RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 21 1 JUNE 1993-I

Multiple Andreev scattering in superconductor–normal metal–superconductor junctions as a test for anisotropic electron pairing

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(Received 2 December 1992)

Current-voltage characteristics of superconductor–normal metal– superconductor junctions due to multiple Andreev reflections at the interfaces are investigated for anisotropic superconductors. Using nonequilibrium time-dependent Bogoliubov–de Gennes equations, it is shown that the presence of gap nodes perpendicular to the interface greatly smear subharmonic gap structures corresponding to voltages $V \leq 2\Delta(\hat{\mathbf{k}})/en$ for $n = 1, 2, 3, \ldots$ for n Andreev reflections as seen in s-wave superconductors, while the overall current due to Andreev reflections is reduced. Further, asymmetric line shapes for dI/dV are predicted. It is suggested that these features could be used to distinguish between superconductors with or without nodes.

Organic, heavy-fermion, and cuprate superconductors all possess features which question the applicability of BCS theory to these materials.[1] It is highly expected that these materials differ from BCS superconductors in regard to the nature of the pairing ground state of the electrons. Power-law temperature dependences of various transport and thermodynamic quantities have led many authors to suggest that the superconducting state of these materials can be described by different pair states other than a BCS s-wave singlet state. While power-law behavior is indicative of additional mechanisms in operation not included in BCS theory, power laws do not in themselves indicate the type of pairing of the ground state due to their sensitivity to the effects of impurity scattering.[2] The problem is that there are very few experiments which *directly* couple to the superconducting order parameter to allow a precise determination of its structure. However, it has recently been suggested that Andreev scattering[3] in point contacts, i.e., scattering of low energy quasiparticles due to a spatially varying order parameter, could in principle be used to identify the symmetry of an anisotropic superconductor.[4] However, such an experiment requires a **k**-dependent current measurement, and problems with electron collimation make an accurate determination of the gap unlikely at present.

However, a somewhat different situation is encountered in superconducting weak links. It has been known for some time that BCS superconductor–normal metal–BCS superconductor junctions provide many sharp features in current-voltage characteristics.[5] These features are mainly due to the bound Andreev states which generate supercurrents at voltages $eV = 2\Delta/(n-1)$ due to n repeated reflections at the superconducting interfaces. An electron (hole) with energy below the gap edge starts its motion in the normal region and is accelerated towards an interface by the applied field. At the interface, electrons (holes) are retroflected (reflected with reversed momentum vector, as opposed to normal reflection where only the momentum component perpendicular to the interface is reversed) into holes (electrons), creating an electron (hole)-like Cooper pair in the condensate which carries away the excess current. The hole (electron) then gets reaccelerated towards the other interface and creates another Cooper pair by retroflecting. This process continues until either the particle suffers an inelastic collision in the normal metal or until it gains enough energy to climb out of the potential well. The sharp features in the I-V characteristic produced by multiple Andreev reflections have recently been observed in Y-Ba-Cu-O break junctions.[6] Previous theories concerning this experimental situation have been confined to the case of BCS s-wave gaps, where a gap exists around the entire Fermi surface. However, in superconductors which possess gap nodes along the Fermi surface, Andreev reflection will not be allowed for those momentum directions corresponding to the position of the gap nodes. Therefore, the peaks in the I-V curves corresponding to multiple Andreev reflections will only occur for a subset of **k**, reducing the overall signal and broadening the sharp structure seen for the s-wave case. The differences in the signal could in principle distinguish between the two cases—superconductors with or without gap nodes. This is the subject of the present paper.

The purpose of the paper is to report calculations based on a microscopic theory for Andreev scattering in superconductor–normal metal–superconductor (SNS) junctions of anisotropic superconductors. Specifically, we obtain an expression for the time and spatially averaged current density in a relaxation time approximation using nonequilibrium Bogoliubov–de Gennes equations, generalized from the BCS-type theory of Kümmel *et al.*[7] to anisotropic superconductors. It is shown for a particular choice of d-wave gap that the current-voltage characteristics for superconductors with gap nodes differ dramatically compared to BCS ones. Namely, the subharmonic gap structure is greatly smeared and the overall signal is reduced. It is suggested that this setup could be used to determine the existence or nonexistence of gap nodes.

The formalism used here is similar to the formalism of Kümmel, Gunsenheimer, and Nicolsky[7] adapted to the case of anisotropic superconductivity. Details can be found in this reference and we therefore will be brief. The theory is applied to the physical situation of two long superconducting banks of length D separated by a normal metal region which exists in the z direction for $|z| < a$, where $a \ll D$. An externally applied bias voltage exists across the normal metal region only, and is defined by a time-dependent vector potential $\mathbf{A}(\mathbf{x},t) = \mathbf{e_z}\Theta(a-|z|)Vct/2a$, where a gauge is chosen so that the pair potential is real and the Josephson equations are obeyed.

Our starting point to describe this setup is the Bogoliubov–de Gennes equations for inhomogeneous anisotropic superconductors in the presence of a time-dependent vector potential:

$$i\hbar\frac{\partial}{\partial t}u_n(\mathbf{x},t) = h_0^+(\mathbf{x},t)u_n(\mathbf{x},t) + \int d\mathbf{x}'\Delta(\mathbf{x},\mathbf{x}')v_n(\mathbf{x}',t), \tag{1}$$

$$i\hbar\frac{\partial}{\partial t}v_n(\mathbf{x},t) = -h_0^-(\mathbf{x},t)v_n(\mathbf{x},t) + \int d\mathbf{x}'\Delta^*(\mathbf{x},\mathbf{x}')u_n(\mathbf{x}',t), \tag{2}$$

along with the self-consistency requirement,

$$\Delta(\mathbf{x},\mathbf{x}') = V(\mathbf{x},\mathbf{x}')\sum_n u_n(\mathbf{x})v_n^*(\mathbf{x}')f(E_n) - v_n^*(\mathbf{x})u_n(\mathbf{x}')f(-E_n). \tag{3}$$

Here u_n, v_n are the electron, hole-like quasiparticle wave functions and $h_0^{\pm}(\mathbf{x},t) = [-i\hbar\nabla \pm e\mathbf{A}(\mathbf{x},t)/c]^2/2m - \mu$ with chemical potential μ. We now rewrite the gap parameter in terms of center of mass $\mathbf{R} = (\mathbf{x}+\mathbf{x}')/2$ and relative coordinate $\mathbf{r} = \mathbf{x}-\mathbf{x}'$. Dividing out the fast oscillations of the wave functions by defining $(\hat{u}_n,\hat{v}_n) = e^{-ik_F\hat{\mathbf{k}}\cdot\mathbf{x}}(u_n,v_n)$ and retaining lowest-order terms in $(k_F\xi_0)^{-1}$, with ξ_0 the coherence length, we obtain the time-dependent Andreev equations,

$$i\hbar\frac{\partial}{\partial t}\hat{u}_n(\mathbf{x},t) = \hat{h}_0^+(\mathbf{x},t)\hat{u}_n(\mathbf{x},t) + \Delta(\hat{\mathbf{k}},\mathbf{x})\hat{v}_n(\mathbf{x},t), \tag{4}$$

$$i\hbar\frac{\partial}{\partial t}\hat{v}_n(\mathbf{x},t) = -\hat{h}_0^-(\mathbf{x},t)\hat{v}_n(\mathbf{x},t) + \Delta^*(\hat{\mathbf{k}},\mathbf{x})\hat{u}_n(\mathbf{x},t), \tag{5}$$

where $\Delta(\mathbf{k},\mathbf{x}) = \int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\Delta(\mathbf{r},\mathbf{x})$ is the Fourier transform of $\Delta(\mathbf{r},\mathbf{x})$ with respect to the relative coordinate, and $\hat{h}_0^{\pm} = e^{-ik_F\hat{\mathbf{k}}\cdot\mathbf{x}}h_0^{\pm}e^{ik_F\hat{\mathbf{k}}\cdot\mathbf{x}}$.[8]

For an exact solution of Eqs. (3)–(6), one must take into account the normal reflection and transmission properties of the interface, the proximity induced coupling in the normal region, and the effect of the interface on the gap parameter. The wave functions are first calculated and then a new potential is obtained, which in turn gives new wave functions. The iteration process is continued until self-consistency is fulfilled. Such a program has been carried out in Refs. 4 and 9. It was found that both effects will tend to reduce the current generated by multiple Andreev reflections. Taking an oversimplified route, we will ignore the effects of self-consistency and replace the barrier by a perfectly reflecting step function, i.e.,

$$\Delta(\hat{\mathbf{k}},\mathbf{x}) = \Theta(|z|-a)\Delta(\hat{\mathbf{k}}). \tag{6}$$

For gaps which have a component perpendicular to the interface the interface is pairbreaking and the gap will become suppressed under self-consistency. However, the interface has little effect on a gap which has no component perpendicular to the interface and thus the approximation of replacing the potential by a step function is not bad in this case.

The solutions to the Andreev equations are given by spinor wave functions in both the normal layer and superconducting banks. Neglecting Fermi wave-vector mismatch, electrons and holes with momentum in the same direction are coupled together by Andreev reflection and are decoupled from electrons and holes with opposite momentum. We refer the reader to technical details contained in Refs. 7 and 4. The result gives the following expression for the average supercurrent density produced by multiple Andreev reflections in a relaxation time approximation:

$$\langle \mathbf{J_{AR}}\rangle = \mathbf{e_z}\frac{e}{m}\int\frac{d\Omega_k}{4\pi}\int dE\sum_{n=1} g(E,\Omega_k)[f_0(E)k_e - f_0(-E)k_h]e^{-2na/l} \times\left(|\,A_n^+(\hat{\mathbf{k}},E+eV/2)\,|^2 - |\,A_n^-(\hat{\mathbf{k}},E-eV/2)\,|^2\right), \tag{7}$$

which represents the current generated by accelerated electrons and holes which climb up the potential well minus the current generated in the opposite direction by the decelerated electrons and holes. At $T=0$ the only contribution to the supercurrent is due to the initial holes. Here l is the inelastic mean free path, which is taken to be a constant independent of energy and temperature, $k_{e,h} = k_{Fz} \pm E/\hbar v_{Fz}$ with $k_{Fz} = k_F - (k_x^2+k_y^2)/2k_F$, and f_0 is a Fermi function. The probabilities for n repeated Andreev reflections $|\,A_n(\hat{\mathbf{k}},E)\,|^2$ limit the energy interval in Eq. (6) to mostly those states below the gap edge. The angular-dependent Andreev reflection probability is given by

$$|\,A_1(\hat{\mathbf{k}},E)\,|^2 = |E - \sqrt{E^2 - |\,\Delta(\hat{\mathbf{k}})\,|^2}|^2/\,|\,\Delta(\hat{\mathbf{k}})\,|^2, \tag{8}$$

which is unity for $|\,E\,| < |\,\Delta(\hat{\mathbf{k}})\,|$ and falls off sharply for higher energies. Ignoring over-the-barrier quantum reflections, we simply approximate the probabilities as a product of step functions, i.e.,

$$|\,A_n^{\pm}(\hat{\mathbf{k}},E)\,|^2 \approx \Theta(\,|\,\Delta(\hat{\mathbf{k}})\,| - |\,E \pm eV/2\,|\,) \times\Theta(\,|\,\Delta(\hat{\mathbf{k}})\,| - |\,E \pm neV \mp eV/2\,|\,). \tag{9}$$

$g(E,\Omega_k)$ is the angular-dependent quasiparticle density of states for an SNS junction, which for thick superconducting banks consists of the spatially quantized bound Andreev states for energies below the gap and those of the anisotropic superconducting density of states for energies above the gap. For thick superconducting banks ($\xi_0 \ll a \ll D$) one has just the bound Andreev states and scattering states and g can be represented by the sum of two independent contributions. The two-dimensional quasiparticle density of states $g(E, \Omega_k)$ rises slowly from zero to a sharp peak corresponding to $E(k_F)$, where the dispersion curve of the spectrum of bound Andreev states flattens out (see Fig. 2 of Ref. 7). While the density of states develops additional features for thin superconducting banks, for very thick banks it can be reasonably approximated by a contribution from the lowest quasiparticle subband

$$g(E, \Omega_k) \approx \frac{2ma}{(\pi\hbar)^2} \frac{E^2}{\sqrt{|\Delta(\hat{\mathbf{k}})|^2 - E^2}} \Theta(E_{\max} - E), \qquad (10)$$

for energies below the gap, where $E_{\max} = \min[(1+\pi)\frac{E_F}{k_F a}, |\Delta(\hat{\mathbf{k}})|]$. With these approximations, the expression for the supercurrent for $T = 0$ reduces to the simple expression

$$\langle \mathbf{J}_{AR} \rangle = \mathbf{e_z} \frac{2ak_F e}{(\pi\hbar)^2} \int \frac{d\Omega_{k_F}}{4\pi} \sum_{n=1}^{n=n_{\max}} e^{-2na/l} \Theta\left(\frac{2\,|\Delta(\Omega_{k_F})|}{eV} - n \right) \times \left(|neV - |\Delta(\Omega_{k_F})|\,|\, neV \sqrt{1 + \frac{2\,|\Delta(\Omega_{k_F})|}{neV}} + |\Delta(\Omega_{k_F})|^2 \arcsin\left| \frac{neV}{|\Delta(\Omega_{k_F})|} - 1 \right| \right), \qquad (11)$$

where $n_{\max} = \frac{2\max\Delta(\Omega_{k_F})}{eV}$. The sum over n is limited by the field voltage eV. A term in the series is lost whenever $eV = \frac{2|\Delta(\Omega_{k_F})|}{n-1}$. This is the basic reason for the subharmonic gap structure and the associated negative differential conductivity. For isotropic gaps, $\Delta(\Omega_{k_F})$ =const, the current-voltage characteristic shows jumps at these voltages, as can be seen in Fig. 1 (see also Fig. 4 in Ref. 7) for two inelastic scattering lengths. The inelastic scattering effects only the "foot" structure and has no effect on the jumps. The foot structure can be attributed to a balance between the number of Andreev reflections at the interfaces and the probability for an electron or hole to undergo an inelastic collision.

We now consider gaps which are zero on a portion of the Fermi surface. For an example we will consider a two-dimensional (2D) hexagonal representation for a d-wave gap, i.e., $\Delta(\Omega_{k_F}) = \Delta_0(\hat{k}_x\hat{k}_y)$, which has nodes that are perpendicular to the interface. However, the results are not sensitive to the actual angular dependence of the gap parallel to the interface, only to the presence of gap nodes. The nodes of the gap function restrict the region of energy space where Andreev reflection occurs, leading to a reduction of the supercurrent generated. This can be seen in Fig. 1, which is a comparison between the current generated by multiple Andreev reflections for s- and d-wave superconductors. The effect of inelastic scattering is the same for this case, namely, the presence of gap nodes does not effect the "foot" structure, which can be seen clearly in Fig. 2. The most notable difference, however, is that the sharp discontinuities are completely smeared for a superconductor with gap nodes, and the jumps turn into cusps. The cusps can be clearly seen in Fig. 2. Therefore it is not possible to tell from the I-V curves the maximum value of the gap as it is in isotropic s-wave superconductors, given that experimental resolution will smear the curves somewhat. The differences can be better seen in the derivative of the I-V curve. While the jumps in the I-V curves for the s-wave case turn into symmetric delta function peaks for the derivative plot, for the d-wave case, asymmetric peaks which show a gradual rise from $eV = 2\Delta_0/n$ to a sharp peak at $eV = 2\Delta_0/(n-1)$ are obtained, as can be noted in Figs. 3 and 4. This asymmetry is not present in the s-wave case and could therefore be a signature for the presence of gap nodes in the superconductor. It can also be noted that the second derivative can be used to test for asymmetry as well.

We now mention the consequences of our approximations. Small superconducting banks relative to the nor-

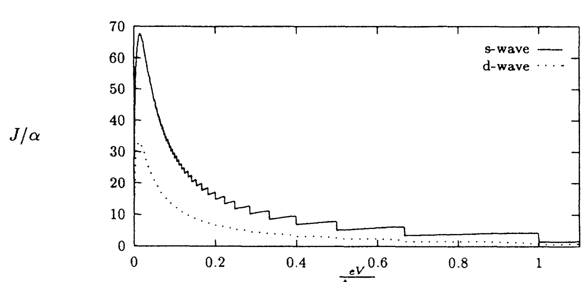

FIG. 1. Comparison between current generated with s-wave superconductors and d-wave superconductors in clean limit ($a/l = 0.01$). Here $\alpha = \frac{2ak_F e}{(\pi\hbar)^2}$.

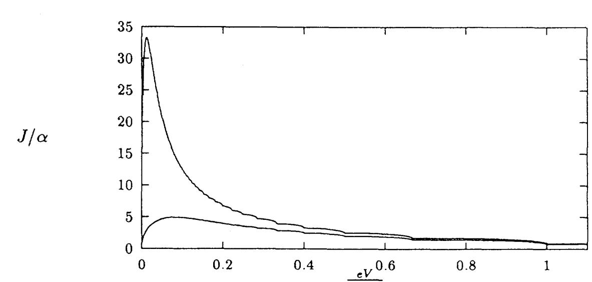

FIG. 2. Results for d-wave case for clean and dirty cases ($a/l = 0.01$, upper curve; $a/l = 0.06667$, lower curve).

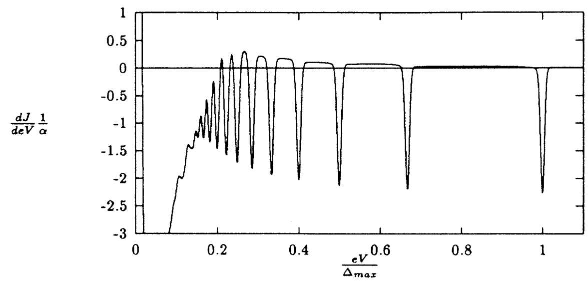

FIG. 3. Derivative of s-wave characteristic convoluted with a Gaussian of fixed width $\Gamma = 0.005 \cdot \Delta_{\rm max}$.

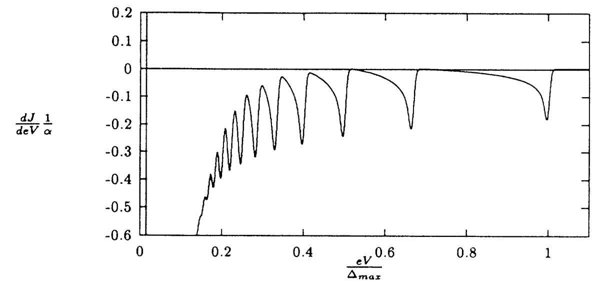

FIG. 4. Derivative of d-wave characteristic convoluted with a Gaussian of fixed width $\Gamma = 0.005 \cdot \Delta_{\rm max}$. Note the asymmetric line shape.

mal metal region, large coherence length superconductors, and gaps which are not mutually aligned or have a component perpendicular to the interface will all lead to a net smearing of the I-V features and minimize the signal seen in the dI/dV curves, making it difficult to extract the Andreev contribution to the current. However, we note that the smearing in these situations will be an overall one which would not obscure the *inherent* asymmetry of the peaks around $eV = 2\Delta_0/(n-1)$.

There has been recent evidence for the presence of Andreev reflections in point contact spectroscopy experiments in Y-Ba-Cu-O (Refs. 6 and 10) and in the heavy fermion superconductor URu_2Si_2.[11] Also, recently supercurrents produced by multiple Andreev reflections in Y-Ba-Cu-O break junctions have been observed.[6] The I-V curves show a "foot" structure at low bias voltages, while no steplike jumps were seen. Also, peaks at a few discrete voltages are observed in the dI-dV characteristic which do show some asymmetry. Both of these features are consistent with findings here for anisotropic superconductors. It may be that there exists tunneling between different grain-grain contacts inside the granular junction that obscures the step features in the I-V curves. However, this explanation seems unlikely since the "foot" structure is clearly seen, and the large smearing necessary to obscure jumps for the s-wave case would also obscure the "foot." Likewise, the difficulty in observing peaks in the dI-dV curves can be attributed to the reduction of the steps to cusps for the anisotropic case. However, more accurate tests will be needed to clearly identify whether the line shapes are intrinsically asymmetric.

We close with a few remarks concerning the effects of conventional anisotropic gaps [with $\Delta(\hat{\mathbf{k}}) \neq 0$ everywhere on the Fermi surface] on the I-V curves. It can be shown that the I-V curves contain features reminiscent of the isotropic s-wave case, but new "jumps" occur in the I-V curves for $eV = 2\Delta_{\rm min}/n$ *and* $eV = 2\Delta_{\rm max}/n$.[12] The curves in this case will be clearly different than those obtained from junctions made with superconductors with gap nodes, and the two cases can be distinguished clearly by the dI/dV signatures.

A lack of "jumps" is common to any superconductor with gap nodes, but the absence of the steps cannot uniquely identify the gap symmetry. We cannot show this way whether $\Delta(\hat{\mathbf{k}})$ including its zeros has the symmetry of the Fermi surface (conventional order parameter) or whether it has a lower symmetry (unconventional order parameter). If angular-dependent Andreev measurements were achieved, i.e., if the position of the steps in the I-V characteristic or the peaks in the derivative curve could be tracked as a function of angle, one could check the symmetry of the nodes on the Fermi surface. The relative sign of $\Delta(\hat{\mathbf{k}})$ on different parts of the Fermi surface separated by nodal lines would still remain unknown from the experiments. However, Andreev reflection measurements could in principle provide important information on the angular dependence of the nodes on the Fermi surface, an important step towards unraveling the nature of the order parameter.

We would like to thank Professor Dr. R. Kümmel, U. Gunsenheimer, and C. Bruder for insightful comments and communications, and thank Professor Dr. F. Steglich, K. Gloos, and G. Zwicknagl for discussions.

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