

Phase separation of edge states in the integer quantum Hall regime

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Coulomb effects on the edge states of a two-dimensional electron gas in the presence of a high magnetic field are studied for different widths of the boundaries. Schrödinger and Poisson equations are self-consistently solved in the integer quantum Hall regime. Regions of flat bands at the Fermi level appear for smooth interfaces in order to minimize the electrostatic energy and are related to the existence of dipoles induced by the magnetic field. These plateaus determine the phase separation in stripes of compressible and incompressible electron liquids.

A high magnetic field perpendicular to a two-dimensional (2D) electron gas produces a spectrum with a continuous part in which states located close to the boundaries are involved. These edge states are essential for the response of the system to external perturbations, particularly in carrying a net electrical current when a bias is applied to the sample.¹ For an empty system (or when electron-electron interaction is neglected), edge states are completely determined by the confining potential. However, when a density of electrons populates the system, the Coulomb interaction changes qualitatively the band structure. Such a problem has been only addressed in the case of a very smooth external potential either by a Thomas-Fermi approach^{2,3} or by solving the Poisson equation in the classical limit.⁴ Spin-splitting effects have also been studied in a variational scheme.⁵ In some cases, for example, in the case of the fractional quantum Hall regime, in which many-body effects are crucial, only general arguments have been given to discuss the importance and behavior of edge states.⁶⁻¹⁰ In all those works, the smoothness of the confining potential is a necessity due to the strong simplifications involved. The actual situations in experiments cover the whole range from abrupt (etching techniques) to smooth (gate techniques) boundaries.¹ To understand the properties of the edge states for any width of the interface region a quantum analysis including electrostatic effects is needed. This implies solving self-consistently the Schrödinger and Poisson equations for electrons confined by a potential defined in a 2D system in the presence of the external magnetic field.

We are interested in studying the properties of electrons confined in the z direction by semiconductor in-

terfaces or quantum wells affected by a magnetic field $\mathbf{B} = B\mathbf{u}_z$. The system has no constrictions in the y direction while in the x direction there are some boundaries with typical widths significantly larger than the extent of wave functions in the z direction. Therefore, we neglect the effects of the width in z of the electron gas and just consider a strictly 2D (xy) system with boundaries in the x direction. Physically, such boundaries are produced by gate potentials which deplete the 2D gas in some regions confining the electrons in the rest of the xy plane. We have a total depletion under the gates by putting infinite barriers in $x = 0$ and $x = W$, while the smoother interface region is the sum of two terms: the electrostatic potential produced by the electronic charge, and a confining potential in the x direction which we take as the one created by a fictitious distribution of positive charge with the trapezoidal shape

$$\rho_+(x) = \frac{n_+}{L} \begin{cases} x, & 0 < x < L \\ L, & L < x < W - L \\ W - x, & W - L < x < W. \end{cases} \quad (1)$$

In this way, the electrons with a 2D density n_+ are smoothly confined in a channel of width W . With this model, the Landau gauge $\mathbf{A} = (0, Bx, 0)$ is adequate to study the problem. The wave functions have the form

$$\Psi_{n,k}(x, y) = \frac{e^{iky}}{\sqrt{L_y}} \phi_{n,k}^\sigma(x), \quad (2)$$

where the wave function $\phi_{n,k}^\sigma(x)$ is an eigenstate, with eigenvalue $\varepsilon_{n,k}^\sigma$, of the one-dimensional Schrödinger equation

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{m^* \omega_c^2 (x - x_0)^2}{2} + eV(x) + \frac{\sigma g^* \mu_B B}{2} \right] \phi_{n,k}^\sigma(x) = \varepsilon_{n,k}^\sigma \phi_{n,k}^\sigma(x), \quad (3)$$

where $\omega_c = eB/m^*$ is the cyclotron frequency, $x_0 = k\ell^2$ is the semiclassical center of the orbit in terms of the magnetic length $\ell = \sqrt{\hbar/eB}$, σ is the spin of the electron, g^* is the effective g factor, and μ_B the Bohr magneton.

The self-consistent electrostatic potential is given by

$$V(x) = \frac{2e}{\epsilon} \int dx' [\rho_+(x') - \rho(x')] \ln|x - x'|, \quad (4)$$

ϵ being the dielectric constant of the medium and the electronic self-consistent charge density being given by

$$\rho(x) = \sum_{\sigma,n} \int^{E_F} d\varepsilon_{n,k}^{\sigma} |\phi_{n,k}^{\sigma}(x)|^2. \quad (5)$$

E_F is the Fermi level which, for given total charge density, must be computed within the iterative procedure to solve the Schrödinger equation. We define the total charge by means of the filling factor ν for the 2D bulk (i.e., the center region of the wide channel). The iterations are started by taking the electronic charge locally equal to the positive background, i.e., with a potential $V(x)$ which is flat in between the two infinite barriers. We neglect the exchange interaction between electrons; therefore, we restrict our calculation to even filling factors where exchange effects are less important.¹¹ Electrostatic effects are the same for any ν corresponding to an incompressible liquid in the bulk, i.e., with a large enough gap at the Fermi level. Therefore, it is possible to extend our conclusions to odd filling factors.

Since it is very interesting to discuss what happens to the different branches of the dispersion relation, we present results for the case $\nu = 6$ and $B = 1$ T. We use typical parameters for GaAs so that g^* is so small that there is no difference between spin-up and spin-down bands. In order to analyze the importance of the boundary smoothness Figs. 1–3 give the dispersion relations $\varepsilon_{n,k} \equiv \varepsilon_n(x_0)$, potential profiles $V(x)$, and charge densi-

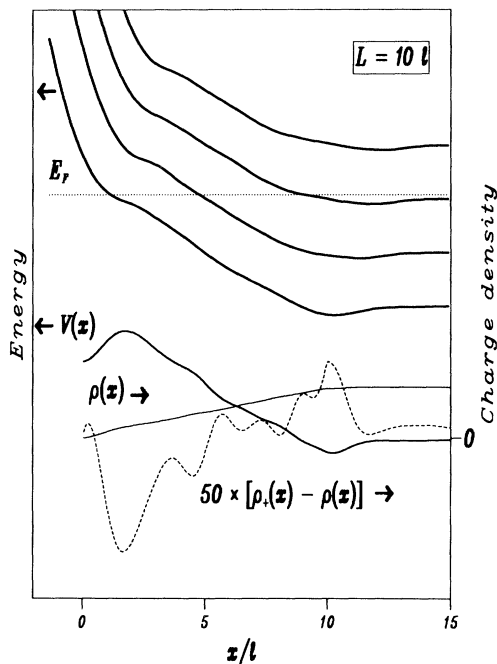


FIG. 1. Dispersion relations $\varepsilon_n(x_0)$ (broad continuous lines), potential profiles $V(x)$ (intermediate continuous line), and charge densities $\rho(x)$ (narrow continuous line) and $50[\rho_+(x) - \rho(x)]$ (dashed line) as a function of the position for a boundary with $L = 10l$. The Fermi level (dotted line) self-consistently computed is also shown.

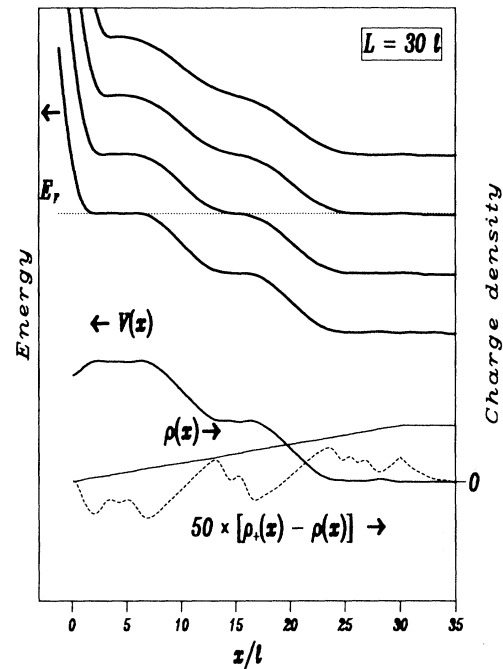


FIG. 2. Dispersion relations $\varepsilon_n(x_0)$ (broad continuous lines), potential profiles $V(x)$ (intermediate continuous line), and charge densities $\rho(x)$ (narrow continuous line) and $50[\rho_+(x) - \rho(x)]$ (dashed line) as a function of the position for a boundary with $L = 30l$. The Fermi level (dotted line) self-consistently computed is also shown.

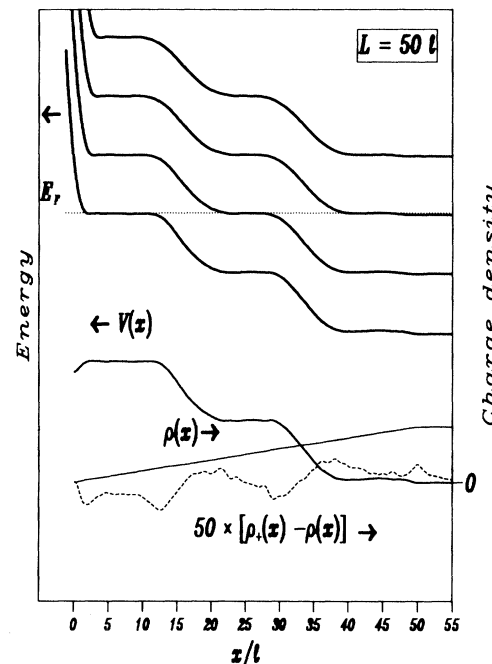


FIG. 3. Dispersion relations $\varepsilon_n(x_0)$ (broad continuous lines), potential profiles $V(x)$ (intermediate continuous line), and charge densities $\rho(x)$ (narrow continuous line) and $50[\rho_+(x) - \rho(x)]$ (dashed line) as a function of the position for a boundary with $L = 50l$. The Fermi level (dotted line) self-consistently computed is also shown.

ties $\rho(x)$ and $\rho_+(x) - \rho(x)$ (multiplied by 50) for the cases $L = 10\ell$, $L = 30\ell$, and $L = 50\ell$, respectively. All the figures give results computed with $W = 2L + 10\ell$ in which the two interfaces are decoupled from each other as we have checked out by increasing the separation (changing the factor 10 by higher numbers) and obtaining the same results. Due to the symmetry with respect to the center of the channel we only present results for $x < W/2$. From Fig. 1, one observes that an interface of ten magnetic lengths can be considered rather abrupt because no plateaus at the edge appear in $V(x)$ or $\varepsilon_n(x_0)$ as suggested for smooth interfaces.⁴ Only some inflections appear for $L = 10\ell$ that become flat regions for $L = 30\ell$ and still broader for the smoother case of $L = 50\ell$. For $L = 30\ell$ and $L = 50\ell$ there are two types of plateaus near the Fermi level which are intrinsically different from each other. Those spatially close to the edges are not completely occupied because they lie in energy exactly at the Fermi level while the plateau in the bulk is clearly below E_F so that its states are fully occupied. Since we work in a single-particle model, our results for the flat regions at the edge present very small oscillations instead of being completely flat so that they accommodate both electrons and holes around the Fermi level. In our calculations, the amplitude of such oscillations tends to zero, and the plateaus can be considered absolutely flat for any physical purpose, in particular for considering these regions as locally compressible liquids corresponding to noninteger filling factors.

Let us try to understand the physical causes for the shape of the dispersion relation by starting with the flat regions close to the edge. Among these plateaus, there are regions of incompressible liquid corresponding to integer local filling factors. When L increases, the compressible regions extend while the incompressible ones remain practically the same. This observation helps us understand the physical origin of the spatial separation of the compressible and incompressible phases. In a region of integer filling factor, the electronic charge density is practically constant and the positive background varies linearly. This implies an electrostatic dipole that increases with the width of the incompressible region. Since that dipole means an increase of the energy, the system reacts to minimize the energy by decreasing the

width of the incompressible region as much as possible, i.e., by forming compressible regions. In this new phase, the electronic charge does not need to be constant any more. Therefore it can have the same shape of the positive background (as shown in the figures) eluding any increase of the electrostatic energy. It is important to stress that this argument is independent of the form of a background which varies with the position so that our results should be completely general for any interface. In the bulk of the sample, the fictitious positive background is constant, the Fermi level lies above the Landau state, giving the well-known incompressible liquid for integer filling factor. The width of the incompressible phases at the interface cannot decrease up to zero because there is a lower limit imposed by the extent of the wave functions. The minimization of the electrostatic energy, which depends on the strength $e^2/\epsilon\ell$ of the interaction, dominates on any variation of kinetic energy, which depends on the cyclotron energy $\hbar\omega_c$.

Disorder induced localization only affects the bulk electronic structure but not the edge states, so our results are valid for actual samples.¹³ The experiments for integer ν made on samples with boundaries created by etching involve abrupt interfaces and the electron liquid must be incompressible. On the contrary, for samples with boundaries defined by gate potentials the interfaces are typically broader than ten times the magnetic length and phase separation should occur. The current will be carried in the compressible regions where the zero velocity of each state compensates the infinite density of states. So, transport does not seem to be a good way to detect the electrostatic effects here studied. Spectroscopic experiments are better candidates to analyze compressible regions because the high density of states must produce strong Fermi edge singularities in absorption and emission of light¹² as well as significant alterations in the spectrum of edge magnetoplasmons.

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¹For a review see, e.g., C. W. J. Beenakker and H. van Houten, in *Solid State Physics: Advances in Research and Applications*, edited by H. Ehrenreich and D. Turnbull (Academic, New York, 1991), Vol. 44, p. 1.

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