Spectrum of relaxation times for Ising-spin clusters in random fields

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Exact results on the single spin-flip Glauber dynamics of six coupled random-field Ising spins with a coordination number of 4 are presented. Two distributions of random fields, a binary distribution (BD) and a Gaussian distribution (GD), are investigated. The effects of the static magnetic field are discussed. In the zero-magnetic-field case, the number of diverging relaxation times is equal to the number of energy minima minus one. This rule is broken in the presence of a magnetic field. The longest relaxation times in the absence of the field verify the Arrhenius law with the energy barrier determined by the energy needed to invert the ground-state spin configuration. At low temperatures, according to the Arrhenius law, the spectrum of relaxation times shows a double-peak distribution on a logarithmic scale. In the GD case, the energy-barrier distribution is continuous while it is quasidiscrete in the BD case.

I. INTRODUCTION

Ising or Ising-like models in random fields are good representation of a large number of impure materials. The random-field Ising (RFI) systems can be realized experimentally by the use of a diluted antiferromagnet in a uniform magnetic field. It has been shown by Aeppli and Bruinsma[1] that for an infinite chain of RFI spins with a binary distribution (BD) of random fields and for sufficiently high temperatures, the probability distribution for various observables is a devil's staircase, and they suggest that these results could be tested experimentally by nuclear-magnetic-resonance or Mössbauer spectroscopy. Other experimental techniques, such as neutron scattering[2] and linear birefringence[3] are currently used to probe RFI systems. The theoretical study of infinite one-dimensional (1D) RFI systems in the presence of a variety of RF's performed by Andelman[4] concluded that the devil's staircase is a special characteristic of discrete distributions and that for continuous ones there are no such nonanalyticities.

The dynamics of RFI systems in two and three dimensions in the presence of a static random field was investigated using a self-consistent method by Vilfan and Stefan.[5] They did not observe the logarithmic time dependence of the correlation length typical for the transition from the metastable to the equilibrium configuration. Little is known about the dynamics of Ising spins (see, e.g., Gawlinski *et al.*,[6] Grant and Guntor,[7] and Forgacs, Mukamel, and Pelcovits[8]). Also Pytte and Fernandez[9] have used Monte Carlo simulations for studying the equilibration of Ising systems in random fields at low temperature.

The Ising model on a triangular lattice with antiferromagnetic nearest-neighbor and ferromagnetic next-nearest-neighbor interactions is investigated by treating the master equation with the molecular-field approximation and the cluster-decoupling approximation.[10,11] The relaxation process is investigated by studying the flow in the space of sublattice magnetization in each approximation. Banavar, Cieplak, and Muthukumar[12] have used the six-spin cluster for studying the dynamics of a spin-glass system.

Since very little about the dynamics of RFI systems is understood, it seems worthwhile to determine dynamical properties of the RFI model with well-understood static properties. This model consists of Ising spins located on the sites of the Sierpenski gasket with all exchange interactions identical and the random fields at the sites distributed according to either binary or Gaussian distributions. The static properties of this model were investigated exactly by Ismail and Salem.[13]

Not only does the model discussed here display a $T=0$ transition, but it also has two advantages. First, it can be analyzed exactly. Second, it allows the study of various distributions of the random fields (H_i). These choices will be summarized in the next section. We shall focus our attention on the binary (BD) and Gaussian (GD) distributions of the RF. In Sec. III, we apply Glauber dynamics[14] to obtain the equations of motion, and we discuss all possible forms of the transition-probability function $W_j(S_j \rightarrow S_j)$. In Sec. IV, we explain the procedure used to derive the relaxation times.

Our main concern in this paper, however, is studying the effects of the static magnetic field on the spectra of relaxation times and also to discuss the corresponding distribution of energy barriers (ΔE). It is also very useful to compare RFI systems, which possess the two basic ingredients, randomness and frustration, with the conventional SG (see, e.g., Cieplak and Ismail[15]). Thus in Sec. V, we discuss the relaxation times in the absence of the external magnetic field, while we investigated the influence of a static field on the relaxation spectrum in Sec. VI. We shall see, in particular, that the number of diverging relaxation times ceases to be related to the number of energy minima.

II. THE MODEL

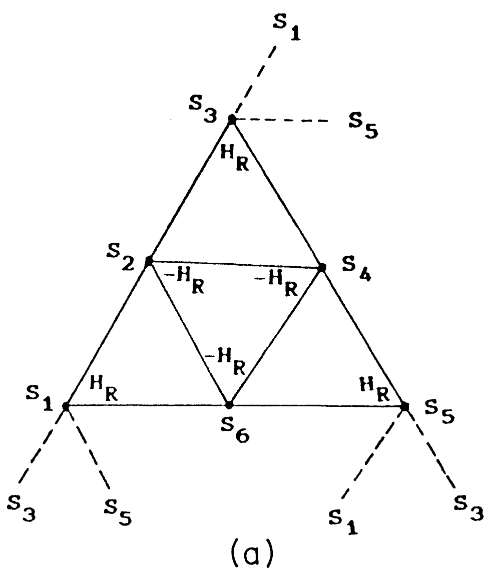
We consider Ising spins $S_i = \pm 1$, which interact in the way described by the usual RF Hamiltonian

$$\mathcal{H} = -J \sum_{\langle ij \rangle}^{N} S_i S_j - \sum_{i=1}^{N} H_i S_i - B \sum_{i=1}^{N} S_i \ . \tag{1}$$

The exchange couplings J's are identical. The sum over j means the sum over the first-nearest neighbors. The local random fields, H_i's, are site dependent. N refers to the number of spins. The magnetic field B consists, in general, of two parts; a static one and an oscillatory one,

$$B = H + H_0 \sin(\omega t) \ . \tag{2}$$

We shall be concerned here with the static part only (i.e., H). We study six spins arranged as shown in Fig. 1 and we consider two different distributions of RF's.



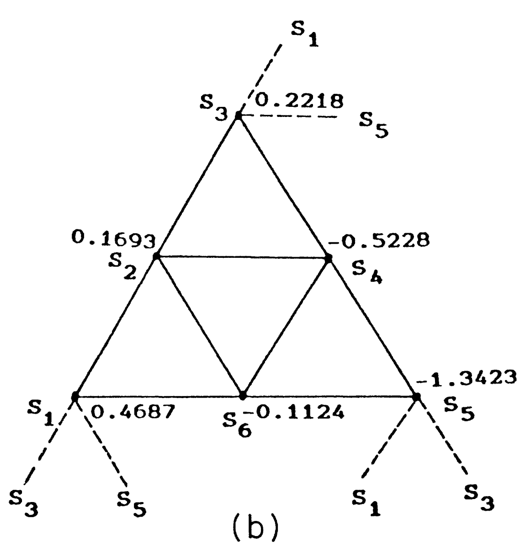


FIG. 1. The six-spin clusters considered in this paper. (a) The $\pm H_R$'s refer to the local random-field components for the BD case. (b) The numbers indicate values of the local random field components for the GD case.

A. The BD case

As an example we take H_i as shown in Fig. 1(a). We have investigated various amplitudes, i.e., H_R of RF. We have obtained three energy minima for $H_R = 0.1$, 0.7, and 1.5. These are the following:

(a) $S_1 = S_2 = S_3 = S_4 = S_5 = S_6$,

(b) $-S_1 = -S_2 = -S_3 = -S_4 = -S_5 = -S_6$,

(c) $S_1 = -S_2 = S_3 = -S_4 = S_5 = -S_6$.

The values of the energy states (E) and the energy barriers (ΔE) for reversing all spins from this state with a sequence of single-spin flips are shown in Table I.

As we see from Table I, the first and second energy states have the same energies but with inversing spin configurations. While the third state has a higher energy with antiparallel spin configuration. For $H_R = 2.1$, we obtain three energy minima as follows:

(a) $S_1 = -S_2 = S_3 = -S_4 = S_5 = -S_6$,

(b) $S_1 = S_2 = S_3 = S_4 = S_5 = S_6$,

(c) $-S_1 = -S_2 = -S_3 = -S_4 = -S_5 = -S_6$.

It is clear that the ground state is a completely bond-frustrated state, while the other two have local RF frustration. For $H_R = 4.1$, there exists only one energetic minimum of energy -24.6 and with antiparallel spin configuration. The smallest energy required to invert spins in this state is equal to $\Delta E = 41.0$.

B. The GD case

As an example, we take H_i as shown in Fig. 1(b). We have investigated two different widths, i.e., W_d of RF and obtained three energy minima. For $W_d = 0.5$ and 1.0, the spin configurations of the energy states are the same and are as follows:

(a) $-S_1 = -S_2 = -S_3 = -S_4 = -S_5 = -S_6$,

(b) $S_1 = S_2 = S_3 = S_4 = S_5 = S_6$,

(c) $S_1 = S_2 = S_3 = -S_4 = -S_5 = -S_6$.

The corresponding energy and energy barriers have values as shown in Table II.

III. EQUATIONS OF MOTION

According to Glauber dynamics,[14] the single-flip dynamics of N coupled Ising spins can be specified in terms of the density matrix $P(S_1, S_2, \ldots, S_N, t)$, which satisfies the master equation

TABLE I. Values of the energy states E and the energy barriers ΔE for the BD case.

H_R							
0.1		0.7		1.5		2.1	
E	ΔE	E	ΔE	E	ΔE	E	ΔE
−12	11.6	−12	9.2	−12	6.0	−12.6	21
−12	8.2	−12	9.2	−12	6.0	−12.0	12.2
−0.6	1.0	−4.2	7.0	−9.0	15	−12.0	3.8

$$\frac{d}{dt}P(S_1,S_2,\ldots,S_N,t)=-\sum_{j=1}^{N}W_j(S_j\rightarrow -S_j)P(S_i,\ldots,S_j,\ldots,S_N,t)+\sum_{j=1}^{N}W_j(-S_j\rightarrow S_j)P(S_i,\ldots,-S_j,\ldots,S_N,t)\ . \tag{3}$$

The probability of finding the system in the configuration $S_1,S_2,\ldots,S_N$ at time t is given by $P(S_1,S_2,\ldots,S_N)$, whereas $W_j(S_j\rightarrow -S_j)$ is the probability of reversing a spin at site j. In equilibrium, the jth spin has the value of S_j with a probability $P_j^{\text{eq}}(S_j)$. The state of canonical equilibrium (which is possible when B is time independent) for the system can be reached by imposing the following detailed balance condition:

$$W_j(S_j\rightarrow -S_j)P_j^{\text{eq}}(S_j)=W_j(-S_j\rightarrow S_j)P_j^{\text{eq}}(-S_j)\ . \tag{4}$$

We found that $W_j(S_j\rightarrow -S_j)$ can be chosen to be one of the following forms:

$$\text{(a)}\ W_j(S_j\rightarrow -S_j)=\left[\frac{1}{2\tau_0}\right](1-S_j\tanh\beta h_j)(1-S_j\tanh\beta H_j)^*\ \ (1-S_j\tanh\beta B)\ , \tag{5a}$$

where

$$h_j=J\sum_i S_i \tag{5a'}$$

is the exchange field acting on S_j, H_j is the local random field, B is the external magnetic field (2), τ_0 is the macroscopic flipping time, and $\beta=1/k_BT$, where T is the temperature and k_B is the Boltzmann constant;

$$\text{(b)}\ W_j(S_j\rightarrow -S_j)=\left[\frac{1}{2\tau_0}\right](1-S_j\tanh\beta h_j')(1-S_j\tanh\beta H_j)\ , \tag{5b}$$

where H_j is defined as in (5a) and h_j' can be written as

$$h_j'=h_j+B\ ; \tag{5b'}$$

$$\text{(c)}\ W_j(S_j\rightarrow -S_j)=\left[\frac{1}{2\tau_0}\right](1-S_j\tanh\beta h_j)(1-S_j\tanh\beta H_j')\ , \tag{5c}$$

where h_j is defined as in (5a') and H_j' has the form

$$H_j'=H_j+B\ ; \tag{5c'}$$

$$\text{(d)}\ W_j(S_j\rightarrow -S_j)=\left[\frac{1}{2\tau_0}\right](1-S_j\tanh\beta h_j'')(1-S_j\tanh\beta B)\ , \tag{5d}$$

where

$$h_j''=h_j+H_j\ ; \tag{5d'}$$

$$\text{(e)}\ W_j(S_j\rightarrow -S_j)=\left[\frac{1}{2\tau_0}\right](1-S_j\tanh\beta h_j''')\ , \tag{5e}$$

where h_j is the total field acting on S_j and is defined as

$$h_j'''=h_j+H_j+B\ . \tag{5e'}$$

In the absence of the external magnetic field ($B=0$), it is clear that the first three transition rate formulas are equivalent and also that the last two (5d) and (5e) are coinciding. The first three forms are not suitable, since we see that the effective exchange field (h_j) is separated from the local random field (H_j). On the opposite side, the forms (5d) and (5e) are more convenient.

For time-dependent correlation of n spins with $n \le N$, Eqs. (3) and (5d) lead to the following equations of motion:

$$\left[n+\tau_0\frac{d}{dt}\right]\langle S_{i_1},S_{i_2},\dots,S_{i_n}\rangle=\sum_{j=1}^{n}\langle S_{i_1},S_{i_2},\dots,V^j,\dots,S_{i_n}\tanh\beta h_j''\rangle$$
$$+h\sum_{j=1}^{n}\left[\langle S_{i_1},S_{i_2},\dots,V^j,\dots,S_{i_n}\rangle-\langle S_{i_1},S_{i_2},\dots,V^j,\dots,S_{i_n}\tanh\beta h_j''\rangle\right], \quad (6)$$

and Eqs. (3) and (5e) provide us with another form for the equations of motion as follows:

$$\left[n+\tau_0\frac{d}{dt}\right]\langle S_{i_1},S_{i_2},\dots,S_{i_n}\rangle=\sum_{j=1}^{n}\langle S_{i_1},S_{i_2},\dots,V^j,\dots,S_{i_n}\tanh\beta h_j'''\rangle, \quad (7)$$

where $h=\tanh(\beta B)$ and V^j denotes that S_j is absent in the correlation under study. The angular brackets denote an average over the density matrix. Putting $n=1$ in Eq. (6), we obtain

$$\left[1+\tau_0\frac{d}{dt}\right]\langle S_i(t)\rangle=\langle\tanh\beta h_i''\rangle+h[1-\langle S_i\tanh\beta h_i''\rangle] \quad (8)$$

and from (7) we have

$$\left[1+\tau_0\frac{d}{dt}\right]\langle S_i(t)\rangle=\langle\tanh\beta h_i'''\rangle. \quad (9)$$

We know that $\tanh\beta h_i''$ is an odd function of h_i'' and can be expressed by an effective polynomial in h_i'', which contains only the odd power of h_i''. For clusters with the coordination number of 4, valid for the geometry of Fig. 1, one gets

$$\tanh\beta h_i''=\gamma_i+\sum_j{}'\gamma_{ij}S_j+\sum_{\substack{j,k\\ j\neq k}}\Gamma_{i,jk}S_jS_k+\sum_{\substack{j,k,l\\ j\neq k\neq l}}{}'\Gamma_{i,jkl}S_jS_kS_l+\sum_{\substack{j,k,l,m\\ j\neq k\neq l\neq m}}{}'\Gamma_{i,jklm}S_jS_kS_lS_m. \quad (10)$$

The summations in Eq. (10) are restricted to nearest neighbors of site i and the coefficients γ_i, γ_{ij}, $\Gamma_{i,jk}$, $\Gamma_{i,jkl}$, and $\Gamma_{i,jklm}$ depend on T and H_i. Then

$$\gamma_i=\tfrac{1}{16}[\tanh\beta(4J+H_i)+4\tanh\beta(2J+H_i)+6\tanh\beta H_i-4\tanh\beta(2J-H_i)-\tanh\beta(4J-H_i)],$$
$$\gamma_{ij}=\tfrac{1}{16}[\tanh\beta(4J+H_i)+2\tanh\beta(2J+H_i)+2\tanh\beta(2J-H_i)+\tanh\beta(4J-H_i)],$$
$$\Gamma_{i,jk}=\tfrac{1}{16}[\tanh\beta(4J+H_i)-2\tanh\beta H_i-\tanh\beta(4J-H_i)], \quad (11)$$
$$\Gamma_{i,jkl}=\tfrac{1}{16}[\tanh\beta(4J+H_i)-2\tanh\beta(2J+H_i)-2\tanh\beta(2J-H_i)+\tanh\beta(4J-H_i)],$$

and

$$\Gamma_{i,jklm}=\tfrac{1}{16}[\tanh\beta(4J+H_i)-4\tanh\beta(2J+H_i)+6\tanh\beta H_i+4\tanh\beta(2J-H_i)-\tanh\beta(4J-H_i)].$$

Typically for $\tanh(\beta h_i'')$ in Eq. (9), it can be expressed in the same way as in (10) but with coefficients γ_i', γ_{ij}', $\Gamma_{i,jk}'$, $\Gamma_{i,jkl}'$, and $\Gamma_{i,jklm}'$.

In the weak external magnetic field, we can neglect all terms of higher order than the first in B. Then these coefficients can be written in terms of γ_i, γ_{ij}, $\Gamma_{i,jk}$, $\Gamma_{i,jkl}$, and $\Gamma_{i,jklm}$ as follows:

$$\gamma_i'=\gamma_i+\frac{h}{16}[\operatorname{sech}^2\beta(4J+H_i)+4\operatorname{sech}^2\beta(2J+H_i)+6\operatorname{sech}^2\beta H_i+4\operatorname{sech}^2\beta(2J-H_i)+\operatorname{sech}^2\beta(4J-H_i)],$$
$$\gamma_{ij}'=\gamma_{ij}+\frac{h}{16}[\operatorname{sech}^2\beta(4J+H_i)+2\operatorname{sech}^2\beta(2J+H_i)-2\operatorname{sech}^2\beta(2J-H_i)-4\operatorname{sech}^2\beta(4J-H_i)],$$
$$\Gamma_{i,jk}'=\Gamma_{i,jk}+\frac{h}{16}[\operatorname{sech}^2\beta(4J+H_i)-2\operatorname{sech}^2\beta H_i+\operatorname{sech}^2\beta(4J-H_i)], \quad (12)$$
$$\Gamma_{i,jkl}'=\Gamma_{i,jkl}+\frac{h}{16}[\operatorname{sech}^2\beta(4J+H_i)-2\operatorname{sech}^2\beta(2J+H_i)+2\operatorname{sech}^2\beta(2J-H_i)-\operatorname{sech}^2\beta(4J-H_i)],$$

and

$$\Gamma_{i,jklm}'=\Gamma_{i,jklm}+\frac{h}{16}[\operatorname{sech}^2\beta(4J+H_i)-4\operatorname{sech}^2\beta(2J+H_i)$$
$$+6\operatorname{sech}^2\beta H_i-4\operatorname{sech}^2\beta(2J-H_i)+\operatorname{sech}^2\beta(4J-H_i)].$$

TABLE II. Values of the energy states E and the energy barriers ΔE for the GD case.

$W_d=0.5$		$W_d=1.0$	
E	ΔE	E	ΔE
-13.118	10.949	-14.235	9.898
-10.882	8.383	-9.765	4.767
-2.837	2.990	-5.675	5.980

In (11) and (12) the symbols j, k, l, and m denote the four neighboring spin of the central site i. If we put $h=0$ we notice that both coefficients in (12) are identical. We shall be concerned here to solve Eq. (9).

In the absence of the external field, the single-spin expectation values depend on one-, two-, three-, and four-spin correlations, the three- and five-spin correlations depend on one-, two-, three-, four-, and five-spin correlations. But for even-spin correlations the two- and four-spin correlations depends on the one-, two-, three-, four-, and five-spin correlations and the six-spin correlations depend on one-, two-, three-, four-, five-, and six-spin correlations.

With the magnetic field operative, the situation is the same; all odd- and even-spin correlations couple to odd- and even-spin correlations as in the absence of the magnetic field, but this time more correlation functions are coupled together and we also have the constant terms in the equation of motion. For N spins there are 2^N-1 different correlations; 2^{N-1} of them are comprised of an odd numbers of spins. In particular, for our six-spin cluster under consideration, there are 32 odd correlations and 31 even correlations.

IV. METHOD OF SOLUTION

In order to solve Eq. (9) we construct a vector $\mathbf{V}(t)$ from all 63 correlation functions. The first six components of $\mathbf{V}(t)$ are $\langle S_1(t)\rangle$ through $\langle S_6(t)\rangle$, the next 20 components are three-spin correlations, and then there are six correlations that contain five spins. The remaining 31 components are 15 two-spin correlations, 15 four-spin correlations, and only one six-spin correlation. With this convention, the equations of motion acquire the compact matrix form

$$\tau_0\frac{d}{dt}[\mathbf{V}(t)]=X\mathbf{V}(t)+\mathbf{Y}+h[x\mathbf{V}(t)+\mathbf{y}] . \quad (13)$$

The matrices X and x and the vectors $\mathbf{Y}$ and $\mathbf{y}$ are built up of blocks represented schematically by

$$X=\begin{bmatrix} A_{32\times 32} & C_{32\times 31} \\ D_{31\times 32} & B_{31\times 31}\end{bmatrix} , \quad (14a)$$

$$x=\begin{bmatrix} a_{32\times 32} & c_{32\times 31} \\ d_{31\times 32} & b_{31\times 31}\end{bmatrix} , \quad (14b)$$

$$\mathbf{Y}=\begin{bmatrix} \mathbf{Y}'_{32} \\ \mathbf{Y}''_{31}\end{bmatrix} , \quad (14c)$$

$$\mathbf{y}=\begin{bmatrix} \mathbf{y}'_{32} \\ \mathbf{y}''_{31}\end{bmatrix} . \quad (14d)$$

The subscripts indicate dimensionalities of the matrices and vectors involved.

We know that the terms $\mathbf{Y}$ and $\mathbf{y}$ in the differential Equation (13) are inhomogeneous parts. The time evolution has no externally driven terms (convection terms) in the static magnetic field so that we can obtain it from the homogeneous part only. The relaxation times τ_v are equal $-1/\lambda_v$, where λ_v are the eigenvalues of the eigenvalue problem

$$\tau_0\frac{d}{dt}[\mathbf{V}(t)]=-Z\mathbf{V}(t) , \quad (15)$$

where

$$Z=-X-hx . \quad (16)$$

It is clear that the 63 eigenvectors form a complete set, and any physical quantity of interest F can be expressed as a linear combination of these eigenvectors. Taking into account the inhomogeneous terms as well, we get

$$F(t)=\sum_{v=1}^{63} f_v(T)e^{-t/\tau_v(T)}+c . \quad (17)$$

This method enables us to study relaxation times in absence and presence of the external magnetic field (h). We point out that the method discussed here will not be applicable to larger-sized systems because of the large dimension of the matrix which has to be diagonalized.

V. RELAXATION TIMES IN ZERO MAGNETIC FIELD

Consider that $h=0$ in Eq. (15). Between the 63 relaxation times, corresponding to the eigenvalues of Z, there are some that diverge when T approaches zero, and the remaining ones are nondivergent.

A. The BD case

For the particular RFI cluster shown in Fig. 1(a), we have two divergent relaxation times τ_1 and τ_2 at sufficiently low temperature for $H_R=0.1$, 0.7, 1.5, and 2.1. Figure 2(a) shows these divergent relaxation times as functions of T. But for $H_R=4.1$ (i.e., greater than $4J$), there are no diverging relaxation times at all. We have found in Sec. II $H_R=0.1$, 0.7, 1.5, and 2.1 clusters that there are three energy minima, while for $H_R=4.1$ there is only one minimum. This means that the number of divergent relaxation times is one less than the corresponding number of energy minima.

The values of the relaxation times t_v depend on the amplitude; H_R of the BD of RF. In Fig. 2(a) we note that the longest relaxation time, τ_1, decreases with increasing the amplitude H_R. In fact, the longest diverging relaxation time can be described by an Arrhenius law:

$$\tau_1=\tau'_0 e^{\varepsilon/k_B T} , \quad (18)$$

where τ'_0 is of the order of τ_0 and ε is essentially independent of T at low temperatures McMillan[16] and Banavar, Cieplak, and Muthukumar[12] have found out that the barriers ε in Eq. (18) should, in general, be of the order of energies ΔE required to invert a system in its energy minimum (in a sequence of single-spin events). Figure 3(a) confirms the validity of Eq. (18) for $H_R=0.1$, 0.7, 1.5, and 2.1. The Arrhenius behavior of τ_1 translates into straight lines on the $\ln(\tau_v/\tau_0)$ versus $1/T$ plot. For $H_R=0.1$, the activation energy ε in (18) is equal to 11.429, for $H_R=0.7$, we got that $\varepsilon=9.091$, while for $H_R=1.5$ it is found to be equal to 5.8. The corresponding ε for $H_R=2.1$ is 5.8. These values for ε are quite close to the characteristic energy scale ΔE for a complete spin reversal of the ground state (compare Table I).

Figure 4(a) shows how the long relaxation times compare to the remaining ones at several values of T. The

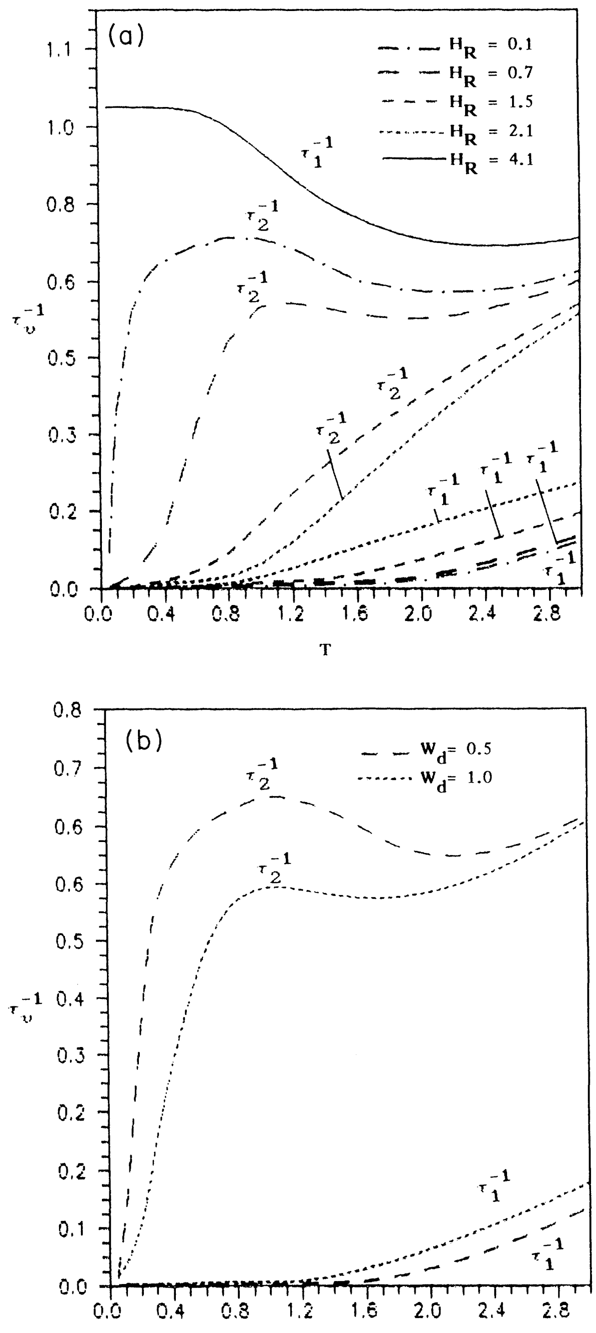


FIG. 2. (a) The inverse of the longest relaxation times for the RFI cluster shown in Fig. 1(a) for different H_R of RF vs T. The magnetic field is set equal to zero. (b) The inverses of the longest relaxation times for RFI cluster shown in Fig. 1(b) for different W_d of RF vs T. The magnetic field is set equal to zero.

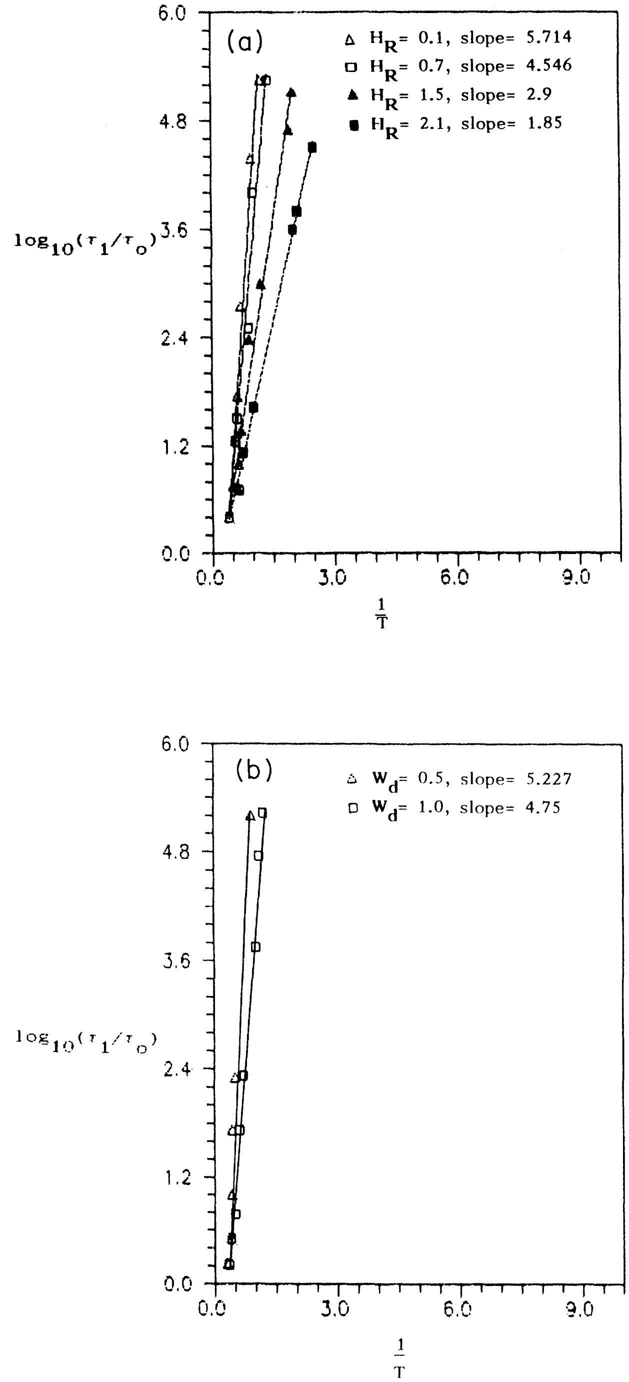


FIG. 3. (a) The Arrhenius plot for τ_1 corresponding to the RFI cluster shown in Fig. 1(a) for different H_R of RF. (b) The Arrhenius plot for τ_1 corresponding to the RFI cluster shown in Fig. 1(b) for different W_d of RF.

spectrum of τ_v's is presented in the logarithmic scale for $H_R = 0.1$, 0.7, and 1.5. As we see, the longest divergent relaxation times τ_1 and τ_2 are very well separated from the rest of the spectrum. At very low temperature, on increasing T, there will be no diverging relaxation time. On the other hand, the nondivergent relaxation times are rather uniformly distributed on the logarithmic scale.

B. The GD case

The divergent relaxation times versus T are shown in Fig. 2(b) for the particular cluster exposed in Fig. 1(b) for $Wd = 0.5$ and 1.0. We have two divergent relaxation times, τ_1 and τ_2, at sufficiently low temperature, while there exist three energy minima in such case as in Table II. Therefore, the relation between the number of divergent relaxation times and the number of energy minima seems to be the same as found in the BD case of RF. We see that the longest diverging relaxation time, τ_1, for $W_d = 0.5$ is longer than that obtained for $W_d = 1.0$.

Figure 3(b) shows the validity of the Arrhenius law (18) for τ_1 for both $W_d = 0.5$ and 1.0. We have found for $W_d = 0.5$ that $\varepsilon = 10.455$, while the corresponding energy barrier ΔE equals 10.949. Also for $W_d = 1.0$, we found that $\varepsilon = 9.898$ and $\Delta E = 9.500$. Therefore, we can say here that the activation energy (ε) is approximately the same as the energy barrier (ΔE) mentioned in Table II. A general conclusion is that only the longest of the diverging relaxation times have a barrier coinciding with the reversal energy of a minimum.

The spectra of relaxation times are plotted at different T on the logarithmic scale for $W_d = 0.5$ and 1.0 as shown in Fig. 4(b). At low temperature, we see that the longest diverging relaxation times are well separated from the nondiverging ones, which are distributed uniformly on the logarithmic scale.

We have studied 400 different six-spin clusters in RF for both cases; BD and GD in the absence of the external magnetic field. We conclude that the number of diver-

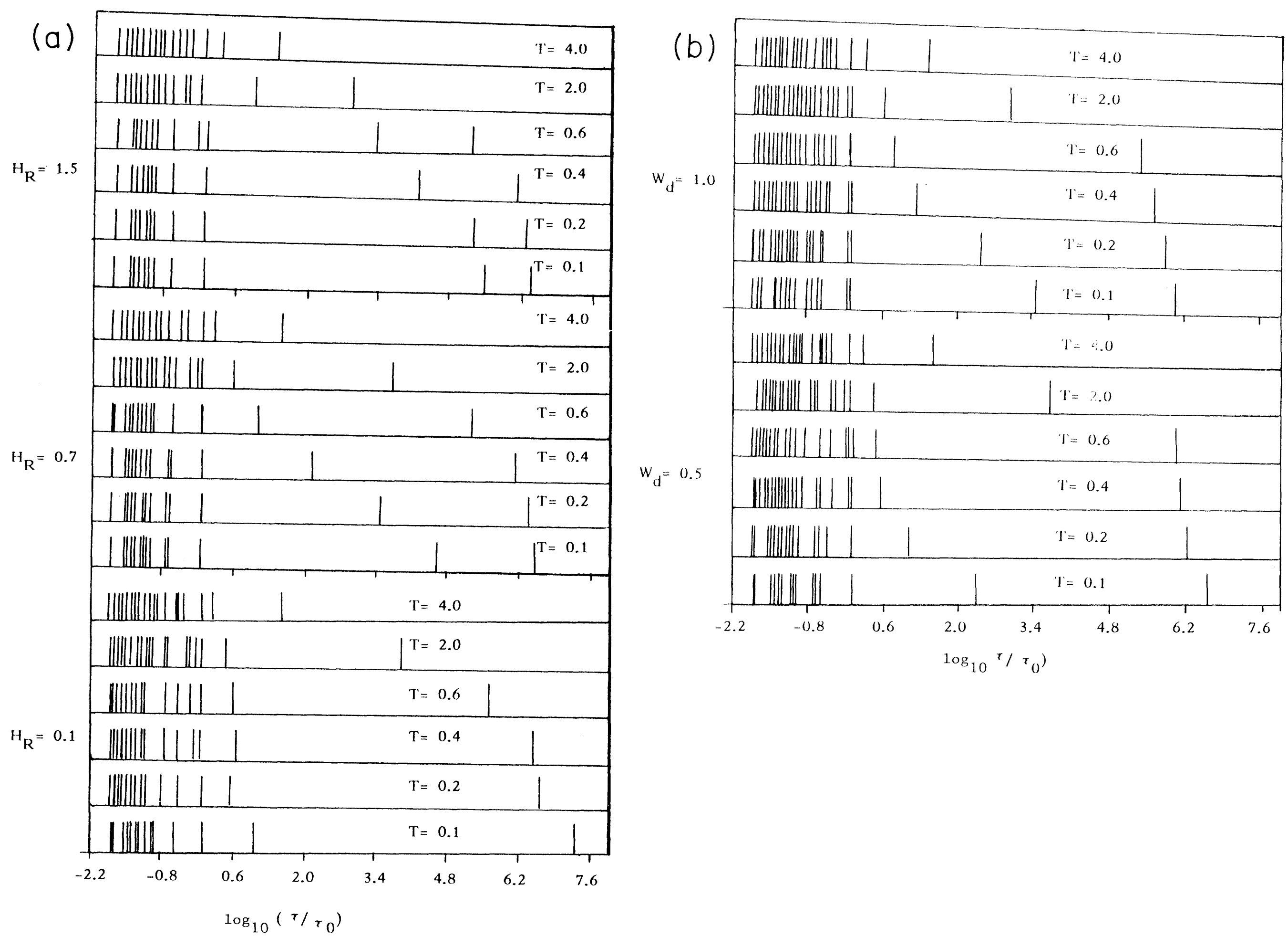


FIG. 4. (a) The distributions of relaxation times of the RFI cluster shown in Fig. 1(a) for different H_R of RF on the logarithmic scale. The value of H_R is declared on the left-hand side. The spectra are shown for six values of T in the absence of any magnetic field. (b) The distributions of the relaxation times of the RFI cluster shown in Fig. 1(b) for different W_d of RF on the logarithmic scale. The value of W_d is declared on the left-hand side. The spectra are shown for six values of T in the absence of any magnetic field.

gent relaxation times is equal to the number of energy minima minus one.

VI. RELAXATION TIMES IN A STATIC MAGNETIC FIELD

The relaxation times of particular RFI clusters shown in Figs. 1(a) and 1(b) are investigated as functions of the static magnetic field (H) at $T=0.3$.

A. The BD case

The longest relaxation times τ_1 are seen to be fairly insensitive to the field up to $H \simeq 0.3$ for $H_R=0.1$ and 0.7 as shown in Fig. 5(a). For $H_R=1.5$ and 2.1, τ_1 is sensitive to H. An increase in H beyond 0.3 causes an initial decrease in τ_1. It is clear that the number of divergent relaxation times is two as long as $H \leq 4$ for $H_R=0.1$, 1.7, 1.5, and 2.1. For $H_R=0.1$, we get three energy minima as long as $H_R \leq 0.1$, but for $0.1 < H < 4$ we have only two minima, and finally one minimum for $H \geq 4$. As we see, the number of minima decreases with increasing H. Also we discover the failure of the relation between the number of diverging relaxation times and the number of energy minima.

B. The GD case

Figure 5(b) shows that the parts of the spectra including nondivergent relaxation times are not affected by the external magnetic field. The divergent relaxation times do depend on H and, in particular, τ_1. It is clear that the longest diverging relaxation time decreases slightly with increasing H. The number of divergent relaxation times is still the same whenever $H \leq 4$ for both $W_d=0.5$ and 1.0. On the other hand, the number of energy minima is affected by the static field. For $W_d=0.5$, we have three energy minima as long as $H \leq 0.1$. For $0.1 < H < 3.5$ we have two minima. But for $H > 3.5$ we obtain only one minimum. For the same cluster, but with $W_d=1.0$, we have three minima for $H \leq 0.2$. For $0.2 < H < 3.0$ we have found two minima reduced to only one for $H > 3.0$.

Generally, in the presence of a static field we conclude that the number of divergent relaxation times is the same as in the absence of the field, whereas the number of energy minima decreases with increasing H. The equality of the number of divergent relaxation times and the number of energy minima minus one is no longer valid in the presence of a static magnetic field. Similar conclusions were reached by Cieplak and Lusakowski[17] for Ising spin glasses, Cieplak, Cieplak, and Łusakowski[18] for spin glassy semimagnetic semiconductors, and by Ismail[19] for 1D random-field Ising systems.

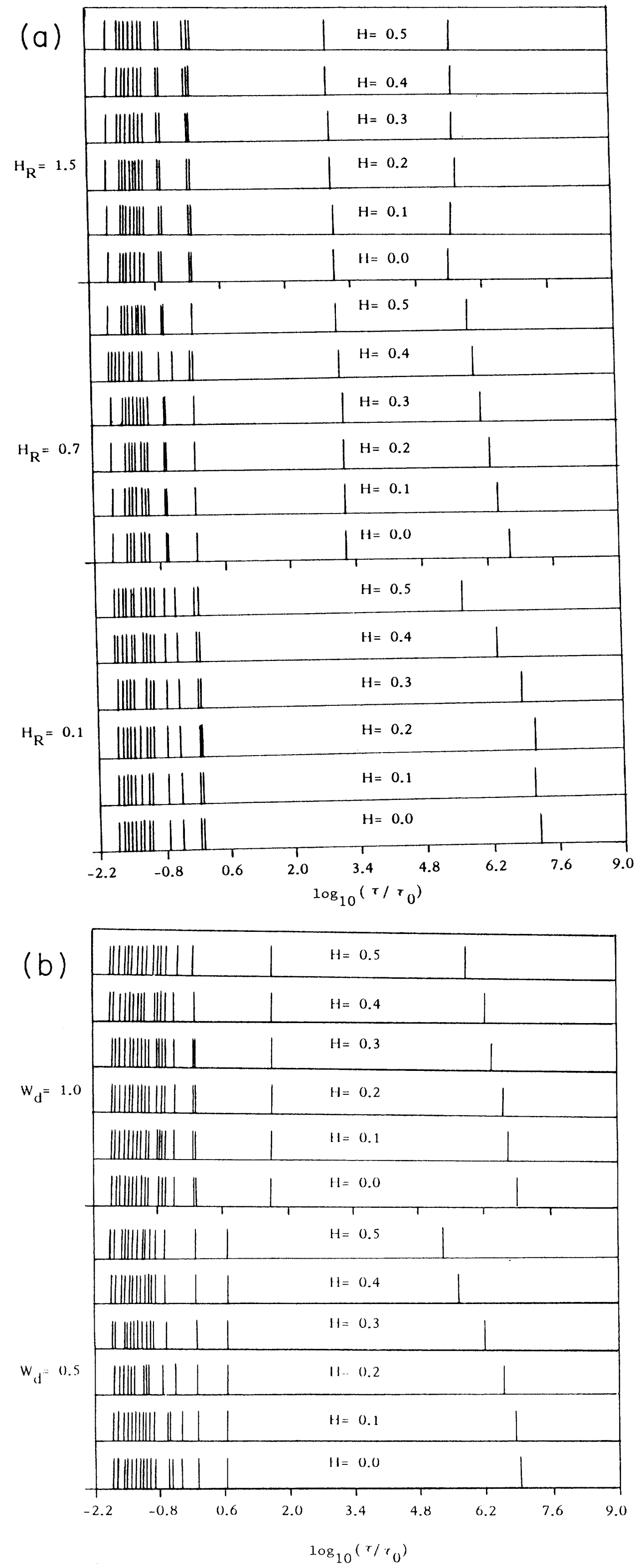


FIG. 5. (a) The same as in Fig. 4(a) but in the presence of a static magnetic H. The spectra are shown at $T=0.3$ for different H. (b) The same as in Fig. 4(b) but in the presence of a static magnetic field H. The spectra are shown at $T=0.3$ for different H.

VII. SPECTRA OF RELAXATION TIMES AT LOW TEMPERATURE

We have calculated the averaged spectra of relaxation times for RFI clusters of six spins in the absence of the external field. These distributions are obtained at $T=0.2$

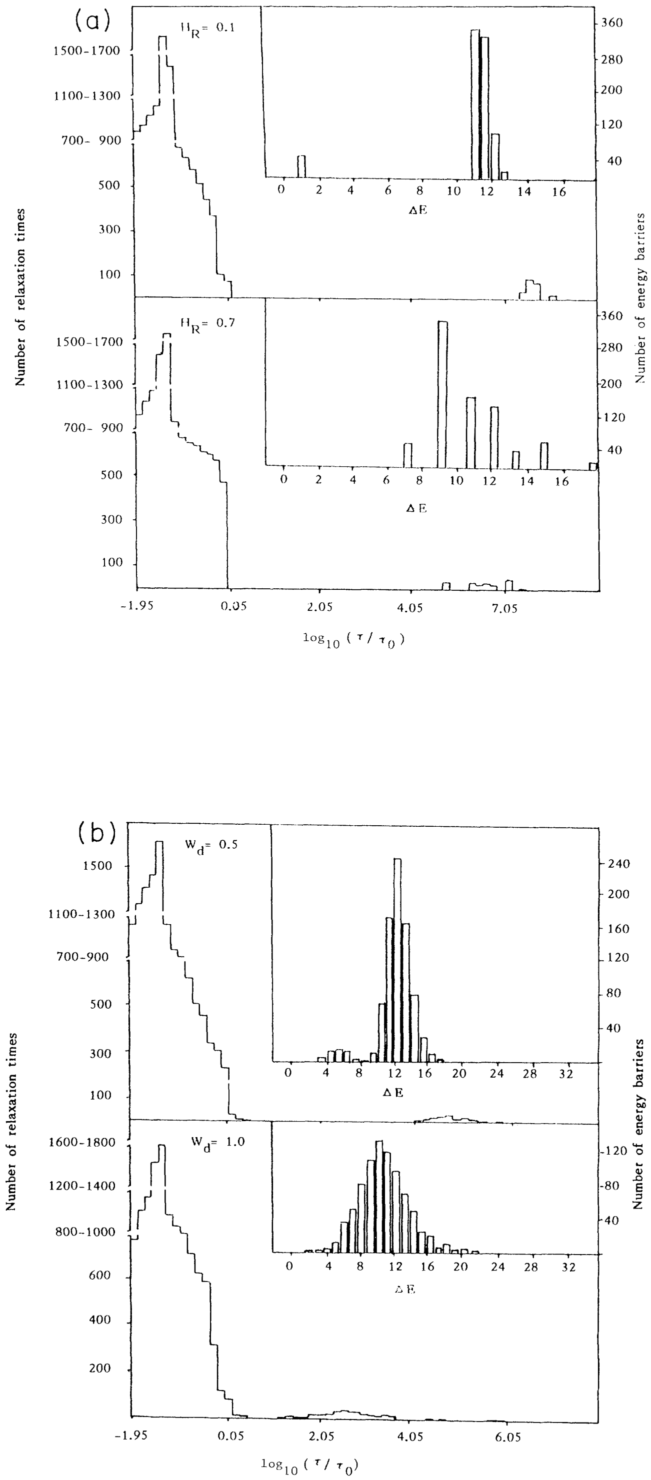


FIG. 6. (a) The main part of the figure as in Fig. 4(a), but for clusters of six spins and for $k_B T = 0.2$. The inset shows the distributions for the energies required to invert energy minima in the BD case of RF for $H_R = 0.1$ and 0.7. (b) The main part of the figure as in Fig. 4(b), but for clusters of six spins and for $k_B T = 0.2$. The inset shows the distributions of the energies required to invert energy minima in the GD case of RF for $W_d = 0.5$ and 1.0.

using Eq. (15) and averaged over 400 samples for $H_R = 0.1$ and 0.7 in the BD case and for $W_d = 0.5$ and 1.0 in the GD case.

A. The BD case

Figure 6(a) shows the spectra of relaxation times for $H_R = 0.1$ and 0.7. The distribution of the logarithms of these times is not uniform at low temperature. As we see, it consists of two separated parts; the high peak that corresponds to the rapid process of the relaxation with time in the order of τ_0 and the small peak is related to the slower process. The short relaxation times are most ubiquitous, while the longest ones are the rarest. The insets show the corresponding distributions of energies required to invert each energy minimum. The two peaks in the $H_R = 0.1$ spectrum reflect the existence of the two different regions in the corresponding energy-barrier distribution. The distribution of ΔE's is quasidiscrete. We found that the various relaxation mechanisms are well separated only at very low temperature. The separation between the regions of the relaxation times shrinks with increasing T. We also see that increasing the amplitude (H_R) of RF, e.g., $H_R = 0.7$ as shown in Fig. 6(a) leads to break the discreteness in the distribution of relaxation times. The corresponding ΔE's distribution is quasicontinuous. A further increase in H_R would make the distribution of relaxation times more and more continuous, consisting of a long tail for rapid times and highly visible peak for short times.

B. The GD case

We have obtained continuous spectra of relaxation times as shown in Fig. 6(b). Each spectrum contains one peak for the rapid process with times of order of τ_0 and a smooth tail corresponding to the slower processes. The corresponding distribution of ΔE's is uniform. A log-uniform distribution of τ_υ should then infer an essentially uniform distribution of ΔE's. As it is apparent in the case of spin glassy semimagnetic semiconductor,[18] the distribution of ΔE's are continuous. These results are in complete agreement with the ones obtained by using Gaussian local random field.

The next step in extending our analysis is to calculate the real and imaginary parts of dynamic susceptibilities to see the connection between the frequency dependence of susceptibility and the spectral analysis of the relaxation process.

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[1]G. Aeppli and R. Bruinsma, Phys. Lett. A **97**, 117 (1983).
[2]H. Yoshizawa, R. A. Cowly, G. Shirane, R. J. Birgeneau, H. J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. **48**, 438 (1982).
[3]D. P. Belanger, A. R. King, and V. Jaccarino, Phys. Rev. Lett. **48**, 1050 (1982).
[4]D. Andelman, Phys. Rev. B **34**, 6214 (1986).
[5]I. Vilfan and J. Stegan, Solid State Commun. **54**, 795 (1985).
[6]E. T. Gawlinski, K. Kaski, M. Grant, and J. D. Guntor, Phys. Rev. Lett. **53**, 2266 (1984).
[7]M. Grant and J. D. Guntor, Phys. Rev. B **29**, 6266 (1984).
[8]G. Forgacs, D. Mukamel, and R. A. Pelcovits, Phys. Rev. B **30**, 205 (1984).
[9]E. Pytte and J. F. Fernandez, Phys. Rev. B **31**, 616 (1985).
[10]T. Uezu and K. Kawasaki, J. Phys. Soc. Jpn. **56**, 918 (1987).
[11]K. Kawasaki and T. Uezu, J. Phys. Soc. Jpn. **57**, 3532 (1988).
[12]J. R. Banavar, M. Cieplak, and M. Muthukumar, J. Phys. C **18**, L157 (1985).
[13]G. Ismail and A. A. Salem (unpublished).
[14]R. J. Glauber, J. Math. Phys. **4**, 294 (1963).
[15]M. Cieplak and G. Ismail, Acta Phys. Pol. A **76**, 513 (1989).
[16]W. L. McMillan, Phys. Rev. B **28**, 5216 (1983).
[17]M. Cieplak and J. Lusakowski, J. Phys. C **19**, 5253 (1986).
[18]M. Cieplak, M. Z. Cieplak, and J. Lusakowski, Phys. Rev. B **36**, 620 (1987).
[19]G. Ismail, Ph.D. thesis, Warsaw University, 1989.