High Landau levels in a smooth random potential for two-dimensional electrons

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We study the density of two-dimensional electronic states for high Landau levels in a perpendicular magnetic field and smooth random potential. The theory developed applies if the correlation radius of a random potential is larger than the magnetic length. Under this condition the exact summation of the diagram series for all the orders of perturbation theory is performed. The density of states represents a system of Gaussian peaks with the width Γ decreasing with energy E as $E^{-1/4}$. The magnetic-field dependence of the width is $\Gamma \propto \sqrt{B}$, if the correlation radius is smaller than the classical Larmour radius. In the opposite case Γ does not depend on B. If the correlation radius is smaller than the magnetic length, the self-consistent Born approximation, generalized to the case of smooth potential, applies.

I. INTRODUCTION

The discovery of the quantum Hall effect has stimulated the study of the electronic states in two-dimensional (2D) systems in a perpendicular magnetic field. One of the principal questions being investigated is the disorder-induced broadening of Landau levels. Theoretical papers on this subject can be conventionally divided into three groups: (i) exact solutions for some models of a short-range random potential;[1-4] (ii) calculation of "tails" of the density of states in the vicinity of some Landau level with the use of an instanton method;[5-10] (iii) calculation of the density of states in the frame of the self-consistent Born approximation (SCBA).[11-22]

In the latter approach, introduced by Ando and Uemura,[11,12] the calculation of the self-energy reduces to the summation of the subsequence of diagrams without self-intersections. In the case of a δ-correlated random potential (white noise) SCBA gives the width of the lowest Landau level of the same order of magnitude as Wegner's exact solution.[1] However, SCBA, which results in a semielliptic shape for the density of states, does not reproduce the Gaussian tail of the exact solution. The correct behavior of the density of states in the tail is provided by the instanton approach.[5,7]

It should be mentioned that the accuracy of SCBA increases with an increase in the number n of Landau level.[10,16,23,24] Indeed, each self-intersection in the diagram results in a phase factor which oscillates rapidly as n becomes large. Therefore, for $n \gg 1$, such diagrams are small compared to those that are taken into account within SCBA approach. This statement is also supported by the calculation of the tails of high Landau levels carried out in Ref. 8. It was shown that if the energy is measured from the boundary of Ando's semiellipse, then the characteristic width of the tail decreases with n as $n^{-1/2}$.

The consideration in Refs. 8, 10, and 16 refers to the case of a short-range random potential. Modification of the SCBA approach to the potential with a finite correlation radius was developed in Refs. 13–15, 18–21, and 24.

It turns out, however, that the SCBA approach is not applicable for large enough correlation radius R_c. This case is investigated in the present paper. As we will show, the criteria for the SCBA to be justified is $R_c \ll l$ where $l = (\hbar c/eB)^{1/2}$ is the magnetic length (B stands for magnetic field). For $R_c \sim l$ the contribution to the density of states from diagrams with self-intersections is of the same order as from SCBA diagrams. However, for $R_c \gg l$ another simplification occurs: the phase factors caused by self-intersections become small. We will show that for high Landau levels this allows one to perform the summation of all the diagrams and to obtain a closed expression for the density of states.

The relevant parameter, which determines the shape of the density of states, as a function of energy E, is the ratio R_c/R_L, where

$$R_L(E) = \left(\frac{2E}{\hbar\omega_c}\right)^{1/2} l \tag{1}$$

is the Larmour radius and $\omega_c = eB/mc$ is the cyclotron frequency (m stands for the electron mass). Note that for high Landau levels $R_L \gg l$. When $R_c \ll R_L$ our result matches the SCBA extended to the finite values of R_c. In the opposite case $R_c \gg R_L$ the result obtained describes a simple quasiclassical picture, where the random potential modulates the bottom of the band and the staircase of Landau levels just follows this modulation.[25,26]

The paper is organized as follows. In Sec. II we investigate the condition of validity of the SCBA approach in the case of a smooth random potential. In Sec. III we derive an expression for the density of states using a technique developed by Efros[25,26] for three-dimensional systems at zero magnetic field. The analysis of the result obtained and the numerical examples are presented in Sec. IV. Section V concludes the paper.

II. SCBA APPROACH IN THE CASE OF A SMOOTH RANDOM POTENTIAL

We start from the conventional expression for the density of states $g(E)$,

$$g(E) = \frac{1}{2\pi^2 l^2}\mathrm{Im}\sum_n \frac{1}{E - E_n - \Sigma_n(E)} = \frac{1}{2\pi^2 l^2}\mathrm{Im}\sum_n G_n(E), \quad (2)$$

where

$$E_n = (n + \tfrac{1}{2})\hbar\omega_c, \quad (3)$$

$G_n(E)$ and $\Sigma_n(E)$ are, respectively, the averaged Green function and the self-energy corresponding to nth Landau level. We study the density of states at energies $E \gg \hbar\omega_c$ corresponding to high Landau levels. For $n \gg 1$ the self-energy $\Sigma_n(E)$ varies slowly with n. Therefore, it is convinient to rewrite Eq. (2) using Poisson's formula,

$$g(E) = \frac{m}{2\pi\hbar^2}\left\{1 + 2\sum_{k=1}^{\infty}\cos\left[2\pi k\left(\frac{E}{\hbar\omega_c} - \frac{1}{2}\right)\right] \times \exp\left(-2\pi k\frac{\mathrm{Im}\Sigma(E)}{\hbar\omega_c}\right)\right\}. \quad (4)$$

In Eq. (4) we omitted the dependence of Σ on n, assuming that $\Sigma(E) = \Sigma_n(E)$, with $n = E/\hbar\omega_c$.

The first diagram for $\sum_n(E)$ is shown in Fig. 1(a). The expression, corresponding to this diagram, reads

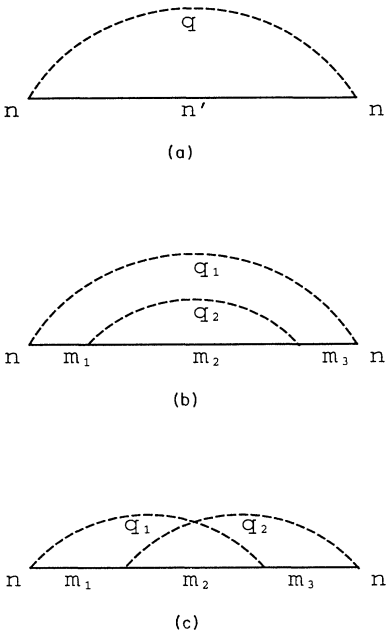

FIG. 1. The Feynman diagrams for the self-energy Σ_n. Solid line corresponds to the electron Green function, dashed line corresponds to the correlator of random potential. (a) The simplest diagram. (b) and (c) The second-order diagrams.

$$\Sigma_n^{(1)}(E) = \sum_{n'p'}\frac{1}{E - E_{n'}}\int\frac{d^2q}{(2\pi)^2}S(q)|\langle np|e^{i\mathbf{qr}}|n'p'\rangle|^2, \quad (5)$$

where $S(q)$ is the Fourier transform of the correlator of the random potential $V(\mathbf{r})$:

$$S(q) = \int d^2r\, e^{-i\mathbf{qr}}K(r), \quad (6)$$

where

$$K(|\mathbf{r} - \mathbf{r}'|) = \langle V(\mathbf{r})V(\mathbf{r}')\rangle, \quad (7)$$

and $\langle\cdots\rangle$ denotes configurational averaging.

The matrix element $\langle np|e^{i\mathbf{qr}}|n'p'\rangle$ calculated from nonperturbed wave functions has the following form:

$$\langle np|e^{i\mathbf{qr}}|n'p'\rangle = \frac{2\pi}{L_y}\delta(p - p' - q_y) \times e^{iq_x(p+p')l^2/2 + i(N-N')(\theta' + \pi/4)}A_{nn'}(ql), \quad (8)$$

where

$$A_{nn'}(ql) = \left(\frac{N'!}{N!}\right)^{1/2}e^{-q^2l^2/4}\left(\frac{ql}{\sqrt{2}}\right)^{N-N'} \times L_{N'}^{N-N'}\left(\frac{q^2l^2}{2}\right), \quad (9)$$

$L_{N'}^{N-N'}$ is the associate Laguerre polinomial, L_y is the normalization length, and N, N', θ' are defined as

$$N = \max(n, n'), \quad N' = \min(n, n'), \quad \theta' = \theta\,\mathrm{sgn}(n - n'), \quad \theta = \arctan(q_y/q_x). \quad (10)$$

Since we are interested in high Landau levels $n, n' \gg 1$, expression (8) can be simplified using the asymptotics of Laguerre polinomials,[27]

$$L_n^{\alpha}(x) \simeq \frac{\Gamma(n + \alpha + 1)}{n!}e^{x/2}\left[\left(n + \frac{\alpha + 1}{2}\right)x\right]^{-\alpha/2} \times J_{\alpha}\left[\sqrt{4x\left(n + \frac{\alpha + 1}{2}\right)}\right], \quad (11)$$

where J_α is the Bessel function. Then we get for $A_{nn'}$,

$$A_{nn'} \simeq J_{|n-n'|}(ql\sqrt{n + n' + 1}) \simeq J_{|n-n'|}(qR_L), \quad (12)$$

where R_L is the Larmour radius defined by Eq. (1) in which we substituted $E = E_n$. Note that the asymptotic (11) is valid if the argument of the Bessel function is less then n. Thus, one gets the condition

$$q \ll \frac{n}{R_L} = \frac{n^{1/2}}{2^{1/2}l}. \quad (13)$$

In Eq. (12) we have replaced n' by n in the argument of the Bessel function. The corresponding correction to the argument qR_L is of the order of $(n - n')qR_L/n$. For

$(n-n') \sim 1$ condition (13) guarantees that this correction is much less than unity. In general, such a correction can be neglected if the condition $|n-n'| \ll n/qR_L$ holds. According to (13), the value of n/qR_L is large. In the next section we will show that the relevant n and n' satisfy this condition.

Substituting (12) into (8) we get the final expression for the matrix element in the limit of large n:

$$\langle np|e^{i\mathbf{qr}}|n'p'\rangle = \frac{2\pi}{L_y}\delta(p-p'-q_y)e^{iq_x(p+p')l^2/2}C_{n-n'}(\mathbf{q}), \tag{14}$$

where the coefficients $C_{n-n'}(\mathbf{q})$ are given by

$$C_{n-n'}(\mathbf{q}) = i^{n-n'}e^{i(n-n')\theta}J_{n-n'}(qR_L). \tag{15}$$

Note that the matrix element depends now only on the difference $n-n'$.

In the frame of the SCBA aproach one should replace $E-E_{n'}$ by $E-E_{n'}-\Sigma_{n'}$ in the denominator of Eq. (5). Then the SCBA equation for the self-energy takes the form

$$\Sigma_n^{\mathrm{SCBA}}(E) = \sum_{n'}\int\frac{d^2q}{(2\pi)^2}S(q)\frac{J^2_{n-n'}(qR_L)}{E-E_{n'}-\Sigma^{\mathrm{SCBA}}_{n'}(E)}. \tag{16}$$

Since the typical value of q in Eq. (16) is of the order of $1/R_c$, condition (13) demands that

$$R_c \gg \frac{R_L}{n} \sim \frac{l}{n^{1/2}}. \tag{17}$$

On the other hand, for large enough n we have $R_c \ll R_L$. Then the argument of the Bessel function is large, so that one can replace $J_{n-n'}$ by its asymptotic expression. We have

$$\int d^2q\, S(q)J^2_{n-n'}(qR_L) \simeq \frac{2}{R_L(E)}\int_0^\infty dq\, S(q). \tag{18}$$

With (18) the solution of Eq. (16) for the imaginary part of the self-energy can be written as

$$\mathrm{Im}\Sigma^{\mathrm{SCBA}}(E) = \frac{1}{2\pi\hbar\omega_c R_L(E)}\int_0^\infty dq\, S(q). \tag{19}$$

This solution is valid if $\mathrm{Im}\Sigma^{\mathrm{SCBA}} \gg \hbar\omega_c$, when a large number of Landau levels contribute to the sum in Eq. (16). If condition (17) is not fulfilled, then, according to (13), the integral over q should be cut off at $q \simeq n/R_L$. Thus, we arrive at $\mathrm{Im}\Sigma^{\mathrm{SCBA}} = mS(0)/4\pi\hbar^2$, which is the SCBA result for the "white-noise" random potential.[11,12]

To investigate the validity of the SCBA approach, let us consider the contribution of the second-order diagrams to the self-energy. These diagrams are shown in Figs. 1(b) and 1(c). Diagram (b) is taken into account by SCBA, but diagram (c) is not. The expressions corresponding to diagrams (b) and (c) have the following form:

$$\Sigma^{(2)}_{n,\mathrm{b}} = \int\frac{d^2q_1}{(2\pi)^2}S(q_1)\int\frac{d^2q_2}{(2\pi)^2}S(q_2)\sum_{m_1m_2m_3}\frac{C_{n-m_1}(\mathbf{q}_1)C_{m_1-m_2}(\mathbf{q}_2)C_{m_2-m_3}(-\mathbf{q}_2)C_{m_3-n}(-\mathbf{q}_1)}{(E-E_{m_1})(E-E_{m_2})(E-E_{m_3})}, \tag{20}$$

$$\Sigma^{(2)}_{n,\mathrm{c}} = \int\frac{d^2q}{(2\pi)^2}S(q_1)\int\frac{d^2q}{(2\pi)^2}S(q_2)\exp[il^2(q_{1x}q_{2y}-q_{1y}q_{2x})] \times\sum_{m_1m_2m_3}\frac{C_{n-m_1}(\mathbf{q}_1)C_{m_1-m_2}(\mathbf{q}_2)C_{m_2-m_3}(-\mathbf{q}_1)C_{m_3-n}(-\mathbf{q}_2)}{(E-E_{m_1})(E-E_{m_1})(E-E_{m_3})}, \tag{21}$$

where the coefficients $C_{n-n'}$ are defined by Eq. (15). As was mentioned above, the additional phase factor in the second diagram results from the intersection of broken lines. The integration over q_1 and q_2 in Eq. (20) can be carried out separately. Using the relation

$$\int\frac{d^2q}{(2\pi)^2}S(q)C_{m_1-m_2}(\mathbf{q})C_{m_3-m_4}(-\mathbf{q}) = \delta_{m_1-m_2,m_4-m_3}I_{m_1-m_2}, \tag{22}$$

where

$$I_{m_1-m_2}\left(\frac{R_c}{R_L}\right) = \int_0^\infty\frac{dq}{2\pi}qS(q)J^2_{m_1-m_2}(qR_L), \tag{23}$$

Eq. (20) can be rewritten as

$$\Sigma^{(2)}_{n,\mathrm{b}} = \sum_{m_1m_2}\frac{I_{n-m_1}I_{m_1-m_2}}{(E-E_{m_1})^2(E-E_{m_2})}. \tag{24}$$

As follows from the previous consideration [see Eq. (18)], the product $I_{n-m_1}I_{m_1-m_2}$ can be estimated as

$$I_{n-m_1}I_{m_1-m_2} \sim \frac{S^2(0)}{R_c^2R_L^2} \sim \frac{S^2(0)}{nR_c^2l^2} \quad \text{for } R_c \gg \frac{l}{n^{1/2}}, \tag{25a}$$

$$I_{n-m_1}I_{m_1-m_2} \sim S^2(0)\left(\frac{n}{R_L^2}\right)^2 \sim \frac{S^2(0)}{l^4} \quad \text{for } R_c \ll \frac{l}{n^{1/2}}. \tag{25b}$$

To estimate $\Sigma^{(2)}_{n,\mathrm{c}}$ let us first perform the integration over the angles in Eq. (21). Then the expression for $\Sigma^{(2)}_{n,\mathrm{c}}$ takes the form

$$\Sigma^{(2)}_{n,c} = \int \frac{d^2q_1}{(2\pi)^2} S(q_1) \int \frac{d^2q_2}{(2\pi)^2} S(q_2) \sum_{m_1 m_2 m_3} J_{n-m_1+m_2-m_3}(l^2 q_1 q_2) \times \frac{J_{n-m_1}(q_1 R_L) J_{m_2-m_3}(q_1 R_L) J_{m_1-m_2}(q_2 R_L) J_{m_3-n}(q_2 R_L)}{(E-E_{m_1})(E-E_{m_2})(E-E_{m_3})}. \quad (26)$$

As in the previous case, the estimate of the integral in the numerator (we denote it with Q) depends on the relation between R_c and $l/n^{1/2}$. If $R_c \ll l/n^{1/2}$, then the upper limits in the integrals over q_1 and q_2 should be chosen as $l/n^{1/2}$. Since $q_1 R_L \sim q_2 R_L \sim n \gg 1$, we get the following estimate for Q:

$$Q \sim \frac{S^2(0)}{R_L^2} \int_0^{n^{1/2}/l} dq_1 \int_0^{n^{1/2}/l} dq_2 J_{n-m_1+m_2-m_3}(l^2 q_1 q_2) = \frac{S^2(0)}{R_L^2} \int_0^{n^{1/2}} \frac{dv}{v} \int_0^v du\, J_{n-m_1+m_2-m_3}(u) \sim \frac{S^2(0)}{R_L^2 l^2} \ln(n). \quad (27)$$

In the opposite case $R_c \gg l/n^{1/2}$ the integration over q_1 and q_2 is cut off at $1/R_c$. Then the estimate of the numerator in Eq. (26) takes the form

$$Q \sim \frac{S^2(0)}{R_L^2} \int_0^{1/R_c} dq_1 \int_0^{1/R_c} dq_2 J_{n-m_1+m_2-m_3}(l^2 q_1 q_2) = \frac{S^2(0)}{R_L^2 l^2} \int_0^{l^2/R_c^2} \frac{dv}{v} \int_0^v du J_{n-m_1+m_2-m_3}(u). \quad (28)$$

It is seen from Eq. (28), that the result depends strongly on the ratio of the correlation radius R_c to the magnetic lenght l. Indeed, if $R_c \ll l$ (but still much greater then $l/n^{1/2}$) we get

$$Q \sim \frac{S^2(0)}{R_L^2 l^2} \ln\left(\frac{l^2}{R_c^2}\right). \quad (29)$$

If $R_c \gg l$ (but still much smaller then R_L) the argument of the Bessel function in Eq. (28) is small. Thus, only the terms with

$$n - m_1 = m_3 - m_2 \quad (30)$$

contribute to the sum in Eq. (26). For these terms the estimate (28) takes the form

$$Q \sim \frac{S^2(0)}{R_L^2 R_c^2}. \quad (31)$$

To summarize the results, let us consider the ratio $\Sigma^{(2)}_{n,c}/\Sigma^{(2)}_{n,b}$ in the different domains of the correlation radius R_c,

$$\frac{\Sigma^{(2)}_{n,c}}{\Sigma^{(2)}_{n,b}} \sim \frac{\ln(n)}{n} \quad \text{for } R_c \ll \frac{l}{n^{1/2}}, \quad (32a)$$

$$\frac{\Sigma^{(2)}_{n,c}}{\Sigma^{(2)}_{n,b}} \sim \frac{\ln\left(\frac{l^2}{R_c^2}\right)}{\left(\frac{l^2}{R_c^2}\right)} \quad \text{for } \frac{l}{n^{1/2}} \ll R_c \ll l. \quad (32b)$$

The result (32a) agrees with that obtained in Ref. 16 for white-noise potential. Both estimates (32) match at $R_c \sim l/n^{1/2}$.

Equations (32) show that the diagrams with self-intersections are still suppressed if the correlation radius is smaller than the magnetic length. Therefore, SCBA is valid in this domain.

For $R_c \gg l$ we have [see Eqs. (25) and (31)] $\Sigma^{(2)}_{n,c} \sim \Sigma^{(2)}_{n,b}$, i.e., both diagrams (b) and (c) are of the same order. The latter conclusion can be drawn directly by comparing the expressions (20) and (21) for both diagrams. Indeed, since the characteristic values of q_1 and q_2 in (21) are $1/R_c$, the phase factor $\exp[il^2(q_{1x}q_{2y} - q_{1y}q_{2x})] \simeq 1$ when $R_c \gg l$. Moreover, it can be easily seen, after neglecting the phase factor in Eq. (21), that both diagrams (20) and (21) are just equal. The same holds for all the orders of the perturbation theory. The reason for this is that under the condition $R_c \gg l$ the "conservation rule" (30) for the numbers of Landau levels applies to each broken line of a diagram, no matter whether it is crossed by the other lines or not.

In the next section we will show that neglecting the phase factors in the diagrams with self-intersections allows one to perform the summation of all the terms in the perturbation series for large n. The explicit expression for the density of states is derived in the region $R_c \gg l$, where the SCBA approach fails.

III. DERIVATION OF THE EXPRESSION FOR THE DENSITY OF STATES

To illustrate the main idea of the derivation, let us first consider the simplest diagram for the trace of the Green function $G(E) = \sum_n G_n(E)$,

$$G^{(2)}(E) = \sum_{nn'} \frac{I_{n-n'}}{(E-E_n)^2(E-E_{n'})}, \quad (33)$$

where $I_{n-n'}$ is defined by Eq. (23). Since $I_{n-n'}$ is positive, the sign of each term in the sum (33) is determined by the sign of $E - E_{n'}$. Let us show that in the limit of large n the terms with $n \neq n'$ cancel each other, so that only the terms with $n = n'$ contribute to the sum. Note that Eq. (33) can be written in the following form:

$$G^{(2)}(E) = \sum_{n \neq n'} \frac{I_{n-n'}}{(E_n - E_{n'})^2} \left(\frac{1}{E - E_{n'}} - \frac{1}{E - E_n} \right) + \sum_{n \neq n'} \frac{I_{n-n'}}{(E_n - E_{n'})(E - E_n)^2} + \sum_n \frac{I_0}{(E - E_n)^3}. \quad (34)$$

It should be emphasized that $I_{n-n'}$ depends only on the absolute value of $n - n'$ [see Eq. (23)]. Then each term in the first sum in Eq. (34) changes sign by the replacement $n \leftrightarrow n'$. Hence, the first sum is zero. The second sum in Eq. (34) can be presented as

$$\frac{1}{\hbar\omega_c} \sum_n \frac{1}{(E - E_n)^2} \sum_{k=-n}^{\infty} \frac{I_k}{k}. \quad (35)$$

Let us show that this sum apears to be much less than the third sum in Eq. (23). Indeed, according to the definition (23), I_k contains the square of the Bessel function $J_k^2(qR_L)$. Note, that for large values of k the Bessel function is exponentially small if its order is larger than its argument.[27] According to (13), we have $qR_L \ll n$, so that the ratio I_n/I_0 is small. This allows us to extend the lower limit in the sum over k in (35) to $-\infty$. Then the terms with $k = k'$ and $k = -k'$ cancel each other (there is no term with $k = 0$).

The fact that only the terms with $k < qR_L$ contribute to the sum (35) justifies neglecting the difference between n and n' in the argument of the Bessel function in Eq. (12). Indeed, the corresponding condition can be written as $|k| \ll n/qR_L$ [see the discussion after Eq. (13)]. For the maximal relevant $k \sim qR_L$ the condition demands that $(qR_L)^2 \ll n$. Note, that since $q \sim R_c^{-1}$ and $R_L \sim n^{1/2}l$, the latter condition is equivalent to $R_c \gg l$—the same condition that allows one to neglect the phase factors in the diagrams.

Finally, we have

$$G^{(2)}(E) = \sum_n \frac{I_0}{(E - E_n)^3}. \quad (36)$$

The cancelation, demonstrated above, takes place also in the higher-order diagrams. This allows one to obtain an explicit expression for the density of states.

To derive the expression for $g(E)$, we extend the technique developed in Refs. 25 and 26 to the case of a two-dimensional system in a magnetic field. The Dyson-Schwinger equation for the self-energy Σ_n [Fig. 2(a)] can be written as follows:

$$\Sigma_n = \sum_{n'} I_{n-n'} G_{n'} \Gamma_{n'n}, \quad (37)$$

where

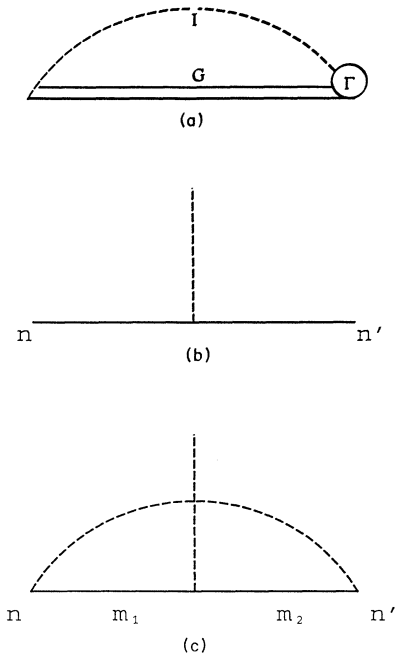

FIG. 2. (a) The graphical representation of the Dyson-Schwinger equation. (b) and (c) The simplest diagrams for the vertex part $\Gamma_{n'n}$.

$$\Sigma_n = E - E_n - G_n^{-1} \quad (38)$$

and $\Gamma_{n'n}$ is the vertex part. The simplest diagrams for $\Gamma_{n'n}$ are shown in Figs. 2(b) and 2(c). It should be emphasized that only neglecting the phase factors, which are caused by self-intersections, makes it possible to perform the momentum integration in each diagram. Then the variables in the Dyson-Schwinger equation depend only on the numbers of Landau levels. Taking into account the phase factors would result in the appearence of the momentum integration in the right-hand side (rhs) of the Dyson-Schwinger equation. Therefore, Eq. (37) is valid under the condition $R_c \gg l$.

To complete the system (37), one should relate the vertex part to the self-energy. In the case $n = n'$ this relation has the form

$$\Gamma_{nn} = \frac{dG_n^{-1}}{dE}. \quad (39)$$

It can be considered as the discrete analogue of the Ward identity. For $n \neq n'$ it can be proved that the following identity holds:

$$\Gamma_{n'n} = \frac{G_n^{-1} - G_{n'}^{-1}}{E_{n'} - E_n}. \quad (40)$$

The proof is given in the Appendix. It extensively uses

the fact that n and n' are large.

Substituting Eqs. (39) and (40) into (37), we get the following system of linear equations:

$$I_0\frac{dG_n}{dE}+(E-E_n)G_n-1=\sum_{n'\neq n}\frac{I_{n-n'}}{E_{n'}-E_n}G_{n'}-\sum_{n'\neq n}\frac{I_{n-n'}}{E_{n'}-E_n}G_n. \quad (41)$$

The second term in the rhs vanishes for the same reason as (35). Then the solution of the system can be presented as

$$G_n(E)=-\frac{i}{\hbar}\int_0^\infty dt\,P(t)\exp\left(\frac{i}{\hbar}(E-E_n)t\right), \quad (42)$$

where $P(t)$ satisfies the equation

$$\frac{dP(t)}{dt}+P(t)\left(\frac{I_0}{\hbar^2}t+\frac{i}{\hbar^2\omega_c}\sum_{k\neq 0}\frac{I_k}{k}e^{-ik\omega_c t}\right)=0 \quad (43)$$

with the boundary condition $P(0)=1$. The solution of Eq. (43) can be easily found to be

$$P(t)=\exp\left[-\frac{I_0}{2\hbar^2}t^2-\frac{4}{\hbar^2\omega_c^2}\sum_{k=1}^{\infty}\frac{I_k}{k^2}\sin^2\left(\frac{k\omega_c t}{2}\right)\right]. \quad (44)$$

Substituting (44) into (42) and performing the summation over n, we get

$$\sum_n G_n(E)=-\frac{i}{\hbar}\int_0^\infty dt\,\exp\left(\frac{i}{\hbar}Et-\frac{I_0}{2\hbar^2}t^2\right)\sum_n\exp\left[-\frac{i}{\hbar}E_nt-\frac{4}{\hbar^2\omega_c^2}\sum_{k=1}^{\infty}\frac{I_k}{k^2}\sin^2\left(\frac{k\omega_c t}{2}\right)\right]. \quad (45)$$

The sum over n in the rhs represents the system of sharp peaks at $t=2\pi p/\omega_c$, where p is an integer. For such t the sum over k in the exponent in Eq. (45) vanishes. Finally we obtain

$$\begin{aligned}G(E)&=-\frac{i}{\hbar}\sum_n\int_0^\infty dt\,\exp\left(\frac{i}{\hbar}(E-E_n)t-\frac{I_0(E)}{2\hbar^2}t^2\right)\\&=-\frac{i}{\hbar}\sum_n\int_{-\infty}^{\infty}\frac{dV}{\sqrt{2\pi I_0(E)}}\int_0^\infty dt\exp\left(-\frac{V^2}{2I_0(E)}+\frac{i}{\hbar}(E-E_n-V)t\right)\\&=\int_{-\infty}^{\infty}\frac{dV}{\sqrt{2\pi I_0(E)}}\exp\left(-\frac{V^2}{2I_0(E)}\right)\sum_n\frac{1}{E-E_n-V}.\end{aligned} \quad (46)$$

The corresponding expression for the density of states takes the form

$$g(E)=\frac{1}{2\pi l^2}\sum_n\frac{1}{\sqrt{2\pi}\Gamma(E)}\exp\left(-\frac{(E-E_n)^2}{2\Gamma^2(E)}\right), \quad (47)$$

with $\Gamma(E)$ given by

$$\Gamma^2(E)=I_0(E)=\int_0^\infty\frac{dq}{2\pi}qS(q)J_0^2[qR_L(E)]. \quad (48)$$

The analysis of this expression is performed in the next section.

IV. DISCUSSION AND NUMERICAL RESULTS

Let us give a qualitative derivation of the main result obtained [Eqs. (47) and (48)]. In the absence of a magnetic field the wave packet, corresponding to a two-dimensional electron moving in a smooth potential, represents a strip with the width of the order of de Broglie wavelength

$$\lambda_d(E)=\frac{2\pi\hbar}{\sqrt{2mE}} \quad (49)$$

with its center following the classical trajectory. Correspondingly, the quasiclassical state in a magnetic field represents a ring with the width λ_d and the radius equal to the Larmour radius R_L. Note that the condition $R_c\gg l/n^{1/2}$, which was extensively used in our consideration [Eqs. (17) and (32)], is nothing but the usual quasiclassical condition $R_c\gg\lambda_d$. It guarantees that the change of the potential across the ring is negligibly small. In this case the effective potential, acting on the ring, can be written as

$$V_{\rm eff}(R_L)=\int_0^{2\pi}\frac{d\phi}{2\pi}V(R_L,\phi), \quad (50)$$

where integration is performed along the ring, ϕ being the polar angle. In the presence of the magnetic field $V_{\rm eff}$ plays essentially the same role as the potential V at zero field. Namely, we can regard it as the potential modulating the bottom of the band, so that the density of states can be written in the following form

$$g(E)=\left\langle\sum_n\delta(E-E_n-V_{\rm eff})\right\rangle. \quad (51)$$

Let us demonstrate that Eq. (51) with $V_{\rm eff}$, defined by Eq. (50), leads to the same result, Eq. (47), as was obtained in the previous section. Since $V_{\rm eff}$ is related linearly to the Gaussian potential V, its distribution function is also Gaussian

$$f(V_{\rm eff}) = \frac{1}{\sqrt{2\pi}\Gamma_{\rm eff}} \exp\left(-\frac{V_{\rm eff}^2}{2\Gamma_{\rm eff}^2}\right), \tag{52}$$

where the width $\Gamma_{\rm eff}$ is given by

$$\Gamma_{\rm eff}^2 = \langle V_{\rm eff}^2 \rangle. \tag{53}$$

To connect $\Gamma_{\rm eff}$ to the correlation function of the potential V, let us rewrite Eq. (50) in the following form:

$$V_{\rm eff} = \frac{1}{2\pi R_L} \int d^2r\, V(\mathbf{r}) \delta(|\mathbf{r}| - R_L). \tag{54}$$

Substituting Eq. (54) into Eq. (53) and averaging with the use of Eqs. (6) and (7), we obtain

$$\Gamma_{\rm eff}^2 = \int \frac{d^2q}{(2\pi)^2} S(q) \left| \int \frac{d^2r}{2\pi R_L} e^{i\mathbf{qr}} \delta(|\mathbf{r}| - R_L) \right|^2. \tag{55}$$

Since the integral over $\mathbf{r}$ is equal to $J_0(qR_L)$, we realize that $\Gamma_{\rm eff} = \Gamma$, where Γ is defined by Eq. (48). Finally, averaging Eq. (51) with the distribution function (52), we arrive at Eq. (47).

If the correlation radius is much larger than the Larmour radius, the difference between $V_{\rm eff}$ and V vanishes. The change of potential along the Larmour orbit is negligibly small. In this case we have $V_{\rm eff} = V$. The broadening of Landau levels is then independent of the number n. Indeed, if $R_c \gg R_L$ the argument of the Bessel function is small and can be replaced by unity. Then we get

$$\langle V_{\rm eff}^2 \rangle = \langle V^2 \rangle = \int_0^\infty \frac{dq}{2\pi} q S(q). \tag{56}$$

Consider now the opposite limit $R_c \ll R_L$. In this case we can divide the Larmour orbit into the segments with length R_c. Within each segment the potential varies slowly and has a random value. Then the mean square potential can be estimated as

$$\langle V_{\rm eff}^2 \rangle \sim \frac{\langle V^2 \rangle}{M}, \tag{57}$$

where $M \sim R_L/R_c$ is the typical number of segments. It is seen from Eq. (57) that in this case the width Γ is proportional to $R_L^{-1/2} \sim E^{-1/4}$. Such a decrease of the width with energy results from the effective averaging out of the random potential by the electron, moving along the Larmour orbit. The analytical expression for Γ emerges from the replacement of the Bessel function by its asymptotics in Eq. (48),

$$\Gamma^2(E) = \frac{1}{2\pi^2 R_L(E)} \int_0^\infty dq\, S(q). \tag{58}$$

We see that $\Gamma^2 \sim R_L^{-1}$ in agreement with the above qualitative arguments.

If the ratio $\Gamma/\hbar\omega_c$ is large, the Landau levels manifest themselves as a weak modulation in the density of states on the top of a constant background. In this case it is convinient to rewrite Eq. (47) in a different form using the Poisson summation formula,

$$g(E) = \frac{m}{2\pi\hbar^2}\left\{1 + 2\sum_{k=1}^{\infty} \cos\left[2\pi k\left(\frac{E}{\hbar\omega_c} - \frac{1}{2}\right)\right] \times \exp\left(-\frac{2\pi^2\Gamma^2(E)}{\hbar^2\omega_c^2}k^2\right)\right\}. \tag{59}$$

It can be seen from Eqs. (58) and (59) that in the region $l \ll R_c \ll R_L$ the amplitude of modulation increases with energy. In the case $\Gamma(E) \gg \hbar\omega_c$ one should only keep the term $k = 1$ in the sum of Eq. (59). Then we have

$$\frac{\delta g}{g} = 2\cos\left[2\pi\left(\frac{E}{\hbar\omega_c} - \frac{1}{2}\right)\right] \times \exp\left(-\frac{1}{\hbar^2\omega_c^2 R_L(E)} \int_0^\infty dq\, S(q)\right). \tag{60}$$

Note that the same expression for $\delta g/g$ follows from the SCBA approach in the region $\lambda_d \ll R_c \ll l$, if the condition $\mathrm{Im}\Sigma^{\rm SCBA} \gg \hbar\omega_c$ is fulfilled [see Eqs. (4) and (19)]. The coincidence of the results can be explained as follows. The condition $\mathrm{Im}\Sigma^{\rm SCBA} \gg \hbar\omega_c$, which guarantees that the modulation in $g(E)$ is weak, allows one to replace the sum over n' in the SCBA equation (16) by the integral. Then it immediately follows that the imaginary part of this integral does not depend on the actual value of $\Sigma^{\rm SCBA}$ in the denominator. Therefore, despite the fact that in the region $l \ll R_c \ll R_L$ the SCBA approach neglects diagrams of the same order as those it takes into account, it provides the correct amplitude of the first harmonics of modulation in $g(E)$. We should emphasize that our result maps on the SCBA, only if the modulation is

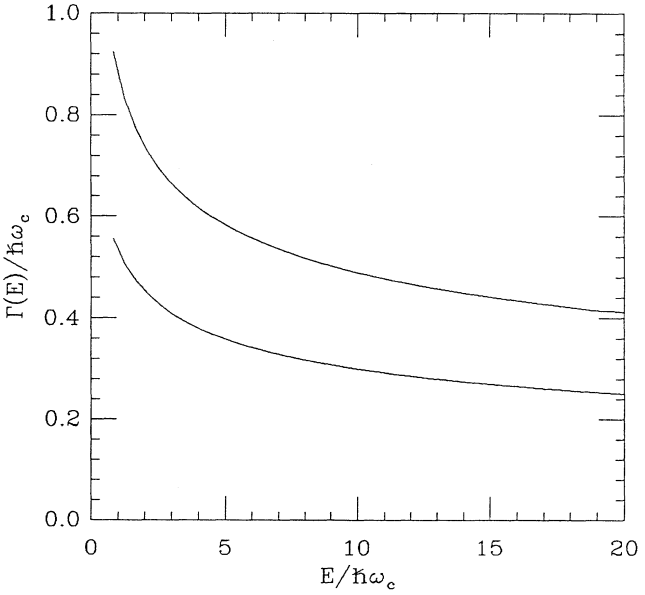

FIG. 3. The width $\Gamma(E)$ is shown with $N = 2\times10^{11}$ cm^{-2}, $B = 1$ T for $z_0 = 50$ Å (upper curve) and $z_0 = 200$ Å.

very weak. Indeed, it is seen that the terms with $k = 2$ in Eqs. (4) and (59) differ exponentially.

Below we present the results of numerical calculations performed for a 2D electron gas, confined in a GaAs heterostructure with the impurity plane separated by a distance z_0 from the 2D electron plane. The random potential results from the spacial fluctuations in the distribution of donors. The Fourier transform $S(q)$ of the correlation function of random potential is then given by[12,28]

$$S(q) = (4\pi)^2 (N a_B^2) E_B^2 \frac{e^{-2qz_0}}{\left(q + \frac{2}{a_B}\right)^2}, \tag{61}$$

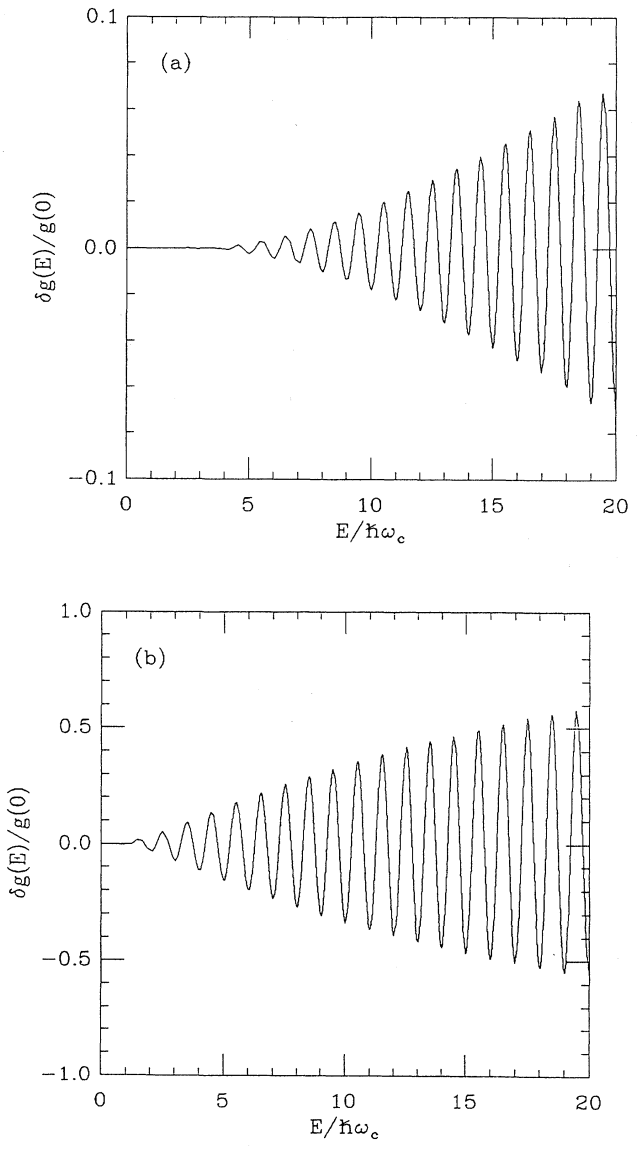

FIG. 4. Modulation of the density of states $\delta g(E)/g(0)$ is shown with $N = 2 \times 10^{11}$ cm^{-2}, $B = 1$ T for (a) $z_0 = 50$ Å, (b) $z_0 = 200$ Å.

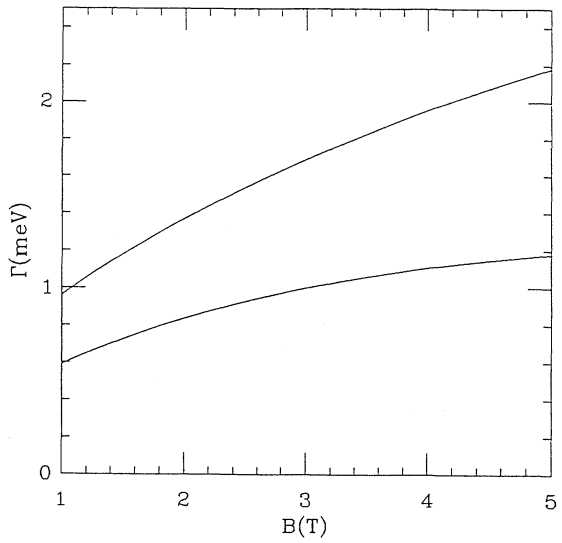

FIG. 5. The width $\Gamma(B)$ is shown with $N = 2 \times 10^{11}$ cm^{-2}, $E = 10$ MeV for $z_0 = 50$ Å (upper curve) and $z_0 = 200$ Å.

where a_B and E_B are correspondingly the Bohr radius and the Bohr energy in GaAs ($a_B = 100$ Å, $E_B = 6$ meV). The concentration of donors was chosen to be $N = 2 \times 10^{11}$ cm^{-2}. In Fig. 3 we plot the dependence of Γ on energy for $z_0 = 50$ and 200 Å. It is seen that Γ varies with E rather slowly at large E. The shape of the density of states versus E is plotted in Fig. 4. Both curves represent a system of peaks on the top of a constant background with the amplitude slowly increasing with energy. It is seen that the modulation of the density of states for $z_0 = 50$ Å is less pronounced than for $z_0 = 200$ Å, since the increase in z_0 supresses the short-range harmonics of random potential.[29–31] In Fig. 5 we present a plot of Γ as a function of magnetic field. With a very good accuracy this dependence is $\Gamma \sim \sqrt{B}$. The magnitude of Γ differs about two times for $z_0 = 50$ and 200 Å.

V. CONCLUSION

In this paper we developed the theory for the density of 2D electronic states with high Landau levels in a smooth random potential. We have shown that the SCBA approach can be extended to the case of finite correlation radius of random potential, up to the values $R_c \leq l$. The corresponding expressions for the density of states and for the self-energy are given by Eqs. (4) and (19). If $R_c \gg l$, the SCBA fails, but the quasiclassical approximation applies. The density of states represents a system of Gaussian peaks [Eq. (47)] with the width Γ, given by Eq. (48). In the region $l \ll R_c \ll R_L$ the width Γ slowly decreases as the energy increases [Eq. (58)]. In the case of weak modulation, the expression for the density of states is given by Eq. (59) and the amplitude of modulation slowly increases with energy. The dependence of Γ on magnetic field is $\Gamma \sim \sqrt{B}$. If $R_c \gg R_L$, the width Γ does

not depend on magnetic field or energy, and is given by Eq. (56).

In our consideration we have assumed that the random potential is Gaussian. If it is created by impurities with the finite radius of the potential r_0, then the Gaussian approach is valid only for high concentrations of impurities N such that $Nr_0^2 \gg 1$. In the opposite case, $Nr_0^2 \ll 1$, we have $R_c \simeq r_0$, and the physical picture depends on the relation between R_c and R_L. For $R_c > R_L$ the motion of the Larmour circle in the potential of an individual impurity should be considered.

The situation is different, however, if impurities are charged. Then the potential at each point represents the sum of long-ranged Coloumb potentials of individual impurities. The condition of validity of the Gaussian approach depends on whether impurities are located in the plane or they are spaced from the plane by some distance z_0. For $z_0 = 0$ one should take into account the screening of the impurity potential by the electron gas. Since the screening radius is equal to the Bohr radius a_B,[12] the condition of validity of the Gaussian approach is $N^{1/2}a_B > 1$. However, for typical experemental concentrations, $N \sim 10^{11} - 10^{12}$ cm^{-2}, the values $N^{-1/2}$ and a_B are of the same order. The situation improves with the increase of z_0, since more impurities contribute to the potential at a given point.[28–31] It is seen that the correlator (61) falls down exponentially as z_0 increases, i.e., the potential becomes smoother. When z_0 is larger than a_B, the condition of validity of the Gaussian approach takes the form $N^{1/2}z_0 > 1$. Under the latter condition $R_c \simeq z_0 > N^{-1/2}$, i.e., the electron "feels" the potential of many impurities. In other words, the discreteness of impurities is not important.

Let us briefly discuss the connection between our results and the experimental data. The 2D density of states has been investigated in a number of experiments by different methods. They are the mesurements of magnetization,[32] heat capacity,[33,34] magnetocapacitance,[35] and the spectra of radiative recombination of electrons.[36] In most experiments[33–36] the width of the Landau levels was found to be an oscillating function of magnetic field. The reason for such a behavior is the change of the random potential with B. Indeed, by changing the filling factor of the Landau levels the magnetic field alters the screening of external potential.[29–31] This effect can be taken into account in our theory if we introduce the B dependence of screening into the correlator $S(q)$. In the experiment[32] the width Γ did not oscillate with B. The best fit of the data was achieved, assuming $g(E)$ to be the sum of Gaussians with the width Γ varying with B as $\sqrt{B}$. Such a dependence on magnetic field can be obtained in the frame of the SCBA approach.[11,12] However, the semielliptic shape of the density of states with a gap between Landau levels is completely inconsistent with the experimental data. Our expression for the density of states results in the Gaussian broadening of Landau levels and provides the $\sqrt{B}$ dependence of Γ, as follows from Eq. (58). The decrease of Γ with energy is rather slow and becomes noticeable only if a large numbers of Landau levels is considered. For a spacer thickness $z_0 = 200$ Å the magnitude of Γ, as it is seen from Fig. 5, is $\Gamma \sim 1$ meV, in agreement with the experimental value $\Gamma \sim 1$–3 meV, reported in Ref. 32. Note, however, that the parameters of the sample, studied in Ref. 32 ($z_0 = 400$ Å, $N = 5.4 \times 10^{11}$ cm^{-2}), are somewhat different from those used in the numerical calculations.

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APPENDIX

In this appendix we give the proof of the Ward identity (40). The self-vertex part $\Lambda_{n'n}$ is defined by the equation

$$\Gamma_{n'n} = 1 + \Lambda_{n'n}, \tag{A1}$$

and contains all the diagrams contributing into $\Gamma_{n'n}$, except the simplest one, shown on Fig. 2(b). In the absence of phase factors [see Eq. (21)] the contribution of all the kth-order diagrams into $\Lambda_{n'n}$ can be written as

$$\Lambda_{n'n}^{(N)} = \int Dq \sum_{\{m\}} \sum_{i=1}^{N-1} \delta_{n'-n,m_i-m_{i+1}} \frac{C_{n'-m_1} \cdots C_{m_{i-1}-m_i} C_{m_{i+1}-m_{i+2}} \cdots C_{m_N-n}}{(E-E_{m_1}) \cdots (E-E_{m_i})(E-E_{m_{i+1}}) \cdots (E-E_{m_N})}, \tag{A2}$$

where $N = 2k$, $C_{n-n'}$ is defined by Eq. (15) and the first sum in (A2) is performed over all internal indexes. The brief notation Dq stands for the following expression:

$$Dq = \sum_{\text{diagrams}} \prod_{l=1}^{N} \frac{d^2q_l}{(2\pi)^2} \prod_{ij} (2\pi)^2 \delta(\mathbf{q}_i + \mathbf{q}_j) S(q_i), \tag{A3}$$

where the product of k δ functions performs the contraction of k pairs of $C_{n-n'}$ forming the lines of a given diagram. The analytical expression for each line is given by Eq. (22). Using the relation

$$\frac{1}{(E-E_{m_i})(E-E_{m_{i+1}})} = \frac{1}{E_{n'}-E_n} \left(\frac{1}{E-E_{m_i}} - \frac{1}{E-E_{m_{i+1}}} \right), \tag{A4}$$

one can rewrite Eq. (A2) in the following form:

$$\Lambda_{n'n}^{(N)} = \int Dq \sum_{\{m\}} \sum_{i=1}^{N-1} \frac{\delta_{n'-n,m_i-m_{i+1}}}{E_{n'}-E_n} \frac{C_{n'-m_1}\cdots C_{m_{i-1}-m_i}}{(E-E_{m_1})\cdots(E-E_{m_{i-1}})} \times \left(\frac{1}{E-E_{m_i}} - \frac{1}{E-E_{m_{i+1}}}\right) \frac{C_{m_{i+1}-m_{i+2}}\cdots C_{m_N-n}}{(E-E_{m_{i+2}})\cdots(E-E_{m_N})}. \tag{A5}$$

Performing summation over m_{i+1} in the first term, and over m_i in the second term in parentheses we get

$$\Lambda_{n'n}^{(N)} = \int Dq \sum_{\{m\}} \sum_{i=1}^{N-1} \frac{1}{E_{n'}-E_n} \left(\frac{C_{n'-m_1}\cdots C_{m_i-m_{i+1}+n-n'}\cdots C_{m_{N-1}-n}}{(E-E_{m_1})\cdots(E-E_{m_{N-1}})} - \frac{C_{n'-m_1}\cdots C_{m_{i-1}-m_i+n-n'}\cdots C_{m_{N-1}-n}}{(E-E_{m_1})\cdots(E-E_{m_{N-1}})} \right), \tag{A6}$$

where we used the notations $m_N = n$, $m_0 = n'$. It is easy to see that only terms with $i = N-1$ in the first sum, and with $i = 1$ in the second sum of (A6) survive. All other terms cancel each other. The remaining expression reads

$$(E_{n'} - E_n)\Lambda_{n'n}^{(N)} = \int Dq \sum_{\{m\}} \frac{C_{n'-m_1}\cdots C_{m_{N-1}-n'}}{(E-E_{m_1})\cdots(E-E_{m_{N-1}})} - (n' \to n). \tag{A7}$$

The rhs of (A7) is nothing but $\Sigma_{n'}^{(N)} - \Sigma_n^{(N)}$. Thus, we arrive at the identity

$$\Lambda_{n'n} = \frac{\Sigma_{n'} - \Sigma_n}{E_{n'} - E_n}, \tag{A8}$$

which with (38) and (A1) leads to Eq. (40). Note that the cancelation of the terms in (A6) is completely due to the fact that coefficients $C_{n-n'}$ depend only on the difference $n - n'$. Therefore, the condition of validity of the idendity (A8) is $n \gg 1$.

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