

Energy loss from hot electrons in a metal film

M. Kanskar, M. N. Wybourne, and Kris Johnson

Department of Physics, University of Oregon, Eugene, Oregon 97403

(Received 12 January 1993)

In this paper we discuss the energy-loss rate from electrons in a metal film heated by an applied electric field. The acoustic-phonon spectrum in the film has been described by a confined three-dimensional phonon density of states that takes into account the finite thickness of the film. The relationships between the energy-loss rate, the electric field, and the electron temperature are shown to depend on the acoustic coupling strength between the film and the underlying substrate. In agreement with previous analysis, with perfect acoustic coupling the electron temperature and electric field are related by a power law that depends on the phonon dimensionality. However, when the acoustic coupling is less than perfect significant departures from the power-law relationship are found.

INTRODUCTION

Low-temperature nonequilibrium-transport measurements have been used for some time to study the energy relaxation of electrons in metal microstructures.¹ In these experiments, a dc electric field applied across the microstructure is used to heat the electrons above the lattice temperature. During the energy relaxation time of the electrons, the power developed in the microstructure by the electric field increases the energy density of the electrons. Therefore, by determining the electron temperature rise a simple heat-capacity argument can be used to estimate the energy relaxation time of the electrons.² In the earliest electron heating experiments, the phonons in the microstructure were assumed to remain at the temperature of the underlying substrate, which is the same as assuming that the electron-phonon-scattering rate is much smaller than the rate of thermal energy loss from the microstructure. Perrin and Budd have analyzed electron heating experiments on thin metal films in terms of a single-particle electron-phonon-scattering rate and the rate of phonon escape from the film.^{3,4} It was shown that the true situation is described more closely by a bottleneck between the electron-phonon scattering and escape rates, with the lowest rate dominating the energy flow from the electrons to the substrate. Recent experiments have shown that the rate of energy loss from a film at low temperatures depends on the details of the phonon escape.^{5,6} These results imply that the phonon escape rate is at least comparable to the electron-phonon-scattering rate, and shows *a priori* that the heat escape mechanism cannot be neglected in electron heating experiments. Other work has addressed the point that when the film thickness becomes comparable to the dominant phonon wavelength, the density of states of the acoustic phonons in the film is expected to be modified.^{7,8} This modification of the density of states by spatial confinement is also anticipated to affect the details of energy relaxation from hot electrons in films.⁹

In this paper, we discuss the connection between the energy relaxation rate of electrons in a metal film and the characteristic times associated with the single-particle

electron-phonon interaction and phonon escape. In the analysis, we have included both a phonon density of states that takes into account the finite film thickness, and different acoustic coupling conditions between the substrate and the film.

MODEL

When a dc electric field \mathbf{E} is applied across a metal film, a current density \mathbf{J} is caused to flow and the power per volume being delivered to the electron system is $\mathbf{J} \cdot \mathbf{E}$. In the steady state, $\mathbf{J} \cdot \mathbf{E}$ is also equal to the power per volume being delivered to the phonon system, and to the substrate. This situation is shown schematically in Fig. 1. If the energy relaxation time of the electron system is $\tau(T)$, the power delivered to the electron system in a time $\tau(T)$ causes the electrons to heat above the lattice temperature of the film. The electron temperature rise can be determined by

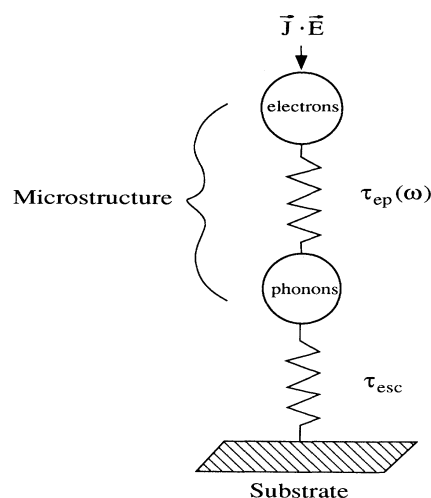


FIG. 1. A schematic representation of an electron heating experiment.

$$\mathbf{J} \cdot \mathbf{E} = \int_{T_s}^{T_e} \frac{\gamma T}{\tau(T)} dT, \quad (1)$$

where $\gamma = \frac{1}{2} \pi^2 k_B^2 N E_F^{-1}$, N is the electron density, E_F is the Fermi energy, and T_s and T_e are the substrate and electron temperatures, respectively. If the acoustic coupling between the film and the underlying substrate is less than perfect, the energy relaxation of the electron system will cause the lattice temperature of the film to be heated above the substrate temperature. In this work, the underlying substrate is assumed to be much larger than the film and to act as a perfect heat bath that remains at T_s . Following the work of Perrin and Budd,^{3,4} in the steady state,

$$\mathbf{J} \cdot \mathbf{E} = \sum_j \int \frac{\hbar \omega D_j(\omega) [n(\omega, T_e) - n(\omega, T_s)]}{\tau_{\text{esc}}^j + \tau_{\text{ep}}^j(\omega)} d\omega, \quad (2)$$

where $D_j(\omega)$ is the phonon density of states for mode j , $n(\omega, T)$ is a Bose distribution evaluated at a temperature T , τ_{esc}^j is the time for phonons of mode j to escape from the film, and $\tau_{\text{ep}}^j(\omega)$ is the single-particle electron-phonon-scattering time for the corresponding mode. We assume the electron-electron-scattering time is much smaller than any other time in the problem, hence the electron system is internally in equilibrium at T_e .

In order to maintain the steady state, the power per volume transferred from the electrons to the phonons, and from the phonons to the substrate, must have the same rate of change with electron temperature. Therefore, by equating the derivatives with respect to the electron temperature of Eqs. (1) and (2), $\tau(T)$ can be expressed in terms of τ_{esc}^j and $\tau_{\text{ep}}^j(\omega)$. Following this procedure, we obtain

$$\tau^{-1}(T_e) = \frac{\hbar^2}{4\gamma k_B T_e^3} \sum_j \int \frac{\omega^2 D_j(\omega)}{\tau_{\text{esc}}^j + \tau_{\text{ep}}^j(\omega)} \times \frac{1}{\sinh^2 \left[\frac{\hbar \omega}{2k_B T_e} \right]} d\omega. \quad (3)$$

The escape time is taken to be of the usual form $\tau_{\text{esc}}^j = 4\eta d/v_j$,¹⁰ where v_j is the velocity of sound for mode j in the film, η is a frequency-independent phonon reflection coefficient that is equal to $1/(2\Gamma)$ in the notation of Little,¹¹ and d is the thickness of the film. In the present work, we assume η to be independent of the phonon mode.

In our analysis we have used the single-particle electron-phonon interaction given by the Pippard model.¹² Within this model, the electron-phonon interaction for the longitudinal mode is given by

$$(\tau_{\text{ep}}^L)^{-1} = \frac{Nm}{\rho_m \tau_e} \left[\frac{1}{3} \frac{q^2 l^2 \tan^{-1}(ql)}{ql - \tan^{-1}(ql)} - 1 \right], \quad (4a)$$

and that for transverse modes is given by

$$(\tau_{\text{ep}}^T)^{-1} = \frac{Nm}{\rho_m \tau_e} \frac{1-g}{g}, \quad (4b)$$

where

$$g = \frac{3}{2q^2 l^2} \left[\frac{(q^2 l^2 + 1)}{ql} \tan^{-1}(ql) - 1 \right]. \quad (4c)$$

In these expressions, m is the electron mass, ρ_m is the mass density, q is the magnitude of the phonon wave vector, τ_e is the elastic scattering time of the electrons, and l is the corresponding elastic mean free path.

The appropriate phonon density of states is expected to be modified from the usual ω^2/v_j^3 dependence when the film thickness becomes comparable to one half of the dominant phonon wavelength, $\lambda_{\text{dom}}/2 \approx \hbar v/4kT$. To evaluate the density of states, we have considered a thin film having a thickness d . The direction normal to the film substrate interface is denoted as the direction z . The thickness d will cause the allowed wave vectors in the z direction to be separated by π/d , that is, the phase space will comprise planes corresponding to $q_z = \pm m\pi/d$, where m is an integer. Each plane corresponds to an acoustic subband in the thin film and has a density of states $\omega/2\pi v_j^2$. As the magnitude of the wave vector reaches π/d , there is an extra component to the total density of states that comes from the next acoustic subband. Thus the overall density of states for the thin film is

$$D_j(\omega) = \sum_m \frac{m\omega_j}{2\pi v_j^2 d} \quad (5)$$

for $(m-1)\omega_{0j} \leq \omega_j < m\omega_{0j}$, where the energy separation between the planes is $\omega_{0j} = \pi v_j/d$.

DISCUSSION

We have calculated numerically the integrals in Eqs. (2) and (3) to obtain τ^{-1} as a function of the T_e , and T_e as a function of the electric field magnitude E at a substrate temperature of 1 K. From these data we have obtained τ^{-1} as a function of E . The model has been applied to a 20-nm-thick gold film that has an acoustic coupling to an underlying substrate characterized by η in the range from 1 to 100, which represents the range from perfect to weak coupling. The parameters for gold have been taken to be as follows: $N = 6 \times 10^{28} \text{ m}^{-3}$, $\rho_D = 19\,300 \text{ Kg m}^{-3}$, and $\rho = 5 \times 10^{-8} \text{ } \Omega\text{m}$, which corresponds to an impurity scattering length $l = 16 \text{ nm}$. From the elastic constants of gold,¹³ the velocities of sound are estimated to be $v_L = 3.42 \times 10^3 \text{ ms}^{-1}$ and $v_T = 1.32 \times 10^3 \text{ ms}^{-1}$. At a substrate temperature of 1 K, $\lambda_{\text{dom}}/2$ is 41 and 15 nm for the longitudinal and transverse modes, respectively. Therefore, the phonon density of states in the 20-nm-thick film is somewhere between the three- (large m) and two- ($m = 1$) dimensional limits of Eq. (5).

The relationship between τ^{-1} and T_e and the dependence of this relationship on the acoustic coupling strength are shown in Fig. 2. It can be seen that as the acoustic coupling decreases, the energy relaxation rate is reduced. For weak coupling ($10 < \eta$), the relaxation rate is almost inversely proportional to η , and, as the acoustic coupling is made weaker, is found to approach a T_e^2 dependence. A simple power-law dependence does not hold for strong acoustic coupling, as seen for the case of

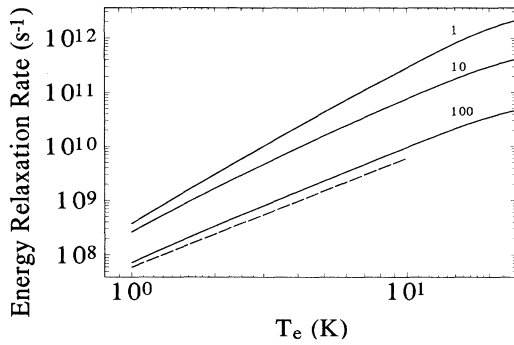


FIG. 2. The energy relaxation rate as a function of the electron temperature calculated for three-dimensional phonons. The three curves are labeled according to the value of η . The dashed line shows a squared dependence.

$\eta=1$ in Fig. 2. For each curve in Fig. 2, we have calculated the corresponding variation of T_e with the applied electric field, as shown in Fig. 3. As expected, the field dependence is strongest for the most weakly coupled film. When η is made larger, it is found that T_e approaches an $E^{1/2}$ dependence, as indicated by the dashed line in Fig. 3, while at stronger acoustic coupling it approaches $E^{2/5}$, as indicated by the dotted line in Fig. 3.

The relationship between electron temperature and electric field has been discussed previously by Anderson, Abrahams, and Ramakrishnan² and Arai.¹⁴ These authors assumed perfect acoustic coupling and a phonon density of states that was strictly either three, two, or one dimensional. They found T_e to be proportional to $E^{2/2+p}$ where p is the phonon dimensionality. The dependence we have found for weak acoustic coupling indicates that $p=2$, yet we have used a confined three-dimensional density of states given by Eq. (5). At the electric fields considered, several longitudinal and transverse subbands will be occupied, therefore p is not equal to 2. This result shows that the parameter p is dependent on η . In the

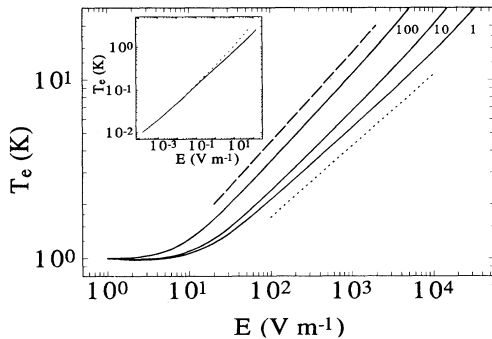


FIG. 3. The electron temperature as a function of the applied electric field. The three curves are labeled according to the value of η . The dashed and dotted lines show an $E^{1/2}$ and an $E^{2/5}$ dependence, respectively. The inset shows the electron temperature as a function of applied electric field calculated for a substrate temperature of 10 mK. The solid curve $\eta=1$ and the dotted curve $\eta=100$.

case of a strongly coupled film ($\eta=1$), the present model is in agreement with the results of Anderson, Abrahams, and Ramakrishnan² and Arai.¹⁴ Therefore, using a confined three-dimensional phonon density of states, we have shown that the acoustic coupling can alter the power law dependence between the electric field and electron temperature over a range that was previously attributed solely to changes in the phonon dimensionality.

At a sufficiently low substrate temperature and applied field, only one acoustic subband is occupied in the steady state. In this case the single-particle electron-phonon interaction is modified slightly, but retains the same frequency dependence over the frequency range considered.^{15,16,9} This indicates that even when the phonons become two dimensional, we still expect T_e to be proportional to $E^{2/5}$ in the strong acoustic coupling limit, as shown in the inset to Fig. 3. This prediction is supported by the mK temperature experiments of Roukes *et al.*¹ who measured electron heating in 100-nm-thick copper films on sapphire using noise thermometry. Taking the average velocity of sound in copper to be $3 \times 10^3 \text{ ms}^{-1}$, the energy spacing between the acoustic subbands in these copper films is about $\omega_0 = 1 \times 10^{11} \text{ s}^{-1}$, from which we expect phonons in the lowest subband to dominate the energy relaxation process at substrate temperatures below about 100 mK. Roukes *et al.*¹ found T_e to be proportional to $E^{2/5}$ at temperatures down to 25 mK, which is in agreement with the present model. At electron temperatures above 200 mK, they observed an increase in the electric-field exponent which again is consistent with the present work, as indicated in the inset to Fig. 3, and confirms that the acoustic coupling between the copper and the sapphire substrate was less than perfect.

The agreement between the data of Roukes *et al.*¹ and the present model poses an interesting issue related to the Eliashberg function $\alpha^2 F(\omega)$. The single-particle electron-phonon-scattering rate we have used implies that $\alpha^2 F(\omega) \rightarrow \omega^3$ at low frequencies.¹⁷ Other work found that in the low-frequency limit $\alpha^2 F(\omega) \rightarrow \omega$,¹⁸ which implies that the electron-phonon interaction would become frequency independent at low frequencies.¹⁷ If this was the

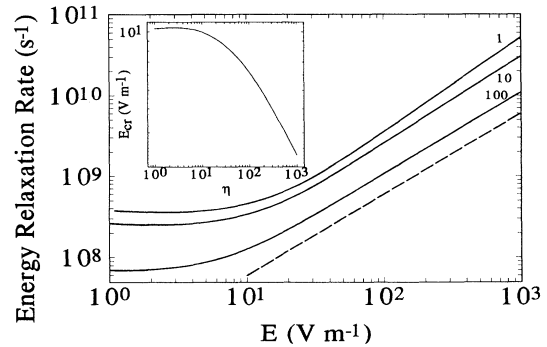


FIG. 4. The energy relaxation rate as a function of the applied electric field. The three curves are labeled according to the value of η . The dashed line indicates a linear relationship. The inset shows the variation of the critical field as a function of the acoustic coupling parameter.

case, in the limit of two-dimensional phonons the present model would predict $T_e \propto E^{2/3}$, which was not observed experimentally.¹

By combining the calculated dependence of T_e with E and that of τ^{-1} with T_e , the relationship between τ^{-1} and E has been obtained for several values of η , as shown in Fig. 4. The weaker the acoustic coupling, the lower the relaxation rate. For weak acoustic coupling and electric fields between about 10 and 10^3 V m^{-1} , τ^{-1} is found to vary almost linearly with the electric field. For more strongly coupled films, the dependence on the electric field over the same field range is slightly stronger and is approximately $E^{1.2}$. Defining a critical field E_{cr} , at which the high-field slope of τ^{-1} intersects the zero-field value, we find that E_{cr} decreases rapidly as the acoustic coupling decreases, as shown in the inset to Fig. 4. This decrease is caused by the electrons heating more rapidly with electric field in films that are less well coupled to the substrate.

CONCLUSION

We have analyzed electron heating in thin metal films using a phonon density of states that takes into account the finite thickness of the film. In the case of perfect coupling, the model is in agreement with previous work.^{2,4} However, if the acoustic coupling is less than perfect, there are significant departures from the previously predicted field dependence of the electron temperature. Thus it is necessary to take account of the acoustic coupling strength when analyzing steady-state electron heating experiments to measure the energy relaxation of electrons in thin metal films.

ACKNOWLEDGMENTS

M.N.W. would like to thank N. Perrin and D. Belitz for many helpful discussions. This work was supported by the National Science Foundation under Grant No. DMR-90-19525, and by a grant from NATO.

¹M. L. Roukes, M. R. Freeman, R. S. Germain, R. C. Richards, and M. B. Ketchen, Phys. Rev. Lett. **55**, 422 (1985).

²P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

³N. Perrin and H. Budd, Phys. Rev. Lett. **28**, 1701 (1972).

⁴N. Perrin and H. Budd, J. Phys. (Paris) Colloq. **33**, C4-33 (1971).

⁵J. C. Nabity and M. N. Wybourne, Phys. Rev. B **42**, 9714 (1990).

⁶J. C. Nabity and M. N. Wybourne, J. Phys. Condens. Matter **2**, 3125 (1990).

⁷G. Bergmann, Wei Wei, Yao Zou, and R. M. Mueller, Phys. Rev. B **41**, 7386 (1990).

⁸Y. K. Kwong, K. Lin, M. S. Issacson, and J. M. Parpia, J. Low Temp. Phys. **88**, 261 (1992).

⁹J. C. Nabity and M. N. Wybourne, Phys. Rev. B **44**, 8990 (1991).

¹⁰W. E. Bron and W. Grill, Phys. Rev. B **16**, 5303 (1977).

¹¹W. A. Little, Can. J. Phys. **37**, 334 (1959).

¹²A. B. Pippard, Philos. Mag. **46**, 1104 (1955).

¹³*Numerical Data and Functional Relationships in Science and Technology*, edited by K.-H. Hellwege and A. M. Hellwege, Landolt-Börnstein, New Series Group III, Vol. 11 (Springer-Verlag, Berlin, 1969).

¹⁴M. R. Arai, Appl. Phys. Lett. **42**, 906 (1983).

¹⁵D. Belitz (private communication).

¹⁶D. Belitz and S. Das Sarma, Phys. Rev. B **36**, 7701 (1987).

¹⁷B. Keck and A. Schmid, J. Low Temp. Phys. **24**, 611 (1976).

¹⁸G. Bergmann, Phys. Rev. B **3**, 3797 (1971).