Slave-particle studies of the electron-momentum distribution in the low-dimensional *t-J* model

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The electron-momentum distribution function in the t-J model is studied in the framework of the slave-particle approach. Within the decoupling scheme used in gauge-field and related theories, we treat formally phase and amplitude fluctuations as well as constraints without further approximations. Our result indicates that the electron Fermi surface observed in high-resolution angle-resolved photoemission and inverse-photoemission experiments cannot be explained within this framework, and the sum rule for the physical electron is not obeyed. A correct scaling behavior of the electron-momentum distribution function near $k \sim k_F$ and $k \sim 3k_F$ in one dimension can be reproduced by considering the nonlocal string fields [Z. Y. Weng *et al.*, Phys. Rev. B **45**, 7850 (1992)], but the overall momentum distribution is still not correct, at least at the mean-field level.

I. INTRODUCTION

The t-J model is one of the simplest models containing the essence of strong correlations, and its implications for oxide superconductivity[1,2] still remain an outstanding problem. The t-J model was originally introduced as an effective Hamiltonian of the Hubbard model in the strong-coupling regime, where the on-site Coulomb repulsion U is very large as compared with the electron-hopping energy t, and therefore the electrons become strongly correlated to avoid double occupancy. In this case, the electron's Hilbert space is severely restricted due to this constraint $\sum_\sigma C^\dagger_{i\sigma}C_{i\sigma} \leq 1$. Anderson[1] and later Zhang and Rice[2] have argued strongly that the basic physics of oxide superconductors can be described by the t-J model.

The normal-state properties of oxide superconductors exhibit a number of anomalous properties in the sense that they do not fit in the conventional Fermi-liquid theory.[3,4] Some properties can be interpreted only in terms of a doped Mott insulator.[3,4] A central question in the theory of these strongly correlated systems concerns the nature of the electron Fermi surface (EFS).[4] High-resolution angle-resolved photoemission and inverse-photoemission experiments[5] demonstrate the existence of a large EFS, with an area consistent with band-structure calculations. Since the band theory is consistent with the Luttinger theorem,[6] this means that the EFS area contains $1-\delta$ electrons per site, where δ is the hole-doping concentration. Although the topology of the EFS is in general agreement with one-electron-band calculations, the Fermi velocity is quite different. This indicates that electron correlations renormalize considerably the results obtained in the framework of a single-particle treatment. It has recently been shown by small-cluster diagonalization that in a two-dimensional (2D) square lattice the EFS within the t-J model is consistent with Luttinger's theorem.[7] Monte Carlo simulations for a nearly-half-filled 2D Hubbard model also support this result.[8] Moreover, the electron-momentum distribution function of a 2D t-J model was studied[9] by using the Luttinger-Jastrow-Gutzwiller variational wave function, which seems to show the existence of an EFS and an algebraic singularity at the Fermi edge. For the one-dimensional (1D) large-U-limit Hubbard model which is equivalent to the t-J model, Ogata and Shiba[10] obtained the electron-momentum distribution function by using the Lieb-Wu exact wave function.[11] Their result also shows the existence of an EFS as well as the singular behavior at $k \sim k_F$ and $k \sim 3k_F$ in the momentum distribution function. Furthermore, Yokoyama and Ogata[12] studied the 1D t-J model by using exact diagonalization of small systems and found a power-law singularity appearing at k_F in the momentum distribution function.

So far, strong correlation effects can be properly taken into account only by numerical methods,[7–10,12] such as the variational Monte Carlo technique,[13] exact cluster diagonalization,[14] and various realizations of the quantum Monte Carlo method.[15] Apart from these numerical techniques, an analytical approach to the t-J model receiving a great deal of attention is the slave-particle

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theory,[4,16] where the electron operator $C_{i\sigma}$ is presented as $C_{i\sigma}=a_i^\dagger f_{i\sigma}$ with $a_i^\dagger$ as the slave boson and $f_{i\sigma}$ as the fermion or vice versa. This way the nonholonomic constraint

$$\sum_\sigma C_{i\sigma}^\dagger C_{i\sigma} \leq 1 \tag{1}$$

is converted into a holonomic one

$$a_i^\dagger a_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1, \tag{2}$$

which means a given site cannot be occupied by more than one particle. A new gauge degree of freedom must be introduced to incorporate the constraint, which means that the slave-particle representation should be invariant under a local gauge transformation $a_i \to a_i e^{i\theta(r_i,t)}$, $f_{i\sigma} \to f_{i\sigma} e^{i\theta(r_i,t)}$, and all physical quantities should be invariant with respect to this transformation. We call such slave-particle approach "conventional." The advantage of this formalism is that the charge and spin degrees of freedom of electrons may be separated at the mean-field level, where the elementary charge and spin excitations are called holons and spinons, respectively, but spinons and holons are strongly coupled by gauge-field (phase) fluctuations[17] or other effects beyond the mean-field approximation. The two fluids of spinons and holons represent the same set of electrons, and they must, on average, flow together. The decoupling of charge and spin degrees of freedom in the large-U-limit Hubbard model is undoubtedly correct in 1D,[10] where the charge degrees of freedom of the ground state are expressed as a Slater determinant of spinless fermions, while its spin degrees of freedom are equivalent to the 1D $S=\frac{1}{2}$ Heisenberg model, so that charge and spin excitations propagate at different velocities. However, the situation is still not clear in 2D.

The 1D t-J model (the large-U-limit Hubbard model) behaves like a Luttinger liquid.[10,12] The separation of the spin and charge degrees of freedom is indeed generic to the universality class of Luttinger liquids.[18] Anderson[19] has hypothesized that the normal state of 2D strongly interacting systems relevant for the oxide superconductors should be described in terms of the theory of Luttinger liquids. Thus it is interesting to understand the global features of the electron-momentum distribution function and EFS in the t-J model where separation of the spin and charge degrees of freedom might be expected. To our knowledge, the global features of the electron-momentum distribution function and EFS have been obtained only by using numerical techniques, and have not been studied systematically by analytical methods.

In this paper, we study analytically the electron-momentum distribution of the t-J model by using the slave-particle approach. In Sec. II, we present a formal study of the electron-momentum distribution function and discuss the sum rule obeyed by it. Under the decoupling scheme commonly used in gauge-field[17] and related theories, we treat formally the phase and amplitude fluctuations, constraints, and other effects without further approximations. The results obtained indicate that the EFS observed in the high-resolution angle-resolved photoemission and inverse photoemission experiments[5] cannot be explained by the conventional slave-particle approach. Moreover, the sum rule of the physical electron is not obeyed, if the corresponding sum rules are imposed on the above particles. Since the total number of electrons is independent of interactions, this result is also valid beyond the decoupling scheme within the slave-particle approach. In Sec. III, we give an example, following Ref. 20, to show that the scaling behavior of the electron-momentum distribution function near $k\sim k_F$ and $k\sim 3k_F$ in 1D can be reproduced by considering nonlocal string fields in the mean-field approximation, but there is still no EFS, and the sum rule for the electron number is still violated. If the Luttinger theorem is obeyed, the Fermi volume is invariant under the interactions. Thus our mean-field result seems to indicate that the correct scaling behavior of the electron-momentum distribution does not guarantee the existence of an EFS (in the sense of global distribution) in the same theoretical framework. Section IV is devoted to a summary and discussions on related problems.

II. FORMAL STUDY OF ELECTRON-MOMENTUM DISTRIBUTION

The slave-particle theory can be the slave-boson or the slave-fermion theories according to statistics assigned to spinons and holons. The slave-boson formulation[21] is one of the popular methods of treating the t-J model. This method, however, does not give a good energy of the ground state.[22] For example, in the half-filled case, where the t-J model reduces to the antiferromagnetic Heisenberg model, the lowest-energy state obtained by this method fails to show the expected long-range Néel order, and the energy is considerably higher than numerical estimates.[13] Also, it does not satisfy the Marshall sign rule.[23] However, this theory provides a *spinon* Fermi surface[17] even in the mean-field approximation. Alternatively, the slave-fermion approach naturally gives an ordered Néel state at half-filling,[24] which obeys the Marshall[23] sign rule. The ground-state energy obtained in this case is much better than the slave-boson case. At the mean-field level, they are quite different, although in principle they should be equivalent to each other. The differences may be reduced by going beyond the mean-field[25] approximation. In the following, we use both approaches to discuss the electron-momentum distribution function and the sum rule obeyed by it, and a direct comparison of the obtained results is made.

First consider the slave-fermion representation in which the electron operator can be expressed as $C_{i\sigma}=e_i^\dagger b_{i\sigma}$, where $e_i^\dagger$ is the slave fermion, while $b_{i\sigma}$ is the Schwinger boson, with the constraint $\sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} + e_i^\dagger e_i = 1$. In this case, the Lagrangian $L_{\rm SF}$ and the partition function $Z_{\rm SF}$ of the t-J model in imaginary time τ can be written as

$$L_{\rm SF} = \sum_{i\sigma} b^\dagger_{i\sigma}\partial_\tau b_{i\sigma} + \sum_i e^\dagger_i \partial_\tau e_i + H + \sum_i \lambda_i \left(e^\dagger_i e_i + \sum_\sigma b^\dagger_{i\sigma} b_{i\sigma} - 1 \right), \tag{3}$$

$$H = -t \sum_{\langle ij \rangle \sigma} e_i e^\dagger_j b^\dagger_{i\sigma} b_{j\sigma} + \frac{J}{4} \sum_{\langle ij \rangle} b^\dagger_{i\alpha} b_{i\beta} b^\dagger_{j\gamma} b_{j\delta} (\sigma_{\alpha\beta}\sigma_{\gamma\delta} - \delta_{\alpha\beta}\delta_{\gamma\delta}) - \mu \sum_i e^\dagger_i e_i, \tag{4}$$

$$Z_{\rm SF} = \int De De^\dagger Db Db^\dagger D\lambda e^{-\int d\tau L_{\rm SF}(\tau)}, \tag{5}$$

where λ_i is the Lagrangian multiplier on site i, μ is the chemical potential, and the summation $\langle ij \rangle$ is carried over nearest neighbors. Following the common practice, the boson operator $b^\dagger_{i\sigma}$ keeps track of the spin, while the fermion operator $e^\dagger_i$ keeps track of the charge; i.e., they should obey the following sum rules:

$$\delta = \langle e^\dagger_i e_i \rangle = \frac{1}{Z_{\rm SF}} \int De De^\dagger Db Db^\dagger D\lambda e^\dagger_i e_i e^{-\int d\tau L_{\rm SF}(\tau)}, \tag{6}$$

$$1 - \delta = \sum_\sigma \langle b^\dagger_{i\sigma} b_{i\sigma} \rangle = \frac{1}{Z_{\rm SF}} \sum_\sigma \int De De^\dagger Db Db^\dagger D\lambda b^\dagger_{i\sigma} b_{i\sigma} e^{-\int d\tau L_{\rm SF}(\tau)}, \tag{7}$$

where δ is the hole-doping concentration, and $\langle \cdots \rangle$ means the thermodynamical average. We assume that there is no Bose condensation of spinons, which means that the temperature[24] of the system is $T = 0^+$. In gauge-field theory,[17,26] one can introduce the SU(2)-invariant Hubbard-Stratonovich transformation and decouple the Lagrangian by using the auxiliary fields. In the present formal study, there is no need to make any transformation for the Lagrangian (3), and we will treat formally phase fluctuations (gauge fields),[17] amplitude fluctuations,[25] constraints, etc. For discussing the electron-momentum distribution function, we define the following Matsubara fermion, boson, and electron Green's functions

$$g(R_i - R_j, \tau - \tau') = -\langle T_\tau e_i(\tau) e^\dagger_j(\tau') \rangle = -\frac{1}{Z_{\rm SF}} \int De De^\dagger Db Db^\dagger D\lambda e_i(\tau) e^\dagger_j(\tau') e^{-\int d\tau L_{\rm SF}(\tau)}, \tag{8}$$

$$D_\sigma(R_i - R_j, \tau - \tau') = -\langle T_\tau b_{i\sigma}(\tau) b^\dagger_{j\sigma}(\tau') \rangle = -\frac{1}{Z_{\rm SF}} \int De De^\dagger Db Db^\dagger D\lambda b_{i\sigma}(\tau) b^\dagger_{j\sigma}(\tau') e^{-\int D\tau L_{\rm SF}(\tau)}, \tag{9}$$

$$G_\sigma(R_i - R_j, \tau - \tau') = -\langle T_\tau C_{i\sigma}(\tau) C^\dagger_{j\sigma}(\tau') \rangle = -\langle T_\tau e^\dagger_i(\tau) e_j(\tau') b_{i\sigma}(\tau) b^\dagger_{j\sigma}(\tau') \rangle = -\frac{1}{Z_{\rm SF}} \int De De^\dagger Db Db^\dagger D\lambda e^\dagger_i(\tau) e_j(\tau') b_{i\sigma}(\tau) b^\dagger_{j\sigma}(\tau') \times e^{-\int d\tau L_{\rm SF}(\tau)}. \tag{10}$$

The spinon-holon scattering contained in Eq. (10) is a four-particle process; therefore the spinon and holon are strongly coupled. At the mean-field level, the holons and spinons are separated completely. However, in many theoretical frameworks, such as the usual gauge theories discussed by many authors,[17,26] where the vertex corrections are ignored, but the random-phase-approximation (RPA) bubbles are included, the spinons and holons are still strongly coupled by phase fluctuations (gauge fields),[17] amplitude fluctuations,[25] and other effects. Nevertheless, in all these cases the electron Green's function $G_\sigma(k, iw_n)$ can be presented as a convolution of the fermion Green's function $g(k, iw_n)$ and boson Green's function $D_\sigma(k, iw_n)$,

$$G_\sigma(k, iw_n) = \frac{1}{N} \sum_q \frac{1}{\beta} \sum_{w_m} g(q, iw_m) D_\sigma(q+k, iw_m+iw_n). \tag{11}$$

The coupling of the gauge field to these particles can be strong, and a partial resummation of the diagrams has been carried out,[17] but the vertex corrections were neglected, being the essence of the decoupling approximation. This is an important but the only approximation apart from the sum rules for slave particles [Eqs. (6) and (7)] in our formal study. In what follows, one will find that many difficulties might appear due to this approximation. The fermion and boson Green's functions can be expressed as frequency integrals of fermion and boson spectral functions as

$$g(k, iw_n) = \int_{-\infty}^{\infty} \frac{dw}{2\pi} \frac{A_e(k, w)}{iw_n - w}, \tag{12}$$

$$D_\sigma(k, iP_n) = \int_{-\infty}^{\infty} \frac{dw}{2\pi} \frac{A_{b\sigma}(k, w)}{iP_n - w}, \tag{13}$$

respectively.[27] Substituting Eqs. (12) and (13) into Eq. (11), we obtain the electron Green's function by summing over the Matsubara frequency iw_m,

$$G_\sigma(k, iw_n) = \frac{1}{N}\sum_q \int_{-\infty}^{\infty} \frac{dw'}{2\pi} \int_{-\infty}^{\infty} \frac{dw''}{2\pi} A_e(q, w') A_{b\sigma}(q+k, w'') \times \frac{n_F(w') + n_B(w'')}{iw_n + w' - w''}, \quad (14)$$

where $n_F(w')$ and $n_B(w'')$ are the Fermi and Bose distribution functions, respectively. The spectral functions $A_e(q, w')$ and $A_{b\sigma}(k, w'')$ obey the sum rules coming from the commutation relations

$$\int_{-\infty}^{\infty} \frac{dw}{2\pi} A_e(q, w) = 1, \quad (15)$$

$$\sum_\sigma \int_{-\infty}^{\infty} \frac{dw}{2\pi} A_{b\sigma}(k, w) = 2. \quad (16)$$

The electron's Hilbert space has been severely restricted, but the fermion and boson themselves are not restricted. By an analytic continuation $iw_n \to w + i\eta$ in the electron Green's function(14), the electron spectral function $A_{c\sigma}(k, w) = -2\mathrm{Im}G_{c\sigma}(k, w)$ can be obtained as

$$A_{c\sigma}(k, w) = \frac{1}{N}\sum_q \int_{-\infty}^{\infty} \frac{dw'}{2\pi} A_e(q, w') A_{b\sigma}(q+k, w+w') \times [n_F(w') + n_B(w+w')]. \quad (17)$$

Therefore, the electron spectral function $A_{c\sigma}(k, w)$ obeys the sum rule

$$\sum_\sigma \int_{-\infty}^{\infty} \frac{dw}{2\pi} A_{c\sigma}(k, w) = 1 + \delta, \quad (18)$$

which is less than 2 since an amount of $(1-\delta)$ of the doubly occupied Hilbert space is pushed to infinity as $U \to \infty$ in deriving the t-J model. Thus the spectral function $A_{c\sigma}(k, w)$ only describes the lower Hubbard band. In deriving Eq. (18), we have used the identities

$$n_{b\sigma}(k) = \int_{-\infty}^{\infty} \frac{dw}{2\pi} n_B(w) A_{b\sigma}(k, w), \quad (19)$$

$$n_e(k) = \int_{-\infty}^{\infty} \frac{dw}{2\pi} n_F(w) A_e(k, w). \quad (20)$$

This is because $n_B(w)A_{b\sigma}(k, w)$ and $n_F(w)A_e(k, w)$ can be interpreted as the probability functions of state k with energy w for boson and fermion, respectively. A similar interpretation is also valid for the electron spectral function $A_{c\sigma}(k, w)$. Thus the number of electrons in state k is obtained by summing over all energies w, weighted by the electron spectral function

$$n_c(k) = \sum_\sigma \int_{-\infty}^{\infty} \frac{dw}{2\pi} n_F(w) A_{c\sigma}(k, w) = 1 - \delta - \frac{1}{N}\sum_{q\sigma} n_{b\sigma}(k+q) n_e(q). \quad (21)$$

In Eq. (21), the first term of the right-hand side, $1-\delta$, is independent of k, and therefore it is true for all k states of the entire Brillouin zone. The value of the second term of the right-hand side is of the order of δ, and hence it is not enough to restore the EFS; i.e., the distribution outside the should-be EFS is still of order 1. Figure 1 shows the mean-field electron-momentum distribution $n_c(k)$ for doping $\delta = 0.125$ in 1D. Beyond the mean-field approximation, but still within the decoupling scheme (11), there are no important corrections for the global features of the electron-momentum distribution and EFS, but the Fermi velocity will be modified. This is because the essential global features of the electron-momentum distribution are dominated by the first term of the right-hand side, $1-\delta$, in Eq. (21), and the second term of the right-hand side in Eq. (21), which is of the order of δ, is not enough to cancel out $1-\delta$ at each k outside the k_F state, i.e., $k_F < k < \pi$, for $k > 0$, and $-\pi < k < -k_F$, for $k < 0$. Since $n_{b\sigma}(k+q) \geq 0$ and $n_e(q) \geq 0$ and a minus sign appears between the first and the second terms of the right-hand side in Eq. (21), then it is impossible to shift the weight $1-\delta$ in state k outside the Fermi points into state k' inside the Fermi points, i.e., $0 < k' < k_F$, for $k' > 0$, and $-k_F < k' < 0$, for $k' < 0$. The above discussions are also true for the 2D case. Therefore, there is no EFS in the standard sense within the decoupling scheme (11) in the conventional slave-fermion approach. If δ holes are introduced into the half-filled system, one might expect that the total electron number per site would be $1-\delta$. However, a surprising result is

$$\frac{1}{N}\sum_k n_c(k) = (1-\delta)^2, \quad (22)$$

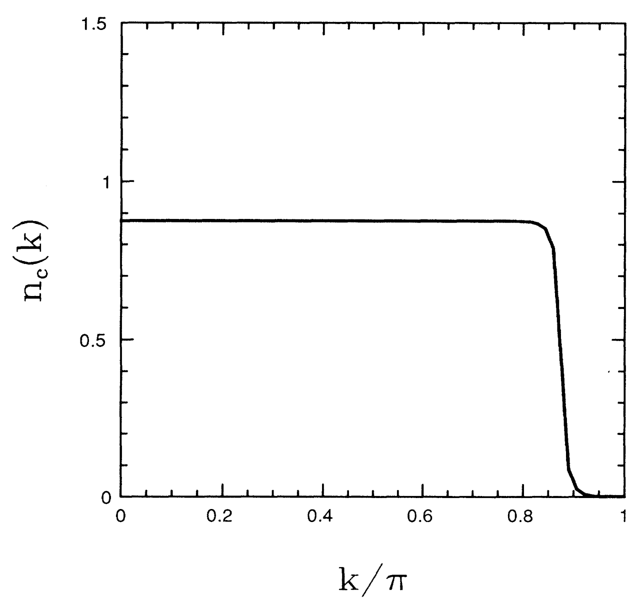

FIG. 1. Mean-field electron-momentum distribution $n_c(k)$ for doping $\delta = 0.125$ in 1D in the conventional slave-fermion approach. The integrated area of the electron-momentum distribution is $1 - \delta - \delta(1-\delta) = (1-\delta)^2$.

which is not the expected value, and violates the sum rule of the electron number.

Alternatively, in the slave-boson representation, the electron operator can be expressed as $C_{i\sigma} = f_{i\sigma} b_i^\dagger$, where $f_{i\sigma}$ is the fermion and $b_i^\dagger$ is the slave boson, with the constraint $\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1$. In this case, the Lagrangian L_{SB} and the partition function Z_{SB} of the t-J model may be written as

$$L_{SB} = \sum_{i\sigma} f_{i\sigma}^\dagger \partial_\tau f_{i\sigma} + \sum_i b_i^\dagger \partial_\tau b_i + H + \sum_i \lambda_i \left(b_i^\dagger b_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} - 1 \right), \tag{23}$$

$$H = -t \sum_{\langle ij \rangle \sigma} b_i b_j^\dagger f_{i\sigma}^\dagger f_{j\sigma} - \mu \sum_{i\sigma} f_{i\sigma}^\dagger f_{i\sigma} + \frac{J}{4} \sum_{\langle ij \rangle} f_{i\alpha}^\dagger f_{i\beta} f_{j\gamma}^\dagger f_{j\delta} (\sigma_{\alpha\beta}\sigma_{\gamma\delta} - \delta_{\alpha\beta}\delta_{\gamma\delta}), \tag{24}$$

$$Z_{SB} = \int Db Db^\dagger Df Df^\dagger D\lambda e^{-\int d\tau L_{SB}(\tau)}, \tag{25}$$

where the fermion operator $f_{i\sigma}^\dagger$ keeps track of the spin, while the boson operator $b_i^\dagger$ keeps track of the charge, i.e.,

$$\delta = \langle b_i^\dagger b_i \rangle = \frac{1}{Z_{SB}} \int Db Db^\dagger Df Df^\dagger D\lambda b_i^\dagger b_i e^{-\int d\tau L_{SB}(\tau)}, \tag{26}$$

$$1 - \delta = \sum_\sigma \langle f_{i\sigma}^\dagger f_{i\sigma} \rangle = \frac{1}{Z_{SB}} \sum_\sigma \int Db Db^\dagger Df Df^\dagger D\lambda f_{i\sigma}^\dagger f_{i\sigma} e^{-\int d\tau L_{SB}(\tau)}. \tag{27}$$

We assume that there is no Bose condensation of holons. Strictly speaking, this is true above the Bose condensation temperature. However, as shown by Lee and Nagaosa,[17] the inelastic scattering of bosons by the gauge field suppresses significantly the Bose condensation temperature. After some formal calculations which are similar to the slave-fermion case, we obtain

$$\sum_\sigma \int_{-\infty}^{\infty} \frac{dw}{2\pi} A_{c\sigma}(k, w) = 1 + \delta, \tag{28}$$

$$n_c(k) = 1 - \delta + \frac{1}{N} \sum_{q\sigma} n_{f\sigma}(k+q) n_b(q), \tag{29}$$

$$\frac{1}{N} \sum_k n_c(k) = 1 - \delta^2. \tag{30}$$

In comparison with Eq. (21), the second term of the right-hand side of Eq. (29) changes sign as an essential difference of the electron-momentum distribution function between the slave-boson and slave-fermion representations. However, the value of the second term of the right-hand side of Eq. (29) is also of the order of δ, and it is also not enough to restore the EFS. Figure 2 shows the mean-field electron-momentum distribution $n_c(k)$ for doping $\delta = 0.125$ in 1D. Beyond the mean-field approximation, but still within the decoupling scheme, there are no important corrections for the global features of the electron-momentum distribution. The reason is almost the same as in the conventional slave-fermion approach. The essential global features of the electron-momentum distribution are dominated by the first term of the right-hand side $1 - \delta$ in Eq. (29). Since $n_{f\sigma}(k+q) \geq 0$ and $n_b(q) \geq 0$, and a plus sign appears between the first and the second terms of the right-hand side in Eq. (29), it is impossible to remove those $1 - \delta$ states beyond the Fermi points. The best situation is that an amount of order of δ is added into each k' state within the Fermi points. The above discussion is also valid for the 2D case. Therefore, there is no EFS within the decoupling scheme in the conventional slave-boson approach. In this case, the sum rule of the electron number is also violated as in the slave-fermion case. The difference between these two approaches is that the electron number is more than the expected value in the slave-boson representation, but less than it in the slave-fermion representation.

These results indicate that there is no real EFS for the electron-momentum distribution function $n_c(k)$ near $k \sim k_F$ within the decoupling scheme in the conventional slave-particle approach. The sum rule of the physical electron number is violated. The scaling behavior of the electron-momentum distribution near k_F is also not cor-

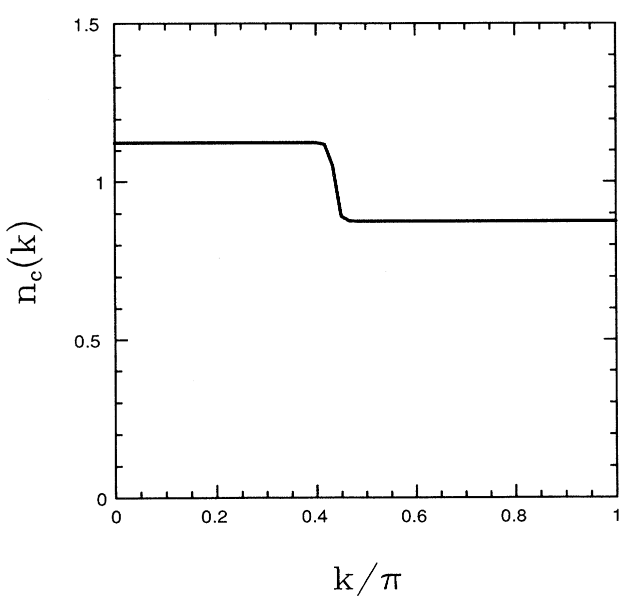

FIG. 2. Mean-field electron-momentum distribution $n_c(k)$ for doping $\delta = 0.125$ in 1D in the conventional slave-boson approach. The integrated area of the electron-momentum distribution is $1 - \delta + \delta(1 - \delta) = 1 - \delta^2$.

rectly described by this scheme. The theory can be applied to both 1D and 2D systems. It was proposed[17,26] that the low-energy physics of the t-J model can be described by a theory of fermions and bosons coupled by a gauge field. Our results also indicate that the gauge fields (phase fluctuations) discussed by many authors[17,26,28] are not strong enough to restore the EFS under the decoupling scheme (11). This is because the effects of phase fluctuations in the slave-particle approach are essentially local in momentum space. On the other hand, it has been shown from experiments[5] that oxide superconducting materials exhibit an EFS, and the EFS obeys the Luttinger theorem,[6] which means the Fermi volume is invariant under interaction. In this sense the EFS of strong correlated systems should also be described by an adequate theory even in the mean-field approximation. In fact, the rearrangements of spin configurations in the electron-hopping process,[10] which are nonlocal effects and beyond the conventional slave-particle approach, play an essential role to restore EFS for a system of decoupled charge and spin degrees of freedom. Thus the electron is not a composite of holon and spinon only, and it should include other fields which describe the nonlocal effects. In the next section, we will see that the scaling behavior of $n_c(k)$ near $k \sim k_F$ and $k \sim 3k_F$ can be obtained in 1D by considering some nonlocal effects. Before going to the next section, we would like to emphasize that the "Fermi surface" obtained previously[29] from the slave-boson theory is a *spinon* Fermi surface, not the real EFS. In order to interpret the high-resolution angle-resolved photoemission and inverse-photoemission experiments,[5] the *spinon* Fermi surface is not enough because experiments have shown the EFS for real electrons.

III. SCALING BEHAVIOR OF THE ELECTRON-MOMENTUM DISTRIBUTION FUNCTION AND THE EFFECTS OF NONLOCAL STRING FIELD IN 1D

Interacting 1D electron systems generally behave like Luttinger liquids[18] in which the correlation functions have power-law decay with exponents which depend on the interaction strength. For the 1D Hubbard model, an exact solution was explicitly obtained by Lieb and Wu.[11] In the limit of $J \to 0$, the t-J model is equivalent to the large-U-limit Hubbard model.[1,2,10] Thus the 1D t-J model provides a good test for various approaches. The asymptotic forms of some correlation functions as well as the single-electron Green's function have been obtained by many authors using different approximations.[20,30] In particular, Ogata and Shiba[10] obtained the electron-momentum distribution function $n_c(k)$ by using the Bethe-Ansatz Lieb-Wu[11] wave function, and their results show the presence of EFS and the correct scaling behavior of $n_c(k)$ at $k \sim k_F$ and $k \sim 3k_F$. It is well established that the Landau Fermi-liquid theory breaks down in 1D, namely, (1) there is no finite jump of the momentum distribution at the Fermi surface, (2) there is no quasiparticle propagation, and (3) the spin and charge are separated. On the other hand, there is still a well-defined Fermi surface at k_F, as one would expect from the Luttinger theorem. It is remarkable that the exact solution of the 1D Hubbard model demonstrates explicitly these two aspects at the same time. In this sense it is important to check whether both aspects remain in any appropriate approximate treatment of the 1D model. To our knowledge, such global features of the electron-momentum distribution function even in 1D were obtained only by using numerical techniques. In this section, we try to study analytically this problem. A CP^1 boson-soliton approach including the effects of the nonlocal string field to study the large-U Hubbard model was recently developed by Weng *et al.*[20] They have shown that the electron is a composite particle of a holon and spinon, together with a nonlocal string field. We will draw heavily on their results, but we try to make the presentation self-contained. The correct scaling form of the electron-momentum distribution function can be obtained even in the mean-field approximation (MFA) if one considers the nonlocal string field.

For convenience, we begin with the t-J model, but consider the limit $J \to 0^+$ which is a fixed point different from $J = 0$. In this case, the t-J Hamiltonian may be written as

$$H = -t \sum_{\langle ij\rangle\sigma} C^\dagger_{i\sigma} C_{j\sigma} + \text{H.c.} \tag{31}$$

As in the CP^1 boson-soliton approach,[20] the electron operator $C_{i\sigma}$ can be expressed in the slave-fermion representation including the effects of nonlocal string fields as

$$C_{i\sigma} = e^\dagger_i e^{-i\frac{\pi}{2}\sum_{l<i} n_l} b_{i\sigma} G_i, \tag{32}$$

with the constraint $\sum_\sigma b^\dagger_{i\sigma} b_{i\sigma} + e^\dagger_i e_i = 1$. Here $n_l = e^\dagger_l e_l$ and the fermion operator $e^\dagger_i$ keeps track of the charge, while the boson operator $b^\dagger_{i\sigma}$ keeps track of the spin. $e^{-i\frac{\pi}{2}\sum_{l<i} n_l}$ is the string field, which describes the effects of rearrangements of spin configurations from $-\infty$ to site i when one electron was removed or added at site i. G_i is a projection operator, which ensures that $b_{i\sigma}$ annihilates the spin σ and is a nonlocal phase shift $G_i = e^{i\frac{\pi}{2}(N - X_i - \sum_{l>i} n_l)}$ (X_i is the lattice site), which describes the effects of rearrangements of the spin configurations from site i to $+\infty$ when one electron was removed or added at site i. As shown by Ogata and Shiba,[10] the "*squeezing*" effect, i.e., the rearrangement of spins when holes are squeezed out, is crucial in recovering the Fermi surface and getting the correct exponents for the correlation functions. The nonlocal "string" proposed in Ref. 20 will partly take into account this effect. In fact the motion of holons is also affected by the rearrangements of the spin configurations.[31] We have neglected this second effect since the kinetic energy t is much larger than the magnetic energy J. One can check easily that the anticommunication relations are the same as for the conventional slave-fermion approach. In this case, the Hamiltonian with contraints can be written as

$$H = -t \sum_{j\sigma} [i b^\dagger_{j\sigma} b_{j+1\sigma} e^\dagger_{j+1} e_j + (-i) b^\dagger_{j+1\sigma} b_{j\sigma} e^\dagger_j e_{j+1}] \\ -\mu \sum_j e^\dagger_j e_j + \sum_j \lambda_j \left(e^\dagger_j e_j + \sum_\sigma b^\dagger_{j\sigma} b_{j\sigma} - 1 \right), \tag{33}$$

where μ is the chemical potential, and λ_j is the Lagrangian multiplier on site j. The factor i in the Hamiltonian is very important, which will shift the holon energy spectrum and will give rise to the correct scaling form of the electron-momentum distribution function. Thus we define the new holon operator as

$$e_j = e^{i\frac{\pi}{2}X_j} h_j, \tag{34}$$

and the Hamiltonian may be rewritten as

$$H = -t\sum_{i\sigma}[b_{i\sigma}^\dagger b_{i+1\sigma} h_{i+1}^\dagger h_i + \text{H.c}] - \mu\sum_i h_i^\dagger h_i + \sum_i \lambda_i\left(h_i^\dagger h_i + \sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} - 1\right). \tag{35}$$

Following Weng *et al.*,[20] we can obtain the asymptotic singular behavior of the electron-momentum distribution by a similar calculation, and the result is in agreement with theirs. In doing so, the major approximation is to drop the G_i factor in Eq. (32), while calculating the asymptotic single-electron Green's function. As pointed out by Weng *et al.*,[20] this factor will contribute an additional power-law decay in the asymptotic single-electron Green's function, and one may neglect it for simplicity if only interested in the leading contribution. The main effect due to the nonlocal string is already present. In the MFA, our situation is very similar. Equation (32) holds exactly in the Néel limit of the spin configuration, and therefore the quantum many-body effects are neglected. This does not affect the leading long-wavelength behavior. In this paper, however, we are mainly interested in the global features of the electron-momentum distribution. If the Luttinger theorem[6] is obeyed, the Fermi volume is invariant under interaction and a strongly interacting system should also show a large EFS even in the mean-field approximation. Thus a mean-field treatment is a useful test for the present approach.

The mean-field approximation to the Hamiltonian (35) amounts to treating λ_i as a constant, independent of position and to decoupling the spinon-holon interaction in a Hartree-like approximation by introducing the order parameters

$$\chi = \sum_\sigma \langle b_{i\sigma}^{A\dagger} b_{i+1\sigma}^B\rangle, \tag{36}$$

$$\phi = \langle h_i^{A\dagger} h_{i+1}^B\rangle, \tag{37}$$

where we have considered two sublattices A and B with $i \in A, i+1 \in B$. The self-consistent equations about λ, χ, ϕ, and μ can be obtained by minimizing the free energy.

Under the same approximation as the one used to obtain the asymptotic single-electron Green's function [i.e., G_i factor in Eq. (32) has been dropped], the single-particle electron Green's function $G_{c\sigma}(k,iw_n)$ in the present mean-field approximation can be obtained as

$$G_{c\sigma}(k,iw_n) = \frac{1}{2N}\sum_{q\pm}\frac{n_B(w_q)+n_F(\varepsilon_{q-k\pm\frac{\pi}{2}(1+\delta)})}{iw_n+\varepsilon_{q-k\pm\frac{\pi}{2}(1+\delta)}-w_q}, \tag{38}$$

where

$$w_k = \lambda - t\phi\gamma_k, \qquad \varepsilon_k = \lambda-\mu-t\chi\gamma_k, \qquad \gamma_k = 2\cos(k). \tag{39}$$

Therefore, one can get the electron spectral and momentum distribution functions

$$A_{c\sigma}(k,w) = \frac{\pi}{N}\sum_{q\pm}[n_B(w_q)+n_F(\varepsilon_{q-k\pm\frac{\pi}{2}(1+\delta)})] \times\delta(w+\varepsilon_{q-k\pm\frac{\pi}{2}(1+\delta)}-w_q), \tag{40}$$

$$A_c(k) = \sum_\sigma\int_{-\infty}^{\infty}\frac{dw}{2\pi}A_{c\sigma}(k,w) = 1+\delta, \tag{41}$$

$$n_c(k) = 1-\delta-\frac{1}{N}\sum_{q\pm}n_B(w_q)n_F(\varepsilon_{q-k\pm\frac{\pi}{2}(1+\delta)}), \tag{42}$$

$$\frac{1}{N}\sum_k n_c(k) = (1-\delta)^2. \tag{43}$$

In comparison with Eq. (21), we find that the holon energy spectrum has been shifted by $\frac{\pi}{2}(1+\delta)$ due to the presence of nonlocal string fields. Figure 3 shows the electron-momentum distribution $n_c(k)$ in the mean-field approximation, including the nonlocal string fields for doping $\delta = 0.125$ in 1D. The singular behavior of the electron-momentum distribution function at k_F and $3k_F$ is in qualitative agreement with the numerical results,[10] but the sum rule for the total electron number is still vi-

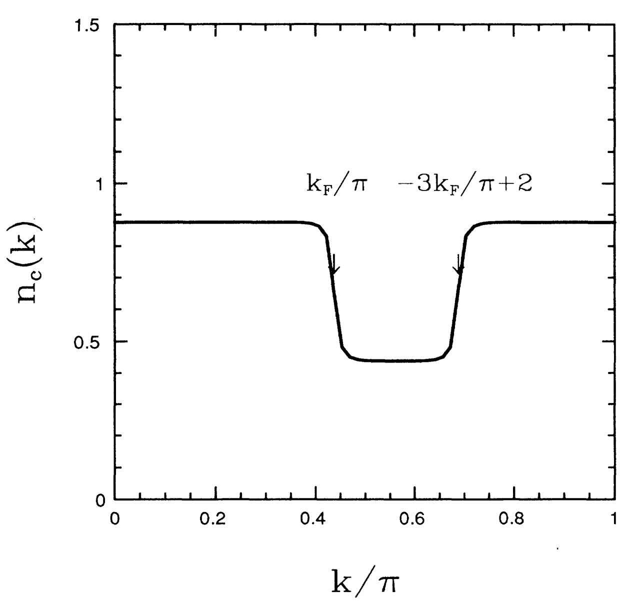

FIG. 3. Mean-field electron-momentum distribution $n_c(k)$ for doping $\delta = 0.125$ in 1D in the slave-fermion approach including the nonlocal string fields. The integrated area of the electron-momentum distribution is $(1-\delta)^2$. Note that, for $\delta = 0.125$, $3k_F$ becomes larger than π and a singularity appears at $3k_F$-2π, while $-3k_F$ is less than $-\pi$ and a singularity appears at $-3k_F+2\pi$. The values of k_F and $-3k_F+2\pi$ are indicated by arrows.

olated and EFS still does not exist in the present framework under the above approximations. Beyond these approximations, the situation is not clear yet. If the Luttinger theorem[6] is obeyed, then there are no important corrections to the global features of the electron-momentum distribution function and no EFS beyond the mean-field approximation in the same theoretical framework. Since the rearrangements of quantum spin configurations in the hopping process play an essential role to restore the EFS,[10] then Eq. (32), which holds exactly[20] in the Néel limit of the spin configuration, is perhaps not enough to describe all of these quantum spin configurations. Thus our feeling is that in this theoretical framework, the correct singular behavior of the electron-momentum distribution may be obtained, but it does not guarantee the existence of the EFS. We also note that only the asymptotic forms were discussed in the previous works by many authors,[30] while the global features of the momentum distribution function have not been considered by their theoretical methods.

IV. SUMMARY AND DISCUSSIONS

We have discussed the electron-momentum distribution function and the sum rule of the electron number in the t-J model by using the slave-particle approach. Under the decoupling scheme (11) used in gauge-field[17] and related theories, we have formally proved that the EFS cannot be restored and the sum rule of electron number is violated in the framework of conventional slave-particle approach. For the 1D t-J model, the correct singularity forms can be reproduced by considering the nonlocal string fields in the special mean-field approximation, but there is still no EFS and the sum rule is still not satisfied.

In the present slave-particle approach to study t-J model, we have pushed out the upper Hubbard band. This means we have neglected the doubly occupied sites in the original slave-particle approach:[21] $C_{i\sigma} = a_i^\dagger f_{i\sigma} + \sigma d_i f_{i-\sigma}^\dagger$, with a constraint $a_i^\dagger a_i + \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + d_i^\dagger d_i = 1$, where $a_i^\dagger$ is the slave boson, $f_{i\sigma}$ is the fermion, and d_i is the boson which describes the doubly occupied sites, or vice versa. In fact in the original slave-particle approach the electron spectral function obeys the sum rule of conventional electron, i.e., $\sum_\sigma \int_{-\infty}^{\infty} \frac{dw}{2\pi} A_{c\sigma}(k,w) = 2$, but the sum rule of the electron number is still violated. We also find that the sum rule of electron number is not violated in the CP^1 representation of electrons, which probably means it is a better choice to study the t-J model.

The decoupling of charge and spin degrees of freedom of electron is undoubtedly correct in the 1D t-J model in the $J \to 0^+$ limit,[10] and the 1D t-J model behaves as a Luttinger liquid.[10,12] Anderson[19] has hypothesized that the normal state of 2D strongly interacting systems relevant for the oxide superconductors might show generalized Luttinger-liquid behavior, and the characteristic separation of charge and spin excitations might be responsible for the experimentally observed temperature dependences of the resistivity[32] and Hall effect.[33] If these properties can be described in the framework of slave-particle approach, then we must be out of the difficulties mentioned in Secs. II and III. A possible way to restore the EFS is to try to include the vertex corrections beyond the decoupling scheme (11). As stated earlier, the coupling to the gauge field can be strong, but so long as the vertex corrections are neglected—i.e., the decoupling scheme is adopted—the difficulty will remain. On the other hand, it is difficult to introduce vertex corrections in the present form of gauge-field theory, because the infrared divergence has not yet been properly handled. Another possibility to avoid this difficulty is that one should make a new interpretation of the constraint (2) and the physical meaning of spinons and holons. In fact the constraint (2) is an operator identity, and one replaces the constraint (2) by Eqs. (6) and (7) in the slave-fermion approach or Eqs. (26) and (27) in the slave-boson case. It is not clear that this is a correct way of imposing the constraint; i.e., the charge is represented by a fermion, while the spin is represented by a boson in the slave-fermion approach and vice versa in the slave-boson version. The crucial point is to implement the local constraint rigorously.[34] Finally, according to the implications of numerical solutions,[10] the electron is perhaps not a composite of holon and spinon only, but it rather should be a composite of a holon and spinon together with something else. This is because the Bethe-ansatz Lieb-Wu's exact wave function[10] for the 1D Hubbard model at the large-U limit may be written as

$$\Psi(x_1,\ldots,x_N) = (-1)^Q \det[\exp(ik_i x_{Q_i})]\Phi(y_1,\ldots,y_M), \tag{44}$$

where the determinant depends only on the coordinates of particles ($x_{Q_1} < \cdots < x_{Q_N}$) but not on their spins. Thus it is the same as the Slater determinant of spinless fermions with momenta k_j's. The spin wave function $\phi(y_1,\ldots,y_M)$ is the same as the Bethe exact solution of the 1D Heisenberg spin system. In the second quantization representation, a spinless fermion operator may be responsible for the Slater determinant of spinless fermions, but a simple fermion or boson operator may be not enough to describe the spin wave function. Perhaps it should be a fermion or boson operator together with something else, such as the Jordan-Wigner's form[35] in 1D, or a map of the three spin operators of the SU(2) algebra onto a couple of canonical boson operators, such as Holstein-Primakoff form,[36] is responsible for the spin wave function. We believe that a successful theoretical framework must include the essential ingredients to give a qualitatively correct description of the global features of the EFS even in the mean-field approximation.

To summarize, we are facing a dilemma: If we would like to split one electron into two particles, one keeping track of the charge, the other keeping track of the spin, and impose corresponding sum rules on them [Eqs. (6) and (7) or Eqs. (26) and (27)] and then use the decoupling scheme for their expectation values, the resulting electron distribution does not satisfy the sum rule and does not show an EFS. In this sense, we have proved in this paper a "no-go" theorem. The alternative would be to give up the attractively simple interpretation of spinons and holons and to look for more complicated charge and spin collective excitations.

Finally we note that the present slave-particle approach is different from those first proposed by Barnes[37] and rediscovered and extended by Coleman,[38] Read and Newns,[39] and Kotliar and Ruckenstein[40] in their works on the mixed-valence problem and heavy-fermion systems. In their formulation, the spin and charge degrees of freedom are not decoupled, where the auxiliary bosons keep track only of the environment by measuring the occupation numbers in each of the possible states for electron hopping. Their theory describes the properties of a Fermi liquid, and is another story.

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[1]P. W. Anderson, in *Frontiers and Borderlines in Many Particle Physics*, Proceedings of the International School of Physics "Enrico Fermi," Course CIV, Varenna, 1987, edited by R. A. Broglia and J. R. Schrietter (North-Holland, Amsterdam, 1987), p. 1; Science **235**, 1196 (1987).

[2]F. C. Zhang and T. M. Rice, Phys. Rev. B **37**, 3759 (1988).

[3]*High Temperature Superconductivity*, Proceedings of the Los Alamos Symposium, 1989, edited by K. S. Bedell, D. Coffey, D. E. Meltzer, D. Pines, and J. R. Schrieffer (Addison-Wesley, Redwood City, CA, 1990).

[4]See, e.g., the review, L. Yu, in *Recent Progress in Many-Body Theories*, edited by T. L. Ainsworth *et al.* (Plenum, New York, 1992), Vol. 3, p. 157.

[5]C. G. Olson *et al.*, Phys. Rev. B **42**, 381 (1990), and references therein; *Physics and Materials Science of High Temperature Superconductors*, edited by R. Kossowsky, S. Methfessel, and D. Wohlleben (Kluwer Academic, Dordrecht, 1990); C. M. Fowler *et al.*, Phys. Rev. Lett. **68**, 534 (1992); J. C. Campuzano *et al.*, *ibid.* **64**, 2308 (1990); B. O. Wells *et al.*, *ibid.* **65**, 6636 (1990); T. Takahashi *et al.*, Phys. Rev. B **39**, 6636 (1989).

[6]J. M. Luttinger, Phys. Rev. **121**, 942 (1961).

[7]W. Stephan and P. Horsch, Phys. Rev. Lett. **66**, 2258 (1991).

[8]D. J. Scalapino, in *High Temperature Superconductivity* (Ref. 3); S. Sorella, E. Tosatti, S. Baroni, R. Car, and M. Parrinello, *Progress in High Temperature Superconductivity* (World Scientific, Singapore, 1988), Vol. 14, p. 457.

[9]R. Valenti and C. Gros, Phys. Rev. Lett. **68**, 2402 (1992).

[10]M. Ogata and H. Shiba, Phys. Rev. B **41**, 2326 (1990); H. Shiba and M. Ogata, Int. J. Mod. Phys. B **5**, 31 (1991).

[11]E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. **20**, 1445 (1968); C. N. Yang, Phys. Rev. Lett. **19**, 1312 (1967).

[12]H. Yokoyama and M. Ogata, Phys. Rev. Lett. **67**, 3610 (1991).

[13]T. K. Lee and Shiping Feng, Phys. Rev. B **38**, 11 809 (1988); C. Gros, *ibid.* **38**, 931 (1988).

[14]See, e.g., the review, E. Dagotto, Int. J. Mod. Phys. B **5**, 77 (1991).

[15]S. Sorella, A. Parola, M. Parrinello, and E. Tosatti, Int. J. Mod. Phys. B **3**, 1875 (1989); J. E. Hirsch and S. Tang, Phys. Rev. Lett. **62**, 591 (1989).

[16]See, e.g., the review, P. A. Lee, in *High Temperature Superconductivity* (Ref. 3).

[17]N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990), and references therein; P. A. Lee and N. Nagaosa, Phys. Rev. B **46**, 5621 (1992); P. A. Lee, Phys. Rev. Lett. **63**, 680 (1990).

[18]F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980); Phys. Lett. **81A**, 153 (1981); J. Phys. C **14**, 2585 (1981); J. Solyom, Adv. Phys. **28**, 201 (1979).

[19]P. W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); **67**, 2092 (1991).

[20]Z. Y. Weng, D. N. Sheng, C. S. Ting, and Z. B. Su, Phys. Rev. B **45**, 7850 (1992); Phys. Rev. Lett. **67**, 3318 (1991).

[21]Z. Zou and P. W. Anderson, Phys. Rev. B **37**, 627 (1988).

[22]S. Liang, B. Doucot, and P. W. Anderson, Phys. Rev. Lett. **61**, 365 (1988).

[23]W. Marshall, Proc. R. Soc. London A **232**, 48 (1955).

[24]D. Yoshioka, J. Phys. Soc. Jpn. **58**, 32 (1989); A. P. Arovas and A. Auerbach, Phys. Rev. B **38**, 316 (1988); C. L. Kane *et al.*, *ibid.* **B41**, 2653 (1990).

[25]Y. M. Li, D. N. Sheng, Z. B. Su, and L. Yu, Phys. Rev. B **45**, 5428 (1992); Mod. Phys. Lett. B **5**, 1467 (1991).

[26]L. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989).

[27]G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981).

[28]L. Ioffe and V. Kalmeyer, Phys. Rev. B **44**, 750 (1991); L. Ioffe and P. Wiegmann, *ibid.* **45**, 519 (1992).

[29]C. A. R. Sá de Melo, Z. Wang, and S. Doniach, Phys. Rev. Lett. **68**, 2078 (1992).

[30]A. Parola and S. Sorella, Phys. Rev. Lett. **64**, 1831 (1990); X. G. Wen, Phys. Rev. B **42**, 6623 (1990); H. J. Schulz, Phys. Rev. Lett. **64**, 2831 (1990); P. W. Anderson and Y. Ren (unpublished).

[31]Z. B. Su, Y. M. Li, W. Y. Lai, and L. Yu, Phys. Rev. Lett. **63**, 1318 (1989); Int. J. Mod. Phys. B **3**, 1913 (1989).

[32]M. Gurvitch and A. T. Fiory, Phys. Rev. Lett. **59**, 1337 (1987).

[33]T. R. Chien, Z. Z. Wang, and N. P. Ong, Phys. Rev. Lett. **67**, 2088 (1991).

[34]Shiping Feng, Z. B. Su, and L. Yu (unpublished).

[35]P. Jordan and E. Wigner, Z. Phys. **47**, 631 (1928).

[36]T. Holstein and H. Primakoff, Phys. Rev. **58**, 1048 (1940).

[37]S. E. Barnes, J. Phys. F **6**, 1375 (1976); **7**, 2637 (1977).

[38]P. Coleman, Phys. Rev. B **29**, 3035 (1984).

[39]N. Read and D. Newns, J. Phys. C **16**, 3273 (1983); N. Read, *ibid.* **18**, 2651 (1985).

[40]G. Kotliar and A. E. Ruckenstein, Phys. Rev. Lett. **57**, 1362 (1986); T. Li, P. Wolfle, and P. J. Hirschfeld, Phys. Rev. B **40**, 6817 (1989).