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Flux-chain buckling in layered superconductors

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When a magnetic field is perpendicular to the c axis of a layered high-T_c superconductor the phenomenon of flux-chain buckling near twin planes has been found. Both numerical simulation and an analytical approach based on vortex-lattice instability give similar conclusions. The period of the buckled structure is estimated and results are in qualitative agreement with experimental observations.

I. INTRODUCTION

In macroscopically homogeneously layered high-T_c superconductors, the vortex lattice can be very anisotropic. If the magnetic field is parallel to the ab plane the Abrikosov unit cell is compressed by the ratio $\lambda_{ab}/\lambda_c \ll 1$ in the direction perpendicular to the ab plane.[1] The London penetration depths λ_{ab} and λ_c correspond to the current direction parallel and perpendicular to the ab plane. In this situation the vortex distribution can be considered as a system of vortex chains in the direction perpendicular to the ab plane, because the distance between vortices of one chain is smaller than the interchain distance.

This vortex structure for the magnetic field parallel to the ab plane has been reported by Dolan *et al.*[2] The Bitter-pattern technique produced by an anisotropic vortex lattice could be treated as a system of rectilinear vortex chains in accordance with the theory. This picture has been observed in dislocation-free samples.

In the presence of a dislocation plane, which is parallel to the c axis and the magnetic-field direction, the vortex distribution is quite different,[2] as shown in Fig. 1. This distribution corresponds to the presence of twin boundaries which appear presumably at the bottom of Fig. 1. The twin edge goes horizontally. The main feature of the vortex distribution in the presence of the twin boundary is the buckling of vortex chains, as can be seen in Fig. 1. This vortex-chain buckling can be imagined as separate segments of a chain making an angle with respect to each other.

The purpose of this paper is to explain the nature of vortex-chain buckling. We will argue that the vortex-lattice instability[3] in the presence of a plane defect leads to the buckling of flux chains. If the magnetic field is parallel to the ab plane, interlayer pinned vortices can slide over the ab direction when the magnetic field varies. If the magnetic field decreases it leads to vortex-lattice in-

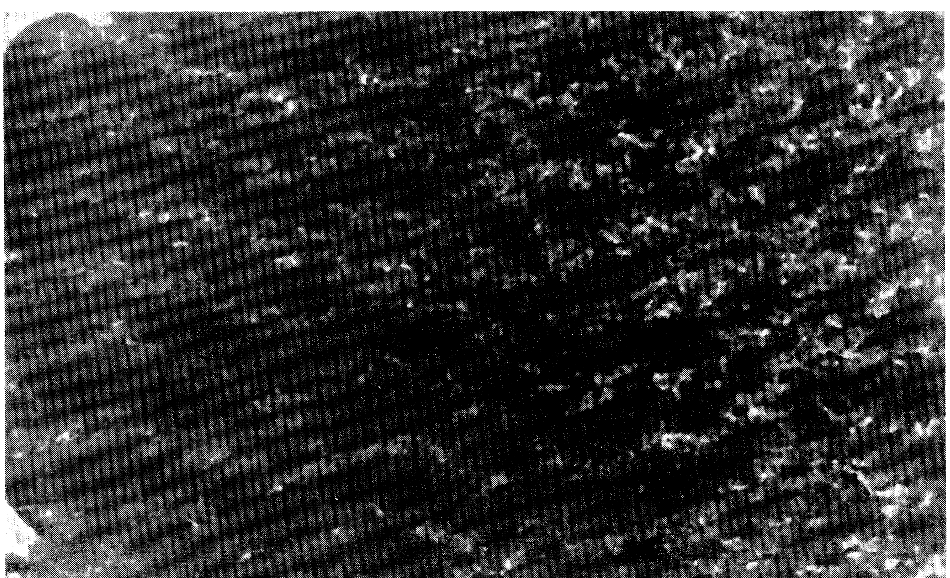

FIG. 1. Pattern observed at a field 8 G perpendicular to c. Individual vortices are not resolved. The horizontal length of the picture represents 37 μm. The twin boundaries in this sample are horizontal and one presumably occurs below the bottom of the picture.

stability, when the vortex chains make a finite angle with respect to the c direction.[3] This angle can have both signs in a three-dimensional homogeneous sample.

A twin boundary plays a double role. Firstly, it attracts vortices,[4] decreasing the local vortex line concentration, which leads to the instability. Secondly, due to boundary conditions on a twin plane, the angle between vortex chains and the c direction alternates its sign, giving rise to the chain buckling observed in Ref. 2.

The analysis of flux-lattice instability[3] has been performed on the basis of the London model. In Ref. 5 the simulation of the vortex distribution in the presence of twin boundaries has been done using the modified Bessel function interaction between vortices. Both approaches, the London model[3] and numerical simulations,[5] give similar results for the shape of vortex chains near the plane defect. Those chains become buckled. In this paper we analyzed both approaches.

II. NUMERICAL SIMULATION

Results of the numerical simulation are shown in Fig. 2.[5] The twin plane is directed parallel to the c and B directions. The magnetic field B is parallel to ab planes. The boundary condition keeps vortices on the twin plane and the interaction between vortices was chosen as the modified Bessel function.

As one can see from Fig. 2, the triangular vortex structure is buckled near the experimental observation by Dolan *et al.*[2] (see Fig. 1). The numerical simulation cannot give an estimate of the buckling period or an expression of this period via other parameters of the problem. In order to do that we develop in the following sections the analytical approach to the buckling problem.

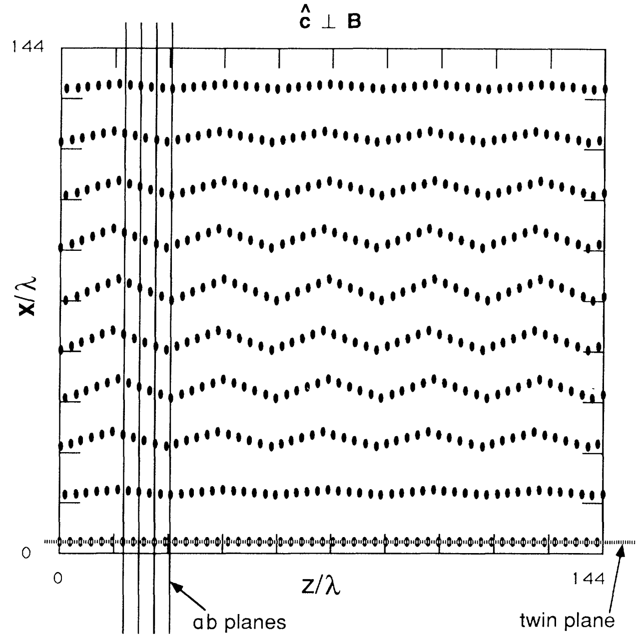

FIG. 2. The numerical simulation of the vortex distribution near twin planes which are parallel to the **c** and B directions ($B \perp$c). Here $\lambda = (\lambda_{ab}\lambda_c)^{1/2}$.

III. FREE ENERGY OF DISTURBED LATTICE

We start with the London model of a vortex structure. For the magnetic field parallel to the ab plane, the London free energy assumes the form

$$F=\frac{1}{8\pi}\int d^3r\left[H^2+\lambda_c^2\left[\frac{\partial H}{\partial x}\right]^2+\lambda_{ab}^2\left[\frac{\partial H}{\partial z}\right]^2\right] . \quad (1)$$

Here $H=H_y$ and the z axis is perpendicular to the ab plane. In the anisotropic case, $\lambda_c/\lambda_{ab}\sim\xi_{ab}/\xi_c\gg 1$. The magnetic field H is determined by the generalized London equation,

$$H-\lambda_c^2\frac{\partial^2 H}{\partial x^2}-\lambda_{ab}^2\frac{\partial^2 H}{\partial z^2}=\Phi_0\sum_{kl}\delta(z-z_{ol})\delta[x-x_0(k+ql+\Psi_{lk})] . \quad (2)$$

The values Ψ_{lk} determine deviations of the vortex system from a perfect lattice. If $\Psi_{lk}=0$ and $q=1/2$, then expression (2) corresponds to the perfect vortex lattice with an isosceles triangle unit cell. The unit-cell area is $x_0z_0=\phi_0/B$. Vortex chains lie parallel to the z axis with the period $x_0/2$.

For our purposes we will consider further only a slowly varying dependence of Ψ_{lk} on l and k. In this case one can put

$$\frac{\partial\Psi}{\partial l}=\frac{\lambda_{ab}p}{2\pi\lambda_c}\frac{\partial u}{\partial z} ,$$
$$\frac{\partial^2\Psi}{\partial l^2}=\left[\frac{p\lambda_{ab}}{2\pi\lambda_c}\right]^{3/2}\left[\frac{\Phi_0}{B}\right]^{1/2}\frac{\partial^2 u}{\partial z^2} , \quad (3)$$
$$\frac{\partial\Psi}{\partial k}=\frac{\partial u}{\partial x} .$$

Here u is a macroscopic displacement of a vortex lattice in the x direction and the parameter p is determined as

$$p=\frac{2\pi z_0}{X_0}\frac{\lambda_c}{\lambda_{ab}}=2\pi z_0^2\frac{\lambda_c B}{\lambda_{ab}\Phi_0} . \quad (4)$$

Following the procedure given in Ref. 3, the free-energy density can be written in the form

$$F=\frac{B^2}{8\pi}\left[1+\left[\frac{\partial u}{\partial x}\right]^2\right]+\frac{B\Phi_0}{32\pi^2\lambda_{ab}\lambda_c}\left[\ln\frac{\Phi_0\lambda_c}{B\lambda_{ab}\xi_{ab}^2}\right]+G\left[p,q+\frac{\lambda_{ab}p}{2\pi\lambda_c}\frac{\partial u}{\partial z}\right]$$
$$+\frac{\Phi_0}{12\pi B}\left[\frac{p\lambda_{ab}}{\lambda_c}\right]^3\left[\frac{\partial^2 u}{\partial z^2}\right]^2\sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\frac{pk^2l^4}{\sqrt{p^2k^2+p/\gamma}}\exp(-l\sqrt{p^2k^2+p/\lambda+2i\pi qkl}\,) , \quad (5)$$

where $\gamma=2\pi\lambda_{ab}\lambda_c B/\Phi_0$ and

$$G(p,q)=\sum_{k=1}^{\infty}\frac{2}{\sqrt{k^2+(\gamma p)^{-1}}}\left[\frac{\sinh\sqrt{p^2k^2+p/\gamma}}{\cosh\sqrt{p^2k^2+p/\gamma}-\cos(2\pi qk)}-1\right]-\ln p+\sqrt{p\gamma}\coth(\sqrt{p/4\gamma})-2\gamma$$
$$+2\sum_{k=1}^{\infty}\left[\frac{1}{\sqrt{k^2+(\gamma p)^{-1}}}-\frac{1}{k}\right]. \qquad (6)$$

IV. VORTEX-LATTICE INSTABILITY

In the absence of the lattice distortion, $u=0$, and the minimum of the vortex-lattice energy F is determined by the minimum of the function $G(p,q)$. This minimum is given by the values $p=\pi/\sqrt{3}$ and $q=1/2$. In the "natural" coordinates $z\sqrt{\lambda_c/\lambda_{ab}}$ and $x\sqrt{\lambda_{ab}/\lambda_c}$, it corresponds to the hexagonal Abrikosov lattice.

The decrease of the magnetic field B in a highly layered material leads to the decrease of the parameter p in (4), because vortices can move only along layers separated by distance z_0 due to strong interlayer pinning. If the parameter p decreases from the equilibrium value $\pi\sqrt{3}$ at some critical value $p=p_c(\lambda)$, the minimum of $G(p,q)$ at $q=1/2$ splits in two minima at $q=q_{1,2}$ so that $q_{1,2}-1/2\sim\pm(p_c-p)$.[3] At $p<p_c$, the position with $q=1/2$ becomes unstable. The plot of $p_c(\gamma)$ is shown in Fig. 3.

At $p<p_c$ each vortex chain in Fig. 2 tends to make an angle θ with the z axis so that

$$\tan\theta=\frac{2\pi\lambda_c}{\lambda_{ab}p}\left[q_{1,2}-\frac{1}{2}\right]. \qquad (7)$$

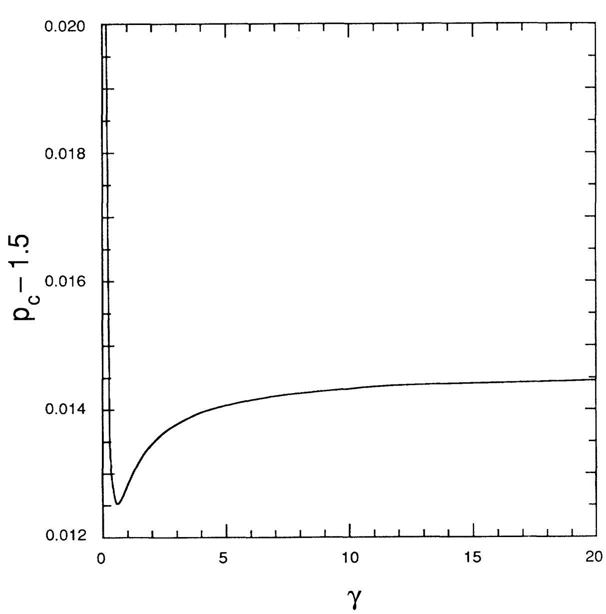

FIG. 3. Plot of the critical value of $p=2\pi z_0^2\lambda_c B/\lambda_{ab}\Phi_0$ vs $\gamma=2\pi\lambda_{ab}\lambda_c B/\Phi_0$. At $p=p_c$, the vortex-lattice instability occurs.

In a bulk homogeneous sample, vortices arrange into a new lattice with some particular value of q corresponding to either q_1 or q_2. This situation changes in the presence of some obstacles directed along the x axis (i.e., twin boundaries) where vortex chains should be parallel to the z axis. In this situation the vortex structure becomes macroscopically inhomogeneous along the z axis. If the values of (p_c-p) is much smaller than p_c the scale of this structure exceeds the unit-cell size and the problem can be considered in terms of the vortex-lattice elasticity theory.

V. EQUATION FOR VORTEX DISPLACEMENT

At small values of (p_c-p) the function $G(p,q)$ can be expanded near the point $q=1/2$. It is convenient to introduce new variables $\tilde z=z/\tilde z_0$, $\tilde x=x/\tilde x_0$, and $\tilde u=u/\tilde u_0$, where

$$\tilde z_0^2=\frac{c\phi_0}{Ba(p_c-p)}\frac{p_c\lambda_{ab}}{2\pi\lambda_c},$$
$$\tilde x_0^2=\frac{16\pi^2 c}{a^2(p_c-p)}\frac{\lambda_c^2}{p_c^2}, \qquad (8)$$
$$\tilde u_0^2=\frac{c}{b}\frac{\phi_0}{B}\left[\frac{2\pi\lambda_c}{\lambda_{ab}p_c}\right].$$

The parameters $a\sim b\sim c\sim 1$ are defined as

$$a=-\frac{1}{p_c-p}\left.\frac{\partial^2 G(p_c,p)}{\partial q^2}\right|_{q=1/2},$$
$$b=\left.\frac{\partial^4 G(p_c,p)}{\partial q^4}\right|_{q=1/2}, \qquad (9)$$
$$c=\frac{4\pi^2}{3}\sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\frac{p(-1)^{kl}}{\sqrt{p_c^2\gamma^2+p_c/\lambda}}k^2l^4 \times\exp(-l\sqrt{p_c^2k^2+p_c/\gamma}).$$

In these units the distortion energy of vortices can be written in the form

$$\delta F=\varepsilon\int d\tilde x\,d\tilde z\left[V\left[\frac{\partial\tilde u}{\partial\tilde z}\right]+\frac{1}{2}\left[\frac{\partial^2\tilde u}{\partial\tilde z^2}\right]^2+\frac{1}{2}\left[\frac{\partial\tilde u}{\partial\tilde x}\right]^2\right], \qquad (10)$$

where $V(y)=\frac{1}{4}y^4-\frac{1}{2}y^2$,

$$\varepsilon=\frac{c\phi_0}{16\pi b}\left[\frac{\phi_0 Ba(p_c-p)}{\pi\lambda_{ab}\lambda_c}\right]^{1/2}. \qquad (11)$$

The equation for the displacement u follows from the minimization of the energy (10),

$$\frac{\partial}{\partial \tilde{z}}\left[V'\left(\frac{\partial \tilde{u}}{\partial \tilde{z}}\right)-\frac{\partial^3 \tilde{u}}{\partial \tilde{z}^3}\right]+\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^3}=0 . \qquad (12)$$

In the absence of the twin boundaries, the solution does not depend on x and it has the formal solution of either $\tilde{u}=\tilde{z}$ or $\tilde{u}=-\tilde{z}$. The twin boundary fixes one row of vortices parallel to the z axis and provides a repulsion of the other rows. It leads to a buckling of the vortex structure with the period L_z along the z axis.

VI. BOUNDARY CONDITION

Suppose there are some twin boundaries which are parallel to the z axis. At each boundary the displacement u should be zero. The approximation used above is valid in the distance $x \gg \lambda_c$ from the twin boundary and for this reason the effective boundary condition for (12) is required. In order to arrive at the boundary condition it is sufficient to consider only an x-dependent displacement $u=u_x$.

In order to model the twin boundary at the point $x=0$, one can introduce the external force $f=\beta\delta(x)$. Then the Fourier component of the total force f_k, including the elastic part, has the form ($k=k_x$)

$$f_k=-k^2 c_{\parallel}(k) u_k+\beta , \qquad (13)$$

where $c_{\parallel}$ is the compression modulus. From the condition $f_k=0$ we have

$$u(x)=\begin{cases}\beta|x|/2c_{\parallel}(0)+u_0 , & x \gg \lambda_c \\ u_0-\beta\delta(x)\left[\lim_{k\to\infty} k^2 c_{\parallel}(k)\right]^{-1} , & x \ll \lambda_c .\end{cases} \qquad (14)$$

For a discrete lattice $\delta(x)=x_0^{-1}$ at $x=0$ and from the condition $u(0)=0$, we obtain

$$\beta=u_0 x_0 \lim_{k\to\infty} k^2 c_{\parallel}(k) \cong \frac{u_0 x_0}{\lambda_c^2}\frac{B^2}{4\pi} . \qquad (15)$$

So at $x \gg \lambda_c$, putting $c_{\parallel}(0)=B^2/4\pi$ we have

$$u(x)=u_0+\frac{x_0|x|}{2\lambda_c^2}u_0 . \qquad (16)$$

This is equivalent to the boundary condition

$$\frac{\partial u}{\partial x}\mathrm{sgn}x=\frac{x_0}{2\lambda_c^2}u , \qquad (17)$$

if we consider the coordinate scale $x \gg \lambda_c$. In the units of (8) the boundary condition (17) gives

$$\frac{\partial \tilde{u}}{\partial \tilde{x}}\mathrm{sgn}\tilde{x}=\frac{4\pi^2\sqrt{c}}{a p_c}\frac{\tilde{u}}{(p_c-p)\sqrt{\gamma}} . \qquad (18)$$

VII. BUCKLED VORTEX STRUCTURE

We consider the limit of large $\gamma \gg 1$. One can consider two limiting cases.

(a) $(p_c-p) \ll \gamma^{-1/2}$. In this case the coefficient on the right-hand side of (18) is large and the boundary condition for (12) is reduced to $u(0)=0$. Neither the boundary condition nor the equation depends on parameters so the period of the buckling along the z axis is $L_z \sim 1$. In ordinary units

$$L_z \sim z_0 (p_c-p)^{1/2} . \qquad (19)$$

(b) $\gamma^{-1/2} \ll (p_c-p) \ll 1$. The scale of u in the x direction is $\tilde{x} \sim 1$ and we can write

$$\int_0^1 d\tilde{x}\left[V'\left(\frac{\partial \tilde{u}}{\partial \tilde{z}}\right)-\frac{\partial^3 \tilde{u}}{\partial \tilde{z}^3}\right] \sim \int \frac{\partial \tilde{u}}{\partial \tilde{x}} d\tilde{z} . \qquad (20)$$

The right-hand side of (20), using formula (18), can be evaluated as

$$\frac{1}{(p_c-p)\sqrt{\gamma}}\int dz \int \frac{\partial u}{dz} dz \sim \frac{L_z^2}{(p_c-p)\sqrt{\gamma}} . \qquad (21)$$

This value should be of order unity and we get $L_z^2 \sim (p_c-p)\sqrt{\gamma}$. In ordinary units,

$$L_z \sim z_0 \gamma^{1/4} . \qquad (22)$$

VIII. DISCUSSION

The decrease of the parameter p leading to instability can be achieved either by decreasing the external magnetic field or by a mechanism of vortice attraction to the twin plane. In the second case, the local vortex concentration diminishes in the vicinity of a twin, and this situation corresponds to that of Ref. 2. The parameter γ for yttrium materials can be evaluated as $\gamma \cong 0.2[B(G)]$. It means that for conventional Bitter-pattern experiments the value of γ is rather large, as it has been chosen in the present consideration.

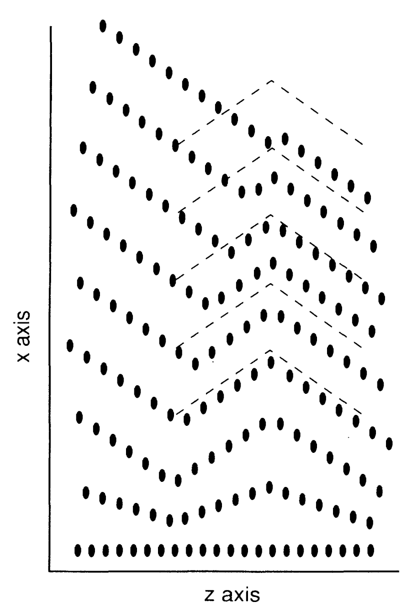

FIG. 4. A qualitative picture of the vortex distribution near single twin planes (bottom line).

The period of the buckled structure is determined by formulas (19) and (22), and in the limit $\gamma \gg 1$, this period exceeds the intervortex distance z_0. If the magnetic field decreases so that $(p_c - p)$ becomes of order unity, or the value of p becomes small, the picture of buckling can be more complicated, because many new minima than appear in the free energy vs p.[3]

The numerical simulation[5] used the value $\gamma \cong 7$ and p was near to p_c. The period of the buckled structure in Fig. 2 is $L_z = 22z_0$. This buckled structure is constricted between two twin planes corresponding to the top and bottom boundaries of Fig. 2. In the case of a single twin plane, the decrease of an external magnetic field leads to the qualitative picture shown in Fig. 4.

IX. CONCLUSIONS

We have found the phenomenon of flux-chain buckling near a twin boundary, when the magnetic field is parallel to the *ab* plane. We have performed both analytical and numerical analysis of the problem and have obtained similar results. The analytical approach allowed us to express the period of the buckled structure through the equilibrium intervortex distance and the magnetic inductance. These results are in agreement with the experimental observations.[2]

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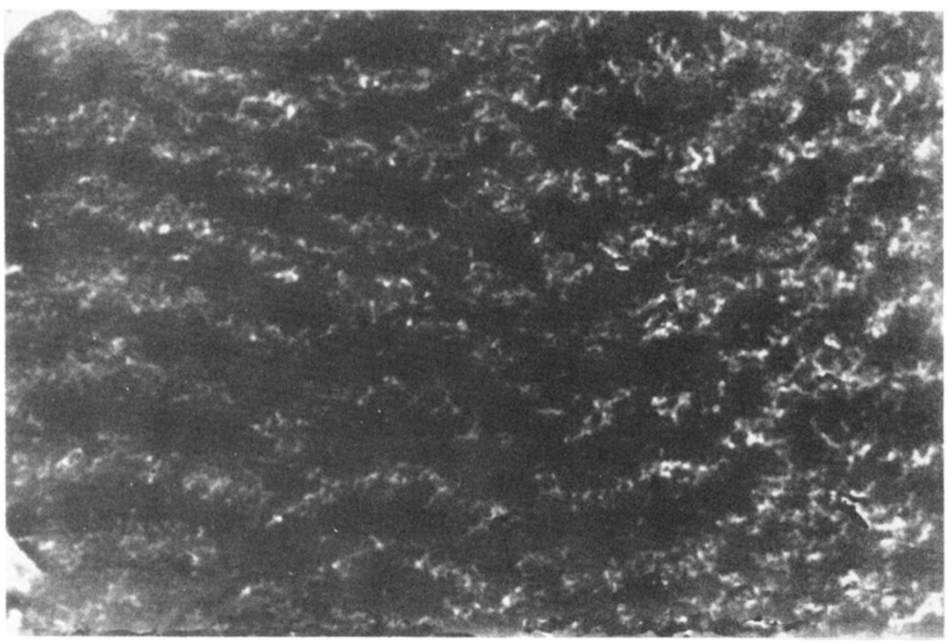

FIG. 1. Pattern observed at a field 8 G perpendicular to c. Individual vortices are not resolved. The horizontal length of the picture represents 37 μm. The twin boundaries in this sample are horizontal and one presumably occurs below the bottom of the picture.