

## Nonlinear response of type-II superconductors in the mixed state in slab geometry

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The nonlinear response of a type-II superconductor of finite thickness arising from vortex motion is investigated. The results of the phenomenological theory extend the complex rf magnetic permeability and conductivity to a specific regime of nonlinear response. Explicit expressions for the complex penetration depths, amplitudes, fields, and densities for the second-harmonic response with various boundary conditions are presented.

### I. INTRODUCTION

Moving vortices are well known to influence the electrodynamic response functions describing the behavior of type-II superconductors.<sup>1-4</sup> These complex-valued response functions include the magnetic permeability, surface impedance, and self- and mutual inductance.<sup>1-7</sup> The vortex motion generally influences both the real and imaginary parts of the response function. In this work a generalization of the complex rf magnetic permeability and conductivity are considered for a specific regime of nonlinear response. In the limit of linear response, the usual permeability is recovered, whose real part characterizes the superconductor screening and whose imaginary part characterizes the dissipation in the superconductor.

We rely on the description of the coupled nonlinear electrodynamics of type-II superconductors in the mixed state presented in Ref. 8. In the framework of this theory, the vortex lattice is effectively treated as an inviscid fluid. (In the vortex continuity equation the vortex areal density is the analog of a fluid mass density per unit volume.) In this context, the wave-vector-dependent tilt and compressional moduli of the flux-line lattice are approximately equal, while the shear modulus is assumed to be negligible. Such an approximation breaks down for large enough wave numbers.<sup>9</sup> The key elements of the theory of Ref. 8 required here are first recalled and then applied to finite-thickness superconductor geometry. We concentrate on that of a superconducting slab of thickness  $d$  with various boundary conditions.

The semi-infinite geometry treated in Ref. 8 is probably best suited to comparison with surface impedance experiments. In particular, the results of Ref. 8 may apply to experiments similar to those described in Ref. 10 when vortices have entered the sample. This description would hold in an intermediate-field region, below a high-field region where hysteresis losses dominate.

By extending the theory to slab geometry it may be easier to relate the results to those of vibrating reed and magnetic permeability experiments. As before, key features of our theory of nonlinear response include the appearance of higher-order harmonics in the time dependence of the electrodynamic fields and of additional complex penetration depths in the spatial dependence of these fields. We recall that the response functions are given in terms of complex penetration depths.<sup>5-7,11-13</sup>

Our theory, which has been shown to be useful for

penetration problems (e.g., Refs. 14 and 15), has recently been applied to transmission problems in linear response.<sup>16,17</sup> In this context the complex response functions include the rf conductivity and transmission and reflection coefficients.

The results of our theory of nonlinear response should be valid in a certain range of microwave field. The driving field should be large enough that a higher harmonic is detectable, yet small enough that the critical state is avoided. Presumably then the driving current density should be well below the critical current density  $J_c$ .

In this paper we concentrate on results for the second harmonic, obtained from a perturbation analysis, in slab geometry. The reasons for this are twofold. Nonlinearities occur in the vortex equation of motion, including the viscous drag coefficient and pinning potential (making the dynamic mobility<sup>5-7</sup> nonlinear). At higher values of the fields, such nonlinearities can be expected to significantly alter the results. In addition, the amplitudes of the higher-order harmonics are expected to decrease rapidly with both the driving and static field strengths.<sup>8</sup> Because of this dependence, the detection of high-order harmonics can be expected to be difficult.

In our theory the governing equations include Maxwell's equations, electric and magnetic constitutive relations, the London equation in the presence of vortices, the two-fluid equation for the total current density, and a vortex equation of motion.<sup>5-7,12</sup> The latter equation takes the form  $\mathbf{v} = \tilde{\mu}_v \mathbf{f}$  where  $\tilde{\mu}_v$  is the complex-valued dynamic vortex mobility<sup>5-7,16</sup> and  $\mathbf{f}$  is the Lorentz force per unit length. We recall that the use of the dynamic mobility allows for pinning, flux flow, and thermal activation of vortices to be included all at once.<sup>5-7,12,13</sup> As an example, in the presence of a viscous drag and a linear pinning restoring force, the expression for the dynamic mobility is<sup>5-7,16</sup>  $\tilde{\mu}_v(\omega, B, T) = (1 + i\kappa_p/\eta\omega)/\eta$ , where  $\eta$  is the viscous drag coefficient and  $\kappa_p$  the pinning force constant.

In Ref. 12 it was shown that not only do the governing equations in our model for linear response close, but that they may be combined into a single governing vector partial differential equation for one of the coupled electrodynamic fields. The closure feature is connected with the self-consistent inclusion of the coupling of the vortex displacement and current density.<sup>5-7,12</sup> In Ref. 8 a similar result was reached for nonlinear response. There, retaining bilinear field nonlinearity in the vortex continuity equation, it was possible to again combine the governing

differential equations. The resulting vector partial differential equation for the total magnetic induction  $\mathbf{B}(\mathbf{x}, t)$ , given below, has only bilinear nonlinearity. In writing this equation we neglect the field dependence of the London penetration depth  $\lambda$  and normal-fluid conductivity  $\sigma_{\text{nf}}=1/\rho_{\text{nf}}$ . The latter quantity provides the connection between the total electric field and normal current density:  $\mathbf{J}_n=\sigma_{\text{nf}}\mathbf{E}$ . We further restrict attention to an isotropic superconductor, although the linear rf response of an anisotropic superconductor has recently been treated with our theory.<sup>18</sup> The single governing equation for  $\mathbf{B}$  in nonlinear response is then<sup>8</sup>

$$\lambda^2 \nabla^2 \dot{\mathbf{B}} - \lambda^2 D_{\text{nf}}^{-1} \ddot{\mathbf{B}} - \dot{\mathbf{B}} = (\phi_0 \bar{\mu}_v / \mu_0) \times \nabla \times (\{ \mathbf{B} - \lambda^2 \nabla^2 \mathbf{B} + D_{\text{nf}}^{-1} \lambda^2 \dot{\mathbf{B}} \} \times [(\nabla \times \mathbf{B}) \times \hat{\mathbf{B}}_0]), \quad (1)$$

where  $\hat{\mathbf{B}}_0$  is the local vortex direction,  $D_{\text{nf}}=1/\mu_0\sigma_{\text{nf}}$  is the normal-fluid diffusion constant, and  $\phi_0$  is the flux quantum. The bilinear terms on the right-hand side of Eq. (1) are due to the motion of the vortices, the right-most factors coming from the Lorentz force.

Due to the inclusion of a normal current-density contribution, our results hold continuously through the transition temperature. When  $T_c$  is reached, the right-hand side of Eq. (1) is no longer present, the London penetration depth diverges, and integrating the left-hand side once with respect to time yields the normal-state diffusion equation. After solving Eq. (1) subject to various boundary conditions we discuss a special case where we expect the nonlinear effect considered here to be most pronounced.

## II. SYMMETRIC TWO-SIDED NONZERO BOUNDARY CONDITION

We consider the superconducting slab to occupy the region  $|x| \leq d/2$ , with vortices along the  $z$  direction,  $\hat{\mathbf{B}}_0=\hat{\mathbf{z}}$ , and  $\mathbf{B}=B(x, t)\hat{\mathbf{z}}$ . Then Eq. (1) reduces to

$$\lambda^2 \partial_{xx} \dot{B} - \lambda^2 D_{\text{nf}}^{-1} \ddot{B} - \dot{B} = -(\phi_0 \bar{\mu}_v / \mu_0) \partial_x \{ [B - \lambda^2 \partial_{xx} B + D_{\text{nf}}^{-1} \lambda^2 \dot{B}] \partial_x B \}, \quad (2)$$

where  $\partial_x \equiv \partial/\partial x$ . The solution of this equation can be developed in infinite-order perturbation theory,<sup>8</sup> but we truncate the corresponding expansion at the second harmonic, as discussed above. With a driving field of equal magnitude applied at both surfaces,

$$B_1(\pm d/2, t) = b_0 \exp(-i\omega t),$$

we take

$$B(x, t) = B_0 + B_1(x, t) + B_2(x, t), \quad (3)$$

where  $B_0 = \text{constant}$ ,

$$B_1(x, t) = b_0 e^{-i\omega t} \frac{\cosh(x/\tilde{\lambda})}{\cosh(d/2\tilde{\lambda})}, \quad (4)$$

and the complex penetration depth in linear response is<sup>5-7,12</sup>

$$\tilde{\lambda} = \frac{[\lambda^2 + (i/2)\delta_{\text{vc}}^2]^{1/2}}{(1 - 2i\lambda^2\delta_{\text{nf}}^{-2})^{1/2}}. \quad (5)$$

In Eq. (5),  $\delta_{\text{vc}}^2 = 2B_0\phi_0\bar{\mu}_v/\mu_0\omega$  is the square of the complex effective skin depth associated with vortex motion and creep and  $\delta_{\text{nf}}^2 = 2\rho_{\text{nf}}/\mu_0\omega$  is the square of the normal-fluid skin depth.<sup>5-7,12</sup> The second harmonic is found from the equation<sup>8</sup>

$$\lambda^2 \partial_{xx} \dot{B}_2 - \lambda^2 D_{\text{nf}}^{-1} \ddot{B}_2 - \dot{B}_2 + (\omega/2)\delta_{\text{vc}}^2 \partial_{xx} B_2 = -(\phi_0 \bar{\mu}_v / \mu_0) \partial_x \{ [B_1 - \lambda^2 \partial_{xx} B_1 + D_{\text{nf}}^{-1} \lambda^2 \dot{B}_1] \partial_x B_1 \}. \quad (6)$$

The solution is sought in the form

$$B_2(x, t) = B_{20} \exp(-2i\omega t) B_2(x), \quad (7)$$

where

$$B_2(x) = \frac{\cosh(2x/\tilde{\lambda})}{\cosh(d/\tilde{\lambda})} - \frac{\cosh(x/\tilde{\lambda}_2)}{\cosh(d/2\tilde{\lambda}_2)} \quad (8)$$

satisfies  $B_2(x = \pm d/2) = 0$ . In (8) the first term is a particular solution of (6) and the second term is a solution of the homogeneous part of this equation. We then find

$$\tilde{\lambda}_2^2 = \frac{\lambda^2 + (i/4)\delta_{\text{vc}}^2}{1 - 4i\lambda^2\delta_{\text{nf}}^{-2}} \quad (9)$$

for the square of the second complex penetration depth and

$$B_{20} = -\frac{b_0^2}{8B_0} \frac{\cosh(d/\tilde{\lambda})}{\cosh^2(d/2\tilde{\lambda})} \frac{\tilde{\lambda}^{-2} \delta_{\text{vc}}^4 (1 - 2i\lambda^2\delta_{\text{nf}}^{-2})}{[3\lambda^2 + (i/2)\delta_{\text{vc}}^2 - 4i\lambda^4\delta_{\text{nf}}^{-2}]} \quad (10)$$

for the second-harmonic amplitude. The ratio of cosh's in Eq. (10) shows the effect of finite superconductor thickness.

Now, via Eq. (3),  $B$  is known, and the current density follows from Ampère's law,  $\mathbf{J} = -(\hat{\mathbf{y}}/\mu_0)\partial B/\partial x$ , where

$$J_y(x, t) = -\frac{b_0}{\mu_0 \tilde{\lambda}} e^{-i\omega t} \frac{\sinh(x/\tilde{\lambda})}{\cosh(d/2\tilde{\lambda})} - \frac{B_{20}}{\mu_0} e^{-2i\omega t} \times \left[ \frac{2}{\tilde{\lambda}} \frac{\sinh(2x/\tilde{\lambda})}{\cosh(d/\tilde{\lambda})} - \frac{1}{\tilde{\lambda}_2} \frac{\sinh(x/\tilde{\lambda}_2)}{\cosh(d/2\tilde{\lambda}_2)} \right]. \quad (11)$$

By use of Faraday's law for this geometry, we obtain the electric field,

$$E_y(x, t) = i\omega \tilde{\lambda} b_0 e^{-i\omega t} \frac{\sinh(x/\tilde{\lambda})}{\cosh(d/2\tilde{\lambda})} + B_{20} 2i\omega e^{-2i\omega t} \times \left[ \frac{\tilde{\lambda}}{2} \frac{\sinh(2x/\tilde{\lambda})}{\cosh(d/\tilde{\lambda})} - \tilde{\lambda}_2 \frac{\sinh(x/\tilde{\lambda}_2)}{\cosh(d/2\tilde{\lambda}_2)} \right]. \quad (12)$$

Equations (11) and (12) show that the relation<sup>16</sup>  $\bar{\sigma} = i/\mu_0\omega\tilde{\lambda}^2$  for the rf complex conductivity no longer holds in nonlinear response.

The average rf magnetic induction in the superconductor is

$$\langle b \rangle = \frac{1}{d} \int_{-d/2}^{d/2} b(x, t) dx, \quad b = B - B_0. \quad (13)$$

We find

$$\langle b \rangle = b_0 e^{-i\omega t} (2\tilde{\lambda}/d) \tanh(d/2\tilde{\lambda}) + B_{20} e^{-2i\omega t} \times \left[ \frac{\tilde{\lambda}}{d} \tanh(d/\tilde{\lambda}) - \frac{2\tilde{\lambda}_2}{d} \tanh(d/2\tilde{\lambda}_2) \right], \quad (14)$$

generalizing the result for the linear response magnetic permeability  $\tilde{\mu}$ .<sup>6</sup> Alternatively, Eqs. (11) and (12) can be

$$u_x(x,t) = -\frac{i}{2} \frac{\delta_{vc}^2}{B_0} \left\{ \frac{b_0}{\tilde{\lambda}} e^{-i\omega t} \frac{\sinh(x/\tilde{\lambda})}{\cosh(d/2\tilde{\lambda})} - \frac{B_{20}}{2} e^{-2i\omega t} \left[ \frac{2}{\tilde{\lambda}} \frac{\sinh(2x/\tilde{\lambda})}{\cosh(d/\tilde{\lambda})} - \frac{1}{\tilde{\lambda}_2} \frac{\sinh(x/\tilde{\lambda}_2)}{\cosh(d/2\tilde{\lambda}_2)} \right] \right\}. \quad (15)$$

The corresponding vortex magnetic field  $B_v(x,t)$  and areal density  $n(x,t)$  can be found from the relation<sup>8</sup>  $B_v = n\phi_0 = B - \lambda^2 \partial_{xx} B + D_{nf}^{-1} \lambda^2 \dot{B}$ . The result is

$$B_v(x,t) = B_0 + b_0 \frac{i}{2} \frac{\delta_{vc}^2}{\tilde{\lambda}^2} e^{-i\omega t} \frac{\cosh(x/\tilde{\lambda})}{\cosh(d/2\tilde{\lambda})} + B_{20} e^{-2i\omega t} \left[ (1 - 4\lambda^2 \tilde{\lambda}^{-2} - 4i\lambda^2 \delta_{nf}^{-2}) \frac{\cosh(2x/\tilde{\lambda})}{\cosh(d/\tilde{\lambda})} - \frac{i}{4} \frac{\delta_{vc}^2}{\tilde{\lambda}_2^2} \frac{\cosh(x/\tilde{\lambda}_2)}{\cosh(d/2\tilde{\lambda}_2)} \right]. \quad (16)$$

The vortex motion induced electric field  $\mathbf{E}_v = \mathbf{B}_v \times \mathbf{v}$  is given by  $E_{vy} = v_x B_v$ .

### III. ONE-SIDED NONZERO BOUNDARY CONDITION

We now turn to the situation where the superconducting slab occupies the region  $0 \leq x \leq d$  and the applied rf field  $B_1(x,t) = B_1(x) \exp(-i\omega t)$  satisfies  $B_1(x=0) = 0$  and  $B_1(x=d) = b_0$ . Then we have for the linear response term

$$B_1(x) = b_0 \sinh(x/\tilde{\lambda}) / \sinh(d/\tilde{\lambda}). \quad (17)$$

Seeking a solution as before in the form of Eqs. (3) and (7) gives for the spatial dependence of the second harmonic

$$B_2(x) = \frac{\cosh(2x/\tilde{\lambda})}{\cosh(2d/\tilde{\lambda})} - \frac{\cosh(x/\tilde{\lambda}_2)}{\cosh(2d/\tilde{\lambda}_2)} + \frac{\sinh(x/\tilde{\lambda}_2)}{\sinh(d/\tilde{\lambda}_2)} \left[ \frac{\cosh(d/\tilde{\lambda}_2)}{\cosh(2d/\tilde{\lambda}_2)} - 1 \right], \quad (18)$$

where now in the amplitude  $B_{20}$  the ratio of cosh's in Eq. (10) is replaced by the ratio  $\cosh(2d/\tilde{\lambda})/\sinh^2(d/\tilde{\lambda})$ . It is seen that Eq. (18) satisfies  $B_2(x=0) = B_2(x=d) = 0$ . The full solution for the magnetic induction is provided by Eqs. (3), (7), (17), and (18). As before, the other elec-

used to compute  $\mathbf{E} \cdot \mathbf{J}$  losses in the superconductor, thereby generalizing the calculation of  $\mu'' = \text{Im}(\tilde{\mu})$  to a non-linear regime.

The vortex velocity field can be found from the relation  $\mathbf{v} = \phi_0 \tilde{\mu}_v \mathbf{J} \times \hat{\mathbf{B}}_0$ . Upon integration, the vortex displacement field is

trodynamic fields can be computed and then various derived quantities can be found.

### IV. ASYMMETRIC TWO-SIDED NONZERO BOUNDARY CONDITION

Lastly, we consider a more general boundary condition for the driving field,

$$B_1(x=0) = \gamma b_0, \quad B_1(x=d) = (1-\gamma)b_0, \quad (19)$$

where  $0 \leq \gamma \leq 1$ . This extends the previous results to less symmetric situations. Here we find for  $B_1(x,t) = \exp(-i\omega t) B_1(x)$

$$B_1(x) = b_c \cosh(x/\tilde{\lambda}) + b_s \sinh(x/\tilde{\lambda}), \quad (20)$$

where the coefficients are given in terms of  $\gamma$ ,  $b_0$ , and  $d$  as

$$b_c = \gamma b_0, \quad b_s = b_0 [1 - \gamma(1 + \cosh(d/\tilde{\lambda}))] / \sinh(d/\tilde{\lambda}). \quad (21)$$

The solution for the second harmonic, Eq. (7), is

$$B_2(x) = (b_c^2 + b_s^2) \cosh(2x/\tilde{\lambda}) + 2b_c b_s \sinh(2x/\tilde{\lambda}) - (b_c^2 + b_s^2) \cosh(x/\tilde{\lambda}_2) + A_2 \sinh(x/\tilde{\lambda}_2), \quad (22)$$

where

$$A_2 = -\{2b_c b_s \sinh(2d/\tilde{\lambda}) + (b_c^2 + b_s^2) [\cosh(2d/\tilde{\lambda}) - \cosh(d/\tilde{\lambda}_2)]\} / \sinh(d/\tilde{\lambda}_2). \quad (23)$$

The amplitude  $B_{20}$  differs from Eq. (10) in the replacement of the ratio of cosh's by 2. With this convention for  $B_{20}$ , it can be checked, for instance, that the results for the one-sided nonzero boundary condition are recovered when  $\gamma = 0$ . Given the solution, Eqs. (3), (7), and (20)–(23), the other fields and densities can be found from various electrodynamic relations.<sup>8</sup>

### V. FLUX-FLOW LIMIT

In order to make some of the above results more transparent, we now consider a special case of the vortex dynamics, the flux-flow dominated limit. We consider temperatures low enough that the response of the normal

fluid and the effect of flux creep can be ignored but the system is driven into the flux-flow regime due to a sufficiently high frequency. (Numerical estimates suited to this regime are given below.) Flux-flow effects can dominate pinning effects when the angular frequency  $\omega > \kappa_p/\eta$ . In this arena the vortex displacements will be reduced and we can expect the vortex convection non-linearity effects to dominate hysteretic effects.

We present the corresponding reductions of the results for the symmetric two-sided boundary condition for the slab geometry. In the absence of flux creep, the complex effective skin depth associated with the vortex motion is  $\tilde{\delta}_v = [\delta_f^{-2} + (i/2)\lambda_C^{-2}]^{-1/2}$ , where  $\delta_f^2 = 2B_0\phi_0/\mu_0\eta\omega$  is the

square of the flux-flow skin depth and  $\lambda_C = (B_0 \phi_0 / \mu_0 \kappa_p)^{1/2}$  is the Campbell (or pinning) penetration depth. Accordingly, in the flux-flow limit,  $\tilde{\delta}_v \rightarrow \delta_f$ , the complex penetration depths given by Eqs. (5) and (9) become

$$\tilde{\lambda}^2 = \lambda^2 + (i/2)\delta_f^2, \quad \tilde{\lambda}_2^2 = \lambda^2 + (i/4)\delta_f^2. \quad (24)$$

In this limit the amplitude of the second harmonic, Eq. (10), becomes

$$B_{20} = -\frac{b_0^2}{8B_0} \frac{\cosh(d/\tilde{\lambda})}{\cosh^2(d/2\tilde{\lambda})} \frac{\delta_f^4}{\lambda^4} \times \frac{1}{[1 + (i/2)\delta_f^2/\lambda^2]} \frac{1}{[3 + (i/2)\delta_f^2/\lambda^2]}. \quad (25)$$

Aside from the geometric (cosh) factor, this amplitude can be appreciable either when  $\delta_f \sim \sqrt{2}\lambda$ , in which case  $\tilde{\lambda}^2$  has equal real and imaginary parts, or when  $\delta_f^2 \gg \lambda^2$ .

Suppose we consider parameters suitable for the high- $T_c$  superconductor Y1:2:3 at a temperature of 10 K.<sup>15</sup> Here we have a pinning force constant in the range  $10^4$ – $10^5$  N/m<sup>2</sup> and a drag coefficient of approximately  $10^{-6}$  Ns/m<sup>2</sup>. We then expect the flux-flow regime to occur for frequencies greater than 10–100 GHz, consistent with recent experiments.<sup>15</sup> An increase of the static magnetic field has competing influences in observing the effect of convective nonlinearity. A higher field (increase in  $\delta_f$ ) will cause flux-flow sooner, but decrease the intervortex spacing. Fields of the order of 0.1–1 T may be suitable in practice.

## VI. SUMMARY

In this paper we considered the nonlinear response of a type-II superconductor of finite thickness in the mixed state. This geometry introduces a new length  $d$  in the problem. Specific results for complex penetration depths, amplitudes, fields, and densities were given for the second harmonic response. The results are expected to be valid when  $d$  is sufficiently large. In the presence of a parallel static field, as considered in this paper,  $d$  should be larger than several intervortex spacings, making our continuum description with density  $n$  valid. In the limit of very large thickness, previous results for both linear and non-

linear response are recovered.

It is possible that the techniques presented here can be modified to provide results relevant to microwave harmonic mixing<sup>19</sup> as applied to type-II superconductors. With this technique, radiation is applied at both the fundamental and second-harmonic frequency. This experimental approach has now been applied to high-temperature superconductors and may give information on the superconducting order parameter.<sup>20</sup>

We considered the London limit of Abrikosov vortices. A possible extension to include variation of the magnitude  $f$  of the order parameter in our theory would be to replace the London equation with vortex term with the Ginzburg-Landau (GL) equations. This extension would have  $f$  appearing in the supercurrent source equation<sup>12</sup> and the first GL equation. Not only would it probably be more difficult to obtain analytical solutions in this augmented theory but the results would be expected to be rigorous only near the transition temperature. Another extension of the theory would be to include the dynamics of Josephson vortices. Presumably in such a treatment the coefficients in the equation of motion would need to be taken as those appropriate to Josephson vortices and the intrinsic penetration depth would need to be replaced with a combination including the Josephson penetration depth.

The theory of nonlinear response continues to include the nonlocality of vortex interaction, wherein the elastic response of the vortex lattice depends on the length scale of the strain. At the transition temperature, quantities such as the effective complex skin depth  $\tilde{\delta}_{vc}$  associated with vortex motion and flux creep and the second-harmonic amplitude  $B_{20}$  vanish, leading to results appropriate to the normal state. At  $T_c$ , the governing vector Eq. (1) itself becomes the normal-state diffusion equation. Our results give the possibility of further analyzing measurements of the complex permeability or conductivity in a restricted regime of nonlinear vortex response.

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