. 4. A weakly dissipative stable regime holds for $0.3 < (E_H/E_s) < 0.35$ if $\eta = 0.7$. The corresponding value for the resistivity ρ_{xx}, as obtained from Eq. (26) and the expression $\rho_{xx} = E_x/j_x \approx j_d/E_H(\sigma_{yx}^0)^2$ is 8×10^{-2} $\Omega/\square$ and compares well with the experimental one 5×10^{-2} $\Omega/\square$ before breakdown as obtained with[4] $s = 2.48\times10^5$ cm/sec, $B = 11.3$ T, $W \approx 200$ μm, $T = 0.95$ K, etc.

Further analysis of Eqs. (11)–(13) and (26) reveals that condition (10) can be satisfied for a variety of combinations of the parameters E_H, E_s, E_e, η^2, and x_T. In general, increasing the thickness l_z leads to smaller breakdown velocities.

Similar results hold for the DA interaction, i.e., depending on the parameters sometimes we have NDC and breakdown, sometimes we do not. For instance, for $\eta^2 x_T^2/2 > 1, \eta < 1$, and $0 < \eta_+^2 - 1 \ll 1$ we obtain

$$j_d \equiv j_{\rm DA}^{\rm II}(E_e, E_H) = (12\Gamma/\tilde{l}^2)\eta_+^2[A_5(\eta x_T) - A_5(\eta_+ x_T)]. \tag{29}$$

From here it is easy to show, for $\eta^2 x_T^2 \gg 15$, that NDC is impossible in contrast with the PA interaction where it is; cf. Eqs. (26)–(28). Another case where NDC is possible is realized for $\eta_- \equiv (E_H - E_e)/E_s > 1, \eta^2 x_T^2/2 > 1$, and $x_T^2 > 2$. Assuming $[\eta_-^2 - 1]^2 \ll 1$ we obtain

$$j_d = \frac{6\Gamma\sqrt{\pi}}{\tilde{l}^2}\frac{\eta_-^2}{[2 + \lambda_+(\eta_-^2 - 1)]^{5/2}}. \tag{30}$$

Here NDC is satisfied even for $l_z = 0$. In the opposite limit, $\eta_-^2 \gg 1$ we obtain, from Eqs. (11)–(13) for $l_z = 0$,

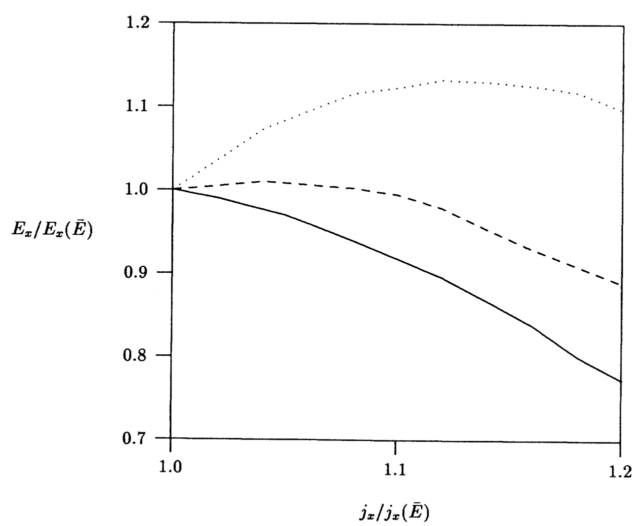

FIG. 4. Current-voltage characteristics for the PA interaction corresponding to Eq. (26) ($x_T = 2.5$, $l_z^2/\tilde{l}^2 = 1$, $\bar{E} = E_s - E_e > 0$); the solid, dashed, and dotted curves correspond to $\eta = 0.6$, 0.7, and 0.8, respectively.

$$j_d = (\Gamma\sqrt{2/\pi}/\bar{l}^2)\eta_-. \tag{31}$$

For $E_H \gg E_e$ the dependence $j_x = j_x(E_x)$ becomes almost Ohmic.

V. BREAKDOWN RELATED TO FLUCTUATIONS IN THE CONFINING POTENTIAL

We now consider the possibility of the breakdown due to smooth fluctuations in the confining potential that depend on the x coordinate. Given the experimental way of producing a 2DEG we expect that these fluctuations are more pronounced in the z direction than in the y direction. We therefore consider only fluctuations of a characteristic scale $\Lambda_x \gg \bar{l}$ which make the ground electric subband E_{z0} depend on x, $E_{z0} = E_{z0}(x)$, but for simplicity we take Ω constant, independent of x. This in turn makes $y_{\rm RE}$ and $y_{\rm LE}$ fluctuate with x. Consequently, $k_{\rm RE} = -k_{\rm LE} = \bar{k} + \Delta k$ with $|\Delta k| \ll \bar{k}$ and $\bar{k}$ representing an average over a segment $l_x \gg \Lambda_x$. We further assume that Δk takes any value in the interval $-\Delta k_0, \Delta k_0$ with equal probability.

We first consider the PA interaction at low temperatures for $x_T^2\eta_+^{-5} \gg 2\eta^{-5}$, $\eta+(x) \equiv [E_e(x) + E_H]/E_s > 1$, and $-\eta_-(x) \equiv [E_e(x) - E_H]/E_s < 1$; we then obtain

$$j_d(x) = (\Delta/W)[E_s^3/E_e^3(x) - \eta_+^{-3}], \tag{32}$$

where $\Delta = |e|\omega_c c' k_B^3 T^3/2\pi^2\hbar^4 s^4 m^*\tilde{\omega}^2$; if $-\eta_-(x) > 1$ the result is

$$j_d(x) = -(\Delta/W)[\eta_-^{-3}(x) + \eta_+^{-3}(x)]. \tag{33}$$

Notice that if j_d is given by only Eq. (32), or (33) the breakdown condition (10) is not fulfilled. For instance, if $E_e \gg E_s$, Eq. (33) shows that NDC is absent for $0 \leq v_D < s(E_e/E_s)$; that is, low-voltage breakdown, such as when $0 < v_d - s \lesssim s$, is impossible. Similar results are obtained for the DA interaction. We suppose that the characteristic region for the transition from Eq. (32) to Eq. (33) is described for fixed x by $\Delta E_H \approx \Delta E \ll \Delta E_0$, where ΔE_0 corresponds to Δk_0; here $\Delta E = \max\{\Omega^2 k_B T/|e|\omega_c s, k_B T m^*\tilde{\omega}^2/\hbar|e|\omega_c\bar{k}\}$. Using the potential V_y' we consider the case $(y_{\rm RE} - y_r), (y_l - y_{\rm LE}) \ll y_r - y_l$, i.e., $W \approx y_r - y_l$. Then it is possible to neglect the x dependence of E_H and W in Eqs. (32) and (33) . The average of $j_d(x)$, denoted by $\bar{j}_d(x)$, is given by

$$\bar{j}_d(x) = \frac{1}{2\Delta E_0}\int_{\bar{E}-\Delta E_0}^{\bar{E}+\Delta E_0} j_d dE_e, \tag{34}$$

where $\bar{E}$ corresponds to $\bar{k}$ and

$$\begin{aligned}\bar{j}_d \equiv \bar{j}_d(x) = \frac{\Delta E_s^3}{4\Delta E_0 W}[\,(\bar{E} - \Delta E_0)^{-2} + E_s^{-2} - (E_s + E_H)^{-2}\\ +(\bar{E} + \Delta E_0 + E_H)^{-2} - (\bar{E} - \Delta E_0 + E_H)^{-2} - (\bar{E} + \Delta E_0 - E_H)^{-2}],\end{aligned} \tag{35}$$

if (i) $\bar{E}+\Delta E_0 > E_s+E_H$ and (ii) $E_s - E_H < \bar{E} - \Delta E_0 < E_s + E_H$. Only this case is favorable for breakdown. In connection with Eqs. (34) and (35) one can see that upon increasing E_H the contributions to $\bar{j}_d$ determined from Eq. (33) diminish while those determined from Eq. (32) increase. This leads to a decrease in $\bar{j}_d$. Apparently, if a region E_H existed for which NDC is possible it would be of width at most $2\Delta E_0$. From Eq. (35) the breakdown condition reads

$$\begin{aligned}(E_s + E_H)^{-3} + (\bar{E} - \Delta E_0 + E_H)^{-3}\\ -(\bar{E} + \Delta E_0 + E_H)^{-3} - (\bar{E} + \Delta E_0 - E_H)^{-3} < 0.\end{aligned} \tag{36}$$

We write $\bar{E} = E_s + \xi\Delta E_0$ with $|\xi|\Delta E_0 \ll E_s$ and assume $\Delta E_0, E_H \ll E_s$. Then from Eq. (36) taking into account (i) and (ii) the breakdown condition becomes

$$\begin{aligned}&(\xi + 3)/2 < E_H/\Delta E_0 < (\xi + 1), \quad 1 < \xi < 5,\\ &(\xi - 1) < E_H/\Delta E_0 < (\xi + 1), \quad \xi > 5.\end{aligned} \tag{37}$$

As can be seen, the region of E_H for NDC is not greater than $2\Delta E_0$, as stated earlier. Since dissipation is connected with processes at the channel boundaries only confining potential fluctuations at the left and right boundaries will be important for NDC; therefore, they can be assumed to be statistically independent. Finally, for the DA interaction an analysis similar to that given above leads to the following conditions for breakdown:

$$\begin{aligned}&(\xi + 14)/7 < E_H/\Delta E_0 < (\xi + 1), \quad 7/6 < \xi < 7/2,\\ &(\xi - 1) < E_H/\Delta E_0 < (\xi + 1), \quad \xi > 7/2.\end{aligned} \tag{38}$$

VI. CONCLUDING REMARKS

The main result of this paper is that the electron-phonon interaction in narrow two-dimensional channels leads to a substantial dissipation at the edges of the channel and consequently to a (low-voltage) breakdown of the dissipationless QHE regime if criterion (10) is satisfied. This can happen for drift velocities v_D smaller or much smaller than the speed of sound s in sharp contrast with wide channels where all previous treatments predict breakdown for $v_D \geq s$. In general, the finite thickness of the channel leads to a v_D smaller than that of the zero thickness channel. A variety of situations with or without NDC (or breakdown) can occur depending on the values of the pertinent parameters. Support for our picture of the breakdown of the QHE as a destruction of a nearly dissipationless regime due to NDC can be found in Refs. 4, 5, 10, and 11 and the review in Ref. 1; in many experiments the onset of dissipation in the QHE is related to instabilities, hysteretic behavior,[4] etc. To our knowledge these results are new. Below we discuss the relevant approximations and assumptions used in obtaining them.

The main assumption was that the term $-eE_x x$ in Eqs. (1) and (5) was small enough that perturbation theory could be used, i.e., that E_x was weak thus limiting the results' validity. This could be satisfied if, e.g., $E_x \ll E_H$. Indeed, this is a sufficient assumption and is reflected in the final results since from Eqs. (6)–(8) we obtain $E_x = \sigma_{yy} E_H/\sigma^0_{yx} \approx E_H \rho_{xx}/\rho_{yx} \ll E_H$. This condition, which is different from the low-voltage one $v_D < s$, is usually satisfied in the experiments.[3–6,10,11,18] Assuming its validity and using the experimental[4] values for ρ_{xx} and ρ_{yx} just before and after the breakdown we find, respectively, $E_x/E_H \approx 10^{-5}$ and 10^{-3}. Therefore the approximation $E_x \ll E_H$ is well justified.

As explained in the text, the assumption about the parabolic confining potential, made for analytic simplicity, was not at all crucial and could be relaxed through the use of more realistic models such as V_y' or the more general $\bar{V}_y$. With regard to the confining frequency Ω we remark that we have not attempted to express it in terms of the channel width W. Model expressions, $\Omega = f(W)$, can be found in Ref. 16.

The principal reason for the importance of the electron-phonon interaction, which leads to dissipation mainly at the edges of the *narrow* channel is that in the latter, in contrast with the *wide* channel, the Landau levels are tilted upwards at the edges and cross the Fermi level; cf. Figs. 1 and 2. Therefore, the interaction and the consequent dissipation will be more important for the electrons that are closer to the Fermi level. This being the case it is then reasonable to expect modifications of the conditions for breakdown on the drift velocity v_D as presented in the text. In fact, a weakly dissipative regime would be established when $v(k_e, k_E) \geq 1$ as the θ function shows in, e.g., Eqs. (14) and (21). It is reassuring that this condition is equivalent to the following one on the drift velocity $v_D(y_e)$ at the channel edges $y_e = y_{\rm RE}$ [$B > 0, E_H > 0$ in Eqs. (14) and (21)]. Assuming a parabolic confining potential we have

$$v_D(y_e) = v_D - \frac{1}{eB}\frac{dV_y}{dy}\bigg|_{y=y_e} = v_D - \frac{m^*\Omega^2 y_e}{eB} \geq s. \quad (39)$$

Since $e < 0$ this means that the dissipation in narrow channels starts when the average channel drift velocity v_D is *smaller* than s; here a small difference between v_x and v_D has been neglected. In contrast, in wide channels the dissipation starts when v_D is *larger* than s as Eq. (39) shows for $\Omega \to 0$ since it is valid for the more realistic potential V_y'. We further notice, with reference to Fig. 2, that at the edges of the channel, $Y_0 \cong Y_{\rm RE}$ or $Y_0 \cong Y_{\rm LE}$, the contribution to the dissipation decreseas with temperature in a power-law fashion if the corresponding drift velocity $v_D(Y_{\rm RE})$ or $v_D(Y_{\rm LE})$ exceeds s. Here the characteristic extent $\delta y \propto T$ of the edge states that contribute to j_d tends to zero when the temperature does. If at the left edge we have $v_D(Y_{\rm LE}) > s$, i.e., for v_D much larger than that of Fig. 2, we can have electron transitions from full states to empty ones only due to phonon emission and hence a finite dissipation at $T = 0$.

Because dissipation occurs mainly at the edges we have been able to use more realistic models for the confining potentials, when $100\ \mu{\rm m} \geq W \gg 1\ \mu{\rm m}$, namely V_y' and $\bar{V}_y$, with well-defined edges. The results as explained in the text agree well with the essential features of those of Ref. 4 previously, to our knowledge, not explained. We mention in particular the agreement between the calculated and observed values of v_D and ρ_{xx}. More important, the current-voltage characteristics, as detailed at the end of Sec. II, matches well the observed one and lends support to the model $\bar{V}_y$ with different confinement at the edges ($\Omega_+ \neq \Omega_-$). We emphasize that the observations of Ref. 4 are not unique since low-voltage breakdown with similar characteristics was observed in Refs. 3 and 6 and was interpreted with a model in which the current flows mainly along the edges. In particular, the breakdown in Ref. 3 was observed in sample 2, 4 μm wide, for $v_D \leq 1.6 \times 10^5$ cm/sec$< s$ near a filling factor $\nu = 2$ ($B = 7$ T) and for $v_D \approx 4 \times 10^4$ cm/sec$\ll s$ ($B \approx 7.5$ T) in agreement with our findings. The subsequent interpretation[3] used the electron-phonon interaction model of Ref. 9.

Our treatment of the results of Ref. 4 involved mainly the intra-Landau-level transitions. The interlevel ones, although present, did not contribute much to the current for the conditions stated. This is not always the case and depends mainly on the parameters. For instance, in Ref. 3 both kinds of transitions occur depending on the value of the Hall voltage ($\propto E_H$). In Ref. 19 the high breakdown current densities (not applicable to our case) are clearly associated with interlevel transitions and again with electron-phonon interaction. Indeed, for the narrowest sample of Ref. 18 ($L = 10.2\ \mu$m, $W = 1\ \mu$m) and the $h/2e^2$ plateau at $B = 6.3$ T the critical current density 29 A/m, obtained in Ref. 19, is close to the experimental one[18] and gives $v_D \approx 5.6 \times 10^6$ cm/sec $\gg s$. It follows that $\exp(-\hbar\omega_c/2m^* v_D^2) \approx 4 \times 10^{-2} > \delta y/W \approx 10^{-2}$; this is opposite to what we assumed after Eq. (10). Notice that for another sample ($L = 14\ \mu$m, $W = 66\ \mu$m) Fig. 5 of the same reference gives $v_D < 4 \times 10^4$ cm/sec $\ll s$.

It is worth emphasizing that in narrow channels we do not always have $v_D \gg s$. Indeed, in Ref. 11 the breakdown for one constriction ($L = 10\ \mu$m, $W \approx 1.5\ \mu$m), before illumination, was observed at $v_D < 3.4 \times 10^5$ cm/sec$\approx 1.4s$ as is easily obtained from curve A of Fig. 1 and the critical currents of Fig. 2 of this work. This is at least one order of magnitude smaller than the critical v_D reported in Refs. 18 and 19 for comparable values of W, L, B, T. In Ref. 20 the critical v_D for the channel with $L = 10\ \mu$m and $W = 4\ \mu$m, before illumination, was 2×10^5 cm/sec$< s$. That is, for rather narrow channels, $W \sim 1\ \mu$m, the breakdown starts usually for $v_d \lesssim s$ but in some cases[18] for $v_D \gg s$.

Within our model we can understand qualitatively two more observations: the increase in v_D, after illumination of the sample, reported in Ref. 11 and the large difference in v_D between Refs. 11 and 18, for rather similar values of W, L, etc., despite the inapplicability of our theory to the results of the latter. The first one is due to an increase in the electron density which leads to an increase in Ω and, consequently, in the parameter E_e that modifies the relevant breakdown conditions. For instance, if $E_e > E_s$ we have no breakdown for $0 < v_D/s < (E_e - E_s)/E_s$, based on Eq. (33); that is, the larger E_e the larger the critical v_D for breakdown [if $(E_e - E_s)/E_s > 1$ we have $v_D > s$]. As for the second observation we simply have to assume that in Ref. 18 we had $E_e \gg E_s$ and in Ref. 11 $E_e \sim E_s$ and refer, e.g., to Eqs. (20) and (33). Notice that for $E_e \gg E_s$ we have $E_H \geq E_e$ (for the region of possible breakdown) and our treatment ceases to apply.

Finally, we have shown that breakdown is possible when in addition to the electron-phonon interaction there exist fluctuations in the confining potential along the z direction. The fluctuations modify the breakdown conditions; cf. Eqs. (36)–(38). Not being aware of any relevant experimental data we cannot test our theory in this respect.

ACKNOWLEDGMENT

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APPENDIX

Using the δ function in Eq. (8) we first carry out the integration over q_z and then that over q_y. Then we obtain

$$j_d = \frac{|e|\omega_c c'}{4\pi^2\hbar s m^* \tilde{\omega}^2 W} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dq_x dk_x q_x [q_x \tilde{v}]^{m+1}\{f[E_0(k_x)] - f[E_0(k_x - q_x)]\} \times \left\{ \frac{1}{1 - e^{[E_0(k_x) - E_0(k_x - q_x)]/k_B T}} - \frac{1}{1 - e^{\hbar s q_x \tilde{v}/k_B T}} \right\} \times I_0[q_x^2 \tilde{l}^2 \lambda_-(\tilde{v}^2 - 1)/4] e^{-q_x^2 \tilde{l}^2 [1+\lambda_+(\tilde{v}^2-1)/2]/2}, \tag{A1}$$

where $E_0(k_x) = E_{0k_x}(E_H = 0) = \hbar\tilde{\omega}/2 + \hbar^2 k_x^2/2\tilde{m} + E_{z0}$ and

$$q_x \tilde{v} > 0, \quad \tilde{v}^2 > 1. \tag{A2}$$

Here $I_0(x)$ is the modified Bessel function and $\tilde{v} = v - (\hbar q_x/2m^* s)(\Omega/\tilde{\omega})^2$. Since $\hbar\tilde{\omega} \gg k_B T$, we have approximately $f[E_0(k_x) - f(E_0(k_x - q_x)] \approx -2k_x q_x \delta(k_x^2 - k_e^2)$. Moreover, because k_e is much larger than the typical q_x we can approximate $\tilde{v}$ by v. Then from Eq. (A1) we obtain, after the integration over k_x, Eq. (11) of the text.

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