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I-V characteristics of high-temperature superconductors with columnar defects

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The vortex glass transition in the presence of columnar defects is studied by Monte Carlo simulations of a vortex loop model, suggested by the analogy to the $T = 0$ superconductor-insulator transition for dirty bosons in (2+1) dimensions. From finite-size scaling analysis of the I-V characteristic we find two dynamical exponents describing the nonequilibrium behavior. We obtain $z_\perp = 6 \pm 0.5$ and $z_\parallel = 4 \pm 0.5$ when the current is applied perpendicular and parallel to the columnar defects, respectively.

Strong thermal fluctuations and quenched disorder give properties to high-temperature superconductors not observed in conventional type-II superconductors. The irreversibility line separates two regions in the mixed state with distinct properties of the penetrating magnetic field lines. Above the irreversibility line, vortex lines are in a liquid state due to the strong thermal fluctuations. M.P.A. Fisher[1] has suggested that in the presence of quenched disorder there is a phase transition into a new superconducting state at the irreversibility line, called the vortex glass.[2] One signature of a vortex glass transition is universal scaling of the nonlinear current-voltage (I-V) characteristic.[1] This has been verified in several experiments,[3–7] and in Monte Carlo simulations.[8–11]

The original vortex glass model assumes pointlike impurities. In recent experiments several groups[12] have found a remarkable shift upwards in the irreversibility line upon heavy-ion irradiation high-T_c samples. The damage tracks from the ions form permanent "columnar defects" with linear dimension comparable to the sample size. These columnar defects strongly enhance flux pinning in the superconductor.

Nelson and Vinokur[13] and Lyuksyutov[14] have studied the case of columnar defects, using an analogy to the Bose glass.[15,16] In this picture, the glass transition is analogous to the zero-temperature superconductor-insulator transition of dirty bosons in two dimensions. The magnetic flux lines through the three-dimensional (3D) bulk superconductor correspond to world lines of bosons in the (2+1)D path integral formula for the partition function. Static disorder for the (2+1)D boson system corresponds to columnar defects for the 3D superconductor. The superconducting phase corresponds to the insulating localized Bose glass phase for dirty bosons, and the resistive vortex fluid phase corresponds to the superconducting phase of the dirty bosons.

A question of principal interest is whether a stable glass phase can actually exist in 3D. This issue was studied by Monte Carlo (MC) simulations of gauge glass models with point disorder.[8,9,17] These simulations found that the lower critical dimension is somewhat smaller than $d = 3$, but so close to $d = 3$ that it is hard to resolve unambiguously. On the other hand, the existence of the glass phase in the presence of columnar defects is clear due to the analogy with the dirty boson problem which has a phase transition which is now well established.[18,19] Furthermore, in these calculations screening of the flux lines, and the anisotropy due to the preferred direction picked out by the applied magnetic field, were not considered.

In this paper we report MC results for the nonlinear I-V characteristic at the glass transition of a model 3D high-temperature superconductor with columnar defects. Screening of the flux lines is taken into account by neglecting long-range vortex interactions altogether and assuming only a short-range repulsion. Our model effectively assumes that the magnetic field fluctuations diffuse more rapidly than the vortices. The case of long-range forces and columnar defects is being considered elsewhere.[20] Dong *et al.*[21] have recently considered the case of a single elastic string moving in a random medium. Here we consider the more difficult case of an interacting gas of such strings.

Let us briefly discuss scaling of the nonlinear I-V characteristics at the glass transition.[1,13] Any configuration of wiggling vortex lines may be described as a constant background of straight lines plus closed loops. The characteristic size of the loops defines the correlation length ξ, which diverges as $\xi \sim |T - T_g|^{-\nu}$. The correlation time is assumed to obey standard dynamical scaling: $\tau \sim \xi^z$, where z is the dynamical exponent which will be computed below. Columnar defects make the system strongly anisotropic, and this is reflected in the divergence of the correlation length. From the analogy to the dirty boson problem we infer that the correlation length in the direction of the columns scales as $\xi_\parallel \sim \xi_\perp^{z_Q}$, where $z_Q = 2$ is the quantum "dynamical exponent" of the dirty bosons at $T = 0$ in 2D.[19]

Consider voltage fluctuations across a correlation volume of size $\xi_\perp \times \xi_\perp \times \xi_\parallel \sim \xi_\perp^{2+z_Q}$. The linear response electric field E, the current density I, and the resistivity ρ are related by $E = \rho I$. From the Josephson and Nyquist relations the resistance R across the correlation volume scales as $R \sim \xi^{-z}$. In the case of a transverse current applied in the direction perpendicular to the columnar defects, the linear resistivity becomes $\rho = R\xi_\perp\xi_\parallel/\xi_\perp \sim \xi_\perp^{z_Q - z}$, and $E \sim \xi_\perp^{z_Q - z} I$. We also consider the case of an applied longitudinal current. Here we find $\rho = R\xi_\perp^2/\xi_\parallel \sim \xi_\perp^{2 - z_Q - z}$, and $E \sim \xi_\perp^{2 - z_Q - z} I$.

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We now have to find the nonlinear I-V characteristics. At finite current density I, vortex lines fall out of equilibrium due to the Lorentz force. The work against the Lorentz force to create a closed loop costs energy $E \sim \pm AI$, where $\pm$ is the orientation and A the area of the loop. For large enough loops the energy shift reaches k_BT. This defines a characteristic "current length" ξ_I beyond which correlations are destroyed.[1] In an isotropic system we would define $\xi_I^2 I = 1$ (dropping a factor of k_BT). With columnar defects and a transverse current we instead take $(\xi_\perp \xi_\parallel)_I I \sim \xi_I^{1+z_Q} I = 1$. The nonlinear I-V characteristic is now expected to have the form $V \sim I^{y_\perp}$ with exponent $y_\perp = (1+z)/(1+z_Q)$, and, in general, the electric field is expected to scale like $E \sim \xi_\perp^{-(1+z)} F_\pm(I\xi_\perp^{1+z_Q})$, where F_+ and F_- are universal scaling functions valid for $T \geq T_g$ and $T < T_g$, respectively.[13] In the case of a longitudinal current we instead have $(\xi_\perp^2)_I I \sim \xi_I^2 I = 1$, and following the same derivation gives $V \sim I^{y_\parallel}$ with $y_\parallel = (z_Q + z)/2$, and $E \sim \xi_\perp^{-(z_Q+z)} F_\pm(I\xi_\perp^2)$.

For a finite system at $T = T_g$ the diverging correlation length is cut off by the system size L. With transverse I we therefore expect $E \sim L^{-(1+z)} F(IL^{1+z_Q})$. Thus a plot of EL^{1+z} versus IL^{1+z_Q} is expected to give a universal, system size independent curve. Notice that the meaning of the scaled quantities is very natural. We have $EL^{1+z} \sim V/\langle V^2 \rangle_{\rm eq}^{1/2}$, where $V = EL$, $\langle V^2 \rangle_{\rm eq}^{1/2}$ is the rms equilibrium voltage fluctuation, and $IL^{1+z_Q} \sim (L/\xi_I)^{1+z_Q}$.

For our simulation we want a minimal model that describes the collective behavior of interacting vortex lines in the presence of thermal fluctuations and columnar defects. We choose the dirty boson action[19]

$$\beta H = \beta \sum_{\mathbf{r}} \left\{ \frac{1}{2} \mathbf{J}^2(\mathbf{r}) - v(\mathbf{r}_\perp) J_z(\mathbf{r}) \right\}, \qquad (1)$$

where $\beta \equiv 1/k_BT$, $\mathbf{J} = (J_x, J_y, J_z)$ are integer variables defined on the links of the lattice, taking values from $-\infty$ to ∞. These integer variables represent the vortex lines, for example, $J_x(\mathbf{r}) = \pm 1$ means that one vortex line with orientation ± 1 goes from $\mathbf{r}$ to $\mathbf{r} + \hat{\mathbf{x}}$. The vortex "current" is constrained to be divergenceless. The first term in H acts as a string tension and as a short-range repulsion. The second term in H describes the columnar defects. $v(\mathbf{r}_\perp)$ is a random site energy with uniform probability distribution on the interval $[0, 1]$. Notice that we do not model the disorder as a set of columns of fixed strength and random location. The spacing of such defects would introduce an additional length scale and worsen our finite-size scaling results. Instead we choose a random-strength (columnar) potential $v(\mathbf{r}_\perp)$ on each site. In the absence of any commensuration with the density of vortices this model will be in the same universality class and so have no effect on the critical behavior.

Due to the anisotropy from the columnar defects, the correlation length in the direction of the columns diverges at T_g as $\xi_\parallel \sim \xi_\perp^{z_Q}$ with $z_Q = 2$. To enable finite-size scaling we must use lattice sizes that scale correspondingly. We use lattices of size $L \times L \times L^2/4$ with periodic boundary conditions in all directions. We use an applied magnetic field such that there is a fixed density of $f = 1/2$ of vortex lines in the direction of the columnar defects, corresponding to half filling in the dirty boson problem. The transition point $T_g = 0.248 \pm 0.002$ for this model is already known from our previous work on the superconductor-insulator transition.[19]

Our MC algorithm consists of attempts to insert closed loops of vortex lines with random sign on the plaquettes of the lattice. The algorithm is ergodic in the sense that all possible configurations of closed loops are accessible. Nonequilibrium effects due to the Lorentz force from the finite current are included by biasing the acceptance of plaquette moves. The energy change of a move includes a term of the form $\Delta\epsilon = (\nabla \times \mathbf{J}) \cdot \mathbf{I}$, where the discrete curl gives the orientation of the plaquette loop.

We do a thermal average over vortex loops for each realization of the columnar disorder, followed by a (quenched) average over the random disorder potential. As an estimate of the required number of MC update sweeps in the thermal averages we use the time to return to steady state after a reversal of the current. The voltage is measured over up to 10^7 sweeps, with up to 10^6 initial sweeps discarded to reach steady state. For large currents a smaller number of sweeps is sufficient. Also the number of disorder realizations necessary to get small statistical error depends on the size of the applied current. At large enough current, so that $\xi_I \ll L$, the system will self-average and a small number of disorder realizations is enough. At small I we use up to 20 realizations of the disorder.

The voltage is given by the rate of phase slip across the system. This is measured by keeping track of the number of plaquette moves in the plane perpendicular to the applied current. The electric field is proportional to

$$E \propto \frac{1}{\Omega} \left[\left\langle \frac{d}{dt} (N_+ - N_-) \right\rangle \right], \qquad (2)$$

where $\Omega = L_x L_y L_z$ is the volume of the system, $N_\pm$ is the number of accepted plaquette moves with positive/negative orientation in the plane perpendicular to the applied current, and the time derivative stands for the change per sweep. The bracket $\langle\ \rangle$ denotes thermal average, and $[\ ]$ denotes quenched disorder average. Equation (2) implicitly assumes heavily overdamped dynamics so that MC time can be equated with real time. The update order in a sweep through the lattice is random rather than sequential, in order to avoid possible systematic error at high current where the acceptance rates are high.

Monte Carlo results for the nonlinear I-V characteristic at the glass transition are shown in Fig. 1. In 1(a) the current density I is transverse to the columns, and in 1(b) I is longitudinal.[22] The dynamical exponent z was adjusted until all data for different lattice sizes collapse on the same curve. Surprisingly, this happens for *different* values of z for a transverse and a longitudinal current. In 1(a) z has been set to $z_\perp = 6$, and in 1(b) z has been set to $z_\parallel = 4$. The MC data have three different regimes: (1) For small current, where $\xi_I > L$, the

MC data points approach the dashed line, whose slope is 1. This is a finite size effect: when the current length is cut off by the system size L the response becomes ohmic. (2) In the nonlinear I-V scaling regime, $1 < \xi_I < L$, the MC data nearly coincide with the solid line, whose slope is given by the scaling prediction for the exponent y in $V \sim I^y$. In (a) $y_\perp = (1+z_\perp)/(1+z_Q) = 7/3$, and in (b) $y_\parallel = (z_Q+z_\parallel)/2 = 3$. (3) At higher current, with $\xi_I < 1$, scaling breaks down as expected (data not shown). Error bars on the MC points represent the standard deviation of the voltage fluctuations between different disorder realizations.

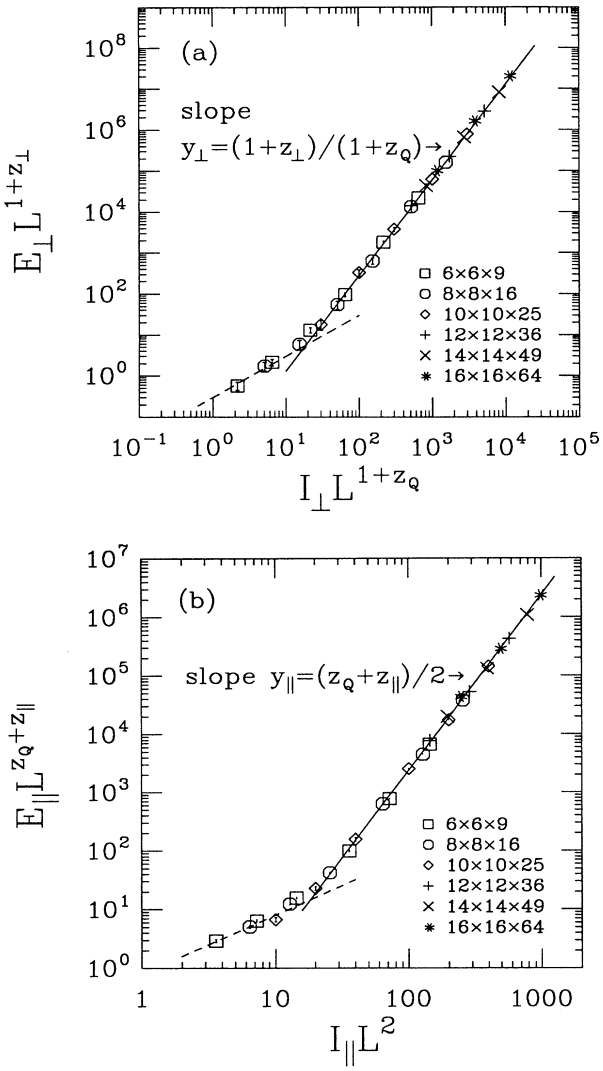

FIG. 1. Log-log plot of Monte Carlo results for the nonlinear I-V characteristic at the vortex glass transition with columnar defects. Plotted in (a) are MC data for EL^{1+z} vs IL^{1+z_Q}, where E is the electric field, and the applied current density I is perpendicular to the columnar defects. Plotted in (b) is EL^{z_Q+z} vs IL^2, where the current density I is parallel to the columns. The dynamical exponent z has been adjusted until all data for different system sizes collapse on the same curve. In (a) z has been set to $z_\perp = 6$, and in (b) $z_\parallel = 4$. For other values of z scaling quickly breaks down. The solid straight line in (a) has slope $y_\perp = (1+z_\perp)/(1+z_Q) = 7/3$ given by scaling (see text). The solid straight line in (b) has slope $y_\parallel = (z_Q+z_\parallel)/2 = 3$. The dashed straight lines in (a) and (b) both have slope 1, and indicate the ohmic region at small current.

What is the significance of the scaling plots in Fig. 1? The critical temperature T_g was determined independently,[19] and is not an adjustable parameter. The figure shows that by adjusting only a *single* parameter, the dynamical exponent z, we simultaneously get *two* things: (1) all MC data collapse on the same universal curve, and (2) the curve is a remarkably good power law over six decades with an exponent which is consistent with the predictions of the scaling analysis. For other choices of z both these properties quickly break down. From the rate at which the data collapse breaks down, we estimate $z_\perp = 6 \pm 0.5$ and $z_\parallel = 4 \pm 0.5$. One would have expected, at least in equilibrium, that a single dynamical exponent would suffice, but our simulation shows that in this nonequilibrium case we need two.[23] Attempts to use a single exponent $z \approx 5$ give a distinctly poorer fit to the data. As a test of universality we changed the aspect ratio of the lattices from $L_z = L_x^2/4$ to both $L_x^2/8$ and L_x^2. We also varied the applied magnetic field from $f = 1/2$ to $f = 1/4$. These did not change the values of the dynamical exponents. We have not yet tested lattice structures other than simple cubic.

The following simple argument suggests how the two different dynamical exponents $z_\perp = 6$ and $z_\parallel = 4$ might arise. First consider an isotropic system, and study the motion of a blob of vortex lines of diameter ξ. As a crude approximation we view the blob as a polymer containing $N \sim \xi^2$ segments. The time it takes each segment to random walk a distance ξ scales as $t_1 \sim \xi^2$. The diffusion coefficient for the center of mass (CM) is down by a factor of N. Hence the time for the CM to move a distance ξ scales as $t \sim \xi^4$, and thus $z = 4$. This argument neglects correlations in the motion of the segments of the vortex lines. The result coincides with the mean field result.[24] When a finite current is applied, the role of the diverging correlation length ξ is taken over by the finite current length ξ_I. By analogy with the above results, we make the simple ansatz for the anisotropic case that the characteristic time is proportional to the square of the cross-sectional area of the correlation volume as viewed in the direction of current flow. This yields $t_\perp \sim \xi_I^{2(1+z_Q)}$, and $z_\perp = 2(1+z_Q) = 6$ for current perpendicular to the columns. For the parallel case, we have $t_\parallel \sim \xi_I^{2(1+1)}$, so that $z_\parallel = 2(1+1) = 4$. Clearly a more detailed analysis is needed to justify this ansatz.

How do our results compare with experiments on systems without deliberately introduced columnar defects? Experiments[25] on single crystals of Y-Ba-Cu-O give $z = 4.5 \pm 1.5$ and $\nu = 2.0 \pm 1$, by fitting data to point-disorder vortex glass scaling laws. In these experiments correlated disorder might be present in the form of dislocations, twin boundaries, etc. If, following Nelson and Vinokur,[13] we instead fit to the Bose glass scaling laws used in this paper for columnar defects, we find $z = 7 \pm 2$ and $\nu = 1.3 \pm 0.5$. These are roughly consistent with our values $z_\perp = 6 \pm 0.5$ and $\nu = 1.0 \pm 0.1$, where ν was calculated in previous

equilibrium simulations of dirty bosons.[19] Simulations of point and twin-boundary disorder are underway.

In summary, we report Monte Carlo simulations of the nonlinear *I-V* characteristic of a model high-temperature superconductor with columnar defects. We find two dynamical exponents describing the nonequilibrium behavior, $z_\perp = 6 \pm 0.5$ and $z_\parallel = 4 \pm 0.5$ in the case of an applied transverse and longitudinal current, respectively. Experimental measurements in systems with deliberately introduced columnar defects would be highly desirable.

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[23] At and below T_g and for infinitesimal longitudinal current, the current may flow only near the surface of a sample containing columnar defects, as it does for the Abrikosov vortex lattice [E. H. Brandt, J. Low Temp. Phys. **42**, 557 (1981); **44**, 33 (1981); **44**, 59 (1981)]. We do not expect this effect to obviate our analysis for finite current densities where the resistivity is finite.

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