Kinetics of a quasi-one-dimensional electron gas in a transverse magnetic field. II. Arrays of quantum wires

I. I. Boiko and V. Sheka
Institute of Semiconductors, Academy of Sciences, 45 Prospekt Nauky, Kiev, Ukraine

P. Vasilopoulos
Department of Physics, Concordia University, 1455 de Maisonneuve Boulevard West, Montréal, Québec, Canada H3G 1M8
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The quantum kinetic equation for the one-electron distribution function is derived for an array of quantum wires in the presence of a magnetic field perpendicular or parallel to the plane of the wires. Screening is treated dynamically in the random-phase approximation. The results are valid for array periods large enough that tunneling between the wires can be neglected. The energy and momentum relaxation frequencies are evaluated when only the lowest level is occupied for scattering by three- or two-dimensional impurities and by acoustical phonons.

I. INTRODUCTION

Currently systems of reduced dimensionality, such as quantum wires, are the subject of numerous investigations due to their potential device applications; cf. Refs. 1–6 and references cited therein. In previous papers[1,2] we have proposed a first-principles transport formalism for the quasi-one-dimensional electron gas (Q1DEG) which we applied to arrays of quantum wires[3,4] and which we extended later on[7] for one Q1DEG in the presence of a magnetic field B. In this paper we consider transport in periodic arrays of quantum wires in a nonzero field B. To our knowledge this subject has not been treated.

The problem of arrays is in some respects qualitatively different from that of the isolated wires because of two important effects: the tunneling between the wires, for relatively short interwire separations, and the mutual screening of the scattering potentials by electrons from different wires. The latter is present even when the interwire separation is long due to the long range of the Coulomb interaction. Here we will consider only the latter effect relegating the treatment of tunneling to a future work.

In the next section we present the general formalism for multilevel occupation. In Sec. III we present simplified results for the one-level occupation and in Sec. IV the corresponding relaxation frequencies evaluated for a degenerate electron gas scattered by impurities or acoustical phonons. Summary and conclusions follow in Sec. V.

II. QUANTUM KINETIC EQUATION AND COLLISION INTEGRAL

We consider a periodic array of quantum wires, of length $L_x \equiv L$, in the xy plane, with period l, as shown in Fig. 1. The wires are parallel to the x axis and indexed by $p = \pm 1, \pm 2, \ldots$. The magnetic field B is parallel to the z axis; later on it will be taken in the xy plane. Taking the vector potential $\mathbf{A} = [-B(y-pl), 0, 0]$ the one-electron Hamiltonian H_p^0 in the pth quantum wire is given by

$$H_p^0 = (\mathbf{P} + e\mathbf{A})^2/2m^* + V_y + V_z + H_s^0, \qquad (1)$$

where $\mathbf{P}$ is the momentum operator, m^* the effective mass, and H_s^0 the standard spin Hamiltonian. The potentials V_y and V_z determine the form of the quantum wells in the corresponding directions. Due to the periodicity we have $V_y = V(y - pl)$ for all p. For simplicity we will take the confining potential V_y parabolic: $V_y = m^*\Omega_1^2 y^2/2 \equiv \hbar^2 q_1^2 y^2/2$. The eigenvalues corresponding to Eq. (1) are then given by

$$E_A^p \equiv E_A \equiv \hbar\omega_A = \hbar\tilde{\omega}(a + 1/2) + \hbar^2 k_\alpha^2/2\tilde{m} + E_{a_z} + E_\sigma, \qquad (2)$$

where $\tilde{\omega} = \sqrt{\omega_c^2 + \Omega_1^2}$, $\tilde{m} = m^*\tilde{\omega}^2/\Omega_1^2$, and ω_c is the cyclotron frequency. The last two terms on the right-hand side are, respectively, the energy along the z direction and the spin energy. Further, $A = (a, a_z, k_\alpha)$ is the set of the quantum indices and $k_\alpha \equiv k_x$ is the wave vector in the x direction. The corresponding normalized wave function has an orbital part given by

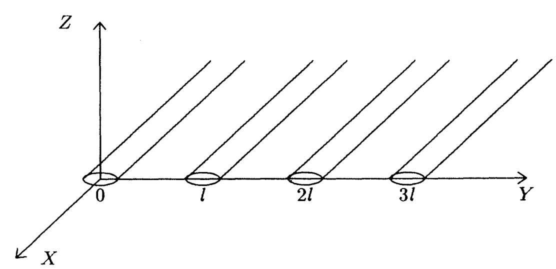

FIG. 1. Geometry of the array of quantum wires; the array period is l.

$$\Psi_a^p(\mathbf{r}) = L^{-1/2} e^{ik_\alpha x} \psi_{a_z}(z) \Phi_a(y - y_\alpha - pl), \tag{3}$$

where $y_\alpha = -\hbar k_\alpha \omega_c/\tilde{\omega}^2 = -k_\alpha/\tilde{q}^2$. In what follows we will incorporate E_σ in E_{a_z} and thus drop the spin index σ.

Equations (1)–(3) pertain to a free electron. We now assume that the electrons interact with each other, with an external potential, e.g., impurities, phonons, etc., and with an external electric field; the matrix element of the total potential is denoted by φ_{AB}. Due to the identity of the wires the diagonal part of the density matrix $f_{AA}^p \equiv f_A^p$ is independent of p, i.e., $f_A^p \to f_A$. The quantum kinetic equation for f_A is obtained from the corresponding many-body Hamiltonian and the equation of motion for the density matrix ρ_{AB}. It has the same form as that for one wire [cf. Ref. 7, Eq. (5)] with its collision integral $\mathrm{St} f_A$ given by

$$\mathrm{St} f_A = -\frac{e}{2\pi^2\hbar} \mathrm{Im} \sum_B \int d\omega \int d\omega' \, e^{-i(\omega+\omega')} \langle [\delta\rho_{AB}^p(\omega), \delta\varphi_{BA}^p(\omega')]_+ \rangle, \tag{4}$$

where $[\,,\,]_+$ denotes half the anticommutator. Here the fluctuations $\delta\rho^p$ and $\delta\varphi^p$ depend on the wire index p, but the collision integral does not. The fluctuating parts appearing in $\langle [\,,\,] \rangle$ are given by

$$\delta\rho_{AB}^p(\omega) = \delta\rho_{pAB}^0(\omega) + eM_{AB}(\omega)\, \delta\varphi_{AB}^p(\omega) \tag{5}$$

and

$$\delta\varphi_{AB}^p(\omega) = (1/2\pi) \int dq_x \, \delta_{k_\alpha - k_\beta, q_x} \, \delta\varphi_{AB}^p(\omega, q_x), \tag{6}$$

with

$$\delta\varphi_{AB}^p(\omega, q_x) = \delta\varphi_{pAB}^0(\omega, q_x) + \sum_{p'A'B'} \delta_{k_{\alpha'} - k_{\beta'}, q_x} \, D_{ABA'B'}^{p-p'}(\omega, q_x) \times \delta\rho_{A'B'}^{p'}(\omega). \tag{7}$$

Here, α stands for the two indices k_α and a,

$$D_{ABA'B'}^{p-p'}(\omega, q_x) = e \int_{-\infty}^{\infty} dq_y \, \frac{e^{-iq_y l(p-p')} \lambda_{\alpha\beta}(q_y) \lambda_{\beta'\alpha'}(-q_y)}{q_\perp \, \epsilon^s_{a_z b_z, a'_z b'_z}(\omega, \mathbf{q}_\perp)} \tag{8}$$

and

$$\lambda_{\alpha\beta}(q_y) = \int_{-\infty}^{\infty} dy \, e^{iq_y y} \, \Phi_{a_y}^*(y - y_\alpha) \, \Phi_{b_y}(y - y_\beta) = \sqrt{a!/b!}\, e^{-[|q_0|^2 + iq_y \mu(k_\alpha + k_\beta)\tilde{q}^{-2}]/2} \times q_0^{b-a} L_a^{b-a}(|q_0|^2), \tag{9}$$

where $\mu = \omega_c/\tilde{\omega}$, $q_0 = [iq_y - \mu(k_\alpha - k_\beta)]/\sqrt{2}\tilde{q}$, $\tilde{q} = \sqrt{m\tilde{\omega}/\hbar}$, and $L_a^b(x)$ is a Laguerre polynomial. Further, the source potentials $\delta\rho^0$ and $\delta\varphi^0$ and the matrix M_{AB} are given in Ref. 7 and $\epsilon^s_{\dots}$ is the dielectric function. The latter is given by

$$\frac{1}{\epsilon^s_{a_z b_z, a'_z b'_z}(\omega, \mathbf{q}_\perp)} = \int dz \int dz' \, \frac{\Psi_{a_z}^*(z) \Psi_{b_z}(z) \Psi_{b'_z}^*(z') \Psi_{a_z}(z')}{\epsilon_s(\omega, \mathbf{q}_\perp, z, z')} = (q_\perp/\pi) \int dq_z \frac{g_{a_z b_z}(q_z) \, g_{b'_z a'_z}(-q_z)}{q^2 \, \epsilon_s(\omega, \mathbf{q})}, \tag{10}$$

if the dielectric function is uniform in the z direction, with

$$g_{a_z b_z}(q_z) = \int_{-\infty}^{\infty} dz \, e^{iq_z z} \Psi_{a_z}^*(z) \Psi_{b_z}(z). \tag{11}$$

The system of Eqs. (5)–(7) is solved in the manner of Ref. 7 and the collision integral has two terms: the first term describes collisions between the electrons themselves and the second one collisions of the electrons with an external system. The latter is denoted by $\mathrm{St}_{\mathrm{es}} f_A$ and has the form

$$\begin{aligned}\mathrm{St}_{\mathrm{es}} f_A = \frac{e^2}{2\pi\hbar^2 L} \sum_{BA'B'A''B''} & \delta_{k_\alpha, k_{\alpha'}} \delta_{k_\alpha, k_{\alpha''}} \delta_{k_\beta, k_{\beta'}} \delta_{k_\beta, k_{\beta''}} \\ & \times \int_{-\infty}^{\infty} dq_y \lambda_{\alpha'\beta'}(q_y) \lambda_{\beta''\alpha''}(-q_y) R_{k_\alpha}^{KK'}(C; lq_y) R_{k_\beta}^{*KK''}(C; lq_y) \\ & \times \Bigg\{ (f_B - f_A) \langle \delta\varphi_s^2 \rangle_{C, q_y}^{K'_z,''} \\ & \quad -i\pi e^2 \hbar \, \frac{f_A(1 - f_B) + f_B(1 - f_A)}{\sqrt{q_y^2 + (k_\alpha - k_\beta)^2}} \left[\frac{1}{\epsilon^s_{K'_z,''}(C, q_y)} - \frac{1}{\epsilon^s_{K''_z,'}(C, q_y)} \right] \Bigg\}. \end{aligned} \tag{12}$$

Here C stands for the arguments ω_{AB} and $k_\alpha - k_\beta$, and $K = (a;b) \equiv (a,b;a_z,b_z)$; further, $K_z^{','' } = a_z^{'}b_z^{'}, a_z^{''}b_z^{''}$ and R is the inverse of the matrix T with respect to the upper indices given by the following expressions:

$$T_{k_\alpha}^{KK'}(D) = \delta_{KK'} - L^{-1}\sum_{k_\alpha^{'}} M_{K'}(k_\alpha^{'},\omega,q_x)D_F(D), \tag{13}$$

$$\delta_I M_{AB}(\omega) = \delta_I M_K(k_\alpha,\omega,q_x), \tag{14}$$

$$\delta_I D_{ABA'B'}(D) = \delta_I D_F(D), \tag{15}$$

$$D_{ABA'B'}(D) = \frac{2\pi e}{l}\sum_{n=-\infty}^{\infty}\frac{1}{\sqrt{q_x^2+q_{yn}^2}} \times \frac{\lambda_{\alpha\beta}(q_{yn})\lambda_{\beta'\alpha'}(-q_{yn})}{\epsilon^s_{a_zb_z,a_z^{'}b_z^{'}}(\omega,q_x,q_{yn})}, \tag{16}$$

with the abbreviations $D \equiv \omega, q_x, lq_y$, $F \equiv k_\alpha - k_{\alpha'}$, $I \equiv k_\alpha - k_\beta, q_x$, and $q_{yn} = q_y - 2\pi n/l$.

The correlator of the scattering potentials $\langle\rangle$ in Eq. (12) is evaluated as in Refs. 2 and 7 using the fluctuation-dissipation theorem and assuming that the scattering system remains at equilibrium with temperature T_s. The result for the collision integral is then given by

$$\mathrm{St}_{\mathrm{es}} f_A = -\frac{e^2}{\hbar^2 L}\sum_B \langle\delta\tilde{\varphi}^2\rangle_C^{AB,AB} \times \left\{ f_A - f_B + \frac{[f_A(1-f_B)+f_B(1-f_A)]}{\coth(\hbar\omega_{AB}/2k_BT_s)}\right\}. \tag{17}$$

The correlator $\langle\rangle$ in Eq. (17) is that of the scattering potentials screened both by the external system and by the particles of the Q1DEG. It is given by

$$\langle\delta\tilde{\varphi}^2\rangle_C^{AB,AB} = \frac{1}{2\pi}\sum_{A'B'A''B''}\delta_{k_\alpha,k_{\alpha'}}\delta_{k_\alpha,k_{\alpha''}}\delta_{k_\beta,k_{\beta'}}\delta_{k_\beta,k_{\beta''}}\,X \times \int_{-\infty}^{\infty} dq_y \lambda_{\alpha'\beta'}(q_y)\lambda_{\beta''\alpha''}(-q_y)R_{k_\alpha}^{KK'}(C;lq_y)R_{k_\beta}^{*KK''}(C;lq_y), \tag{18}$$

where

$$X = \langle\delta\varphi_{s,0}^2\rangle_{C,q_y}^{K_z^{','' }} = \frac{i\pi e^2\hbar}{q_\perp}\coth\left(\frac{\hbar\omega}{2k_BT_s}\right)\left[\frac{1}{\epsilon^{s,0}_{K_z^{'},''}(C)} - \frac{1}{\epsilon^{*s,0}_{K_z^{''},'}(C)}\right]. \tag{19}$$

The contribution to screening by the Q1DEG is embodied in the functions R. The periodicity aspects of the system are expressed in R as well: electrons from different wires participate in the screening of the scattering potentials that electrons "see" in any wire that is within the screening length.

In the presence of an electric field **E** directed along the axis of the wires the quantum kinetic equation has the standard form with the collision integral as described above. The momentum and energy balance equations are derived from it in the manner described earlier[1,7] and from them the relaxation frequencies. The second part of the collision integral describing electron-electron collisions does not contribute to these frequencies.[1,7] If we assume that the drift velocity **u** is the same for all energy levels, the expressions (33)–(35) of Ref. 7 can be taken over and the only change, reflecting the periodicity of the present system, will be through the function R that enters the expression for the correlators.

III. ONE-LEVEL RESULTS

A. Perpendicular magnetic field

All expressions given above simplify considerably if the electrons occupy only the lowest level as is expected to be the case for most of the experimentally accessible densities. Then we can take $a = a_z = 0$. If we further consider only the lowest spin sublevel the diagonal density matrix is labeled only by the continuous wave vector $k \equiv k_x$ and the collision integral takes the form

$$\mathrm{St}_{\mathrm{es}} f_k = \frac{e^2}{2\pi\hbar^2}\sum_{k'}\int_{-\infty}^{\infty} dq_x\,\delta_{k-k',q_x}\int_{-\infty}^{\infty} dq_y\, e^{-q_y^2/2\tilde{q}^2}\left\{(f_{k'}-f_k)\langle\delta\varphi_s^2\rangle_{\omega_{kk'},\mathbf{q}_\perp} + \frac{2\pi\hbar}{q_\perp}[f_k(1-f_{k'})+f_{k'}(1-f_k)]\mathrm{Im}\Lambda(\omega_{kk'},\mathbf{q}_\perp)\right\} \times |S(\omega_{kk'},q_x;q_y)|^{-2}. \tag{20}$$

Here, for a uniform external system we have

$$\Lambda(\omega,\mathbf{q}_\perp)=\frac{q_\perp}{\pi}\int dq_z\frac{|\int dz|\Psi_0(z)|^2e^{iq_zz}|^2}{q^2\epsilon(\omega,\mathbf{q}_\perp)};\tag{21}$$

the screening factor $S()$ is given by

$$S(\omega,q_x;lq_y)=1+\frac{\Delta\epsilon(\omega,q_x)}{\epsilon_s(\omega,q_x;lq_y)},\tag{22}$$

and the dielectric function by

$$\frac{1}{\epsilon_s(\omega,q_x;lq_y)}=\frac{2\pi}{l}\sum_{n=-\infty}^{\infty}\frac{\Lambda(\omega,q_x;q_{yn})}{\sqrt{q_x^2+q_{yn}^2}}e^{-q_{yn}^2/2\tilde{q}^2}.\tag{23}$$

$\Delta\epsilon()$ is given by the standard expression.[2,7] As for the correlator $\langle\rangle$ of the scattering potentials it can be evaluated along the lines of Refs. 2 and 7 and will be given below for particular cases.

Equation (20) has the same structure as Eq. (28) of Ref. 7 valid for an isolated wire. The only difference is in the screening factor $S()$, which embodies the periodicity aspects of the array. In the limit $l\to\infty$ the dielectric function given above coincides with Eq. (38) of Ref. 7 and so does the collision integral. As for $\mathrm{St}_{ee}f_k$, which describes electron-electron collisions, it has a form of the same structure as Eq. (29) of Ref. 7. For the assumed one-level dispersion law, $\omega_k=\hbar k^2/2\tilde{m}$+const, it vanishes identically.

B. Parallel magnetic field

For a magnetic field along the y axis, $\mathbf{B}=B\hat{\mathbf{y}}$, the vector potential is $\mathbf{A}=(Bz,0,0)$. Now the wave functions in the z direction and the corresponding eigenvalues of the one-electron Hamiltonian depend not only on the discrete quantum number a_z but also on the continuous wave vector along the x direction.

The following consideration shows that there is no principal difference between this case and that of the preceding subsection. If the confining potentials V_y and V_z are both parabolic and have the *same* curvature we can take over all one-level results with $\tilde{q}$ replaced by $q_1=\sqrt{m^*\Omega_1/\hbar}$. If, however, the curvature is different, we must take into account the curvature of V_z in the expression for $\tilde{m}$.

IV. ONE-LEVEL RELAXATION FREQUENCIES

We take the magnetic field normal to the plane of the array and consider a parabolic form for V_z, i.e., $V_z=\hbar^2q_2^4z^2/2m^*$. Using the lowest-level wave function and Eq. (21) we obtain

$$\Lambda(\omega,\mathbf{q}_\perp)=\epsilon_L^{-1}\mathrm{erfc}(q_\perp/\sqrt{2}q_2)e^{q_\perp^2/2q_2^2},\tag{24}$$

where we have assumed that the dielectric function $\epsilon_s(\omega,\mathbf{q})=\epsilon_L$ is approximately constant.

The momentum ν^m and energy ν^T relaxation frequencies are obtained in the manner described earlier[1,7] from the balance equations. The result is

$$\begin{pmatrix}\nu^m\\ \nu^T\end{pmatrix}=\frac{1}{\pi^2n\hbar}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}dq_x\int_{-\infty}^{\infty}dq_ye^{-|\bar{q}|^2}\begin{pmatrix}k_BT_sq_x^2/\tilde{m}\omega^2\\ \omega^2\end{pmatrix}\frac{(\hbar\omega/2k_BT_s)^2}{\sinh(\hbar\omega/k_BT_s)}\times\langle\delta\varphi_{s,0}^2\rangle_{\omega,\mathbf{q}_\perp}\mathrm{Im}\Delta\epsilon^{eq}(\omega,q_x)|S(\omega,q_x;lq_y)|^{-2}.\tag{25}$$

Here $S()$ is given by Eqs. (22) and (23) and $\Delta\epsilon^{eq}(\omega,q_x)$ by Eq. (44) of Ref. 2. We now evaluate the relaxation frequencies for some specific cases of the external scattering system as characterized by the potential correlator $\langle\rangle$.

A. Scattering by volume impurities

The electrons are assumed to be scattered by randomly distributed charged impurities of three-dimensional density $n_I^{(3)}$. Using Eq. (A1) of Ref. 7 we obtain

$$\langle\delta\varphi_{s,0}^2\rangle_{\omega,\mathbf{q}_\perp}=\frac{8\pi^3e^2n_I^{(3)}}{\epsilon_L^2q_\perp^3}G(q_\perp/\sqrt{2}q_2)\;\delta(\omega),\tag{26}$$

where

$$G(x)=2/\sqrt{x}+(1-2t^2)\;\mathrm{erfc}(x)\;e^{x^2}.\tag{27}$$

The result for the momentum relaxation frequency is

$$\nu^m=\frac{8e^4n_I^{(3)}\tilde{m}}{\epsilon_L^2\hbar^3nq_2^2}\int_0^\infty\frac{dx}{(\lambda^2+x^2)^{3/2}}G(\sqrt{\lambda^2+x^2})\times e^{-x^2\eta^2}S_B^{-2}(\lambda,\gamma,x),\tag{28}$$

where

$$S_B(\lambda,\gamma,x)=1+\frac{e^2\lambda}{4\epsilon_LE_Fl}\ln\left(\frac{8E_F}{k_BT_s}\right)\times\sum_{p=-\infty}^{\infty}\frac{\mathrm{erfc}[\sqrt{\lambda^2+(x-p\gamma)^2}]}{\sqrt{\lambda^2+(x-p\gamma)^2}}\times e^{\lambda^2+(1-\eta^2)(x-p\gamma)^2}.\tag{29}$$

Here $E_F=(\hbar\pi n)^2/2\tilde{m}$ is the Fermi energy, n is the electron density, $\lambda=\sqrt{2}\pi n/q_2$, $\gamma=\sqrt{2}\pi/lq_2$, and $\eta=q_2/\tilde{q}$. As before[7] we can construct an interpolation formula for $S_B()$ which is exact in the limits $\tilde{q}l\to0$ and $\tilde{q}l\to\infty$; it reads

$$S_B(\lambda,\gamma,x)=1+\frac{e^2\tilde{m}}{\epsilon_L\pi^2\hbar^2n}\ln\left(\frac{4\pi^2\hbar^2n^2}{\tilde{m}k_BT_s}\right)\times\left[\ln\left(\frac{\sqrt{2}\tilde{q}}{\pi n}\right)+\frac{\pi}{\sqrt{2}q_2l}\frac{\mathrm{erfc}[\sqrt{\lambda^2+x^2}]}{\sqrt{\lambda^2+x^2}}\times e^{\lambda^2+(1-\eta^2)x^2}\right].\tag{30}$$

Notice that the quantities $n, \lambda, \tilde{m}$, and η depend on the magnetic field B; for the B dependence of $n \equiv n(B)$, Eq. (47) of Ref. 7, rewritten here for convenience, reads

$$n(B) = [(n^0)^2 + (\tilde{m}/\pi^2\hbar)(\Omega_1 - \tilde{\omega} + 2\mu_\sigma B/\hbar)]^{1/2}, \tag{31}$$

where $\mu_\sigma = ge\hbar/2m_0$ is the magnetic moment of the electron and n^0 the one-dimensional electron density at zero magnetic field. For $\lambda \ll 1$ and $\eta \leq 1$ the frequency ν^m is given approximately by

$$\nu^m \approx \frac{4e^4\tilde{m}n_I^{(3)}}{\epsilon_L^2\hbar^3\pi^2 n^3}\widetilde{M}_B^{-2}, \tag{32}$$

where the screening factor $\widetilde{M}$ is

$$\widetilde{M}_B = 1 + \frac{e^2\tilde{m}}{\epsilon_L\hbar^2\pi^2 n}\ln\left(\frac{4\pi^2\hbar^2 n^2}{\tilde{m}k_B T_s}\right)\left[\frac{1}{\sqrt{8}nl} + \ln\left(\frac{\sqrt{2}\tilde{q}}{\pi n}\right)\right]. \tag{33}$$

In Figs. 2 and 3 we plot, respectively, the relaxation frequency as a function of the magnetic field $\nu^m(B)$ and of the array period $\nu^m(l)$ using for its evaluation Eq. (25). The B and l dependences are expressed by the dimensionless variables $\tilde{B} = \omega_c/\Omega_1 = m^*\omega_c/\hbar q_1^2$ and $\mathcal{L}=q_1 l$, respectively. Further, we take $q_1 = q_2 = 2 \times 10^8/m$, considering parabolic confinement in both the y and z directions, $2\mu_\sigma B/\omega_c = 1/3, \epsilon_L = 13\epsilon_0, T_s = 4$ K, and define $c = \pi n^0/q_1$. In Fig. 2 the curves 1, 2, and 3 correspond to $\mathcal{L}$ =3, 10, and 30, respectively, and $c = 0.2$. In Fig. 3 the curves 1, 2, and 3 correspond to $\tilde{B} = 0.0, 0.7$, and 1.4, respectively, and $c = 1$. For low B the decrease, in Fig. 2, of ν^m with B is related to the motion of the lower spin level as expressed by the term $-\mu_\sigma B/\hbar$ in Eq. (31). The increase for high B is related to the decrease of the kinetic energy in the x direction and the tendency of the level for depopulation. When the level is depopulated ($n \rightarrow 0$) the relaxation frequency tends to infinity. The increase, in Fig. 3, of ν^m with l reflects the weakening of screening and is similar to that reported earlier[3,4] in the absence of magnetic field. As $l \rightarrow \infty$ the mobility of each wire is determined by the scattering potential screened by electrons only of this wire.

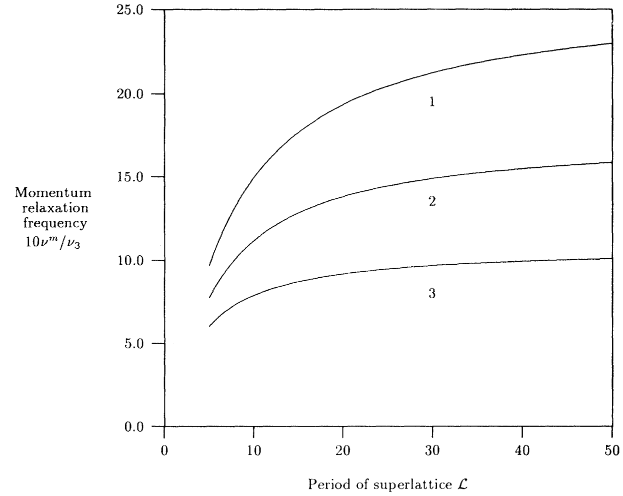

FIG. 3. Momentum relaxation frequency as a function of the superlattice period ($\mathcal{L}=q_1 l$) for scattering by three-dimensional impurities. The curves labeled 1, 2, and 3 correspond to $\tilde{B} = 0.0, 0.7$, and 1.4, respectively, and $c = 1.0$.

B. Scattering by sheet impurities

In this case denoting the impurity density by $n_I^{(2)}$ and using Eq. (A1) of Ref. 4 and Eq. (24) we obtain

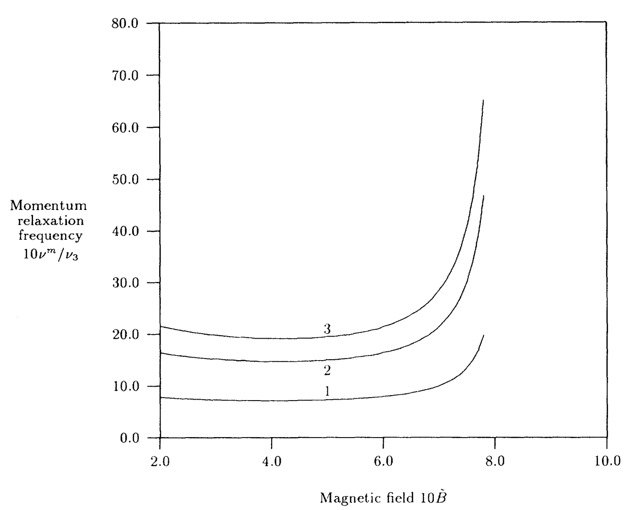

FIG. 2. Momentum relaxation frequency as a function of the magnetic field ($\tilde{B} = \omega_c/\Omega_1$) for scattering by three-dimensional impurities. The curves labeled 1, 2, and 3 correspond to $\mathcal{L}$ =3, 10, and 30, respectively, and $c = 0.2$.

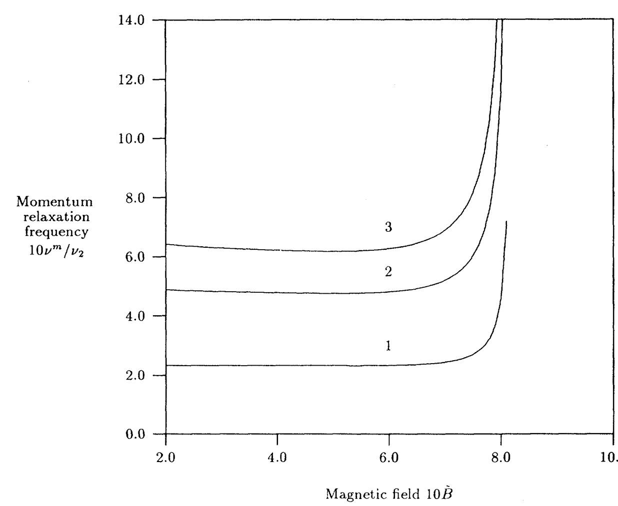

FIG. 4. Momentum relaxation frequency as a function of the magnetic field for scattering by two-dimensional impurities. The curves are labeled as in Fig. 1; $c = 0.2$.

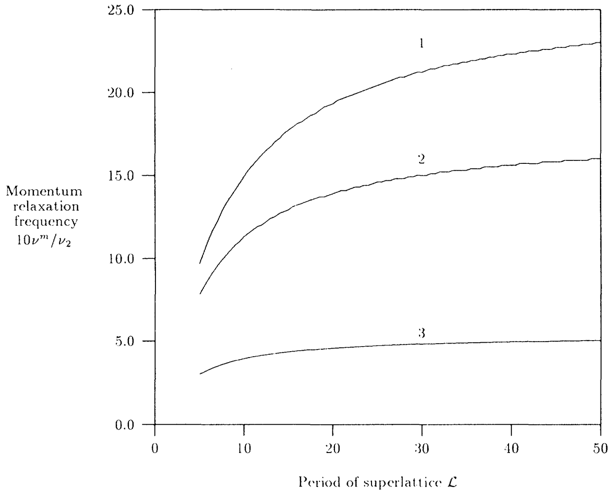

FIG. 5. Momentum relaxation frequency as a function of the superlattice period. The curves are labeled as in Fig. 3; $c = 1.0$.

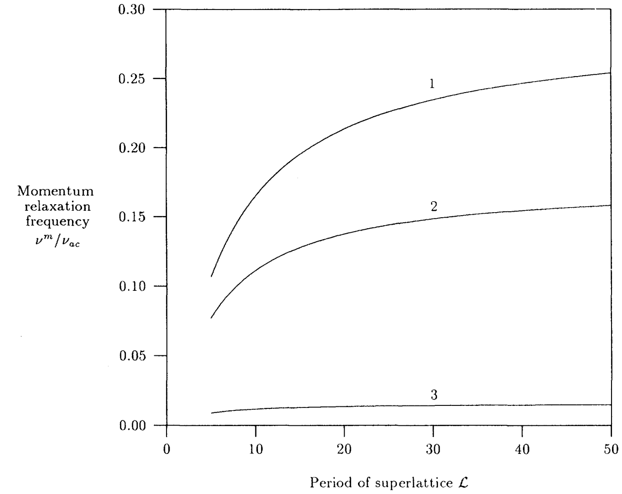

FIG. 6. Momentum relaxation frequency as a function of the superlattice period for scattering by acoustical phonons. The curves are labeled as in Figs. 3 and 5.

$$\langle \delta\varphi^2_{s,0}\rangle_{\omega,\mathbf{q}_\perp} = \frac{8\pi^3 e^2 n_I^{(2)}}{\epsilon_L^2 q_\perp^2}\,[\mathrm{erfc}(q_\perp/2q_2)]^2\, e^{q_\perp^2/2q_2^2}. \tag{34}$$

Inserting this in Eq. (25) we obtain

$$\nu^m = \frac{2\sqrt{2}e^4\tilde{m}n_I^{(2)}}{\epsilon_L^2\hbar^3 n q_2}\int_0^\infty \frac{dx}{\lambda^2+x^2}e^{x^2(1-\eta^2)}[\mathrm{erfc}(x/\sqrt{2})]^2 \times S_B^{-2}(\lambda,\gamma,x). \tag{35}$$

For $\lambda \ll 1$, which corresponds to the one-level occupation, and $\eta \leq 1$ we have approximately

$$\nu^m \approx \frac{4e^4 n_I^{(2)}\tilde{m}}{\epsilon_L^2\hbar^3 n^2}\widetilde{M}_B^{-2}. \tag{36}$$

In Fig. 4 we plot the frequency ν^m as a function of the magnetic field and in Fig. 5 the same quantity as a function of the array period $\nu^m(l)$. The values of c and $\mathcal{L}$, in Fig. 4, and of c and $\tilde{B}$, in Fig. 5, are the same as the corresponding ones in Figs. 2 and 3, respectively. As can be seen the dependences of ν^m on magnetic field and on array period are similar to those in Figs. 2 and 3, respectively.

C. Scattering by acoustical phonons

In this case the potential correlator is obtained along the lines of Ref. 2. The result is

$$\langle \delta\varphi^2_{s,0}\rangle_{\omega,\mathbf{q}_\perp} = \frac{E_1^2\omega^2\hbar}{e^2 2\rho s^3}\cot\left(\frac{\hbar\omega}{2k_BT_s}\right)\frac{e^{(Y/sq_2)^2/2}\theta(Y^2)}{Y}, \tag{37}$$

where $Y = \sqrt{\omega^2 - s^2q_\perp^2}$. For $k_BT_s > \pi\hbar ns$ the relaxation frequencies are given approximately by

$$\begin{pmatrix}\nu^m\\ \nu^T\end{pmatrix} \approx \frac{16E_1^2k_B^3T_s^3\tilde{m}}{\pi^2\rho s^4 n\hbar^5} \times \begin{pmatrix}(6/\pi^2+2\sqrt{X/\pi})^{-1}\\ (30/\pi^4+4\sqrt{X^3/\pi})^{-1}\dfrac{2\tilde{m}k_BT_s}{\pi^2\hbar^2n^2}\end{pmatrix}\overline{M}_B^{-2}, \tag{38}$$

where $X = (k_BT_s/\hbar s)^2(q_2^{-2}+\tilde{q}^{-2})$ and

$$\overline{M}_B = 1 + \frac{e^2\tilde{m}}{\pi^2\hbar^2 n\epsilon_L}\ln\left[\frac{4\pi^2\hbar^2n^2}{\tilde{m}k_BT_s}(1+\sqrt{X})\right] \times\left[\frac{1}{\sqrt{8nl}}+\ln\left(\frac{\sqrt{2}\tilde{q}}{\pi n}\right)\right]. \tag{39}$$

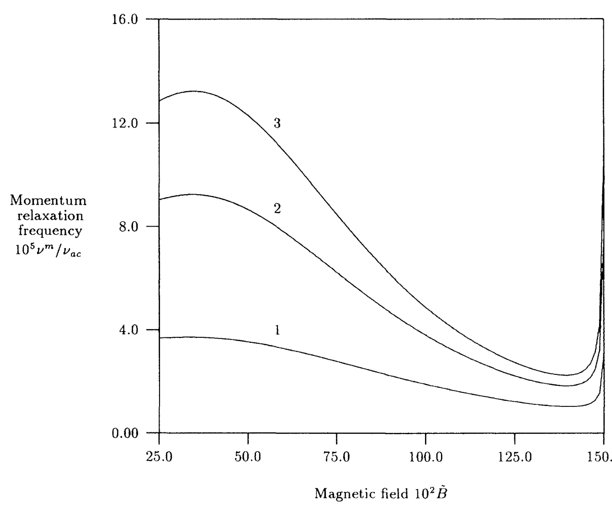

FIG. 7. Momentum relaxation frequency as a function of the magnetic field for scattering by acoustical phonons. The curves are labeled as in Figs. 2 and 4.

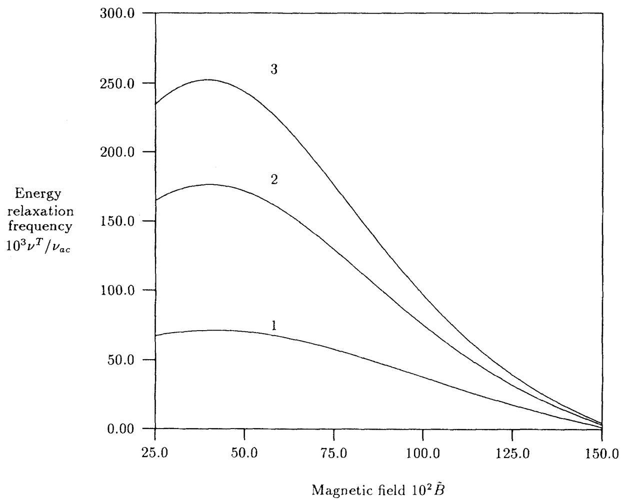

FIG. 8. Energy relaxation frequency as a function of the magnetic field for scattering by acoustical phonons. The curves are labeled as in Figs. 2 and 4.

Here s is the speed of sound, ρ the density of the material, and E_1 the deformation-potential constant. As in Ref. 7, both relaxation frequencies depend strongly on the temperature; this is typical for inelastic scattering of the highly degenerate electron gas since the phase-space volume available for transitions is limited in a narrow region, of width k_BT, around the Fermi surface. The numerical results for $\nu^m(l), \nu^m(B)$, and ν^T, evaluated from Eq. (38) are shown, correspondingly, in Figs. 6, 7, and 8. The parameters used are $k_BT_s/\hbar q_2 s = 4, \rho = 5 \times 10^3$ kg/m^3, and $E_1 = 10$ eV.

V. SUMMARY AND CONCLUDING REMARKS

In this paper we have evaluated the collision integral and the dielectric function for an array of Q1DEG's in the presence of a magnetic field B treating screening in the random-phase approximation. Moreover, we have evaluated the momentum and energy relaxation frequencies when only the lowest spin level is occupied for scattering by impurities or acoustical phonons. This is an extension of the work pertinent for one Q1DEG presented in Ref. 7 for $B \neq 0$ as well as of that for an array when B is absent.[3] The results are valid for relatively large array periods since we have neglected tunneling between the wires. In this respect the results constitute only a partial extension of the work presented in Ref. 4, which takes into account both screening and tunneling but in the absence B. A full extension of the latter work, valid for $B \neq 0$, will be reported elsewhere.

Our numerical results for the relaxation frequencies show a nontrivial dependence on the magnetic field B. We have regions of positive and negative magnetoresistance, as is easily seen using $\sigma = e^2n/\tilde{m}\nu^m$, and at certain values of B the electron density [cf. Eq. (31)] and the conductivity vanish; when this happens the level is depopulated. Similar depopulation effects, for $a > 0$, have been observed[8] for *one* Q1DEG; we are not aware of any experimental results pertinent to arrays that we have treated.

The results for the frequencies also show a nontrivial dependence on the array period l. Their increase with increasing l, or correspondingly the decrease of the mobility, reflects the corresponding weakening of screening. The latter is very important for Q1DEG's due to the singularity of the dielectric function for $\omega \to 0$ and $T \to 0$ at $q = 2q_F$, the wave vector at the Fermi level. For elastic scattering the value $q = q_F$ gives the major contribution to the frequencies. It is understood that the neglect of screening overestimates the mobility considerably.

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