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Lifshitz points in uniaxial ferroelectrics

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The critical behavior of Lifshitz points in systems with short range and uniaxial dipolar interaction is calculated. These multicritical points belong to a new universality class and may be relevant in the phase diagram of $Sn_2P_2(Se_xS_{1-x})_6$. A variety of combinations of spatially anisotropic critical behavior is possible. The critical effects in the elastic constants are briefly mentioned.

I. INTRODUCTION

Recently the behavior of the proper uniaxial ferroelectric $Sn_2P_2(Se_xS_{1-x})_6$ for various concentrations x has been extensively studied.[1,2] The phase diagram contains a line of second-order phase transitions from para- to the ferroelectric phase for $x<0.28$ and a Lifshitz point[3] at $x=0.28$. For $x>0.28$ a transition into an incommensurate phase is observed. On approaching the Lifshitz point the behavior, namely, an increase of the specific-heat anomaly and a decrease of the critical exponent for the order parameter were discussed by considering the contributions of defects to the anomalies in the physical quantities.[4] We want to propose another explanation,[5] which does not involve defects but explains the specific behavior as crossover from the classical behavior with logarithmic corrections for the usual ferroelectric transition to a nonclassical behavior at the Lifshitz point. This Lifshitz point, however, is of a new type, possible in systems with uniaxial dipolar interactions.[6,7] In ferroelectrics with strong uniaxial dipolar interaction fluctuations are strongly suppressed; on the other hand, at the Lifshitz point the fluctuation effects are enhanced. The interplay of these two competing effects determines the critical behavior of a specific spatially anisotropic system.

II. PLANAR LIFSHITZ POINT

The usual transition to the ferroelectric state in a uniaxial ferroelectric is described by a Ginzburg-Landau-Wilson Hamiltonian[8]

$$\mathcal{H}=\int d^3k\frac{1}{2}\left[r_0+c_0\mathbf{p}^2+d_0\mathbf{p}^4+g_0^2\frac{q^2}{\mathbf{p}^2}\right]P_{0k}P_{0-k}+u_0\int d^3k_1d^3k_2d^3k_3P_{0k_1}P_{0k_2}P_{0k_3}P_{0-k_1,-k_2,-k_3}\ . \quad (1)$$

We have decomposed the $\mathbf{k}$ vector into the two-dimensional component $\mathbf{p}$ and the one-dimensional component q ($\equiv k_z$) and irrelevant terms have already been neglected. The parameters r_0, c_0, g_0^2 (we assume this coefficient to be always positive[9]) and u_0 may depend on temperature and/or other physical parameters. For the physical system we have in mind it is the concentration of Se on which these parameters depend. An instability appears if one of these coefficients or their renormalized counterparts become negative. At the ferroelectric critical point only r_0 goes to zero with temperature distance from T_c; all other coefficients stay finite. In this case the $d_0\mathbf{p}^4$ term is irrelevant; however, at the multicritical point, where both, r_0 and c_0 go to zero, this term becomes relevant. In the region where $c_0<0$ and $d_0>0$ a second-order phase transition into an incommensurate phase characterized by a nonzero wave vector $\mathbf{k}_0=(\mathbf{p}_0,0)$ takes place. The ferroelectric phase and the incommensurable phase are separated by a line of first-order phase transitions. All three lines meet at the Lifshitz point (for a review, see Ref. 10).

In order to study the critical behavior at this "dipolar" Lifshitz point one may apply the standard renormalization procedure. For the generalization to d dimensions we fix m the number of components of $\mathbf{p}$ to two and let $\mathbf{q}$ become $(d-2)$ dimensional. By simply scaling the wave vectors $\mathbf{p}$ and $\mathbf{q}$ differently, fixing the coefficients of the $\mathbf{p}^4$ term and the $\mathbf{q}^2/\mathbf{p}^2$ term and then requiring the fourth-order coupling u to be marginal, the upper critical dimension turns out to be $d_c=4$. This is in contrast to the pure ferroelectric transition where we have $d_c=3$ (Ref. 8) and to the $m=2$ type Lifshitz point with short-range interaction only where $d_c=5$.[3] Thus at $d=3$ we have nonclassical critical behavior.

The following renormalizations are needed to compensate the poles in the loop expansion of the vertex functions (we use the same field theoretic renormalization scheme, dimensional regularization, and minimal subtraction, as in Ref. 11 apart from making the dipolar coupling dimensionless) $P_k=Z_P^{-1/2}P_{0k}$, $r=Z_r^{-1}r_0$,

$$u=\kappa^{d-8}Z_u^{-1}Z_P^2A_du_0\ ,$$

and $g=\kappa^{-2}Z_P^{-1/2}g_0$ with κ an arbitrary reference wave number and A_d an appropriate dimension dependent factor (see the Appendix). There are no pole terms in the $\mathbf{q}^2/\mathbf{p}^2$ part of the inverse propagator and therefore there is no new independent Z factor in g. This leads to the following renormalization-group equations for the renormalized vertex functions, representing the specific heat, susceptibility, and order parameter (we have set $d=3$):

$$\left\{\kappa\frac{\partial}{\partial\kappa}+\beta_u\frac{\partial}{\partial u}-\left[\frac{1}{2}\zeta_P+2\right]g\frac{\partial}{\partial g}+\begin{bmatrix}0\\1\\-\frac{1}{2}\end{bmatrix}\zeta_P+\zeta_r\left[\begin{bmatrix}2\\0\\0\end{bmatrix}+r\frac{\partial}{\partial r}\right]\right\}\begin{bmatrix}C\\\chi^{-1}\\m\end{bmatrix}(r,g,u,\kappa)=\begin{bmatrix}4B(g,u)\kappa^5\\0\\0\end{bmatrix}. \quad (2)$$

As usual we have defined $\zeta_i=\kappa(\partial\ln Z_i^{-1}/\partial\kappa)$, $i=P$, r and $\beta_u=\kappa(\partial u/\partial\kappa)$. $B(g,u)$ is related to the additive renormalization of the specific heat. Perturbation theory suggests the change from the coupling u to the effective coupling $\tilde{u}=u/g$. The one-loop order result for the corresponding ζ and β functions reads

$$\zeta_r=12\tilde{u}, \quad \zeta_P=0, \quad (3)$$

$$\beta_{\tilde{u}}=-3\epsilon\tilde{u}+\tfrac{1}{4}\tilde{u}^2. \quad (4)$$

With this variable then the solutions of the renormalization-group equation in $d=3$ is found as

$$\chi(t)_{\pm}^{-1}=(\kappa l)^4\exp\left[\int_1^l\zeta_P[\tilde{u}(z)]\frac{dz}{z}\right]\hat{\chi}_{\pm}^{-1}[\tilde{u}(l)], \quad (5)$$

$$C_{\pm}(t)=(g\kappa^2)^{-1}(\kappa l)^{-3}\exp\left[\int_1^l\left[\frac{1}{2}\zeta_P[\tilde{u}(z)]+2\zeta_r[\tilde{u}(z)]\right]\frac{dz}{z}\right]$$
$$\times\left\{\hat{C}_{\pm}[\tilde{u}(l)]-\int_1^l\frac{dx}{x}4B[\tilde{u}(x)]\exp\left[\int_l^x\frac{dz}{z}\left[\frac{1}{2}\zeta_P[\tilde{u}(z)]+2\zeta_r[\tilde{u}(z)]-3\right]\right]\right\}, \quad (6)$$

$$m(t)=(g\kappa^2)^{-1/2}(\kappa l)^{1/2}\exp\left[-\frac{1}{4}\int_1^l\zeta_P[\tilde{u}(z)]\frac{dz}{z}\right]\hat{m}[\tilde{u}(l)]. \quad (7)$$

$r(x),\tilde{u}(x)$ are the solutions of the corresponding flow equations. The initial condition for $r(0)\sim t$ introduces the relative temperature distance $t=|T-T_c|/T_c$ into the solutions via the flow parameter l, which is determined by the condition $r(l)/\kappa^4l^4=1$. From the asymptotic behavior ($t\rightarrow 0$) of the solutions we then find the critical exponents in one-loop order ($\epsilon=4-d$)

$$\alpha=\epsilon/4, \quad (8)$$

$$\beta=\tfrac{1}{2}-\epsilon/4, \quad (9)$$

$$\gamma=1+\epsilon/4. \quad (10)$$

The anisotropy of the harmonic part of the interaction in **k** space leads to a static **k**-dependent susceptibility which is characterized by two correlation lengths diverging differently. These can be extracted experimentally from the half height contour in **k** space, where $\chi(t,\mathbf{k})=\frac{1}{2}\chi(t,0)$ (with the appropriate limits for $\mathbf{k}\rightarrow 0$). The form of this contour is shown in Fig. 1(b) in comparison to the uniaxial dipolar case [Fig. 1(a)]. The ratio of the dimensions of the disk parallel and perpendicular to the q-axis has increased for the Lifshitz point. The exponents for the longitudinal correlation length $\xi_{\parallel}$ (its inverse is proportional to the thickness of the disk) and the transverse correlation length $\xi_{\perp}$ (its inverse is proportional to the diameter of the disk) are found to be

$$\nu_{\parallel}=\tfrac{3}{4}(1+\epsilon/4), \quad (11)$$

$$\nu_{\perp}=\tfrac{1}{4}(1+\epsilon/4). \quad (12)$$

At T_c the decay of the correlation is algebraic and determined by the exponent η, defined by

$$\chi(q=0,p)\sim p^{-4+\eta}. \quad (13)$$

Since there are no divergent contributions of the form q/p to the two-point vertex function, no second exponent η exists in the dipolar system, even at the Lifshitz point. This is in contrast to the usual Lifshitz point where a new independent exponent in statics appears. In one-loop order, however, all those exponents η are zero.

Since the number of independent exponents depends on the renormalization factors, we have in our case two independent exponents. From the asymptotic solution of the renormalization-group equation for the suitable vertex functions we find the scaling laws

$$\nu_{\parallel}=\nu_{\perp}(3-\eta/2), \quad (14)$$

$$\gamma=\nu_{\perp}(4-\eta), \quad (15)$$

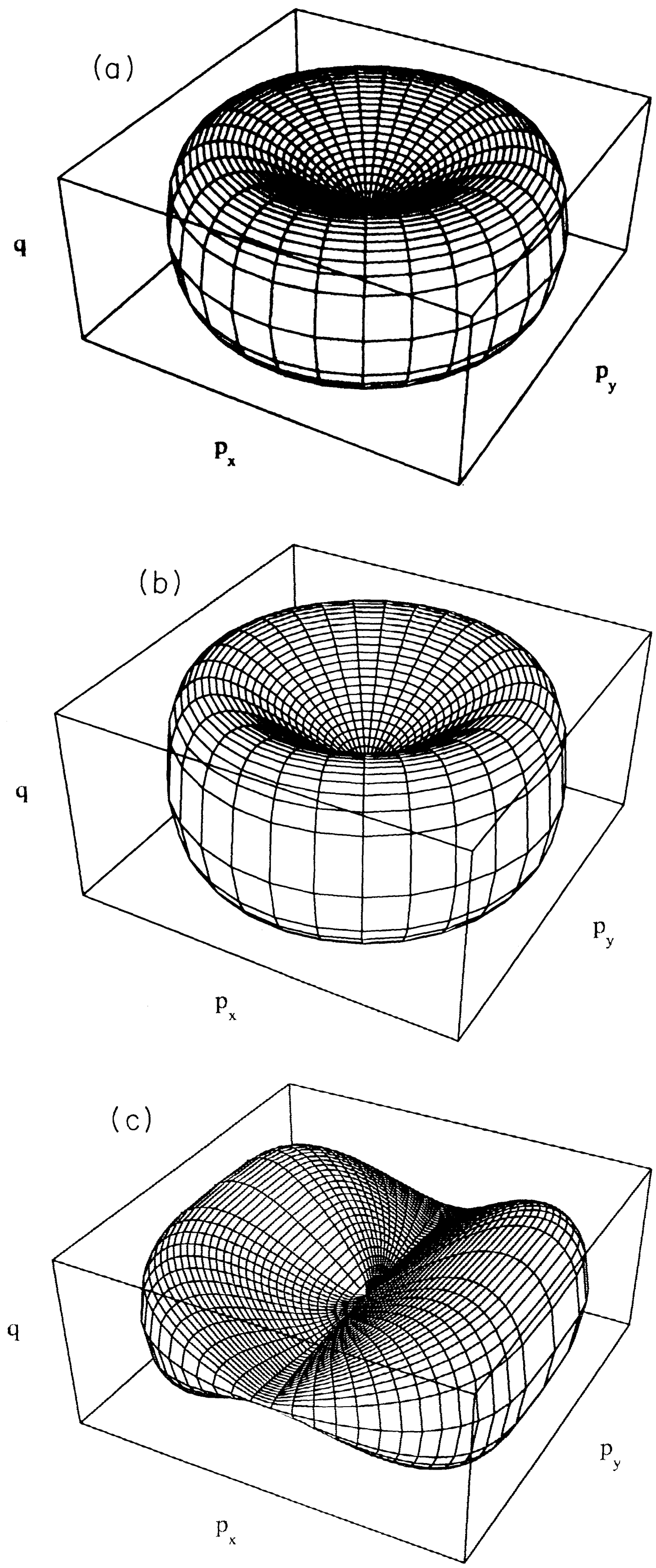

FIG. 1. Regions of critical fluctuations in reciprocal space as given by the half contours of the order parameter susceptibility at the phase transitions in different models: (a) uniaxial ferroelectrics with dipolar coupling, (b) same at the $m=2$ Lifshitz point, and (c) at the $m=1$ Lifshitz point.

$$2-\alpha=2\nu_\perp+(d-2)\nu_\parallel \,. \tag{16}$$

These scaling relations are consistent up to $\mathcal{O}(\epsilon)$ with the results of Eqs. (8)–(12). Note that in the scaling relation Eq. (16) d must be replaced by $4-\epsilon$ if one calculates, e.g., α from the other exponents to some order in ϵ and expands in that order consistently. The main difference to the usual Lifshitz point with $m=2$ in the notation of Ref. 5 can be traced back to the different divergence of the two correlation lengths, even in zeroth loop order. A comparison of the new values for the exponents with other cases is compiled in Table I. It indicates the weakening of the pure Lifshitz point behavior by the uniaxial dipolar forces.

We also have calculated the amplitude functions for the susceptibility and the specific heat above and below T_c. They can be obtained from the renormalized nonasymptotic expressions for the respective correlation functions. One needs the scaling functions of the thermodynamic response functions. These are the inverse two-point vertex function, the $\frac{1}{2}P^2\frac{1}{2}P^2$-correlation function and the equation of state, they give in one-loop order:

$$\hat{\chi}_+^{-1}=1, \quad \hat{C}_+=-\tfrac{3}{8}, \quad B=\tfrac{3}{2} \,, \tag{17}$$

$$\hat{\chi}_-^{-1}=1+27\tilde{u}(t), \quad \hat{C}_-=\frac{1}{8\tilde{u}(t)}-\frac{3}{2} \,, \tag{18}$$

$$\hat{m}^2=\frac{1}{8}\tilde{u}(t) \,, \tag{19}$$

where subscript $+$ or $-$ represents the functions above or below the critical temperature. The amplitude ratios of the inverse susceptibility and the specific heat also can be determined from the solutions of the renormalization-group equations ($\tilde{u}^*=\epsilon/12$)

TABLE I. Compilation of the critical dimension d_c and exponents for systems with one component order parameter and with and without uniaxial dipolar forces in first order of ϵ at $d=3$. The asterisk indicates that one has logarithmic corrections to the power laws. (U: uniaxial dipolar; L: Lifshitz; T: tricritical; and m: dimension of Lifshitz subspace.)

System	d_c	α	β	γ	Ref.
U	3	0^*	$\frac{1}{2}^*$	1^*	8
T	3	$\frac{1}{2}^*$	$\frac{1}{4}^*$	1^*	13
UT	$2\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	1	
$L,\ m=1$	$4\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$1\frac{1}{4}$	3
$L,\ m=2$	5	$\frac{1}{3}$	$\frac{1}{6}$	$1\frac{1}{3}$	3
$LT,\ m=1$	$3\frac{1}{2}$	$\frac{105}{164}$	$\frac{1}{7}$	$1\frac{1}{28}$	12
$LT,\ m=2$	4	$\frac{31}{40}$	$\frac{3}{80}$	$1\frac{3}{40}$	12
$UL,\ m=1$	$3\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$1\frac{1}{6}$	
$UL,\ m=2$	4	$\frac{1}{4}$	$\frac{1}{4}$	$1\frac{1}{4}$	
$ULT,\ m=1$	3	$\frac{1}{2}^*$	$\frac{1}{4}^*$	1^*	15
$ULT,\ m=2$	$3\frac{1}{3}$				15

$$\frac{\chi^+}{\chi^-}=2^\gamma\left[1+\frac{9\epsilon}{4}\right], \quad (20)$$

$$\frac{C^+}{C^-}=2^\alpha\frac{1}{2}+\mathcal{O}(\epsilon) . \quad (21)$$

Ferroelectrics usually have a strong coupling to the elastic degrees of freedom. For the dipolar Lifshitz point with $m=2$ the induced critical behavior in the elastic constants has been considered in Ref. 7. The result was that only for hexagonal systems and the uniaxial direction in the direction of the C_6 axis, the phase transition stays second order. The critical fluctuations couple to the elastic displacements in the planar space perpendicular to the C_6 axis. Then the elastic constant c_{11} goes to zero as $c_{11}\sim t^{1/4}$ and c_{12} is finite with a subleading contribution proportional to c_{11}. Without dipolar interaction the exponent changes and the elastic constant c_{11} goes to zero as $c_{11}\sim t^{1/3}$. The temperature behavior of all other elastic constants remains classical.

III. AXIAL LIFSHITZ POINT

However, the system $Sn_2P_2(Se_xS_{1-x})_6$ is monoclinic and the modulation of the incommensurate phase is in one direction only. Therefore, we generalize the model Hamiltonian Eq. (1) to include anisotropic behavior in the $m=$two-dimensional subspace also (the anisotropy-axes are taken to be perpendicular to each other). Then the Ginzburg-Landau-Wilson Hamiltonian reads (again we write the asymptotic form, so that only the p_y^2 term survives in the denominator of the dipolar term)

$$\mathcal{H}=\int d^3k\frac{1}{2}\left[r_0+c_0p_x^2+d_0p_y^4+g_0^2\frac{q^2}{p_y^2}\right]P_{0k}P_{0-k}$$
$$+u_0\int d^3k_1d^3k_2d^3k_3P_{0k_1}P_{0k_2}P_{0k_3}P_{0-k_1,-k_2,-k_3} . \quad (22)$$

Fixing the dimension of p_x and p_y to one and that of $\mathbf{q}$ to $(d-2)$ in the generalization to higher dimensions, we find the critical dimension $d_c=3\frac{2}{3}$. We have now allowed for different scaling of p_x, p_y, and $\mathbf{q}$. Note that r again scales as p_y^4 (apart from u^2 contributions) as in the former case. Because of the change in the uniaxial dipolar term in the denominator the half contour of the susceptibility is now drastically changed to the former case, where we had a disk squeezed in the origin parallel to the uniaxial axis. Now we have a squeezing along the whole p_y axis [see Fig. 1(c)].

The renormalization proceeds as in the case before apart from the new independent renormalization of the coefficient of the p_x^2 term $c=\kappa^{-2}Z_c^{-1}Z_Pc_0$

$$\left\{\kappa\frac{\partial}{\partial\kappa}+\beta_u\frac{\partial}{\partial u}-\left[\frac{1}{2}\zeta_P+2\right]g\frac{\partial}{\partial g}+\begin{bmatrix}0\\1\\-\frac{1}{2}\end{bmatrix}\zeta_P+\zeta_r\left[\begin{bmatrix}2\\0\\0\end{bmatrix}+r\frac{\partial}{\partial r}\right]-(2+\zeta_P-\zeta_c)c\frac{\partial}{\partial c}\right\}\begin{bmatrix}C\\\chi^{-1}\\m\end{bmatrix}(r,c,g,u,\kappa)=\begin{bmatrix}4B(c,g,u)\kappa^5\\0\\0\end{bmatrix} . \quad (23)$$

The vertex functions exhibit the same poles if one uses the appropriate definition of the geometric factor A_d (see the Appendix). The solutions of the renormalization-group equations at $d=3$ read

$$\chi(t)_\pm^{-1}=(\kappa l)^4\exp\left[\int_1^l\zeta_P[\tilde{u}(z)]\frac{dz}{z}\right]\hat{\chi}_\pm^{-1}[\tilde{u}(l)] , \quad (24)$$

$$C_\pm(t)=(g\kappa^2)^{-1}(c\kappa^2)^{-1/2}(\kappa l)^{-2}\exp\left[\int_1^l\{\zeta_P[\tilde{u}(z)]+2\zeta_r[\tilde{u}(z)]-\tfrac{1}{2}\zeta_c[\tilde{u}(z)]\}\frac{dz}{z}\right]$$
$$\times\left[\hat{C}_\pm[\tilde{u}(l)]-\int_1^l\frac{dx}{x}4B[\tilde{u}(x)]\exp\left[\int_l^x\frac{dz}{z}\{\zeta_P[\tilde{u}(z)]-\tfrac{1}{2}\zeta_c[\tilde{u}(z)]+2\zeta_r[\tilde{u}(z)]-2\}\right]\right] , \quad (25)$$

$$m(t)=(g\kappa^2)^{-1/2}(c\kappa^2)^{-1/4}(\kappa l)\exp\left[-\frac{1}{4}\int_1^l\zeta_c[\tilde{u}(z)]\frac{dz}{z}\right]\hat{m}[\tilde{u}(l)] , \quad (26)$$

where a new effective fourth-order coupling $\tilde{u}=u/g^{1/2}c^{1/4}$ has been introduced. With respect to the new upper critical dimension ($\epsilon=3\frac{2}{3}-d$) we find the exponents given by the same equations as before, Eqs. (8)–(10). For the values at $d=3$ see Table I. As one would have expected for the $m=1$ case and the form of the half contour of the susceptibility the exponents as $d=3$ are more mean field like than in the $m=2$ case.

We now have three correlation lengths ξ_x, ξ_y, and $\xi_\parallel$ diverging differently. At T_c the correlations decay as a power law for $q=0$ with two different exponents η_x and η_y:

$$\chi(q=0,p_x,p_y=0)\sim p_x^{-2+\eta_x} , \tag{27}$$

$$\chi(q=0,p_x=0,p_y)\sim p_y^{-4+\eta_y} . \tag{28}$$

The scaling laws read accordingly

$$\nu_x=\nu_y\frac{4-\eta_y}{2-\eta_x} , \tag{29}$$

$$\nu_\parallel=\nu_y\left[3-\frac{\eta_y}{2}\right] , \tag{30}$$

$$\gamma=\nu_x(2-\eta_x)=\nu_y(4-\eta_y) , \tag{31}$$

$$2-\alpha=\nu_x+\nu_y+(d-2)\nu_\parallel . \tag{32}$$

The amplitude functions have the same form as Eqs. (17)–(19) and the ratios are given with the corresponding ϵ.

The critical behavior of the elastic constants can be found by generalizing the considerations of Ref. 7 to $m=1$. We now must assume orthorombic symmetry for the elastic system since we have three unequivalent directions. As one may expect the relevant coupling of the fluctuations is to the displacements in direction of the Lifshitz axis. Therefore the elastic constant $c_{22}\sim t^{1/6}$ shows nonclassical critical behavior. Without dipolar forces hexagonal symmetry is restored and the exponent of the elastic constant is increased to $c_{22}\sim t^{1/4}$. More details will be given elsewhere.

In comparing the measurements with the results of the models presented so far, one has to observe in the phase diagram of $Sn_2P_2(Se_xS_{1-x})_6$ the possible nearby location of a tricritical point. At tricriticality of the fourth-order term in the Hamiltonian is absent and a sixth-order term in the order parameter P has to be included. Tricriticality reduces the upper critical dimension, enhances the exponent for the specific heat and reduces that for the order parameter. From the condition that the sixth-order term is marginal we find the upper critical dimension $d_c=3$ for $m=1$ and $d_c=3\frac{1}{3}$ for $m=2$. So we expect exponents near the tricritical mean-field values, where for $m=1$ logarithmic corrections and for $m=2$ power laws with exponents very near to the mean-field exponents, to be predicted. We want to note, however, that the logarithmic corrections in all three cases (U, T, ULT $m=1$) appearing in Table I are different.

IV. CONCLUSION

We have calculated the critical exponents at Lifshitz points in systems with uniaxial dipolar interaction. The results show that the usual rule of thumb, namely, a shift of the upper critical dimension d_c without uniaxial dipolar forces to a lower value by one $d_c^{\text{dip}}=d_c-1$ for the case with uniaxial dipolar force, holds for $m=2$ but not for $m=1$. However, if the specific shift is known, the critical exponents in one-loop order can be found by just inserting the corresponding value of ϵ. This is due to the unchanged dependence of the exponents on ϵ in one loop order. In the case of tricritical behavior such a shift of the critical dimension becomes useless since the functional form of the critical exponents on ϵ is changed.

In order to elucidate the influence of the uniaxial dipolar interaction on the critical behavior, a measurement of critical exponents alone seems to be less conclusive because of small differences in these exponents with and without uniaxial dipolar forces. The best way would be to look for zero-loop effects of the spatial anisotropy. This could be done by a measurement of the half contour of the susceptibility in a neutron-scattering experiment similar to the magnetic case of $LiTbF_4$.[14]

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APPENDIX: INTEGRALS AND POLES

For the general case we separate the wave-vector space into the components $\mathbf{k}=(\mathbf{p}_x,\mathbf{p}_y,\mathbf{q})$ with $\dim(\mathbf{p}_x)=\tilde{m}$, $\dim(\mathbf{p}_y)=m$, and $\dim(\mathbf{q})=d-m-\tilde{m}$ we have $d_c=\frac{8}{3}+\frac{2}{3}m+\frac{1}{3}\tilde{m}$. Apart from this change in the definition of $\epsilon=d_c-d$, the one-loop integrals can formally cast into a form that leads to the same pole terms (renormalization factors) as for the isotropic short-range P^4th model

$$\tilde{D}_1(r_0,c_0,g_0)=c_0^{-\tilde{m}/2}g_0^{-(d-\tilde{m}-m)}D_1(r_0) = \int d^{\tilde{m}}p_x\int d^{m}p_y\int d^{d-\tilde{m}-m}q\frac{1}{r_0+c_0p_x^2+p_y^4+g_0^2(q^2/p_y^2)} \tag{A1}$$

$$D_1(r_0)=-\frac{A_d}{\epsilon}r_0^{1-3\epsilon/4},\quad D_2(r_0)=-\frac{\partial}{\partial r_0}D_1(r_0) , \tag{A2}$$

and the m and $\tilde{m}$ dependence is absorbed in the geometric factor

$$A_d=\Gamma\left[\frac{4}{3}-\frac{m}{6}-\frac{\tilde{m}}{3}-\frac{\epsilon}{2}\right]\Gamma\left[\frac{2}{3}-\frac{m}{6}-\frac{\tilde{m}}{6}-\frac{\epsilon}{4}\right]\Gamma\left[\frac{\tilde{m}}{2}\right]\Gamma\left[1+\frac{3\epsilon}{4}\right]\frac{\Omega_{d-m-\tilde{m}}\Omega_m\Omega_{\tilde{m}}}{12(1-3\epsilon/4)(2\pi)^d} . \tag{A3}$$

Then the one-loop contributions to the ζ and β functions Eqs. (3) and (4) are independent of m and $\tilde{m}$ and of the form of the isotropic short-range P^4th model. The factor 3 in the zero-loop contribution of the β function comes from the naive dimension $\kappa^{3\epsilon}$ of the new coupling $\tilde{u}$. The exponents in one-loop order have therefore the same functional form in the corresponding ϵ as given in Eqs. (8)–(10), which are known from the standard P^4th model. The scaling functions are also independent from m and $\tilde{m}$ but different from the P^4th model and given by Eqs. (17)–(21).

[1] Yu. M. Vysochanskii, V. G. Furtsev, M. M. Khoma, A. A. Grabar, M. I. Gurzan, M. M. Maior, S. S. Prechinskii, V. M. Rizak, and V. Yu. Slivka, Zh. Eksp. Teor. Fiz. **91**, 1384 (1986) [Sov. Phys. JETP **64**, 816 (1986)].

[2] Yu. M. Vysochanskii and V. U. Slivka, Usp. Fiz. Nauk **162**, 139 (1992) [Sov. Phys. Usp. **35**, 123 (1992)].

[3] R. M. Hornreich, M. Luban, and S. Shtrikman, Phys. Rev. Lett. **35**, 1678 (1975).

[4] A. P. Levanyuk, V. V. Osipov, A. S. Sigov, and A. M. Sobyanin, Zh. Eksp. Teor. Fiz. **76**, 345 (1979) [Sov. Phys. JETP **49**, 176 (1979)].

[5] R. Folk and G. Moser, in *Ferroelektrizität 88,* edited by G. Schmidt and A. Rost (Wissenschafliche Beiträge, Martin-Luther-Universität Halle-Wittenberg, Halle, 1989).

[6] R. Kretschmer and K. Binder, Z. Phys. B **34**, 375 (1979).

[7] G. Moser and R. Folk, Solid State Commun. **57**, 707 (1986).

[8] A. Aharony, Phys. Rev. B **8**, 3363 (1973).

[9] A. Aharony and M. Fisher, Phys. Rev. B **8**, 3323 (1973).

[10] W. Selke, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, New York, 1992), Vol. 15.

[11] R. Folk and G. Moser, Phys. Lett. A **120**, 39 (1987).

[12] R. Dengler, Phys. Lett. **108A**, 269 (1985); A. Aharony, E. Domany, and R. M. Hornreich, Phys. Rev. B **36**, 2006 (1987).

[13] I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, New York, 1984), Vol. 9.

[14] J. Als-Nielsen, Phys. Rev. Lett. **37**, 1161 (1976).

[15] Calculations of the logarithmic corrections for $m=1$ and ϵ corrections in $m=2$ are in progress.