Velocity-field characteristics of selectively doped GaAs/$Al_xGa_{1-x}As$ quantum-well heterostructures

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Recent experimental observations indicating poor correlation between the saturation velocity and Ohmic mobility lead us to propose a theory for a velocity-limiting mechanism in quantum-well heterostructures. The theory is based on the distribution function which takes into account the electron-drift anisotropy introduced by the high electric field for a degenerately doped quantum well. The drift velocity is shown to be limited by the Fermi velocity; a result which indicates that the saturation velocity is independent of the low-field mobility which is strongly controlled by momentum-randomizing scattering events. The dominance of optical-phonon emission at high electric field lowers the saturation velocity below the Fermi velocity. Excellent agreement is obtained between the theoretical and experimental results on the velocity-field characteristics of GaAs/$Al_xGa_{1-x}As$ quantum-well heterostructures.

I. INTRODUCTION

Recent developments in the programs of very-large-scale integration (VLSI) and very-high-speed-integrated circuits (VHSIC) have indicated the ever-increasing importance of high-field effects which limit the velocity of carriers and hence impose an intrinsic limit on device speed. There have been numerous attempts to identify the mechanism responsible for this saturation of the carrier velocity in high electric fields (see Refs. 1 and 2 for a review). Often a higher mobility is cited in the published literature to give rise to higher saturation velocity. But, recent experiments tend to indicate a poor correlation between saturation velocity and mobility.[3,4] This is further confirmed by recent experimental observations,[5,6] where the high-frequency performance of metal-semiconductor field-effect transistor (MESFET) or metal-insulator-semiconductor field-effect transistor (MISFET) is shown to be comparable to or superior than that achieved by modulation-doped heterostructures. Independently determined decreases in mobility and increases in electron velocity under similar conditions are observed, and are consistent with the earlier theoretical predictions on bulk semiconductors,[1,2,7] but do not conform to the energy-balance theory of Shockley.[8] In spite of the large amount of information available on bulk semiconductors, limited success has been obtained in interpreting velocity-field characteristics in quantum-well heterostructures.

Hirakawa and Sakaki[3] did an extensive and systematic study of velocity-field characteristics under a wide variety of conditions in GaAs/$Al_xGa_{1-x}As$ quantum-well heterostructures. The main conclusion of their experimental findings is that the saturation velocity does not sensitively depend on Ohmic mobility and the carrier concentration. Their attempt to interpret these results in the wake of Monte Carlo and energy balance theories was not successful. The experimental results in bulk semiconductors over the years have clearly shown the saturation velocity to be comparable to the thermal velocity (typically, 10^7 cm/s). On the other hand, Monte Carlo procedures predict the saturation velocity as scattering limited. If that is the case, the saturation velocity should improve with the increase in low-field mobility which is also scattering limited. That was the original impetus behind the push for GaAs technology which has mobility five to six times higher than silicon. But, all recent experiments, to the contrary, show that the higher Ohmic mobility does not necessarily lead to higher saturation velocity. In this paper we explain the velocity-limiting mechanism in a quantum well in the light of theory[1] which has been extremely successful when applied to bulk semiconductors.

II. DISTRIBUTION FUNCTION

The understanding of the distribution function is essential in any carrier-transport study. The equilibrium distribution function is well known to be the Fermi-Dirac distribution function. In equilibrium, the bands are flat unless there is a built-in electric field. The electrons are moving at random with the average velocity in each of the three Cartesian directions equal to zero. In external fields, the Fermi energy (electrochemical potential) and bands are tilted parallel to each other.[1] An excellent description of the carrier transport with tilted-band diagram is given by Böer,[9] who examines the effect of a built-in as well as the applied electric field on the distribution of carriers. The carrier distribution is determined relative to the Fermi level. In zero bias, the distribution is independent of position and the Fermi level (chemical potential) is constant. In an external field, however, the Fermi level and bands are tilted. The carrier distribution is now a function of spatial coordinates. When electrons are accelerated in the field, they can dissipate their net additional energy to the lattice by emitting phonons, thereby causing lattice heating, or by momentum-randomizing collisions. The carriers drop close to the conduction-band edge and the motion is repeated. From intuitive arguments, as suggested by Böer,[9] a limiting velocity comparable to the random thermal velocity at lattice temperature is expected. The carrier motion in an external field is, therefore, not random. It has a finite component in the field direction. This makes distribution of carriers asymmetric in the direction of the applied electric field. The electrons will roll down the energy hill and holes will bubble up the energy hill. The electric field thus tries to organize the otherwise completely random motion. Understandably, the Fermi level will be affected by the applied electric field. All these features are contained in the distribution function, extensively discussed in Ref. 1, which is given below:

$$f(\varepsilon_{nk},E,\theta)=\frac{1}{e^{[\varepsilon_{nk}-\zeta+eEl(\varepsilon_{nk})]/k_BT}+1} . \tag{2.1}$$

ε_{nk} is the quantized energy of an electron in quantum well:

$$\varepsilon_{n,k_x,k_y}=n^2\varepsilon_0+\frac{\hbar^2}{2m^*}(k_x^2+k_y^2) ,\ n=1,2,3,\ldots . \tag{2.2}$$

$l(\varepsilon_{nk})=\tau v$ is the energy-dependent mean free path. This distribution function is consistent with the ansatz proposed several years ago in Ref. 10. The distribution function has a very simple interpretation. The electrochemical potential ζ during the free flight of carriers changes by $\pm eEl$ as electrons tend to sink and holes tend to float on the tilted-energy-band diagram. This observation may suggest that the applied electric field tends to organize the otherwise completely random motion. Electric dipoles el due to quasifree motion of the carriers tend to organize in the direction of the electric field for holes and in the opposite direction for electrons. The high-field effects thus become important when the dipole energy eEl is comparable to the thermal energy for nondegenerate electrons or the Fermi energy for degenerate electrons. The collisions tend to bring the electron closer to the conduction-band edge. If the electric field is strong, this undirectional motion will give carrier drift comparable to the thermal velocity which is average of the magnitude of electron velocity v. A quasiballistic behavior of the carriers thus follows in a strong electric field.

The distribution function of Eq. (2.1) may give the appearance of the directed Maxwellian with the hidden characteristic that the Fermi energy is now a function of the electric field and hence needs renormalization to keep the carrier concentration constant. This is quite convenient because we need not invoke the concept of hot-electron temperature for which energy-balance formulation[11] will otherwise be necessary. In degenerate semiconductors, the mean free path can be taken to be constant. Therefore, all the complicated scattering events are absorbed in the mean free path, which can be easily evaluated from the Ohmic mobility for which reliable data are available. All scattering events change the carrier momentum. However, only some of them, the inelastic-scattering events, change the energy of the carriers. Usually several scattering events are followed by one inelastic scattering, usually by generating a phonon. But, in the high electric field, inelastic scattering can become frequent, comparable to the momentum scattering rate. The scattering length l_{op} associated with the emission of an optical phonon can be obtained by equating the energy gained by an electron in a mean free path to that of an optical phonon $\hbar\omega_0$:

$$l_{\mathrm{op}}=\hbar\omega_0/eE . \tag{2.3}$$

The total scattering length is then given by

$$l^{-1}=l_0^{-1}+l_{\mathrm{op}}^{-1} , \tag{2.4}$$

where l_0 is the low-field scattering length obtained from the Ohmic mobility.

III. DRIFT VELOCITY

The drift velocity for the electron gas in a quantum well, with the distribution function of Eq. (2.1) in the quantum limit ($n=1$), is obtained as

$$v_d=\frac{\sqrt{\pi}}{2}v_{\mathrm{th}}\frac{\int_0^{2\pi}\cos(\theta)F_{1/2}[H(\theta)]d\theta}{\int_0^{2\pi}\ln(1+e^{H(\theta)})d\theta} , \tag{3.1}$$

where

$$H(\theta)=\eta-\delta\cos(\theta) \tag{3.2}$$

is the quasi-Fermi-level which is direction dependent and

$$\delta=eEl/k_BT , \tag{3.3}$$

$$v_{\mathrm{th}}=(2k_BT/m^*)^{1/2} , \tag{3.4}$$

$$\eta=\frac{\zeta-\varepsilon_0}{k_BT} , \tag{3.5}$$

ε_0 is the ground-state energy and $F_{1/2}(x)$ is the Fermi-Dirac integral.[9] η is evaluated from the normalization condition

$$n=\lambda_D^{-2}\int_0^{2\pi}\ln(1+e^{H(\theta)})d\theta . \tag{3.6}$$

In the nondegenerate limit, Eq. (3.1) simplifies to

$$v_d=\frac{\sqrt{\pi}}{2}v_{\mathrm{th}}\frac{I_1(\delta)}{I_0(\delta)} , \tag{3.7}$$

where $I_n(\delta)$ is the modified Bessel function of order n. Taking the limit of Eq. (3.7) as $\delta\rightarrow\infty$, the saturation velocity in the nondegenerate case is

$$v_{d\,\mathrm{sat}}=\frac{\sqrt{\pi}}{2}v_{\mathrm{th}} . \tag{3.8}$$

No such simple expression is obtainable under strongly degenerate conditions, as degeneracy increases in the antiparallel direction to the field and decreases in the parallel direction as electrons transfer under its influence.[1]

Figure 1 shows the comparsion of theoretical results obtained from Eq. (3.1) with the experimental data of Hirakawa and Sakaki.[3] Considering the experimental uncertainties in the high-field measurements, the agreement is extremely good. The values of the mean free path extracted from the Ohmic mobility at 292, 77, and 4.2 K are $l_0=104$, 1998, and 7600 nm, respectively. The Fermi velocity is 2.9×10^5 m/s. The experimental results could not be explained by Monte Carlo theory or by an approach based on an electron temperature model.[3] The results are also consistent with the measurements of

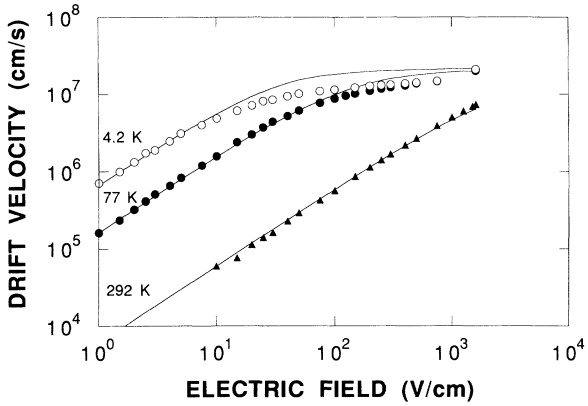

FIG. 1. Velocity-field characteristics of the $GaAs/Al_xGa_{1-x}As$ quantum-well heterostructure at 292, 77, and 4.2 K. The points are the experimental results of Hirakawa and Sakaki (Ref. 3).

Masselink *et al.*[12] who have shown a lack of dependence of the saturation velocity on Ohmic mobility. Under strong degenerate conditions, the velocity tends to a value of 2.0×10^7 cm/s, both for 77 and for 4.2 K. The highest room-temperature velocity reported in Ref. 3 is 0.73×10^7 cm/s, which is considerably lower than the predicted saturation velocity of 1.8×10^7 cm/s, as shown in Fig. 2. Figure 2 clearly shows the predominant role of optical-phonon emission in limiting the electron velocity. Sakaki[13] has proposed engineering the quantum-well wire and quantum box array structures in order to obtain a miniband structure so that the optical-phonon emission is eliminated. The elimination of optical-phonon emission is expected to give considerable improvement in device speed. In such microstructures, velocity at least twice as high is expected from these theoretical considerations. Also shown in Fig. 2 are the results obtained from the empirical relation commonly used in modeling a transistor:[14,15]

$$v_D=\frac{\mu_0 E}{1+\frac{E}{E_c}}\ , \tag{3.9}$$

with

$$E_c=v_{\text{sat}}/\mu_0\ . \tag{3.10}$$

For the sample considered, $E_c=3.1$ kV/cm. This shows that this relation gives a good description of velocity-field characteristics for modeling field-effect transistors.

The experiments of Ketterson *et al.*[16] first showed that the velocity at room temperature is higher than that at 77 K. This indicated that the velocity-limiting mechanism is not controlled by the mobility of the sample. In the light of this theory, under nondegenerate conditions, the saturation velocity is limited by the random thermal velocity which is higher at room temperature. At room temperature, saturation is noticeable at much higher electric fields (Fig. 2) as compared to low-temperature conditions (Fig. 3) because the ratio of the Fermi velocity to the thermal velocity is much higher at cryogenic temperatures.[17]

In Fig. 3 we show the velocity-field characteristics for strongly degenerate gas at 4.2 K for five samples with varying mobility and carrier concentration, with phonon emission (solid lines) and without phonon emission (dashed lines). For all samples considered, the saturation velocity is approximately 2.0×10^7 cm/s. In the absence of phonon emission, the saturation velocity is comparable to the Fermi velocity. In a recent paper by Peatman, Crowe, and Shur,[17] the idea of obtaining saturation velocity closer to the Fermi velocity, which should be as high as $(5-10)\times10^7$ cm/s, has been utilized in designing a novel Schottky diode for millimeter and submillimeter wave applications. Similarly, the recently reported experimental observations of Mizutani and Maezawa[18] on

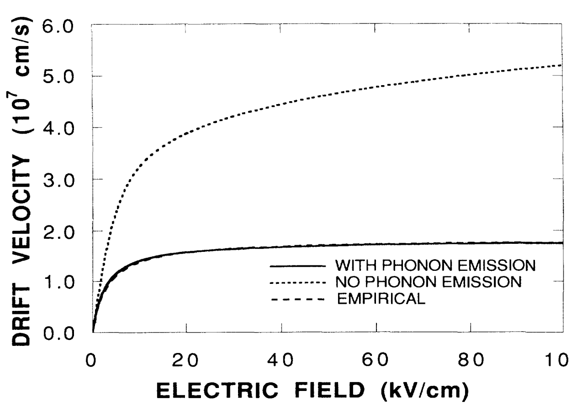

FIG. 2. Extrapolation of velocity-field characteristics at 292 K to higher electric fields. The solid curve is with optical-phonon emission included and the dotted curve is when it is ignored. The dashed curve is the empirical relation commonly used in modeling a transistor [Eq. (3.9)].

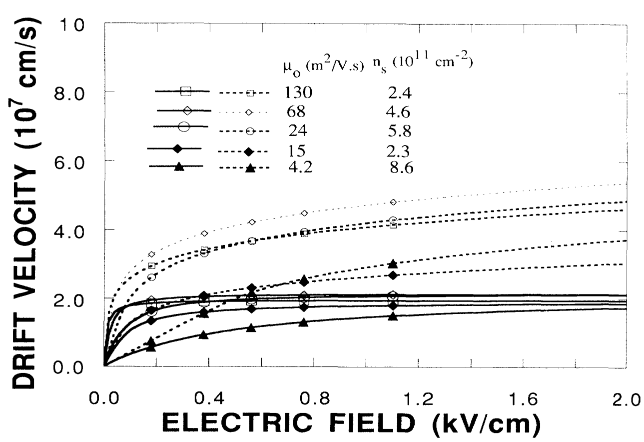

FIG. 3. The velocity-field characteristics at 4.2 K for five samples of varying mobility and carrier concentration. Solid curves, with optical-phonon emission included; dashed curves, with phonon emission ignored.

$Al_xGa_{1-x}As/In_xGa_{1-x}As$ can be easily interpreted in the framework of this theory. For high-mobility materials, the approach towards saturation is much faster than for low-mobility materials. Thus high-mobility materials are more vulnerable to high-field effects than are low-mobility materials as increasingly higher fields are encountered. The saturation velocity does not sensitively depend on carrier concentration or on low-field mobility, in agreement with experimental deductions.[3]

IV. CONCLUSIONS

In conclusion, we have identified a mechanism for velocity saturation as electrons transfer from parallel to antiparallel directions of the electric field, and have interpreted the experimental results on $GaAs/Al_xGa_{1-x}As$ quantum wells under a wide variety of experimental conditions. We therefore hope that the results presented here will prove useful in understanding the world of future quantum devices on a miniaturized scale.

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