Mesoscopic persistent current in small rings

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Small normal-metal rings threaded by a constant magnetic flux have been shown to carry a mesoscopic persistent current at low temperatures. The current is a few-electron effect and its sign and amplitude depend on the microscopic configuration of disorder. Assuming a Gaussian current distribution, we characterize the effect by three quantities, the rms or typical total current $I^{\rm typ}=\langle I^2\rangle_D^{1/2}$, the average current $I^{\rm av}=\langle I\rangle_D$, and the typical single-level current $i^{\rm typ}=\langle i^2\rangle_D^{1/2}$. Specifically, we review and extend the analytical calculations for the typical total and single-level currents focusing on the case of noninteracting electrons in disordered rings in the regime of diffusive transport. We calculate and discuss those current-current correlation functions that describe the dependences of the persistent current on filling, flux, and disorder configuration. Only the single-electron contribution discussed in this paper is known to contribute to the first, ϕ_0-periodic harmonic of the total current in a single ring. The second harmonic also contains an interaction-induced contribution proposed by Ambegaokar and Eckern that survives the disorder average. The Thouless correlation energy E_c is the characteristic energy scale for the amplitude of the total current and its dependences on filling, temperature, and inelastic scattering. The persistent current is sensitive to changing the position of a single impurity. We compare our results with the recent single-ring experiment by Chandrasekhar *et al.*

I. INTRODUCTION

Quantum persistent currents[1] in small nonsuperconducting rings threaded by a magnetic flux φ have aroused considerable interest over the past few years.[2–13] Recently, the first experimental evidence for their existence has been reported.[4,12] Interest in these currents reaches back to the 1960s,[14,15] and work on their mesoscopic nature[16,17] began in 1983 for strictly one-dimensional loops,[1,18–20] and later for ballistic[21] and metallic multichannel rings.[2] A closely related phenomenon exists in mesoscopic SNS junctions,[22,23] where the phase difference of the order parameter across the SNS junction plays the role analogous to that of the flux.

Persistent currents are an equilibrium property of a ring that encloses a static magnetic flux $\varphi=\phi/\phi_0$. They are periodic with the flux quantum $\phi_0=hc/e$ and can be defined in terms of the thermodynamic potential[14,24]

$$I=-\frac{e}{h}\frac{\partial\Omega}{\partial\varphi}\ . \tag{1.1}$$

Mesoscopic persistent currents require that the phase-coherence length be of the order of the ring circumference. In this paper we present a detailed microscopic theory of the effect for noninteracting electrons applicable to single-ring experiments.

For a single ring the current is highly sample specific. In an ideal ring without disorder the current is a periodic function of filling,[21] i.e., the number of electrons N in the ring or the chemical potential μ. The corresponding period is proportional to the number of transverse channels M or $M\Delta_M$, respectively, where Δ_M is the level spacing at the Fermi energy. The current amplitude depends sensitively on geometrical details.[21,25,26] For short cylinders each channel (characterized by its transverse momentum) carries a current of order $I_0=ev_F/L$, where L is the circumference of the ring. Since the signs of the channel currents are uncorrelated the typical amplitude of the total current is of order $M^{1/2}I_0$.[21]

In disordered rings, specifically in the diffusive regime, the current depends sensitively not only on the degree of filling but also on the disorder configuration.[2] For a particular disorder configuration the current fluctuates as a function of N (or μ). The average period of these fluctuations is given by the effective number of channels $M_{\rm eff}$ (or the correlation energy E_c). This indicates that the highest $M_{\rm eff}$ occupied levels determine the total current. In this sense the persistent current is a few-electron effect. It is well known from the theory of universal conductance fluctuations[27] (UCF) that phase-coherent mesoscopic phenomena can be extremely sensitive to changes in the impurity potential.[28–30] For typical experimental parameters moving one or a few impurities over a distance of order of the Fermi wavelength k_F^{-1} has essentially the same effect on the persistent current as redistributing all impurities.

A complete statistical description requires knowledge of the probability distribution for the current. We characterize the persistent current by the disorder averaged current $I^{\rm av}=\langle I\rangle_D$ and the variance of the current $\langle\delta I^2\rangle_D=\langle I^2\rangle_D-\langle I\rangle_D^2$ as appropriate for a Gaussian current distribution. Although there is some numerical evidence for this in the diffusive regime,[6,31] this question deserves further study. The average current may be measured directly in a multiring experiment (if the total signal is proportional to the sum of the signals from the individual rings). Measurement of the current variance requires a single-ring setup.

The current in the diffusive regime is determined by two relevant energy scales,[2] the level spacing $\Delta_M=1/[VN(0)]$, where V is the volume of the system and $N(0)$ the density of states at the Fermi energy, and

the correlation or Thouless energy E_c,

$$E_c=\pi^2\frac{\hbar D}{L^2}\propto h/\tau_D\propto M_{\rm eff}\Delta_M\ . \tag{1.2}$$

Here, τ_D is the diffusion time around the ring, D is the diffusion constant, and $M_{\rm eff}=Ml_{\rm el}/L$ is the effective number of channels.[32,33] E_c determines the range of spectral correlations which exist between the slopes of the energy levels, $i_n=-(e/h)\partial e_n/\partial\varphi$, as well as between level spacings. E_c varies from $M\Delta_M$ close to the ballistic regime to Δ_M near the localized regime.

For noninteracting electrons the average current depends on the averaging procedure.[20] Whereas the average current per ring for a disorder ensemble of many macroscopically identical rings is extremely small if the disorder average is performed at a constant chemical potential,[2,19] it exhibits positive and nonzero even harmonics in φ if the number of electrons N is fixed in each ring.[3,8–11,13,20,34] (N is allowed to vary from ring to ring.) However, a contribution to the average current due to electron-electron interactions[5,23] is much larger. The interaction-induced current is $\phi_0/2$ periodic and its amplitude is proportional to $\lambda(e/\tau_D)$, where λ is a renormalized, dimensionless coupling constant.[35,36] Using a simple (but possibly oversimplified) estimate for λ,[36] this amplitude appears to fall short of explaining the experimental current measured by Lévy *et al.*[4]

Only a single-electron contribution to the variance of the current is known.[37] Its flux dependence is ϕ_0 periodic. In contrast to the average current the variance of the single-electron persistent current is independent of the averaging procedure, i.e., $\langle\delta I^2(N)\rangle_D=\langle\delta I^2(\mu)\rangle_D$.[2] Its amplitude is of order $\langle\delta I^2\rangle_D^{1/2}\propto(e/\tau_D)$.[2] Recently, Chandrasekhar *et al.*[12] reported an observation of persistent-current effects directly in a single gold ring. In Sec. III D we compare their results with theoretical predictions. It is a well-known result that electron-electron interactions do not contribute to UCF [to leading order in $(k_F l_{\rm el})^{-1}$] apart from giving rise to an inelastic scattering cutoff.[27] The influence of interactions on the variance of the persistent current may be considerably more complicated because of the thermodynamic-ensemble questions involved.

In the following we focus attention on noninteracting electrons and single-ring experiments. Under these conditions the average current is negligible compared to the variance of the current.[2,8–11] Thus, it is convenient to introduce the typical total current $I^{\rm typ}=\langle I^2\rangle_D^{1/2}$ instead of the variance. For noninteracting electrons one may also define a typical single-level current $i^{\rm typ}=\langle i^2\rangle_D^{1/2}$.

It has been mentioned that the typical current amplitude is "easily" determined by the Thouless argument.[32] That misses an important point. The Thouless argument is usually applied to single-level quantities. Quite distinctly, the total current is determined not just by the single-level currents but also by the correlations between them. However, the result for the typical total current implies a variant of the Thouless argument, $I^{\rm typ}\propto(\Delta_M/\phi_0)g$, in terms of the dimensionless conductance.[2]

In this paper we give a detailed account of the analytical theory for persistent currents in the diffusive regime of metal rings, as it applies to single-ring experiments. In Sec. II we describe the model and present the calculation of the typical total current $I^{\rm typ}$ and the typical single-level current $i^{\rm typ}$. We show that diagrams involving two diffusion and Cooperon propagators, which were omitted in Ref. 2, also contribute to leading order. In Sec. III we study current-current correlation functions and discuss our results for the dependences of the current on flux, disorder, filling, and temperature with an emphasis on physical interpretation. We also apply the theoretical results to the single-ring experimental setup and compare them to the experimental results of Ref. 12. Section IV contains a brief summary.

II. PERSISTENT CURRENT THEORY

A. Model

We consider a thin-walled normal metal ring (e.g., Cu or Au) of circumference L and transverse area $A=L_yL_z$ threaded by a static magnetic flux $\varphi=\phi/\phi_0$, as shown in Fig. 1. We restrict ourselves to short cylinders for which $L\gg L_y,L_z$. We introduce the number of transverse channels $M=k_F^2A/4\pi$. For noninteracting electrons, the system may be described by the Hamiltonian

$$\hat H=\frac{1}{2m}\left[\hat{\mathbf p}-\frac{2\pi\hbar}{L}\frac{\phi}{\phi_0}\mathbf e_x\right]^2+V(\hat{\mathbf r})\ , \tag{2.1}$$

where $\mathbf e_x$ denotes the unit vector in the longitudinal direction. Unless otherwise stated we consider spinless electrons. We take the disorder potential $V(\mathbf r)$ to be Gaussian white noise, $\langle V(\mathbf r)V(\mathbf r')\rangle_D=\gamma\delta(\mathbf r-\mathbf r')$. The strength of the random potential γ is related to the elastic mean free path by $l_{\rm el}=\hbar v_F/[2\pi\gamma N(0)]$.[38] Alternatively, one may think of averaging over the positions of randomly distributed δ-function scatterers.

Neglecting the curvature, we model the ring geometry by periodic boundary conditions in the longitudinal direction. In the ballistic regime the specific shape of the loop influences the nature of the spectrum[21,25,26] and thus the persistent current. We believe that this sensitivity is negligible in the diffusive regime. In the transverse directions one may choose either periodic or specular hard-wall boundary conditions.

The flux can be eliminated from the Hamiltonian by a gauge transformation, which results in flux-periodic boundary conditions in the longitudinal direction,[14]

$$\psi_n(x+L,y,z)=e^{i2\pi\varphi}\psi_n(x,y,z)\ . \tag{2.2}$$

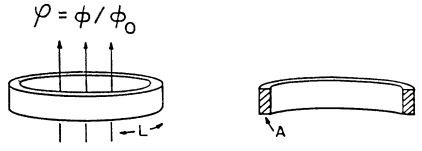

FIG. 1. Thin-walled ring of circumference L and cross-sectional area A threaded by a magnetic flux $\varphi=\phi/\phi_0$, where $\phi_0=hc/e$ is the flux quantum.

This demonstrates that the energy levels and therefore all thermodynamic quantities of the system are periodic functions of flux. With these boundary conditions one may use the usual plane-wave basis with discrete $\mathbf{k}$ and shifted longitudinal momentum eigenvalues, $k_x=(2\pi/L)(n_x-\varphi)$. The current associated with a particular energy level $e_n(\varphi)$ is given by $i_n=-(e/h)\partial e_n/\partial\varphi$. At $T=0$, the total current is given by the sum over all occupied levels, $I=\sum_n i_n$. The total current at finite temperatures can be obtained from the thermodynamic potential, and flux periodicity implies that the current can be characterized by its harmonics I_p,

$$I=-\frac{e}{h}\frac{\partial\Omega}{\partial\varphi}=\sum_{p=1}^{\infty} I_p\sin(2\pi p\varphi) . \tag{2.3}$$

In the diffusive regime the characteristic length scales satisfy $l_{\rm el}<L<\xi$, where $\xi\propto Ml_{\rm el}$ is the localization length. Finite temperatures may be characterized in terms of the thermal diffusion length $l_T=(hD/k_BT)^{1/2}$, where D denotes the diffusion constant $D=\frac{1}{3}v_Fl_{\rm el}$. We assume that the electronic phase-coherence length $l_\Phi(T)$ exceeds L. In the ballistic regime one has $L<l_{\rm el}$ and in the localized regime $\xi<L$.

Within the same model for geometry and disorder the effects of electron-electron interaction on the persistent current can be included.[5,39] Again, analogous results were obtained for SNS junctions.[23] The self-inductance of the ring may be neglected because of the very small magnitude of the persistent current. A magnetic field penetrating the ring will lead to Zeeman splitting and smear out the perfect flux periodicity of the energy levels $e_n(\varphi)$ and consequently of the persistent current. The size of these effects depends on the strength of the magnetic field and on the aspect ratio of the ring. A more complete treatment may also include surface scattering. However, this might only lead to small changes in the effective number of channels.

B. Typical total current

Starting from the current operator in second quantization, one readily derives the following Green's-function formula for the thermodynamic equilibrium current:

$$I=I_0\frac{1}{\beta}\sum_{\omega} e^{i\omega\eta}\sum_{k}\frac{k_x}{k_F}\mathcal{G}(\mathbf{k},\mathbf{k};\omega) . \tag{2.4}$$

Here η is a positive infinitesimal and ω the (Fermion) Matsubara frequencies, $\omega=(2m+1)\pi/\beta$. The finite-temperature Green's function for a particular impurity configuration is denoted by $\mathcal{G}(\mathbf{k},\mathbf{k}';\omega)$. We are working in the gauge with flux-periodic boundary conditions and longitudinal momentum values $k_x=(2\pi/L)(n_x-\varphi)$. Squaring and impurity averaging the current expression (2.4) yields for the typical total current,

$$\langle I^2\rangle_D=I_0^2\frac{1}{\beta^2}{\sum_{\omega,\xi}}'\sum_{\mathbf{k},\mathbf{k}'}\frac{k_xk_x'}{k_F^2}\langle\mathcal{G}(\mathbf{k},\mathbf{k};\omega)\mathcal{G}(\mathbf{k}',\mathbf{k}';\xi)\rangle_D . \tag{2.5}$$

The prime on the Matsubara sum stands for the exponential factors. We employ the usual diagrammatic method for calculating impurity averages. Figure 2 shows the diagrams contributing to this average to leading order in the diffusive regime.[40] The disorder averaged Green's function is approximated by[38]

$$G(\mathbf{k},\omega)=\langle\mathcal{G}(\mathbf{k},\mathbf{k};\omega)\rangle_D=\frac{1}{i\omega-E_k+\mu+iB\,\mathrm{sgn}\omega} . \tag{2.6}$$

Since the impurity averaged system is translationally invariant, G is diagonal in the momentum representation, E_k is the spectrum for the clean system, and impurity scattering mixes $\mathbf{k}$ states over an energy range $B=1/(2\tau_{\rm el})$. We use units with $\hbar=1$ in the remainder of this section. The particle-hole ladder diagram represents the diffusion propagator $D(\mathbf{q}_-;\omega,\zeta)$ of the form[38]

$$D(\mathbf{k}-\mathbf{k}';\omega,\zeta)=\frac{1}{2\pi N(0)\tau_{\rm el}^2}\frac{\theta(-\omega\zeta)}{|\omega-\zeta|+D(\mathbf{k}-\mathbf{k}')^2} . \tag{2.7}$$

The θ function restricts the Matsubara frequencies ω and ζ to opposite sides of the Fermi surface. The electron diffusion represented by the diffusion propagator $D(\mathbf{q}_-;\omega,\zeta)$ is flux insensitive, as reflected by the cancellation of the flux in $\mathbf{q}_-=\mathbf{k}-\mathbf{k}'$. The maximally crossed diagram represents the Cooperon propagator $K(\mathbf{q}_+;\omega,\zeta)$ that has the same form as (2.7) but with $\mathbf{q}_-=\mathbf{k}-\mathbf{k}'$ replaced by $\mathbf{q}_+=\mathbf{k}+\mathbf{k}'$. The quantum interference corrections represented by the Cooperon are flux sensitive, leading to the flux-dependent longitudinal momentum variable $(q_+)_x=k_x+k_x'=(2\pi/L)(n-2\varphi)$. The physical interpretation of the Cooperon in terms of time-reversed electron paths has been extensively discussed in the literature.[38] The length and energy scales introduced by disorder and temperature enter the calculation via the pole structure of G, D, and K. The information about geometry and flux is included in the momentum basis.

Although the diffusion contributions are flux independent, it is important to keep them when calculating typi-

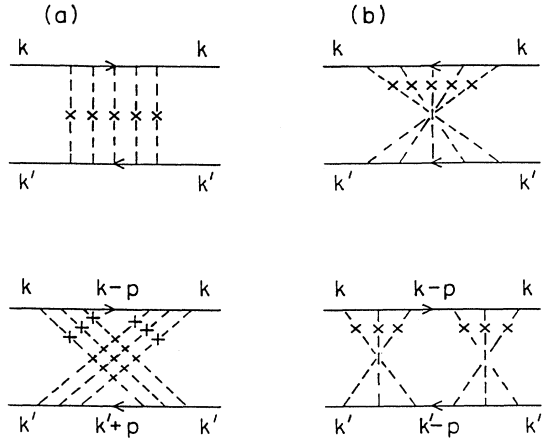

FIG. 2. (a) Diffusion and (b) Cooperon diagrams contributing to the typical current (2.5) to leading order. The solid lines refer to the disorder-averaged electron Green's function (2.6). The dashed lines with crosses represent the averaged disorder scattering.

cal current quantities to obtain the proper flux dependence. This will become apparent in Eq. (2.13). The diagrams in Fig. 2 translate into

$$\langle I^2\rangle_D = I_0^2 \frac{1}{\beta^2} \sum_{\omega,\zeta}{}' \sum_{\mathbf{k},\mathbf{k}'} \frac{k_x k_x'}{k_F^2} G^2(\mathbf{k},\omega) G^2(\mathbf{k}',\zeta) \times \Big\{ D(\mathbf{k}-\mathbf{k}';\omega,\zeta) + \sum_{\mathbf{p}} G(\mathbf{k}-\mathbf{p},\omega) G(\mathbf{k}'+\mathbf{p},\zeta) D^2(\mathbf{k}-\mathbf{k}'-\mathbf{p};\omega,\zeta) + K(\mathbf{k}+\mathbf{k}';\omega,\zeta) + \sum_{\mathbf{p}} G(\mathbf{k}-\mathbf{p},\omega) G(\mathbf{k}'-\mathbf{p},\zeta) K^2(\mathbf{k}+\mathbf{k}'-\mathbf{p};\omega,\zeta) \Big\} . \tag{2.8}$$

We eliminate the $\mathbf{p}$'s from the D^2 and K^2 terms by shifting $\mathbf{k}'\to\mathbf{k}'\mp\mathbf{p}$ and introducing $\mathbf{p}'=\mathbf{k}'\mp\mathbf{p}$, and then obtain

$$\langle I^2\rangle_D = I_0^2 \frac{1}{\beta^2} \sum_{\omega,\zeta}{}' \sum_{\mathbf{k},\mathbf{k}'} \frac{k_x}{k_F} G^2(\mathbf{k},\omega) G(\mathbf{k}',\zeta) \times \Big\{ \frac{k_x'}{k_F} G(\mathbf{k}',\zeta) D(\mathbf{k}-\mathbf{k}';\omega,\zeta) + \sum_{\mathbf{p}'} \frac{p_x'}{k_F} G(\mathbf{k}-\mathbf{k}'+\mathbf{p}',\omega) G^2(\mathbf{p}',\zeta) D^2(\mathbf{k}-\mathbf{k}';\omega,\zeta) + \frac{k_x'}{k_F} G(\mathbf{k}',\zeta) K(\mathbf{k}+\mathbf{k}';\omega,\zeta) + \sum_{\mathbf{p}'} \frac{p_x'}{k_F} G(\mathbf{k}+\mathbf{k}'-\mathbf{p}',\omega) G^2(\mathbf{p}',\zeta) K^2(\mathbf{k}+\mathbf{k}';\omega,\zeta) \Big\} . \tag{2.9}$$

The diffusion poles emphasize small $\mathbf{q}_\pm=\mathbf{k}\pm\mathbf{k}'$, which justifies expanding the Green's functions to first nonzero order in $\mathbf{q}_\pm$. In the terms linear in D and K, this implies $\mathbf{k}=\pm\mathbf{k}'$ everywhere except in the diffusion poles. In the terms quadratic in D and K, the unsquared Green's functions need to be expanded to first order in $\mathbf{q}_\pm$. The $\pm$ in $\mathbf{q}_\pm$ emphasizes the different flux dependences of the two quantities, i.e., $(q_+)_x=(2\pi/L)(n-2\varphi)$ and $(q_-)_x=(2\pi/L)n$. In the following, $\sum_{\mathbf{q}}$ is understood to mean summation over these quantum numbers. The expansion yields

$$\langle I^2\rangle_D = I_0^2 \frac{1}{\beta^2} \sum_{\omega,\zeta}{}' \sum_{\mathbf{k}} \frac{k_x^2}{k_F^2} G^2(\mathbf{k},\omega) G^2(\mathbf{k},\zeta) \times \Big\{ \sum_{\mathbf{q}} [D(\mathbf{q}_-;\omega,\zeta) - K(\mathbf{q}_+;\omega,\zeta)] - \sum_{\mathbf{p}'} \frac{p_x'^2}{m} G^2(\mathbf{p}',\omega) G^2(\mathbf{p}',\zeta) \sum_{\mathbf{q}} \Big[\frac{(q_-)_x^2}{m} D^2(\mathbf{q}_-;\omega,\zeta) - \frac{(q_+)_x^2}{m} K^2(\mathbf{q}_+;\omega,\zeta) \Big] \Big\} . \tag{2.10}$$

One can now perform the $\mathbf{k}$ and $\mathbf{p}'$ sums by replacing them by integrals. Note that the integrals are nonzero only if $-\omega\zeta>0$. This approximation is equivalent to neglecting the flux dependence of the Green's functions. Using the Poisson summation formula one shows that these flux-dependent terms are of order $\exp(-L/2l_{\mathrm{el}})$. They give rise to correlations between different harmonics, which therefore are "exponentially small" for the diagrams considered.[41] The momentum sums yield

$$\langle I^2\rangle_D = I_0^2 \frac{1}{\beta^2} \sum_{\omega,\zeta}{}' \frac{\pi}{6} \frac{N(0)}{B^3} \Big\{ \sum_{\mathbf{q}} [D(\mathbf{q}_-;\omega,\zeta) - K(\mathbf{q}_+;\omega,\zeta)] - \frac{\pi}{3} \frac{\mu N(0)}{B^3} \sum_{\mathbf{q}} \Big[\frac{(q_-)_x^2}{m} D^2(\mathbf{q}_-;\omega,\zeta) - \frac{(q_+)_x^2}{m} K^2(\mathbf{q}_+;\omega,\zeta) \Big] \Big\} . \tag{2.11}$$

For short cylinders, nonzero transverse $\mathbf{q}_\pm$'s are suppressed by a factor of approximately $\exp[-2\pi pL/\max\{L_y,L_z\}]$. The sum over q_x (i.e., n) is most easily performed using the Poisson summation formula. We find

$$\langle I^2\rangle_D = I_0^2 \frac{1}{3B} \sum_{p=-\infty}^{\infty} \frac{1}{\beta^2} \sum_{\zeta,\nu}{}' \int_{-\infty}^{\infty} dx\, e^{i2\pi px} \Big\{ \Big[\frac{1}{4E_c x^2+|\nu|} - \frac{1}{4E_c(x-2\varphi)^2+|\nu|} \Big] - \frac{2}{3} \frac{\mu}{B} \frac{1}{m} \Big[\frac{2\pi}{L} \Big]^2 \Big[\Big[\frac{x}{4E_c x^2+|\nu|} \Big]^2 - \Big[\frac{x-2\varphi}{4E_c(x-2\varphi)^2+|\nu|} \Big]^2 \Big] \Big\} , \tag{2.12}$$

using the (Boson) Matsubara frequencies $\nu=\omega-\zeta$. We do not exhibit the factor $\theta[-\zeta(\zeta+\nu)]$, which is implicit and indicated by the prime on the Matsubara sum. After shifting the integration variable in the Cooperon terms to eliminate the flux, one can combine the linear and quadratic contributions into a derivative,[42]

$$\langle I^2\rangle_D = I_0^2 \frac{1}{6E_c B}\sum_{p=1}^{\infty}\sin^2(2\pi p\varphi) \times \frac{1}{\beta^2}\sum_{\zeta,\nu}{}' \int_{-\infty}^{+\infty} dx\, e^{-i2\pi px}\frac{\partial^2}{\partial x^2} \times \ln\left[x^2+\frac{|\nu|}{4E_c}\right] . \tag{2.13}$$

Note that the Cooperon and diffusion terms have combined to give the correct flux dependence, as mentioned above Eq. (2.8). Correlations between different harmonics are absent to this order.[43] The restricted sum over ζ produces a factor of $\beta|\nu|/2\pi$. After performing the x integral, we find

$$\langle I^2\rangle_D = I_0^2 \frac{4\pi}{3B}\sum_{p=1}^{\infty} p\sin^2(2\pi p\varphi) \times \frac{1}{\beta}\sum_{\nu}\frac{|\nu|}{4E_c}\exp\left[-2\pi p\left[\frac{|\nu|}{4E_c}\right]^{1/2}\right] . \tag{2.14}$$

In the limit of zero temperature, the sum over ν turns into an integral $\sum_\nu \to (\beta/2\pi)\int d\nu$, which is easily evaluated. This yields for the typical total current at $T=0$,[44]

$$\langle I^2\rangle_D = \sum_{p=1}^{\infty}\langle I_p^2\rangle_D \sin^2(2\pi p\varphi) = 24\left[\frac{I_0}{M}\right]^2\left[\frac{M_{\rm eff}}{3\pi}\right]^2\sum_{p=1}^{\infty}\frac{1}{p^3}\sin^2(2\pi p\varphi) . \tag{2.15}$$

Note that the current amplitude does not depend on M; the harmonics are $\langle I_p^2\rangle_D^{1/2}=1.04p^{-3/2}I_0(l_{\rm el}/L) = 3.1p^{-3/2}e/\tau_D$ including a factor 2 for spin. For finite temperatures, Eq. (2.14) may be expressed as

$$\langle I^2(T)\rangle_D = \sum_{p=1}^{\infty} g\left[p^2\frac{k_B T}{E_c}\right]\langle I_p^2(T=0)\rangle_D \sin^2(2\pi p\varphi) \tag{2.16}$$

with $g(0)=1$ and

$$g(x)=\frac{\pi^6}{3}x^2\sum_{n=1}^{\infty} n\exp[-(2\pi^3 nx)^{1/2}] . \tag{2.17}$$

Equation (2.17) is further evaluated and discussed in Sec. III A.

C. Current-current correlation function and typical single-level current

In the Introduction we remarked that the total persistent current for a particular impurity configuration is a sensitive function of filling μ. Information about this dependence in the diffusive regime can be obtained from the correlation function

$$C(\delta E)=\langle I(\mu)I(\mu+\delta E)\rangle_D . \tag{2.18}$$

The calculation of this quantity is analogous to that for the typical total current. The difference in the chemical potential at which the two currents are calculated leads to an additional term $i\delta E\,\mathrm{sgn}(\omega-\zeta)$ in the denominators of the diffusion poles (2.7). Keeping the same diagrams as above one obtains instead of Eq. (2.14),

$$C(\delta E)=I_0^2\frac{4\pi}{3B}\sum_{p=1}^{\infty} p\sin^2(2\pi p\varphi)\,2\,\mathrm{Re}\frac{1}{\beta}\sum_{\nu>0}\frac{\nu}{4E_c}\exp\left[-2\pi p\left[\frac{\nu}{4E_c}+i\frac{|\delta E|}{4E_c}\right]^{1/2}\right] . \tag{2.19}$$

At $T=0$, the sum over ν turns into an integral which yields

$$C(\delta E)=8\left[\frac{I_0}{M}\right]^2\left[\frac{M_{\rm eff}}{3\pi}\right]^2\sum_{p=1}^{\infty}\frac{1}{p^3}\sin^2(2\pi p\varphi)\left[3-x\frac{\partial}{\partial x}\right]\{e^{-x}[(1+x)\cos x+x\sin x]\} , \tag{2.20}$$

where $x=2\pi p\sqrt{|\delta E|/8E_c}$. This result is further discussed in Sec. III A.

At zero temperature the correlation function (2.18) may be related to the current carried by the levels in an energy range δE, $i(\varphi,\delta E)=I(\mu+\delta E)-I(\mu)$,

$$C(\delta E)=\langle I^2\rangle_D-\tfrac{1}{2}\langle i^2,\delta E)\rangle_D . \tag{2.21}$$

Of particular interest is $\langle i^2(\varphi,\delta E)\rangle_D$ with $\delta E=\Delta_M$. Since typical quantities are found to be insensitive to averaging at constant N or constant μ, we argue that

$$\langle[I(\mu+\Delta_M)-I(\mu)]^2\rangle_D \approx \langle[I(N+1)-I(N)]^2\rangle_D . \tag{2.22}$$

This implies that $\langle i^2(\varphi,\Delta_M)\rangle_D$ is a good approximation of the typical single-level current $\langle i^2(\varphi)\rangle_D$. From Eqs. (2.15) and (2.20) one obtains

$$\langle i^2(\varphi)\rangle_D = 16\left[\frac{I_0}{M}\right]^2\left[\frac{M_{\rm eff}}{3\pi}\right]^2\sum_{p=1}^{\infty}\frac{1}{p^3}\sin^2(2\pi p\varphi)\left[3-u\frac{\partial}{\partial u}\right]\{1-e^{-u}[(1+u)\cos u+u\sin u]\}\ , \tag{2.23}$$

with $u=2\pi p(\Delta_M/8E_c)^{1/2}=\frac{1}{2}p(3\pi/M_{\rm eff})^{1/2}$. It is interesting to note the following scaling property of the typical single-level current at constant flux:[45]

$$\langle i^2(\varphi)\rangle_D=\left[\frac{I_0}{M}\right]^2 X\left[\left[\frac{3\pi}{M_{\rm eff}}\right]^{1/2};\varphi\right]. \tag{2.24}$$

The two factors reflect the two basic energy scales of the problem. $I_0/M=\Delta_M/\phi_0$ is determined by the level spacing Δ_M and the scaling function X involves the correlation energy $E_c\propto M_{\rm eff}\Delta_M$. A similar but less stringent scaling property is valid for the harmonics of the single-level current,

$$\langle i_p^2\rangle_D=\left[\frac{I_0}{M}\right]^2\left[\frac{M_{\rm eff}}{3\pi}\right]^{1/2} Y\left[\frac{p}{2}\left[\frac{3\pi}{M_{\rm eff}}\right]^{1/2}\right]. \tag{2.25}$$

Recalling that the harmonics of the energy levels e_p and the single-level currents are related by $i_p=(e/h)2\pi p e_p$, one establishes from Eq. (2.25) that the combination $M\langle e_p^2\rangle_D^{1/2}M_{\rm eff}^{1/4}$ as a function of the variable u should

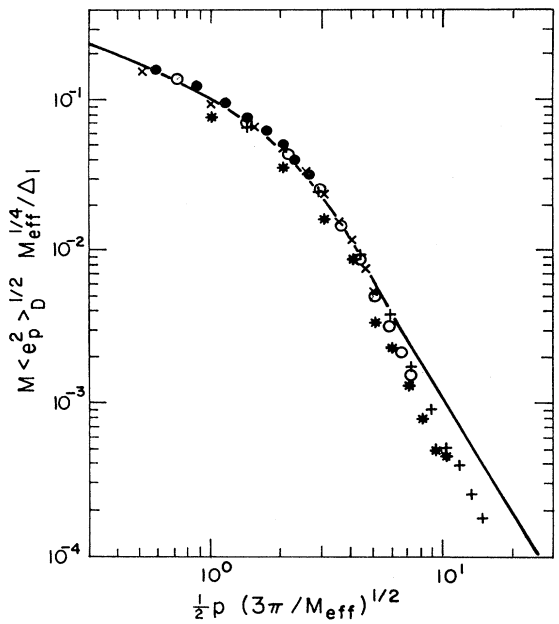

FIG. 3. Scaling plot of the harmonics of the typical single-level energies, $M\langle e_p^2\rangle_D^{1/2}M_{\rm eff}^{1/4}\Delta_1^{-1}$ vs $\frac{1}{2}p(3\pi/M_{\rm eff})^{1/2}$, as discussed in Sec. II C. The solid curve is the analytical result obtained from Eq. (2.25). The numerical results of Ref. 13, Fig. 1, for the Anderson model are also shown. The symbols correspond to those in Ref. 13 and refer to system size and disorder parameter: Stars $64\times4\times4$, $W/t=1.4$; plusses $64\times4\times4$, $W/t=2$; crosses $64\times8\times8$, $W/t=1.4$; circles $64\times8\times8$, $W/t=2$; black circles $64\times14\times14$, $W/t=1.4$. The deviations for large arguments reflect the breakdown of perturbation theory for $p\gtrsim M_{\rm eff}^{1/2}$.

scale. The same scaling behavior was found in numerical calculations.[13] In Fig. 3 we exhibit the scaling function obtained from (2.25) together with the corresponding numerical data of Fig. 1 in Ref. 13. The numerical data were obtained for the Anderson model with hopping matrix element t and random on-site disorder in the range $[-W/2,W/2]$. In terms of these parameters the elastic mean free path is given as $l_{\rm el}=A(t/W)^2$, where the constant A is not well known. In view of such uncertainties in relating the parameters of the numerical and analytical models, we fit the numerical data by independently scaling both coordinate axes by a constant factor. On the double-logarithmic scale, this merely shifts the position of the numerical curve. The good agreement gives us confidence that the replacement (2.22) is a reasonable approximation for calculating the typical single-level current. For large values of $u=\frac{1}{2}p(3\pi/M_{\rm eff})^{1/2}$ the numerical data points fall below the analytical result reflecting the breakdown of perturbation theory for $p\gtrsim M_{\rm eff}^{1/2}$.

For large $M_{\rm eff}$, one may replace the sum over p in Eq. (2.23) by an integral. In particular, for $\varphi=0.25$, the integral may be evaluated analytically and yields[2]

$$\langle i^2(\varphi=0.25)\rangle_D=\frac{\pi}{2}\left[\frac{I_0}{M}\right]^2\left[\frac{M_{\rm eff}}{3\pi}\right]. \tag{2.26}$$

The fact that the typical single-level current is $i^{\rm typ}\propto I^{\rm typ}/M_{\rm eff}^{1/2}$ corroborates the conjecture that effectively $M_{\rm eff}$ electrons contribute to the total current.

III. DISCUSSION OF THEORETICAL RESULTS

A. Dependence on filling, temperature, and inelastic scattering

The dependence of the total current on filling can be inferred from the correlation function $C(\delta E)=\langle I(\mu)I(\mu+\delta E)\rangle_D$, which is shown in Fig. 4 for $T=0$ and flux $\varphi=0.25$, cf. Eq. (2.20). The current-current correlations decrease exponentially with $\delta E^{1/2}$ on a scale given by the Thouless energy E_c and change sign for $\delta E\approx E_c$. This implies that the total current $I(\mu)$ for a particular impurity configuration changes sign with an average period of order E_c. The sign change of the correlation function reflects the anticorrelations in the spectrum, due to which effectively only the highest $M_{\rm eff}$ levels contribute to the total current. Similarly, one infers from Eq. (2.20) that the harmonics of the total current change sign with average period E_c/p^2.

This behavior of $C(\delta E)$ is consistent with the result that the characteristic temperature for the amplitude reduction of the typical total current (2.16) is the correlation energy E_c. For temperatures $k_BT\gtrsim E_c/p^2$, the

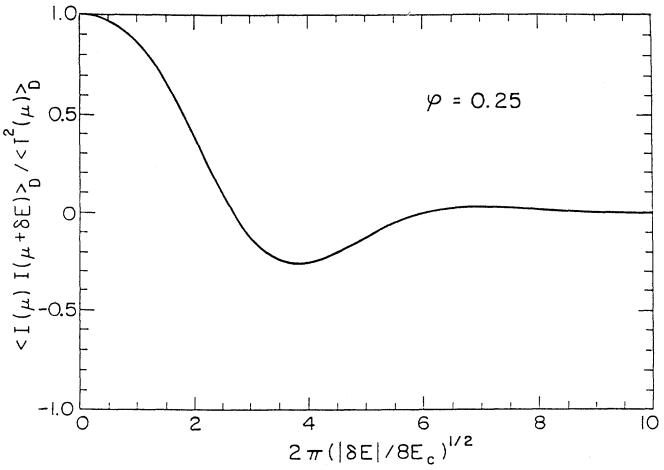

FIG. 4. Zero-temperature current-current correlation function $C(\delta E)/C(0)$ vs difference in filling $2\pi(|\delta E|/8E_c)^{1/2}$, from Eq. (2.20) for flux $\varphi=0.25$. The current changes sign as function of filling with an average period E_c.

Matsubara sum (2.17) is dominated by the first term resulting in

$$\langle I_p^2(T)\rangle_D \propto \langle I_p^2(T=0)\rangle_D T^2 \times\exp[-(2\pi^3 p^2 k_B T/E_c)^{1/2}] .$$

Note that $(2\pi^3 p^2 k_B T/E_c)^{1/2}=2\pi pL/l_T$, in terms of the thermal diffusion length l_T. In Fig. 5 we show the harmonics of the typical total current for an experimentally relevant temperature range. In this regime one needs to consider the complete sum (2.17) and finds numerically that the result may be well approximated by an exponential, when $0\lesssim k_B T\lesssim 2E_c/p^2$,

$$\langle I_p^2(T)\rangle_D \approx \langle I_p^2(T=0)\rangle_D \exp\left[-2p^2\frac{k_B T}{E_c}\right] , \quad (3.1)$$

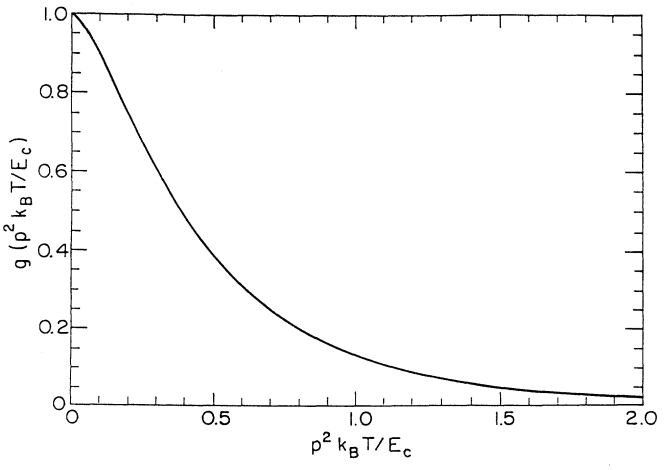

FIG. 5. Temperature dependence of $\langle I_p^2(T)\rangle_D/\langle I_p^2(T=0)\rangle_D$ from Eqs. (2.16) and (2.17). Over this range the temperature dependence is well approximated by an exponential, cf. Eq. (3.1).

with $\langle I_p^2(T=0)\rangle_D$ given by Eq. (2.15). Within this range of exponential temperature dependence the current falls off by more than one order of magnitude from its $T=0$ value. From Fig. 5 one also sees that at very small temperatures, $T\lesssim E_c/(20p^2)$, the curve flattens out. The scale of the temperature dependence of the pth harmonic is set by E_c/p^2, resulting in an increasingly rapid suppression of the higher harmonics. In an experiment one would therefore expect that only very few harmonics contribute significantly.

Inelastic scattering reduces the persistent current amplitude by destroying phase coherence. This can be taken into account phenomenologically by an additional $1/\tau_\Phi$ term in the denominators of the diffusion poles (2.7).[38] Here, τ_Φ denotes the phase breaking time. A straightforward extension of the calculation in Sec. II B shows that this leads to

$$\langle I_p^2\rangle_D^{1/2}=\sqrt{24}\frac{I_0}{M}\frac{M_{\rm eff}}{3\pi}\left[1+p\frac{L}{l_\Phi}+\frac{p^2}{3}\frac{L^2}{l_\Phi^2}\right]^{1/2} \times\exp\left[-p\frac{L}{2l_\Phi}\right] . \quad (3.2)$$

Here we introduced the phase-coherence length $l_\Phi=(D\tau_\Phi)^{1/2}$ appropriate for the diffusive regime. The pth harmonic contributes significantly only if the electronic phase-coherence length exceeds pL. In contrast to most weak-localization effects[38] the temperature dependence of the persistent current is due mainly to temperature averaging. Since the characteristic temperature for inelastic scattering is much larger than the Thouless energy E_c, inelastic scattering leads roughly to a temperature-independent reduction of the current amplitude below temperatures of order E_c/k_B. For this reason we neglected thermal averaging in Eq. (3.2).

B. Flux dependence

The flux dependence of the total and the single-level currents, $I(\varphi)$ and $i(\varphi)$, can be deduced from the corresponding correlation functions

$$F(\varphi,\varphi')=\langle I(\varphi)I(\varphi')\rangle_D=\sum_{p=1}^{\infty}\langle I_p^2\rangle_D\sin(2\pi p\varphi) \times\sin(2\pi p\varphi') , \quad (3.3)$$

$$f(\varphi,\varphi')=\langle i(\varphi)i(\varphi')\rangle_D=\sum_{p=1}^{\infty}\langle i_p^2\rangle_D\sin(2\pi p\varphi) , \times\sin(2\pi p\varphi') , \quad (3.4)$$

where we used that only equal harmonics are correlated. Inserting the Fourier amplitudes from Eqs. (2.15) and (2.23) and performing these sums numerically, one obtains the results shown in Figs. 6 and 7.

The behavior of $\langle I^2(\varphi)\rangle_D$ and $F(\varphi,\varphi')$ in Figs. 6(a) and 7(a) is dominated by the first harmonic, which is due to the $1/p^3$ decay of the higher harmonics. Therefore, a single-ring experiment that measures the total current for a particular impurity configuration should exhibit the fundamental ϕ_0-periodic harmonic, only slightly modified

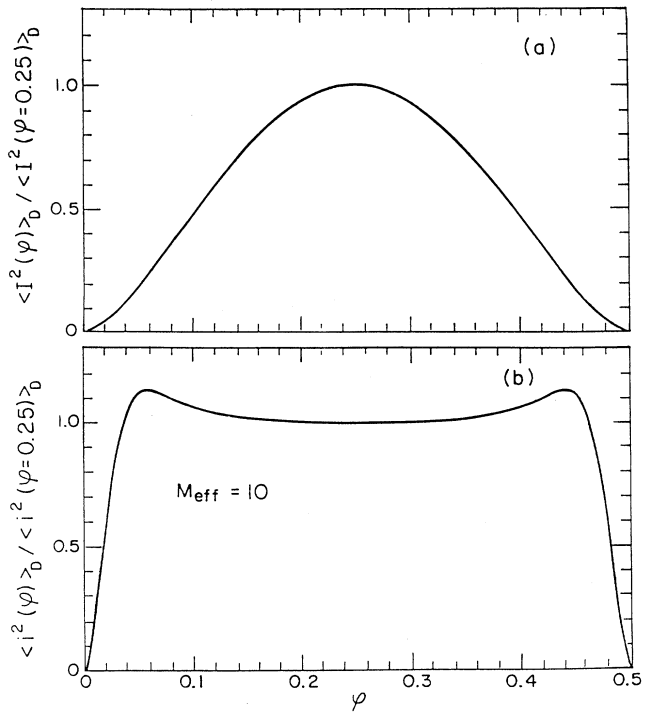

FIG. 6. Flux dependence of (a) typical total current (2.15) and (b) typical single-level current (2.23), normalized by the currents at $\varphi=0.25$.

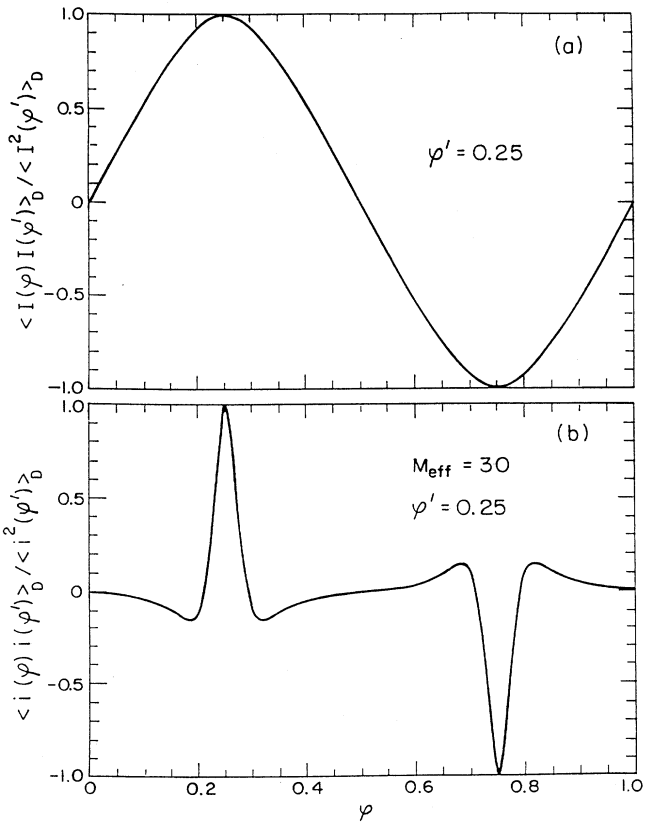

FIG. 7. Correlation function for (a) total and (b) single-level currents as functions of flux φ as defined by Eqs. (3.3) and (3.4), with $\varphi'=0.25$. The width of the peaks in (b) scales as $1/M_{\rm eff}^{1/2}$.

by higher harmonics. At finite temperatures, temperature corrections and inelastic scattering lead to a more rapid suppression of the higher harmonics, since the relevant energy scales as E_c/p^2 as discussed in the previous section.

The flux dependence of the single-level current is of theoretical interest because it gives information about level correlations. The flux dependence of the typical single-level current is plotted in Fig. 6(b) and shows structure on a scale $\Delta\varphi \propto 1/M_{\rm eff}^{1/2}$ near the symmetry points $\varphi=0,\frac{1}{2},1,\ldots$, where the single-level current is pinned to $i(\varphi)=0$. This is due to the crossover of the harmonics from $\langle i_p^2\rangle_D/I_0^2 \propto p$, when $p \ll M_{\rm eff}^{1/2}$, to $\langle i_p^2\rangle_D/I_0^2 \propto p^{-3}$, when $p \gg M_{\rm eff}^{1/2}$. This crossover is also the reason for the peak structure of the correlation function $f(\varphi,\varphi')$ shown in Fig. 7(b). The peak occurs at $\varphi=\varphi'$. Its width depends solely on $M_{\rm eff}$ and scales as $1/M_{\rm eff}^{1/2}$. This indicates that the single-level current as function of flux changes sign approximately $M_{\rm eff}^{1/2}$ times within one period ϕ_0. By contrast, in the weak-disorder limit the single-level current oscillates M times. The sign of $i(\varphi)$ is not correlated on flux scales larger than $1/M_{\rm eff}^{1/2}$. This explains the flux dependence of $\langle i^2(\varphi)\rangle_D$ in Fig. 6(b). Sufficiently far from the symmetry points the zeros of $i(\varphi)$ occur at random values of flux which depend on the particular disorder configuration. This gives rise to $\langle i^2(\varphi)\rangle_D=$const, when $\varphi > 1/M_{\rm eff}^{1/2}$. Residual correlations near the symmetry points lead to the two maxima.

It is interesting to note that the scaling variable for the harmonics is given by $p^2/M_{\rm eff}$. This suggests that higher harmonics might be scaling towards localized behavior (corresponding to $M_{\rm eff}\approx 1$). In the localized regime one expects exponential behavior as function of disorder in contrast to the power-law behavior found in the diffusion regime. For the higher harmonics one might then expect that this sets in already in the diffusive regime. Indeed, a simple argument using diffusion constant renormalization,[46] $D \to D/(1+iD/\omega\xi^2)$, yields an exponential reduction for the pth harmonic of the form $\exp(-p/M_{\rm eff})$.[2] However, a more complete treatment is needed.[47] The results in this section are qualitatively unaffected by this question as long as the crossover to localized behavior occurs for $p \gg M_{\rm eff}^{1/2}$.

C. Sensitivity to variations of the impurity potential

The persistent current is extremely sensitive to changes in the impurity configuration. Here we apply the theoretical approach developed in the context of the universal conductance fluctuations[28–30] (UCF) to the persistent current.

The sensitivity to changes in the disorder potential can be inferred from the correlation function between the currents $I(V)$ and $I(V')$ for impurity potentials V and V', respectively. The two potentials that differ by moving a certain number of impurities δN_i on average over a distance δR. The important diagrams that contribute to this correlation function are those in Fig. 2. Whereas the Green's functions are still given by Eq. (2.6), the denomi-

nators of the diffusion and Cooperon propagators (2.7) involve an additional term $\alpha/\tau_{\rm el}$.[28] Here,[28,30]

$$\alpha \approx \frac{\delta N_i}{N_i} f(k_F \delta R) , \tag{3.5}$$

where N_i is the number of impurities in the system and $f(x)$ is a function that is zero, when $x=0$, and approximately 1, when $x>1$. In terms of the parameter $\kappa=\alpha(L/l_{\rm el})^2$, one obtains for the correlation function

$$\langle I(V)I(V')\rangle_D = \sum_{p=1}^{\infty} [1+p(3\kappa)^{1/2}+p^2\kappa] \times \exp[-p(3\kappa)^{1/2}]\langle I_p^2\rangle_D \times \sin^2(2\pi p\varphi) . \tag{3.6}$$

The correlation function depends *exponentially* on the parameter κ. This is in contrast to the corresponding result for UCF,[28-30] where the conductance correlation function shows a power-law dependence on κ. In this sense the single-electron persistent current is even more sensitive to changes in the disorder configuration than UCF.

The parameter κ has the following physical interpretation.[30] Picture the diffusive motion of the electron as a random walk of step size $l_{\rm el}$. While traveling around the ring once, the electron visits $(L/l_{\rm el})^2$ sites of which a fraction $\alpha \propto \delta N_i/N_i$ is inequivalent between potentials V and V'. Thus, κ gives the number of sites visited by the electron that differ between the two potentials.

Finally, κ can be estimated in terms of physical parameters.[29] Using $1/l_{\rm el} \approx (N_i/V)\sigma$, where V is the volume of the system, and approximating the cross section σ of a single scatterer by $\sigma \approx 4\pi k_F^{-2}$, one finds

$$\kappa \approx \left[\frac{\delta N_i}{N_i}\right]\left[\frac{L}{l_{\rm el}}\right]^2 f(k_F\delta R) \approx \frac{\delta N_i}{M_{\rm eff}}\left[\frac{L}{l_{\rm el}}\right] f(k_F\delta R) . \tag{3.7}$$

Thus for parameters characteristic of recent persistent-current experiments,[4,12] we conclude that $\kappa \approx \delta N_i f(k_F\delta R)$ up to a numerical prefactor of order 1. Hence, the correlation function $\langle I(V)I(V')\rangle_D$ is strongly suppressed even if the impurity configurations V and V' differ only by one impurity (or a few impurities, depending on the numerical prefactor) moved over a distance of order k_F^{-1}. Consequently, the signs of the persistent currents for two such configurations are essentially uncorrelated.

It is appropriate to include a note of caution. The calculation was done for the current at constant chemical potential. There could be correlations in the spectrum that make the current at constant N less sensitive to changes in the disorder configuration. We believe that this is not significant in the diffusive regime.

D. Application to experiment

The persistent-current effect in a single-ring experiment is expected to contain both the single-electron and the interaction-induced contribution. However, the two contributions can be partially separated due to their different flux periodicities.

Only the single-electron persistent current contributes to the first, ϕ_0-periodic harmonic (generally, all odd harmonics).[37] Thus the typical amplitude of the first harmonic of the persistent current is given by the first harmonic of the typical total current $\langle I_{p=1}^2\rangle_D^{1/2}$ in (2.15). The first harmonic of the persistent current is a pure fluctuation effect and consequently its sign is a random function of impurity configuration and filling (for changes of N larger than $M_{\rm eff}$.) The extreme sensitivity of the single-electron persistent current to changes in the impurity configuration [cf. Eq. (3.6)] suggests that sign measurements on the same sample before and after thermal cycling to room temperature are statistically independent. Within the model presented here, the first harmonic of the typical total current $\langle I_{\rm p=1}^2\rangle_D^{1/2}$ can be expressed as a one-parameter formula,

$$\langle I_{p=1}^2\rangle_D^{1/2} = 2(E_c/\phi_0)\exp(-k_BT/E_c) , \tag{3.8}$$

where we have included a factor of 2 for spin [cf. Eq. (3.1)].

Both single-electron and interaction-induced persistent currents contribute to the second harmonic (generally, all even harmonics) of the persistent current. The disorder average of the second harmonic is dominated by the interaction-induced effect. (Here we neglect the small disorder average of the single-electron current in the canonical ensemble.[8-11]) The average current is paramagnetic (diamagnetic) for small flux values if the electron-electron interaction is repulsive (attractive).[5] The single-electron contribution [cf., Eq. (3.1)] leads to sample-specific fluctuations around this average current. The relative size of single-electron and interaction-induced contributions to the second harmonic is determined by the coupling constant λ (cf., Sec. I). According to one estimate for λ,[36] one expects the single-electron contribution to be larger than the interaction-induced current. The signs of the first and the second harmonics of the current are uncorrelated.

Recently, a single-ring experiment has been performed by Chandrasekhar *et al.*[12] Whereas the measured temperature dependence of the first harmonic is consistent with the theoretical prediction of the model discussed in this paper, the amplitude appears to be larger than theoretically expected by one to two orders of magnitude. At this time the experiment does not make a definite statement about the sign of the effect. Two sign measurements on different samples yielded paramagnetic behavior at small flux values. The sign did not change upon thermal cycling to room temperature, which was performed once.[48] Accepting the sign measurements upon different cool downs from room temperature as statistically independent, a positive sign has been found three times. This cannot yet exclude the possibility that the measured current has a random sign as expected according to current theory.

The experiment[12] was performed on two gold rings of diameters 2.4 and 4 μm and a gold square loop of dimen-

sions 1.4×2.6 μm. All loops had a linewidth of approximately 90 nm and a thickness of 60 nm. In Ref. 12 the elastic mean free path l_{el} was estimated by measuring the resistance per square $R_\square$ of gold films fabricated by the same procedure as the loops. We note that via the Einstein relation for conductivity, one can directly express the diffusion constant in terms of $R_\square$, $D=[e^2N(0)R_\square L_z]^{-1}$. Here $L_z=60$ nm is the thickness of the film. Using the measured value $R_\square=0.2$ Ω one finds $D=3.25\times10^{-2}$ m^2/s. There has been some confusion about which dimensionality d should enter the diffusion constant $D=v_F l_{el}/d$. Determining D directly circumvents that problem. (One should use $d=3$.) Thus, Eq. (3.8) for $T=0$ predicts 0.28, 0.12, and 0.25 nA, respectively, for the typical magnitude of the ϕ_0-periodic contribution to the currents of the three loops. The corresponding experimental values[12] are 30 ± 15, 3 ± 2, and 6 ± 2 nA, measured at the lowest experimental temperature of $T=4.5$ mK. These are larger than the theoretical predictions by factors between 25 and 100. One may note that for the multiring experiment of Lévy *et al.*,[4] it was also found that theoretical estimates for the interaction-induced contribution to the average current were smaller by an order of magnitude than the experimental result; however the latter theoretical estimate appears less definitive because of the difficulties in determining the effective coupling constant λ accurately.

IV. SUMMARY

We have presented a model for mesoscopic persistent currents carried by small normal-metal rings threaded by a constant magnetic flux. The theory applies to single-ring experiments. The approach applies to the quantum diffusive regime and utilizes the diffusion and Cooperon approximation. We have focused on the case of noninteracting electrons in disordered rings.

Assuming a Gaussian current distribution in the diffusive regime, we have characterized the magnitude of the effect by three quantities, the average and typical total currents $\langle I\rangle_D$ and $\langle I^2\rangle_D^{1/2}$, which are of experimental relevance, and the typical single-level current $\langle i^2\rangle_D^{1/2}$, which is of theoretical interest. In this paper we considered the latter two quantities. We have also studied those current-current correlation functions which describe the dependences of the persistent current on flux, filling, and disorder configuration. We infer that before disorder averaging the total current as function of filling μ (number of electrons N) exhibits sign changes with an average period $E_c\propto M_{\rm eff}\Delta_M$ ($M_{\rm eff}$) versus period $\Delta_1\propto M\Delta_M$ (M) in the ballistic regime. This implies that the persistent current is effectively a few-electron effect to which of the order of $M_{\rm eff}$ electrons contribute. The sign of the total current becomes uncorrelated for differences in filling larger than E_c. The flux dependence in a single-ring experiment is dominated by the ϕ_0-periodic first harmonic. The signs of the single-electron persistent currents for two disorder configurations that differ by displacing a few impurities by a distance of order of the Fermi wavelength are essentially uncorrelated.

The results are relevant to persistent-current experiments on single mesoscopic metal rings. Within our model the first ϕ_0-periodic harmonic of the persistent current in a single ring is solely due to the single-electron effect. Both the collective and the single-electron effect contribute to the second harmonic of the current. Thus, whereas the first harmonic is a pure fluctuation effect, the second harmonic contains a component that survives the disorder average. The discrepancies between the theoretical predictions of the model presented in this paper and the experimental results of the single-ring experiment by Chandrasekhar *et al.* emphasize the need for further experimental and theoretical work. It would be important to establish whether the sign of the experimental current is indeed sensitive to minute changes in the impurity configuration. Theoretically the influence of electron-electron interactions on the variance of the current needs to be understood.

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[1]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[2]H. F. Cheung, E. K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989); E. K. Riedel, H. F. Cheung, and Y. Gefen, Phys. Scripta **T25**, 357 (1989).

[3]H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989).

[4]L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990); L. P. Lévy, Physica B **169**, 245 (1991).

[5]V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).

[6]G. Montamblaux, H. Bouchiat, D. Sigeti, and R. Friesner, Phys. Rev. B **42**, 7647 (1990).

[7]E. K. Riedel, in *Quantum Coherence in Mesoscopic Systems,* edited by B. Kramer (Plenum, New York 1991), p. 261.

[8]Y. Imry, in *Quantum Coherence in Mesoscopic Systems,* (Ref. 7), p. 221.

[9]A. Schmid, Phys. Rev. Lett. **66**, 80 (1991).

[10]F. von Oppen and E. K. Riedel, Phys. Rev. Lett. **66**, 84 (1991).

[11]B. L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).

[12]V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen,

W. J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).
[13]H. Bouchiat, G. Montambaux, and D. Sigeti, Phys. Rev. B **44**, 1682 (1991).
[14]N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).
[15]See, for example, F. Bloch, Phys. Rev. **137**, A787 (1965); **166**, 415 (1968); M. Schick, *ibid.* **166**, 401 (1968); L. Gunther and Y. Imry, Solid State Commun. **7**, 1391 (1969); R. Landauer (unpublished); Y. Imry and L. Gunther (unpublished).
[16]Y. Imry, in *Directions in Condensed Matter Physics,* edited by G. Grinstein and G. Mazenko (World Scientific, Singapore, 1986), p. 101.
[17]S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).
[18]See also, M. Büttiker, Phys. Rev. B **32**, 1846 (1985); R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985); M. Büttiker, Ann. N.Y. Acad. Sci. **480**, 194 (1986).
[19]O. Entin-Wohlman and Y. Gefen, Europhys. Lett. **8**, 477 (1989).
[20]H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih, Phys. Rev. B **37**, 6050 (1988).
[21]H. F. Cheung, Y. Gefen, and E. K. Riedel, IBM J. Res. Dev. **32**, 359 (1988).
[22]B. L. Altshuler and B. Z. Spivak, Zh. Eksp. Teor. Fiz. **(2**, 609 (1987) [Sov. Phys. JETP **65**, 343 (1987)].
[23]B. L. Altshuler, D. E. Khmelnitskii, and B. Z. Spivak, Solid State Commun. **48**, 841 (1983).
[24]F. Bloch, Phys. Rev. B **2**, 109 (1970).
[25]M. Ya. Azbel and O. Entin-Wohlman, J. Phys. A **22**, L957 (1989); Phys. Rev. B **41**, 395 (1990).
[26]R. L. Schult, M. Stone, H. W. Wyld, and D. G. Ravenhall, Superlatt. Microstruct. **11**, 73 (1992).
[27]P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[28]B. L. Altshuler and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 553 (1986) [JETP Lett. **42**, 447 (1986)].
[29]S. Feng, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. **56**, 1960 (1986); **56**, 2772(E) (1986).
[30]S. Feng, in *Mesoscopic Phenomena in Solids,* edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p. 107.
[31]H. F. Cheung and E. K. Riedel, cf. Ref. 7; (unpublished).
[32]D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977); Phys. Rep. **132**, 249 (1974).
[33]Y. Imry, Europhys. Lett. **1**, 249 (1986).
[34]H. Bouchiat suggested on the basis of Figs. 9 and 10 in Ref. 21 that the result may also apply to multichannel rings (private communication).
[35]B. L. Altshuler, A. G. Aronov, and A. Y. Zyuzin, Zh. Eksp. Teor. Fiz. **84**, 1525 (1983) [Sov. Phys. JETP **57**, 889 (1983)]; B. L. Altshuler, D. E. Khmelnitskii, and B. Z. Spivak, Solid State Commun. **48**, 841 (1983).
[36]U. Eckern, Z. Phys. B **82**, 393 (1991).
[37]An attempt to obtain a contribution to the variance of the current due to electron-electron interactions was made by U. Eckern and A. Schmid, Europhys. Lett. **18**, 457 (1992). However, we find that these authors do not include all relevant diagrams and that the contribution of their diagram is cancelled to leading order in $\hbar/(\mu\tau_{el})$. The diagram that has been omitted represents disorder-induced correlations between the second-order Hartree contribution and the single-electron contribution to the persistent current.
[38]See, e.g., B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 4; G. Bergmann, Phys. Rep. **107**, 1 (1984); P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[39]The interaction-induced contribution to the average current (Ref. 5) can also be obtained from the impurity-averaged version of our Eq. (2.4) by keeping the leading Hartree-Fock corrections to the Green's function.
[40]The quadratic terms in D and K were not considered in Ref. 2.
[41]This does not apply to higher-order diagrams.
[42]Expression (2.13) can be viewed as a second derivative of the variance of the free energy with respect to flux. The diagrams for the free energy involve only a single diffusion or Cooperon.
[43]This has also been inferred from numerical simulations, see Ref. 13.
[44]Strictly speaking, this expression is only valid for harmonics with $p \lesssim M_{\rm eff}^{1/2}$. Higher harmonics require a nonperturbative treatment of the so-called zero mode. For the average current such a calculation has been reported in A. Altland, S. Iida, A. Müller-Groeling, and H. A. Weidenmüller, Europhys. Lett. **20**, 155 (1992).
[45]F. von Oppen and E. K. Riedel, Superlatt. Microstruct. **11**, 179 (1992).
[46]See, e.g., D. Vollhardt and P. Wölfle, Phys. Rev. B **22**, 4666 (1980); K. B. Efetov, Adv. Phys. **32**, 53 (1983).
[47]I. V. Lerner, in *Quantum Coherence in Mesoscopic Systems* (Ref. 7), p. 279.
[48]V. Chandrasekhar (private communication).