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Calculation of the self-energy in a layered two-dimensional electron gas

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The self-energy of a layered two-dimensional (2D) electron gas has been investigated using the Fermi-liquid approach. The zero- and finite-temperature calculations including both single-particle and plasmon excitations have been carried out. It is found that when the interlayer distance is large compared to twice the effective Bohr radius a_B, the interlayer-coupling effects are small and the results correspond to those of a pure 2D electron gas. When the interlayer distance is smaller than $2a_B$, the results are similar to those of a three-dimensional (3D) system. At intermediate interlayer distances, the results show an interesting mixture of 2D and 3D behaviors. The linear temperature dependence of the plasmon contribution to the quasiparticle damping at high temperatures in a layered 2D electron gas, obtained in this paper, may have some relevance to the explanation of the normal-state properties of the high-T_c superconductors.

I. INTRODUCTION

It has been suggested that some normal-state features of a high-T_c superconductor, such as a linear temperature dependence of the inverse lifetime, may be attributed to the collective excitations of the charge or spin density.[1] In a pure two-dimensional (2D) material the inverse lifetime has been shown to have a $(T^2/\varepsilon_F)\ln(T/\varepsilon_F)$ behavior[2-4] which obviously cannot explain a linear temperature dependence of the resistivity in the normal state of the cuprate superconductors. However, our calculation of the plasmon contribution to the imaginary part of the self-energy for a layered 2D electron gas shows that at high temperature the inverse lifetime of the quasiparticles can indeed have a linear temperature dependence.

In the case of a material in which 3D behavior dominates the energy-exchange mechanism, the possibility of the decay of the low-energy or thermal quasiparticles by plasmon excitations is inhibited because the bulk plasmon frequency is so high that conservation of energy and momentum cannot be satisfied. In a material in which 2D properties are exhibited, plasmons can be excited with very low energy, and the decay of these quasiparticles by plasmon excitations is permitted. Furthermore, in a layered 2D electron gas which has properties which, in part, simulate the layered cuprate superconductors, the plasmon dispersion relation has been found to have both a quasiacoustic and a quasioptical nature.[5] In this paper we calculate the quasiparticle damping for this situation and show that the linear temperature dependence of the inverse inelastic lifetime can be explained in terms of plasmon-mediated electron-electron scattering. We specialize to the three different cases of the dimensionality related properties and show that when the interlayer separation is larger than twice the effective Bohr radius a_B, the results basically agree with those of a strictly 2D sample. This is in accord with results obtained by Giuliani and Quinn (GQ).[2] When the interlayer distance is less than $2a_B$ the results are similar to those of a 3D system. In the intermediate range of interlayer distances an interesting mixture of 2D and 3D behaviors is shown to be exhibited by the model.

In Sec. II, we develop the expressions for the real and imaginary parts of the self-energy of the quasiparticles due to electron-electron interactions on the basis of the Fermi-liquid theory. In Sec. III the assumptions used here are applied to a layered 2D electron gas. Results in various limits for both zero and finite temperatures are then presented. Section IV provides some discussion and the possible connection between our results and the normal-state behavior of the high-T_c superconductors.[1]

II. MODEL

The calculations are performed using the standard Fermi-liquid approach[6] in a layered metal with a cylindrical topology of the Fermi surface. The effective interaction between particles, including retardation effects associated with virtual single-particle and plasmon excitations, is taken as the screened Coulomb interaction in random-phase approximation (RPA).[7] In order to describe this retarded electron-electron interaction we introduce the boson Green's function[6]

$$D(\mathbf{k},\omega)=v(\mathbf{k})\left[\frac{1}{\varepsilon(\mathbf{k},\omega)}-1\right] \tag{1}$$

which includes both the single-particle and plasmon excitations. Here $v(\mathbf{k})$ is the bare Coulomb interaction in a layered crystal and can be written as[5]

$$v(\mathbf{k})=v_0(\mathbf{k}_\parallel)f(\mathbf{k}) , \tag{2}$$

where

$$f(\mathbf{k})=\frac{\sinh c\mathbf{k}_\parallel}{\cosh c\mathbf{k}_\parallel-\cos c\mathbf{k}_z} . \tag{3}$$

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Here $\mathbf{k}_\parallel(k_x,k_y)$ and k_z are the momentum components in the plane and normal to the plane, respectively. c is the separation between successive layers of the layered 2D sample, and $v_0(\mathbf{k}_\parallel)=2\pi e^2/k_\parallel \varepsilon_i$ is the Coulomb interaction in a pure 2D case where ε_i is the dielectric constant of the background lattice. The quantity $\varepsilon(\mathbf{k},\omega)$ in (1) is the Fourier wave vector ($\mathbf{k}$) and frequency (ω) component of the dielectric function of the layered 2D electron gas and is given by

$$\varepsilon(\mathbf{k},\omega)=1+v(\mathbf{k})\Pi(\mathbf{k}_\parallel,\omega)\ , \tag{4}$$

where $\Pi(\mathbf{k}_\parallel,\omega)$ is the polarization propagator for the 2D electron spectrum.[8]

Since the Kramers-Kronig relation is valid for the response function $\varepsilon^{-1}(\mathbf{k},\omega)$ for any $\mathbf{k}$ and ω, the boson Green's function also satisfies the dispersion relation

$$D(\mathbf{k},\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}D(\mathbf{k},\omega')}{\omega'-\omega-i\delta}d\omega' \quad (\delta\to+0)\ . \tag{5}$$

The self-energy Σ of a quasiparticle, associated with retarded electron-electron interaction, can be written at the absolute temperature T in the following form[6] ($\hbar=k_B=1$):

$$\Sigma(\mathbf{p},\omega)=-2\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega''}{2\pi}\frac{\mathrm{Im}G(\mathbf{p}+\mathbf{k},\omega'')\,\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega''-\omega-i\delta}\left[\tanh\frac{\omega''}{2T}+\coth\frac{\omega'}{2T}\right]\ . \tag{6}$$

Replacing the imaginary part of the electron Green's function

$$G(\mathbf{k},\omega)=[\omega-\xi_{\mathbf{k}}-\Sigma(\mathbf{k},\omega)]^{-1}$$

(here energies $\xi_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\mu$ are measured relative to the chemical potential μ, and $\varepsilon_{\mathbf{k}}=\mathbf{k}^2/2m^*$ is the 2D single-particle energy) by an approximate δ function and carrying out the angular integration in (6), one obtains the self-energy for a layered system as

$$\Sigma(p,\pm\omega)=\frac{2}{(2\pi)^4 v_p}\int_{-\pi}^{\pi}d\tilde{k}_z\left[\int_{\mp\varepsilon_p}^{0}d\omega''\int_{k_-(\pm\omega'')}^{k_+(\pm\omega'')}dk_\parallel+\int_{0}^{\pm\infty}d\omega''\int_{-k_-(\pm\omega'')}^{k_+(\pm\omega'')}\right]F_\mp(k_\parallel,\omega'')$$
$$\times\int_{-\infty}^{\infty}d\omega'\frac{\mathrm{Im}D(\mathbf{k},\omega')}{\omega'+\omega''\mp i\delta}\left[\tanh\frac{\omega''+\omega}{2T}+\coth\frac{\omega'}{2T}\right]\ , \tag{7}$$

where

$$F_\pm(k_\parallel,\omega)=\left[1-\frac{k_\parallel^2}{4p^2}\left[1\pm\frac{2m^*\omega}{k_\parallel^2}\right]^2\right]^{-1/2}$$

and

$$k_\pm(\omega)=p[1\pm\sqrt{1+\omega/\varepsilon_p}]\ .$$

Here $\tilde{k}_z=k_z c$, $v_p=p/m^*$ is the velocity of the quasiparticle, and $\omega=\xi_p=\varepsilon_p-\mu$, ε_p being the 2D single-particle energy defined above.

We divide $\Sigma(p,\omega)$ into an odd part and an even part in the dependence on ω, the odd part being

$$\Sigma'(p,\omega)=\tfrac{1}{2}[\Sigma(p,\omega)-\Sigma(p,-\omega)]$$

and the even part being

$$\tilde{\Sigma}(p,\omega)+i\Sigma''(p,\omega)=\tfrac{1}{2}[\Sigma(p,\omega)+\Sigma(p,-\omega)]\ . \tag{8}$$

Here $\Sigma'(p,\omega)$ and $\Sigma''(p,\omega)$ describe the renormalization of the quasiparticle energy and the damping of the quasiparticle, respectively, and $\tilde{\Sigma}(p,\omega)$ gives a correction to the chemical potential μ due to the electron-electron interaction.

Since all quantities in (7) other than the Green's function $D(\mathbf{k},\omega)$ are independent of k_z, we introduce an average over k_z as

$$\bar{D}(k_\parallel,\omega)=\frac{1}{2\pi}\int_{-\pi}^{\pi}D(k_\parallel,\tilde{k}_z,\omega)d\tilde{k}_z\ . \tag{9}$$

Using (9) in (7) and (8) and the condition that $\omega<\varepsilon_p$ and $p\approx p_F$, we obtain the real and imaginary parts of the self-energy as

$$\Sigma'(\omega)=\frac{1}{2(2\pi)^2 v_F}\int_0^\infty d\omega'\int_{\omega'/v_F}^{2p_F}[F_+(k_\parallel,\omega')+F_-(k_\parallel,\omega')]\,\mathrm{Re}\bar{D}(k_\parallel,\omega')dk_\parallel\left[\tanh\frac{\omega+\omega'}{2T}+\tanh\frac{\omega-\omega'}{2T}\right] \tag{10}$$

and

$$\Sigma''(\omega)=\frac{1}{2(2\pi)^2 v_F}\int_0^\infty d\omega'\int_{\omega'/v_F}^{2p_F}[F_+(k_\parallel,\omega')+F_-(k_\parallel,\omega')]\,\mathrm{Im}\bar{D}(k_\parallel,\omega')dk_\parallel\left[2\coth\frac{\omega'}{2T}-\tanh\frac{\omega'+\omega}{2T}-\tanh\frac{\omega'-\omega}{2T}\right]\ . \tag{11}$$

An analysis of (7) indicates that expressions (10) and (11) are valid for both T and ω less than ε_F.

Equation (11) is connected with the inelastic lifetime τ_{ee} of a quasiparticle near the Fermi surface by the relation[7,9]

$$\tau_{ee}^{-1} = -2\Sigma''(p_F,\omega) . \tag{12}$$

The real part of the self-energy $\Sigma'(p_F,\omega)$ defines the renormalizations of the effective mass m^* and the Fermi velocity v_F by

$$\tilde{m}^* = m^* \left[1 - \frac{\partial \Sigma'}{\partial \omega} \right]$$

and (13)

$$\tilde{v}_F = p_F / m^* \left[1 - \frac{\partial \Sigma'}{\partial \omega} \right] .$$

III. SELF-ENERGY IN A LAYERED 2D METAL

In this section we present the calculation of the self-energy in a layered 2D metal using (10) and (11). The average boson Green's function appearing in (10) and (11) is first obtained by inserting (1) in (9) and using (2)–(4) as

$$\bar{D}(k_\parallel,\omega) = \frac{v_0(k_\parallel)}{\pi} \int_0^\pi d\tilde{k}_z f(\tilde{k}_z) \times \left[\frac{1}{1+v_0(k_\parallel) f(\tilde{k}_z) \Pi(k_\parallel,\omega)} - 1 \right] . \tag{14}$$

In the region of single-particle excitations defined by $\omega < v_F k$ ($v_F = p_F/m^*$ is the Fermi velocity) and $k < 2p_F$ [where $p_F = (2\pi n_s)^{1/2}$ is the Fermi momentum, and n_s is the 2D density of quasiparticles] the polarization propagator takes a form in which the real and imaginary parts are

$$\mathrm{Re}\Pi(k_\parallel,\omega) \cong m^*/\pi ,$$
$$\mathrm{Im}\Pi(k_\parallel,\omega) \cong \frac{m^{*2}\omega}{\pi p_F k_\parallel} \Theta(k_\parallel v_F - |\omega|) . \tag{15}$$

In the plasmon region, that is in the high-frequency limit $\omega > v_F k$, the polarization propagator $\Pi(k_\parallel,\omega)$ has the form

$$\mathrm{Re}\Pi(k_\parallel,\omega) \cong -\frac{p_F^2 k_\parallel^2}{2\pi m^* \omega^2} \tag{16}$$

with $\mathrm{Im}\Pi(k_\parallel,\omega)=0$.

Using (16) in (14), the plasmon contribution to the average Green's function for a layered 2D system can be represented as

$$\bar{D}_{\mathrm{pl}}(k_\parallel,\omega) = \frac{v_0(k_\parallel)\omega_p^2(k_\parallel)}{\pi} \int_0^\pi \frac{f^2(\tilde{k}_z) d\tilde{k}_z}{\omega^2 - \omega_p^2(k_\parallel) f(\tilde{k}_z) + i\delta} , \tag{17}$$

where $\omega_p(k_\parallel)$ is the plasmon frequency for a pure 2D system with the approximate dispersion relation

$$\omega_p^2(k_\parallel) \cong \frac{e^2}{\varepsilon_i} p_F v_F k_\parallel .$$

After integrating (17) over k_z we obtain for the real part of the average boson Green's function as

$$\mathrm{Re}\bar{D}_{\mathrm{pl}}(k_\parallel,\omega = -v_0(k_\parallel) + \frac{v_0(k_\parallel)\omega^2}{\sqrt{(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)}} \times [\Theta(\omega-\omega_+) - \Theta(\omega_- - \omega)] , \tag{18}$$

where $\Theta(x)$ is the step function, $\omega_+ = \omega_p(k_\parallel)(\coth ck_\parallel/2)^{1/2}$ is the purely optical plasmon frequency, and $\omega_- = \omega_p(k_\parallel)(\tanh ck_\parallel/2)^{1/2}$ is the proper acoustic plasmon frequency.[5] Considerations similar to (18) give the imaginary part of the average Green's function as

$$\mathrm{Im}\bar{D}_{\mathrm{pl}}(k_\parallel,\omega) = -\frac{v_0(k_\parallel)\omega^2}{\sqrt{(\omega_+^2-\omega^2)(\omega^2-\omega_-^2)}} , \quad \omega_- \le \omega \le \omega_+ . \tag{19}$$

The region of excitation of plasmons for the layered system is shown in Fig. 1, where the optical (ω_+) and acoustic (ω_-) limits are shown by solid curves (1) and (2), respectively. The dashed curve [$\omega_* = \omega_p(k_\parallel)(\tanh ck_\parallel)^{1/2}$] in Fig. 1 represents the curve which distinguishes between the optical and acoustic nature of the plasmon band.

Inserting expressions (15) for the polarization propagator into (14), the real and imaginary parts of the single-particle contribution to the average boson Green's function can be obtained as

$$\mathrm{Re}\bar{D}_{\mathrm{sp}}(k_\parallel) = -v_0(k_\parallel) \left[1 - \left[1 + \frac{\chi_e^2}{k_\parallel^2} + \frac{2\chi_e}{k_\parallel} \coth ck_\parallel \right]^{-1/2} \right] \tag{20}$$

and

$$\mathrm{Im}\bar{D}_{\mathrm{sp}}(k_\parallel,\omega) \cong -v_0^2(k_\parallel)\,\mathrm{Im}\Pi(k_\parallel,\omega)\,\sinh^2 ck_\parallel \frac{B}{(B^2-1)^{3/2}} \left[1 - \frac{A^2(3+2B^2)}{2(B^2-1)^2} \right] , \tag{21}$$

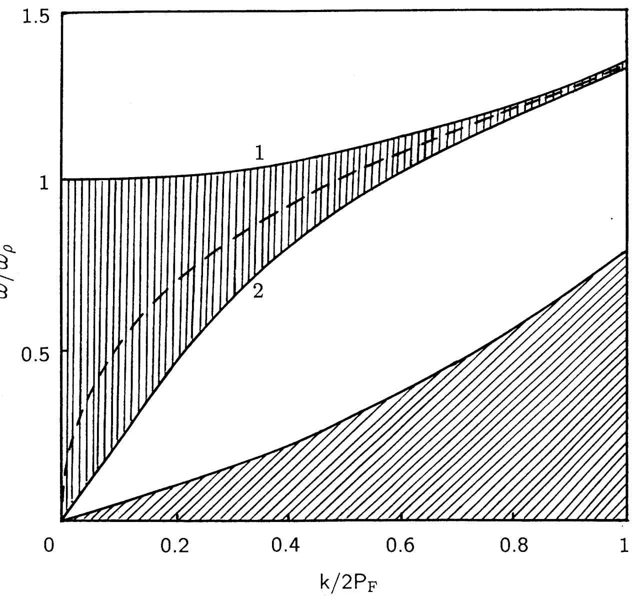

FIG. 1. Representation of the pseudo-optical plasmon band (vertically hatched region) because of the plasmon-mediated effective electron-electron interaction in a layered 2D electron gas. The upper solid curve (1) is the pure optical limit and the lower solid curve (2) represents the pure acoustic limit. The dashed curve represents the frequency above (below) which the plasmon has optical (acoustic) nature. The region of the low-frequency single-particle excitations is shown by the slanted hatched lines.

where

$$A = v_0(k_\parallel)\,\mathrm{Im}\Pi(k_\parallel,\omega)\sinh ck_\parallel \ ,$$

$$B = \cosh ck_\parallel \left[1+\frac{\chi_e}{k_\parallel}\tanh ck_\parallel\right] ,$$

and $\chi_e = 2/a_B = 2e^2m^*/\varepsilon_i$ is the screening parameter.

A. Imaginary part of the self-energy: Plasmon contribution

In this subsection we calculate the plasmon contribution to the imaginary part of the self-energy. Inserting (19) into (11) we get

$$\Sigma''(\omega) = -\frac{\alpha}{2\pi\omega_c^2}\int_0^\infty \omega'^2 d\omega' F\left[\frac{\omega'}{\omega_c}\right] H(\omega,\omega') \ , \qquad (22)$$

where

$$H(\omega,\omega') = \coth\frac{\omega'}{2T} - \frac{1}{2}\tanh\frac{\omega'+\omega}{2T} - \frac{1}{2}\tanh\frac{\omega'-\omega}{2T}$$

and

$$F(x) = \int_x^{\min\{x_c,2/\alpha\}} [\Phi_+(x,z)+\Phi_-(x,z)]\Psi(x,z)dz$$

with

$$\Phi_\pm(x,z) = \left[z^2 - \left[\frac{\alpha}{2}z^2 \pm x\right]^2\right]^{-1/2}$$

and

$$\Psi(z,x) = \left[\left[\frac{z}{\tanh z/g} - x^2\right](x^2 - z\tanh z/g)\right]^{-1/2} . \qquad (23)$$

Here $x_c = k_c a_B > 1$ and $g = 2a_B/c$, while $a_B = \varepsilon_i/e^2m^*$ is the effective Bohr radius, $\alpha = e^2/v_F\varepsilon_i$ is the electron-electron interaction constant, $\omega_c = \omega_p(1/a_B) = e^2p_F/\varepsilon_i$ is the plasmon frequency for a pure 2D case with wave vector $k = 1/a_B$. Also k_c is the critical wave vector above which the plasmon production is precluded in a pure 2D system. It can be shown to be $k_c = 2p_F\alpha^{2/3}$ for $\alpha \ll 1$ (high density limit) and $k_c = p_F(2\alpha)^{1/2}$ for $0 \ll \alpha \lesssim 1$ (intermediate density limit). Unless otherwise stated, the results presented in this paper are valid in the high density limit where the Fermi-liquid theory is most valid. Parameter g in (23) characterizes the strength of the interlayer plasmon coupling. In the remainder of this subsection we consider $\Sigma''_{\mathrm{pl}}(\omega)$ for three different cases: (1) large interlayer separation, (2) short interlayer separation, and (3) intermediate interlayer separation.

1. Large interlayer separation or weak interlayer coupling ($g \ll 1$)

At large interlayer distance ω_+ goes to ω_- and we have the case of the weak interlayer plasmon coupling. In this case the function $\Psi(z,x)$ takes the form

$$\Psi(z,x) \cong \pi\delta(x^2 - z) \ . \qquad (24)$$

In the high density limit ($\alpha \ll 1$), we insert (24) into (22) to obtain

$$\Sigma''(\omega) = -\alpha\int_{\omega_c}^{\omega_{\max}} \frac{\omega' d\omega'}{\sqrt{\omega'^2 - \omega_c^2}} H(\omega,\omega') \ , \qquad (25)$$

where $\omega_{\max}$ is the maximum plasmon excitation energy and is given by $\omega_{\max} = \omega_p(k_c)$ for $k_c \le 2p_F$.

(a) Zero-temperature ($T=0$) case. At $T=0$ we have $H(\omega,\omega') = 1$ and in the high density limit we obtain from (25) the form

$$\Sigma''(\omega) = -\alpha\sqrt{\omega^2 - \omega_c^2} \ , \quad \omega_c \le \omega \le \omega_{\max} \ . \qquad (26)$$

Here ω_c plays the role of the finite threshold for damping due to plasmon excitation. Note that for ω slightly larger than ω_c ($\omega \gtrsim \omega_c$), Eq. (26) yields the result previously obtained by GQ:

$$\Sigma''(\omega) \cong -\sqrt{2}\alpha\omega_c\left[\frac{\omega - \omega_c}{\omega_c}\right]^{1/2} . \qquad (27)$$

Thus, it appears that our result for electron damping due to virtual plasmon excitation in a pure 2D sample is more general than that of GQ, who calculate Σ'' for $\omega \sim \omega_c$. However, as ω becomes much larger than ω_c ($\omega_c \ll \omega < \omega_{\max}$), Eq. (26) gives

$$\Sigma''(\omega) \cong -\alpha\omega(1 - \omega_c^2/2\omega^2) \ . \qquad (28)$$

Thus, the plasmon contribution to the quasiparticle damping has a linear frequency dependence when ω is larger than excitation energy threshold ω_c. As ω becomes greater than $\omega_{\max}$, we find that Σ'' assumes the constant value

$$\Sigma''(\omega)=-\alpha\sqrt{\omega_{\max}^2-\omega_c^2}\ ,\quad \omega>\omega_{\max}\ .$$

Next we consider the intermediate density limit $(0\ll\alpha\le 1)$. In this case at $T=0$ we obtain from (22)

$$\Sigma''(\omega)\cong-\alpha(\omega-\omega_c)\ , \tag{29}$$

which also has a linear dependence on the frequency.

(b) Finite-temperature ($T\neq 0$) case. We now turn to the analysis of quasiparticle damping at finite temperature. At finite temperature $(\omega\ll T)$ and high density $(\alpha\ll 1)$ from Eq. (22) we obtain the imaginary part of the self-energy as

$$\Sigma''(T)=-2\alpha\int_{\omega_c}^{\omega_{\max}}\frac{\omega' d\omega'}{\sqrt{\omega'^2-\omega_c^2}}\sinh^{-1}\frac{\omega'}{T}\ . \tag{30}$$

From (30) it follows that at high temperature $(\omega_c,\omega_{\max}\ll T)$ one has the limiting form

$$\Sigma''(T)\cong-2\alpha T\ln\frac{2\omega_{\max}}{\omega_c}=-\alpha T\ln 4a_B k_c\ . \tag{31}$$

Thus, at high temperature the plasmon contribution to the quasiparticle damping takes a linear temperature dependence.

For the intermediate values of T $(\omega_c\ll T\ll\omega_{\max})$, we have

$$\Sigma''(T)\cong-2\alpha T\ln(2T/\omega_c)\ . \tag{32}$$

At low temperature $(\omega_{\max},\omega_c\gg T)$, Eq. (30) gives

$$\Sigma''(T)\cong-4\alpha\omega_c K_1\left[\frac{\omega_c}{T}\right]$$
$$\approx-2\alpha\sqrt{2\pi\omega_c T}\,\exp(-\omega_c/T)\ , \tag{33}$$

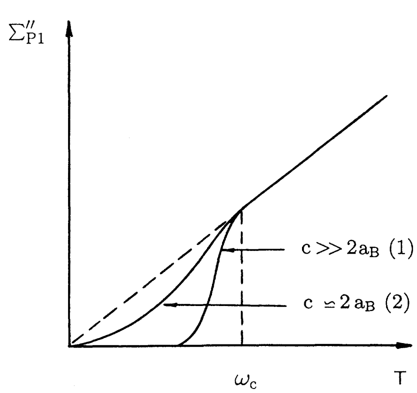

FIG. 2. The temperature dependence of the quasiparticle damping due to the inelastic electron-plasmon interaction. Curve (1) represents the result for the case where the interlayer separation is large $(c\gg a_B)$. The critical frequency ω_c corresponds to the threshold for plasmon excitation in a pure 2D case. Curve (2) corresponds to the result for the intermediate values of the interlayer separation $(c\approx 2a_B)$.

where $K_1(x)$ is MacDonald's function and the asymptotic limit has been used. Figure 2 shows the temperature dependence of the plasmon contribution to the quasiparticle damping in the high density limit $(\alpha\ll 1)$ given by (31)–(33).

In the intermediate density limit when $(0\ll\alpha\le 1)$ the plasmon contribution to the quasiparticle damping assumes the form

$$\Sigma''\sim-4\alpha T\exp(-\omega_c/T)\ .$$

In this subsection we conclude that when the interlayer distance c is much larger than twice the effective Bohr radius, the imaginary part of the self-energy takes the same form as in a pure 2D case in the $T=0$ limit.[2] Since GQ did not consider the temperature dependence of the plasmon contribution to the self-energy we do not have a basis for comparison of our finite-temperature results.

2. *Short interlayer separation or strong interlayer coupling ($g>1$)*

When the interlayer distance c is smaller than $2a_B$ (i.e., $g>1$), from (23) and (22) we obtain

$$\Sigma''(\omega)\ \cong-\frac{\alpha}{\pi}K(\sqrt{1-g^{-1}})\int_0^{\tilde\omega_{\max}}\frac{\omega' d\omega'}{\sqrt{\omega_{\rm pl}^2-\omega'^2}}H(\omega,\omega')\ , \tag{34}$$

where $\tilde\omega_{\max}\sim\omega_c/g^{1/2}$, $K(x)$ is the elliptic integral of the first kind, and $\omega_{\rm pl}=(4\pi e^2 n_s/\varepsilon_i m^* c)^{1/2}$ is the bulk plasmon frequency for a layered 2D system.

(a) Zero-temperature ($T=0$) case. At very small interlayer separation $(g\gg 1)$ we can make the further approximation

$$K[(1-g^{-1})^{1/2}]\sim\tfrac{1}{2}\ln(16g)=\tfrac{1}{2}\ln(32a_B/c)\ .$$

At $T=0$ from (34) we obtain

$$\Sigma''(\omega)\cong-\frac{\alpha}{2\pi}\ln\left[\frac{32a_B}{c}\right]\left[\omega_{\rm pl}-\sqrt{\omega_{\rm pl}^2-\omega^2}\right]\ ,\quad \omega<\tilde\omega_{\max}\ . \tag{35}$$

If $\omega_{\rm pl}\gg\omega$, Eq. (33) gives

$$\Sigma''(\omega)\cong-\frac{\alpha}{4\pi}\ln\left[\frac{32a_B}{c}\right]\omega_{\rm pl}\left[\frac{\omega}{\omega_{\rm pl}}\right]^2\ ,\quad \omega<\tilde\omega_{\max}\ . \tag{36}$$

If ω is larger than $\tilde\omega_{\max}$ $(\ll\omega_{\rm pl})$, $\Sigma''(\omega)$ assumes the constant value

$$\Sigma''(\omega)\cong-\frac{\alpha}{4\pi}\ln\left[\frac{32a_B}{c}\right]\omega_{\rm pl}\left[\frac{\tilde\omega_{\max}}{\omega_{\rm pl}}\right]^2\ ,\quad \omega>\tilde\omega_{\max}\ .$$

Thus, for a very small interlayer distance $(g\gg 1)$ at $T=0$, the quasiparticle damping has a quadratic dependence on the frequency, i.e., $\Sigma''\sim\omega^2/\omega_{\rm pl}$ for small ω, but it assumes a constant value at large frequencies $\omega\gg\tilde\omega_{\max}$.

(b) Finite-temperature ($T\neq 0$) case. At finite tempera-

tures ($\omega \ll T$), Σ'' given by (34) takes the form

$$\Sigma''(T) \cong -\frac{\pi\alpha}{2} \ln\left[\frac{32a_B}{c}\right]\omega_{\rm pl}\left[\frac{T}{\omega_{\rm pl}}\right]^2, \quad T < \tilde{\omega}_{\max}, \quad (37)$$

which obviously has a T^2 dependence. Thus, at short interlayer distances, the system simulates the three-dimensional case, and we get $\Sigma'' \sim \omega^2/\omega_{\rm pl}$ at $T=0$, and $\Sigma'' \sim T^2/\omega_{\rm pl}$ at finite temperatures.

At high temperatures ($\tilde{\omega}_{\max} \ll T \ll \varepsilon_F$), however, the plasmon contribution to the quasiparticle damping has a linear temperature dependence

$$\Sigma''(T) \cong -\frac{\alpha}{\pi} \ln\left[\frac{32a_B}{c}\right]\frac{\tilde{\omega}_{\max}}{\omega_{\rm pl}} T .$$

3. *Intermediate interlayer separation ($g \approx 1$)*

Let us now consider the case where the interlayer distance has an intermediate value ($c \approx 2a_B$) corresponding to an intermediate value for the interlayer plasmon coupling strength. In this case we see a transition from the quadratic dependence to a linear dependence on both ω and T for the quasiparticle damping.

(a) Zero-temperature ($T=0$) case. At $T=0$ some straightforward manipulation of Eqs. (23) and (22) yields

$$\Sigma''(\omega) = \Sigma_1''(\omega) + \Sigma_2''(\omega) , \quad (38)$$

where

$$\Sigma_1''(\omega) \cong -\frac{\alpha}{2}[\omega_c - \mathrm{Re}\sqrt{\omega_c^2 - \omega^2}] ,$$

$$\Sigma_2''(\omega) \cong -\alpha\, \mathrm{Re}\sqrt{\omega^2 - \omega_c^2} , \quad \omega < \tilde{\omega}_{\max} .$$

However, for $\omega > \tilde{\omega}_{\max}$, $\Sigma''(\omega)$ assumes the same constant value as shown by the equation below (28).

(b) Finite-temperature ($T \neq 0$) case. At finite temperatures, from Eqs. (22) and (23) we get

$$\Sigma''(T) = \Sigma''(T < \omega_c) + \Sigma''(T > \omega_c) , \quad (39)$$

where

$$\Sigma''(T < \omega_c) \cong -\alpha\omega_c \frac{\pi^2}{2}\left[\frac{T}{\omega_c}\right]^2 ,$$

$$\Sigma''(T > \omega_c) \cong -\frac{\pi\alpha}{2} T - 2\alpha T \ln\left[\frac{2\omega_{\max}}{\omega_c}\right] .$$

Thus, we see that while at low temperatures ($T < \omega_c$) Σ'' has a quadratic dependence on T, at high temperatures ($T > \omega_c$) it again assumes a linear T dependence. This temperature dependence of $\Sigma''(T)$ is shown by curve (2) in Fig. 2.

B. Imaginary part of the self-energy: Single-particle contribution

Now we consider single-particle contribution to the imaginary part of the self-energy. It can be obtained by first inserting (21) into (11) and then carrying out the integrations in (11). Here we consider two different cases: (1) large interlayer separation and (2) short interlayer separation.

1. *Large interlayer separation ($g \ll 1$)*

For large interlayer separation ($c \gg 2a_B$), expression (11) takes the form of a strict 2D case (GQ) and the imaginary part of the self-energy at $T=0$ and at finite temperatures ($T \gg \omega$) assumes the following well-known forms:

$$\Sigma_{\rm sp}''(\omega) \cong -\frac{\varepsilon_F}{8\pi}\left[\frac{\omega}{\varepsilon_F}\right]^2 \ln\frac{\varepsilon_F}{\omega} ,$$
$$\Sigma_{\rm sp}''(T) \cong -\frac{\pi}{4}\varepsilon_F\left[\frac{T}{\varepsilon_F}\right]^2 \ln\frac{\varepsilon_F}{T} . \quad (40)$$

We can see from (40) that for large interlayer distances the single-particle contribution to the quasiparticle damping $\Sigma_{\rm sp}''$ does not depend on α and agrees with the familiar results for a 2D case, i.e., it has the form $(\omega^2/\varepsilon_F)\ln(\omega/\varepsilon_F)$ or $(T^2/\varepsilon_F)\ln(T/\varepsilon_F)$ which is in agreement with the familiar result of GQ.

2. *Short interlayer separation ($g \gg 1$)*

In the case of a small interlayer distance ($c \ll 2a_B$), (21) takes the simple form

$$\mathrm{Im}\bar{D}_{\rm sp}(k_\parallel, \omega) \cong -\frac{\pi e^2}{\sqrt{2}k_\parallel}\frac{\omega}{\omega_{\rm pl}} \quad (41)$$

and at $T=0$ we obtain

$$\Sigma_{\rm sp}''(\omega) \cong -\frac{\alpha}{4\sqrt{2\pi}}\omega_{\rm pl}\left[\frac{\omega}{\omega_{\rm pl}}\right]^2 \ln\frac{\varepsilon_F}{\omega} . \quad (42)$$

At finite temperatures ($\omega \ll T$) we get

$$\Sigma_{\rm sp}''(T) \cong -\frac{\pi\alpha}{2\sqrt{2}}\omega_{\rm pl}\left[\frac{T}{\omega_{\rm pl}}\right]^2 \ln\frac{\varepsilon_F}{T} . \quad (43)$$

Note that (42) and (43) show that the single-particle contribution to Σ'' of the layered sample has a combination of the two- and three-dimensional natures.

C. Real part of the self-energy

Now we consider the real part of the self-energy Σ' of a quasiparticle in a layered 2D system. First we calculate the contribution of the unscreened Coulomb interaction to Σ' which represents a shift of the excitation energy of the quasiparticle in a layered 2D system. This contribution is given by

$$\Sigma_0'(\mathbf{p}) = -\sum_{\mathbf{k}} v(\mathbf{p}-\mathbf{k}) n_F(\mathbf{k}) ,$$

where $v(\mathbf{k})$ is given by (2) and $n_F(\mathbf{k})$ represents the Fermi distribution function. Since the single-particle energy $\varepsilon_{\mathbf{k}}$ does not depend on k_z, the expression for $\Sigma_0'(\xi, T)$ does not depend on the layered nature of the system and has the same form as that of a strict 2D sample

$$\Sigma_0' \cong -\frac{2}{\pi}\omega_c + \frac{\alpha}{\pi}\left[\xi_p\left[\ln\frac{16\varepsilon_F}{T}-1\right]-T\Gamma\left[\frac{\xi_p}{T}\right]\right], \quad (44)$$

where

$$\Gamma(x)=\frac{1}{2}\int_0^\infty \ln z\, dz\left[\tanh\frac{z+x}{2}-\tanh\frac{z-x}{2}\right].$$

Next we calculate the plasmon contribution to Σ' substituting (18) into (10) and integrating over $k_\parallel$. For short interlayer separation ($g \gg 1$) we find

$$\Sigma_{\rm pl}'(\omega) \cong -\frac{\alpha}{2\pi}\int_0^{\omega_{\max}} d\omega'\left[\ln\frac{16\varepsilon_F}{\omega'}+\frac{\omega'}{\omega_{\rm pl}}K(\sqrt{g}\,)\right] \times\left[\tanh\frac{\omega'+\omega}{2T}-\tanh\frac{\omega'-\omega}{2T}\right].$$

From this expression we obtain the following limiting results:

$$\Sigma_{\rm pl}'(\omega) \propto -\frac{\alpha}{\pi}\omega\ln\frac{\varepsilon_F}{\omega} \quad (T=0),$$
$$\Sigma_{\rm pl}'(T) \propto -\frac{\alpha}{\pi}\omega\ln\frac{\varepsilon_F}{T} \quad (T\gg\omega). \quad (45)$$

We now consider the single-particle contribution to Σ' which can be obtained by using (20) in (10) and then carrying out the integrations in (10). We obtain the limiting values

$$\Sigma_{\rm sp}'(\omega) \cong -\frac{\alpha}{\pi}\omega\left[\ln\frac{16\varepsilon_F}{\omega}+1\right] \quad (T=0),$$
$$\Sigma_{\rm sp}'(T) \cong -\frac{\alpha}{\pi}\omega\ln\frac{32\gamma_E\varepsilon_F}{\pi T} \quad (T\gg\omega), \quad (46)$$

where $\gamma_E=1.78$ is the Euler constant. Our analysis shows that $\tilde{\Sigma}$ defined by (8) does not affect the renormalizations of the effective mass and the Fermi velocity (13), since it is an even function of ω.

Using the total Σ', obtained by summing (44)–(46), in (13) we get

$$\tilde{m}^* \approx m^*\left[1+\frac{\alpha}{\pi}-\frac{\partial\Sigma_{\rm pl}'}{\partial\omega}\right],$$
$$\tilde{v}_F \approx p_F/m^*\left[1+\frac{\alpha}{\pi}-\frac{\partial\Sigma_{\rm pl}'}{\partial\omega}\right]. \quad (47)$$

Using (45), the expression for the plasmon contribution to the real part of the self-energy, in (47) we find that the electron-plasmon interaction introduces a logarithmic divergence, in both frequency and temperature, in the renormalized effective mass and Fermi velocity. This implies that inclusion of the plasmon contribution to the quasiparticle self-energy for a layered 2D system may lead to a behavior which is similar to that of the marginal Fermi-liquid theory proposed by Varma *et al.*[1] for explaining the anomalous normal-state behavior of the high-T_c superconductors.

IV. DISCUSSION

In this paper we have calculated, within the Fermi-liquid approach, the frequency and temperature dependence of the self-energy of a quasiparticle in a layered 2D electron gas. We have shown that in a layered 2D system interlayer plasmon exchange fundamentally changes the frequency and temperature dependence of the quasiparticle damping.

In the case where the interlayer distance is large ($c \gg 2a_B$) and the strength of the interlayer plasmon coupling is small, we obtain the results similar to those of the strict 2D case. Our finding complements the earlier results of GQ on the single-particle contribution to the quasiparticle damping and leads to new results for the plasmon contribution to the quasiparticle damping of a strict 2D electron gas. We find that at $T=0$ the quasiparticle damping has linear frequency dependence beyond a finite threshold energy ω_c. Very close to this threshold our result has a square-root dependence on the frequency which is in agreement with the GQ result. At finite temperature, $\Sigma_{\rm pl}''(T)$ has a linear temperature dependence with a threshold energy of ω_c. Near the threshold energy, $\Sigma_{\rm pl}''(T)$ has an exponential dependence on temperature [see (33)]. Thus, we can consider that the threshold energy ω_c acts as the activation energy for the quasiparticle damping.

In the opposite limit, when the interlayer plasmon exchange is strong and g is much larger than one ($c \ll 2a_B$), our results are similar to those of the three-dimensional case, i.e., quasiparticle damping is given by $\Sigma_{\rm pl}'' \sim -\alpha T^2/\omega_{\rm pl}$ or $\sim -\alpha\omega^2/\omega_{\rm pl}$ for small ω or T, respectively. However, as T increases ($T>\omega_{\max}$) $\Sigma_{\rm pl}''(T)$ again assumes a linear dependence on T.

For intermediate values of the interlayer distance ($c \approx 2a_B$), the quasiparticle damping $\Sigma_{\rm pl}''$ has a similar behavior, i.e., at low frequencies or temperatures it has a quadratic dependence on both ω and T. When the frequency or the temperature becomes comparable with the threshold energy ω_c, the quasiparticle damping undergoes a transition from the quadratic to linear dependence on frequency or temperature. The point to emphasize here is that for all values of the interlayer distance, the plasmon contribution to the quasiparticle damping assumes a linear dependence on T, when T is greater than a certain threshold value.

As expected the single-particle contribution to the quasiparticle damping has the behavior of a strict 2D system when interlayer distance is large compared to $2a_B$ and it has a behavior similar to that of a 3D system for $c \ll 2a_B$. The transition from the 3D to 2D behavior takes place near $c \approx 2a_B$.

Finally let us now briefly discuss our results with regard to their applicability to the normal-state properties of the high-T_c superconductors. First, the linear temperature dependence of the resistivity in the normal state of these superconductors can be explained if we consider that the main contribution to the damping process responsible for the resistivity is connected with the inelastic electron-plasmon interaction in a layered 2D system. Combining Eqs. (12), (32), and (37), an estimate of the resistivity is obtained as

$$\rho \propto 4\pi/\tau_{\rm pl}\omega_{\rm pl}^2 \propto T^2 \quad \text{for } T < \omega_c/\sqrt{g}$$
$$\propto T \quad \text{for } T > \omega_c/\sqrt{g} \; .$$

Second, we wish to point out that from (47) it follows that the renormalized effective mass of the quasiparticle due to electron-plasmon interaction has a logarithmic divergence in both frequency and temperature, which is similar to the marginal Fermi-liquid behavior as discussed by Varma *et al.*[1]

Our theory can also explain some peculiarities observed in the optical properties of high-temperature superconductors in the normal state. For example, it follows from expression (19) that for a layered 2D system

$$\mathrm{Im}\frac{1}{\varepsilon_{\rm pl}(k_\parallel,\omega)}\bigg|_{k_1\to 0} = -\frac{1}{v_0(k_\parallel)}\,\mathrm{Im}\bar{D}_{\rm pl}(k_\parallel,\omega)\bigg|_{k_1\to 0} = \frac{\omega}{\sqrt{\omega_{\rm pl}^2-\omega^2}} \; , \tag{48}$$

i.e., $\mathrm{Im}(1/\varepsilon)$ is a linear function of ω for $\omega < \omega_{\rm pl}$.

Expression (48) combined with the Drude contribution, in our opinion, can explain the smooth line shape of the Raman scattering[10] and the optical conductivity[11,12] of the high-T_c superconductors. One can represent the total optical conductivity as[1]

$$\sigma(\omega) = \sigma_D(\omega) + \sigma_{\rm pl}(\omega) \; . \tag{49}$$

Here $\sigma_D(\omega) = (\omega/4\pi)\,\mathrm{Im}\varepsilon_D(\omega)$ is the Drude contribution to the optical conductivity with

$$\varepsilon(\omega) = \varepsilon_i \left[1 - \frac{\omega_{\rm pl}^2}{\omega^2 \tilde{m}^*/m_0 + i\omega/\tau} \right] , \tag{50}$$

where $\tilde{m}^*$ is the renormalized mass defined by (47) and τ is the total inelastic lifetime of a quasiparticle defined by (12). In (49), $\sigma_{\rm pl}(\omega) = (\omega/4\pi)\,\mathrm{Im}\varepsilon_{\rm pl}(\omega)$ is the plasmon contribution to the optical conductivity with $\varepsilon_{\rm pl}(\omega)$ obtained from (48). Thus, the total optical conductivity obtained by using (48)–(50) will have a tail due to the plasmon contribution for a wide range of values of the frequency $(0 < \omega < \omega_p)$. This conclusion agrees with the results reported by Timusk *et al.*[11] and Schlessinger *et al.*[12] Finally, we remark that our theory can also be used to calculate the transport and optical properties of semiconductor superlattices (InAs-GaSb, GaAs-AlAs, etc.).[13]

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