Interface optical-phonon modes in a four-layer heterostructure of polar crystals

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The equations of motion for the p-polarization field in a four-layer heterostructure (FLHS) of polar crystals are solved exactly for the interface optical-phonon modes. The eigenvectors and the interface charge densities are obtained explicitly. The dispersion relations and their plots for a FLHS and its special cases, an asymmetric trilayer heterostructure (asymmetric single quantum well) and a step quantum well, are given and discussed. We find that there are six (not eight) frequency solutions for the interface optical-phonon modes in a FLHS and that, in the long-wavelength limit, the longitudinal and transverse modes in the two side materials 1 and 4 with frequencies ($\omega_{L1}, \omega_{L4}, \omega_{T1}$, and ω_{T4}) are forbidden (the four-layer structure comprises layers 1–4); two frequency solutions are obtained in their stead. These results are due to the asymmetry of the structure. Moreover, we also find that the situation in an asymmetric trilayer heterostructure is similar to that of a FLHS. This work can be regarded as a generalization of the formalism of Chen, Lin, and George [Phys. Rev. B **41**, 1435 (1990)].

I. INTRODUCTION

Recent progress in semiconductor growth techniques has led to widespread interest in the physics of low-dimensional systems. Semiconductor heterostructures have become the objects of extensive investigation. In particular, much attention has been focused on asymmetric structures such as step quantum wells[1] and asymmetric trilayer heterostructures (also called asymmetric single quantum wells).[2] The presence of interfaces in heterostructures necessarily alters the phonon modes and their interaction with electrons. The existence of the interface phonons in heterostructures has been well recognized experimentally.[3] These interface modes are markedly different from those in bulk materials.[4–6] The interface modes have been found to play a dominant role in electron-phonon coupling and electron relaxation in quantum wells as the well width becomes small.[7] The interface phonons are also found[8] to be responsible for the pinning phenomenon observed in the transition-energy measurements[9] of magnetopolarons bound to hydrogenic impurities in quantum wells. Significant effects of interface modes on the polaron mobility and magnetopolaron resonance have also been obtained in theoretical calculations.[10] Thus, to describe fully the properties of the electron-phonon interaction in heterostructures, the contribution of the interface phonons needs to be seriously considered.[11] Furthermore, the importance of interface phonons in electron-phonon interaction and electron relaxation depends strongly on the potential parameters and boundary conditions,[7,12] hence the study of interface phonons and their interaction with electrons in different symmetric and asymmetric heterostructures is imperative for analyzing experiments and for device applications of systems with these structures.

Recently, Wendler[13] and Farias, Degani, and Hipólito[14] generalized the results of the ionic slab of Fuchs and Kliewer,[4] of Licari and Evrard,[15] and of Liang, Gu, and Lin[16] and studied the electron-phonon interaction and polaron effects in polar semiconductor bilayer systems. More recently, within the framework of the dielectric continuum model of Born and Huang,[17] solutions of the interface optical-phonon modes in a single quantum-well structure were obtained by Chen, Lin, and George,[18] by Liang and Wang,[19] and by Shi and Pan.[20] These works[18–20] generalize the results of bilayer systems of Wendler[13] and Farias, Degani, and Hipólito[14] and are necessary for further study of the electron-phonon interaction and polaron effects in a single quantum-well structure.[21] However, the previous works, except for the one on bilayers,[13,14] are all about symmetric structures.[4,15,16,18–21] To date, little work has been done about the interface phonon modes in the recently advanced asymmetric heterostructures such as asymmetric single quantum wells and step quantum wells which are of great practical importance. It is helpful to calculate and analyze the interface phonons and their interaction with electrons in these asymmetric structures. Since such structures do not have inversion symmetry, their interface phonon modes are neither symmetric nor antisymmetric about the centers of the systems. This property has a significant influence on the electron-phonon interaction because the commonly used selection rules are strictly true only for a perfectly symmetric quantum-well potential and any asymmetry in the potential will lead to a breakdown of these rules.[12] In this paper, we study a kind of asymmetric structure, i.e., a four-layer heterostructure (FLHS) which can be regarded as a generalized structure of both asymmetric single quantum wells[2] and step quantum wells.[1] Incidentally, although the interface optical phonons in a semiconductor FLHS have not been studied, similar four-layer heterostructures composed of different materials have been used in other areas of physics.[22]

In the following, we will mainly study the interface optical-phonon modes in a general FLHS. As important practical examples we will give the dispersion relations for step quantum wells and for trilayer heterostructures. We expect the interface phonons to play an important role in electron-phonon coupling and electron relaxation in these systems. The paper is organized as follows. The coupled integral equations of the p-polarization field are solved exactly for the interface optical-phonon modes in Sec. II. The dispersion relation is discussed in detail and a numerical calculation is given in Sec. III. Finally, we summarize the results obtained in this paper in Sec. IV.

II. INTERFACE OPTICAL-PHONON MODES IN A FOUR-LAYER HETEROSTRUCTURE

We consider a FLHS composed of four different polar crystals as shown in Fig. 1. Layers of materials 1, 2, 3, and 4 are located at $z<-a$, $-a\le z<0$, $0\le z<b$, and $z\ge b$, respectively. Here we take the z axis to be perpendicular to the interfaces, located at $z=-a$, 0, and b.

Taking the dispersion of the LO and TO modes into account, the dielectric function is modified as follows,[23]

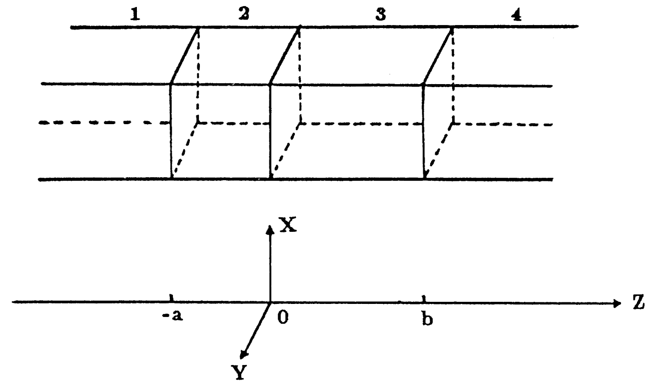

FIG. 1. Geometry of a four-layer heterostructure (FLHS) composed of materials 1, 2, 3, and 4.

$$\epsilon_\nu(\omega)=\epsilon_{\infty\nu}\frac{\omega_{L\nu}^2-\omega^2}{\omega_{T\nu}^2-\omega^2}\ , \tag{1}$$

with

$$\begin{aligned}\omega_{L\nu}^2&=\omega_{L\nu}^2(0)-v_{L\nu}^2q^2\ ,\\ \omega_{T\nu}^2&=\omega_{T\nu}^2(0)-v_{T\nu}^2q^2\ ,\end{aligned} \tag{2}$$

where $\omega_{L\nu}(0)$ and $\omega_{T\nu}(0)$ are the Brillouin-zone-center frequencies of the LO and TO modes, $v_{L\nu}$ and $v_{T\nu}$ are the corresponding acoustic velocities, q is the wave number, and ν is the material index ($\nu=1,2,3,4$). This modification, i.e., Eq. (2) is in agreement with experimental results[24] and theoretical calculations.[25]

The equations of motion for the p-polarization field have been derived as follows:[18–20]

$$4\pi\begin{bmatrix}X_\nu^{-1}(\omega) & 0\\ 0 & X_\nu^{-1}(\omega)\epsilon_\nu(\omega)\end{bmatrix}\begin{bmatrix}P_k(\mathbf{k},z)\\ P_z(\mathbf{k},z)\end{bmatrix}=-2\pi k\int_{-\infty}^{\infty}dz'e^{-k|z-z'|}\begin{bmatrix}1 & i\theta(z-z')\\ i\theta(z-z') & -1\end{bmatrix}\begin{bmatrix}P_k(\mathbf{k},z')\\ P_z(\mathbf{k},z')\end{bmatrix}\ , \tag{3}$$

where $\theta(z)$ is the step function

$$\theta(z)=\begin{cases}+1, & z>0,\\ -1, & z<0,\end{cases} \tag{4}$$

$X_\nu(\omega)$ is the isotropic dielectric susceptibility related to the dielectric function by $X_\nu(\omega)=\epsilon_\nu(\omega)-1$, $\mathbf{k}$ is the two-dimensional wave vector in the xy plane, and the x axis is chosen to lie along $\mathbf{k}$.

Differentiating Eq. (3) with respect to z once and at the same time requiring

$$\begin{vmatrix}X_\nu^{-1}(\omega) & 0\\ 0 & X_\nu^{-1}(\omega)\epsilon_\nu(\omega)\end{vmatrix}\neq 0\ , \tag{5}$$

one can obtain

$$\begin{aligned}\frac{d}{dz}P_k(\mathbf{k},z)&=ikP_z(\mathbf{k},z)\ ,\\ \frac{d}{dz}P_z(\mathbf{k},z)&=-ikP_k(\mathbf{k},z)\ .\end{aligned} \tag{6}$$

Differentiating Eq. (6) with respect to z once, we have the following equations:

$$\begin{aligned}\frac{d^2}{dz^2}P_k(\mathbf{k},z)&=k^2P_k(\mathbf{k},z)\ ,\\ \frac{d^2}{dz^2}P_z(\mathbf{k},z)&=k^2P_z(\mathbf{k},z)\ .\end{aligned} \tag{7}$$

The solutions of Eqs. (6) and (7) take the following forms. We have in the $z<-a$ region

$$\begin{aligned}P_k(\mathbf{k},z)&=iA_1e^{kz}\ ,\\ P_z(\mathbf{k},z)&=A_1e^{kz}\ ;\end{aligned} \tag{8}$$

in the $-a \le z < 0$ region

$$\begin{aligned} P_k(\mathbf{k},z) &= i[A_2 e^{kz} + B_2 e^{-kz}] , \\ P_z(\mathbf{k},z) &= A_2 e^{kz} - B_2 e^{-kz} ; \end{aligned} \tag{9}$$

in the $0 \le z < b$ region

$$\begin{aligned} P_k(\mathbf{k},z) &= i[A_3 e^{kz} + B_3 e^{-kz}] , \\ P_z(\mathbf{k},z) &= A_3 e^{kz} - B_3 e^{-kz} ; \end{aligned} \tag{10}$$

and in the $z \ge b$ region

$$\begin{aligned} P_k(\mathbf{k},z) &= -iB_4 e^{-kz} , \\ P_z(\mathbf{k},z) &= B_4 e^{-kz} . \end{aligned} \tag{11}$$

Substituting Eqs. (8)–(11) into the integral equation (3), we obtain a set of homogeneous equations for the amplitudes A_1, A_2, B_2, A_3, B_3, and B_4 of p polarization. The condition for the existence of a nontrivial solution then leads to the following dispersion relation:

$$(r_1-r_2)(r_3-r_4)(r_2r_3-1)+(r_2-r_3)(r_4-r_3)(1-r_1r_2)e^{2ka} + (r_1-r_2)(r_3-r_2)(1-r_3r_4)e^{2kb}+(r_2r_3-1)(1-r_3r_4)(1-r_1r_2)e^{2k(a+b)}=0 , \tag{12}$$

where

$$r_\nu \equiv \frac{\epsilon_\nu(\omega)+1}{\epsilon_\nu(\omega)-1}, \quad \nu=1,2,3,4 . \tag{13}$$

An equivalent expression for the dispersion relation can be derived from Eq. (12) as follows:

$$\xi = \frac{r_4-r_3}{1-r_3r_4} \frac{(r_1-r_2)(r_2r_3-1)+(r_3-r_2)(1-r_1r_2)e^{2ka}}{(r_1-r_2)(r_3-r_2)+(r_2r_3-1)(1-r_1r_2)e^{2ka}} , \tag{14}$$

where

$$\xi \equiv e^{2kb} . \tag{15}$$

The polarization amplitudes are found to satisfy the following relations:

$$\begin{aligned} A_1 &= \frac{\Delta_1}{\Delta} B_4 , \\ A_2 &= \frac{\Delta_2}{\Delta} B_4 , \\ B_2 &= \frac{\Delta_3}{\Delta} B_4 , \\ A_3 &= \frac{\Delta_4}{\Delta} B_4 , \\ B_3 &= \frac{\Delta_5}{\Delta} B_4 , \end{aligned} \tag{16}$$

where Δ, Δ_1, Δ_2, Δ_3, Δ_4, and Δ_5 are defined as follows:

$$\begin{aligned} \Delta &= (r_1-r_2)e^{-2ka}\left[r_2r_3\left[1-\frac{1}{\xi}\right] - \left[r_3^2-\frac{1}{\xi}\right]\right] \\ &\quad + (1-r_1r_2)\left[r_3\left[1-\frac{1}{\xi}\right] - r_2\left[r_3^2-\frac{1}{\xi}\right]\right] , \\ \Delta_1 &= (1-r_2^2)(1-r_3^2)\frac{1}{\xi} , \\ \Delta_2 &= (r_3^2-1)(r_1r_2-1)\frac{1}{\xi} , \\ \Delta_3 &= (1-r_3^2)(r_2-r_1)e^{-2ka}\frac{1}{\xi} , \\ \Delta_4 &= [(1-r_1r_2)(1-r_2r_3)+(r_1-r_2)(r_2-r_3)e^{-2ka}]\frac{1}{\xi} , \\ \Delta_5 &= [(r_1-r_2)(r_2r_3-1)e^{-2ka}+(r_3-r_2)(1-r_1r_2)]\frac{1}{\xi} . \end{aligned} \tag{17}$$

Equation (16) is equivalent to the boundary conditions that the wave functions have to satisfy at the interfaces. Substituting Eq. (16) into Eqs. (8)–(11) we find the eigenvectors for the interface optical-phonon modes of p polarization as follows:

$$\pi(\mathbf{k},z) = \begin{cases} \frac{\Delta_1}{\Delta} B_4 e^{kz}[i,1], & z < -a, \\ \frac{1}{\Delta} B_4[i(\Delta_2 e^{kz}+\Delta_3 e^{-kz}),(\Delta_2 e^{kz}-\Delta_3 e^{-kz})], & -a \le z < 0, \\ \frac{1}{\Delta} B_4[i(\Delta_4 e^{kz}+\Delta_5 e^{-kz}),(\Delta_4 e^{kz}-\Delta_5 e^{-kz})], & 0 \le z < b, \\ B_4 e^{-kz}[-i,1], & z \ge b. \end{cases} \tag{18}$$

Here $\pi(\mathbf{k},z)$ is the two-dimensional vector defined as

$$\pi(\mathbf{k},z)\equiv(P_k(\mathbf{k},z),P_z(\mathbf{k},z)) \ . \tag{19}$$

The coefficients B_4 in Eq. (18) can be determined by the normalization condition of eigenvectors. In this paper we take the orthonormality condition as follows:[13,18,20]

$$\int_{-\infty}^{\infty} dz \frac{\eta_\nu^{1/2}(\omega_i)\eta_\nu^{1/2}(\omega_j)}{\omega_{p\nu}^2}\pi_j^*(\mathbf{k},z)\cdot\pi_i(\mathbf{k},z)=\delta_{ij} \ , \tag{20}$$

with $\eta_\nu^{1/2}(\omega)$ being given by

$$\eta_\nu^{1/2}(\omega)\equiv\frac{1}{1+\alpha_\nu n_\nu(\lambda_{0\nu}-\lambda_\nu)} \ , \tag{21}$$

where α_ν is the electronic polarizability per ion pair, n_ν is the number of ion pairs per unit volume, and $\lambda_{0\nu}$ and λ_ν are, respectively, given by

$$\lambda_{0\nu}\equiv\frac{4\pi\omega_{0\nu}^2}{\omega_{p\nu}^2} \ ,$$

$$\lambda_\nu\equiv\frac{4\pi\omega^2}{\omega_{p\nu}^2} \ ,$$

with $\omega_{0\nu}$ being the frequency associated with the short-range force between ions and $\omega_{p\nu}$ the ion plasma frequency. According to Eq. (20), the normalization constant B_4 is given by

$$B_4=\left[\frac{k}{\Lambda}\right]^{1/2} \ , \tag{22}$$

where Λ is defined as

$$\Lambda\equiv\frac{\eta_1}{\omega_{p1}^2}\frac{\Delta_1^2}{\Delta^2}e^{-2ka}+\frac{\eta_2}{\omega_{p2}^2}\frac{1}{\Delta^2}[\Delta_2^2(1-e^{-2ka})-\Delta_3^2(1-e^{2ka})] + \frac{\eta_3}{\omega_{p3}^2}\frac{1}{\Delta^2}[\Delta_4^2(e^{2kb}-1)-\Delta_5^2(e^{-2kb}-1)]+\frac{\eta_4}{\omega_{p4}^2}e^{-2kb} \ . \tag{23}$$

The interface optical-phonon modes have no connection with bulk polarization charges because $\nabla\cdot\mathbf{P}=0$. They are accompanied by the surface charges at the interfaces. These interface charge densities σ can easily be determined by the following formula:

$$\sigma=-\mathbf{n}\cdot(\mathbf{P}_2-\mathbf{P}_1) \ , \tag{24}$$

where $\mathbf{n}$ is a unit vector perpendicular to the interface from medium 1 to 2, and $\mathbf{P}_1$ and $\mathbf{P}_2$ are polarization vectors in mediums 1 and 2, respectively. Substituting Eq. (18) into Eq. (24) we can easily obtain the interface charge densities to be

$$\sigma=\begin{cases}\dfrac{1}{\Delta}\left[\dfrac{k}{\Lambda}\right]^{1/2}[(\Delta_1-\Delta_2)e^{-ka}+\Delta_3 e^{ka}], & z=-a,\\[2ex] \dfrac{1}{\Delta}\left[\dfrac{k}{\Lambda}\right]^{1/2}[\Delta_2-\Delta_3-\Delta_4+\Delta_5], & z=0,\\[2ex] \left[\dfrac{k}{\Lambda}\right]^{1/2}\left[\dfrac{\Delta_4}{\Delta}e^{kb}-\left[\dfrac{\Delta_5}{\Delta}+1\right]e^{-kb}\right], & z=b.\end{cases} \tag{25}$$

III. DISCUSSION AND NUMERICAL CALCULATION FOR THE DISPERSION RELATION

In the above section we have obtained the dispersion relation of the interface optical-phonon modes in a FLHS, i.e., Eq. (12) or (14). In this section we will further discuss in detail the dispersion relation for some limiting cases.

In the case that material 1 is the same as material 4 we obtain from Eq. (14) a dispersion relation for commonly used step quantum wells[1] as follows:

$$e^{2kb}=\frac{r_1-r_3}{1-r_1r_3}\frac{(r_1-r_2)(r_2r_3-1)+(r_3-r_2)(1-r_1r_2)e^{2ka}}{(r_1-r_2)(r_3-r_2)+(r_2r_3-1)(1-r_1r_2)e^{2ka}} \ . \tag{26}$$

In the case of $a=b$, we obtain from Eq. (12) the following dispersion relation:

$$e^{2ka}=\frac{(r_3-r_2)[(r_4-r_3)(1-r_1r_2)+(r_2-r_1)(1-r_3r_4)]}{2(r_2r_3-1)(1-r_3r_4)(1-r_1r_2)} \pm\frac{\{(r_2-r_3)^2[(r_4-r_3)(1-r_1r_2)+(r_2-r_1)(1-r_3r_4)]^2-4(r_2r_3-1)^2(1-r_3r_4)(1-r_1r_2)(r_1-r_2)(r_3-r_4)\}^{1/2}}{2(r_2r_3-1)(1-r_3r_4)(1-r_1r_2)} \ . \tag{27}$$

In the case when $r_2=r_3$, $r_1=r_4=0$, and $k\to-k$, the above equation is reduced to

$$\omega^2=\omega_0^2+\frac{\omega_p^2}{2}\frac{\frac{1}{3}\mp e^{-2ka}}{1+2\pi\alpha n(\frac{1}{3}\mp e^{-2ka})} \ , \tag{28}$$

which is the same as Eq. (4.23b) in Ref. 15.

In the case when $a=b$, $r_2=r_3$, and $r_1=r_4$, our model reduces to the model of Refs. 18 and 19. We obtain from Eq. (12) the following dispersion relation:

$$\frac{\epsilon_1(\omega)-\epsilon_2(\omega)}{\epsilon_1(\omega)+\epsilon_2(\omega)}=\pm e^{2ka} \ , \tag{29}$$

which is the same as Eq. (23) in Ref. 18 and Eq. (12) in Ref. 19. From Eq. (29) we can derive all the consequent results of Ref. 18.

In the long-wavelength limit ($k\to0$), we obtain from Eq. (12) the following results:

$$\omega=\omega_{L2},\omega_{L3},\omega_{T2},\omega_{T3} \ , \tag{30a}$$

and

$$\omega_\pm=\left\{\frac{\epsilon_{\infty 1}(\omega_{L1}^2+\omega_{T4}^2)+\epsilon_{\infty 4}(\omega_{L4}^2+\omega_{T1}^2)}{2(\epsilon_{\infty 1}+\epsilon_{\infty 4})}\right.$$
$$\left.\pm\frac{\{[\epsilon_{\infty 1}(\omega_{L1}^2+\omega_{T4}^2)+\epsilon_{\infty 4}(\omega_{L4}^2+\omega_{T1}^2)]^2-4(\epsilon_{\infty 1}+\epsilon_{\infty 4})(\epsilon_{\infty 1}\omega_{L1}^2\omega_{T4}^2+\epsilon_{\infty 4}\omega_{L4}^2\omega_{T1}^2)\}^{1/2}}{2(\epsilon_{\infty 1}+\epsilon_{\infty 4})}\right\}^{1/2} . \quad (30b)$$

From the above two equations we can see that the frequencies of the longitudinal and transverse modes in two side materials 1 and 4 (ω_{L1}, ω_{L4}, ω_{T1}, and ω_{T4}) are forbidden; two new frequency solutions $\omega_\pm$ are obtained in their stead (the other two solutions $\omega=-\omega_\pm$, which are physically meaningless, have been abandoned). This result is completely due to the asymmetry of the FLHS. As stated above, when $a=b$, $r_1=r_4$, and $r_2=r_3$, our model reduces to that of Chen, Lin, and George, and in such a symmetric structure, the frequencies $\omega_{L1}=\omega_{L4}$ and $\omega_{T1}=\omega_{T4}$ are allowed.[18] Furthermore, if we let $\epsilon_{\infty 4}=\epsilon_{\infty 1}$, $\omega_{L4}=\omega_{L1}$, and $\omega_{T4}=\omega_{T1}$, then Eq. (30b) reduces to $\omega=\omega_{L1}, \omega_{T1}$. Thus, in this case, Eq. (30) gives $\omega=\omega_{L1}, \omega_{L2}, \omega_{L3}, \omega_{T1}, \omega_{T2}, \omega_{T3}$.

In the limit $b\to 0$ with a being finite, we obtain from Eq. (12) a dispersion relation for asymmetric trilayer heterostructures[2] as follows:

$$e^{2ka}=\frac{(r_1-r_2)(r_4-r_2)}{(1-r_2r_4)(1-r_2r_1)} . \quad (31)$$

In the long-wavelength limit ($k\to 0$), we obtain from the above equation the following result:

$$\omega=\omega_{L2},\omega_{T2},\omega_\pm , \quad (32)$$

where $\omega_\pm$ is given by Eq. (30b). This result shows that there are only four (not six) frequency solutions for an asymmetric trilayer heterostructure and that the frequencies of two side materials 1 and 4 (ω_{L1}, ω_{L4}, ω_{T1}, and ω_{T4}) are replaced by two new frequencies $\omega_\pm$. This behavior is completely similar to that of a FLHS mentioned above. In the case of $r_1=r_4$, from Eq. (31) we can again obtain the same result as Eq. (23) in Ref. 18 and Eq. (12) in Ref. 19.

In the limit $a\to\infty$ with b being finite, we obtain from Eq. (14) the following dispersion relation:

$$e^{2kb}=\frac{[\epsilon_2(\omega)-\epsilon_3(\omega)][\epsilon_4(\omega)-\epsilon_3(\omega)]}{[\epsilon_2(\omega)+\epsilon_3(\omega)][\epsilon_3(\omega)+\epsilon_4(\omega)]} . \quad (33)$$

In the case of $a\to 0$ with b being finite, the results are analogous to those of the case when $b\to 0$ with a being finite. In the case of $b\to\infty$ with a being finite, the situation is similar to that of the case when $a\to\infty$ with b being finite.

We have performed a numerical calculation for dispersion relation (12). The calculated results for a FLHS is shown in Fig. 2. Figures 3 and 4 give the plots of the dispersion relations for a step quantum well and for an asymmetric single quantum well, respectively. The parameters used in the calculations are as follows. The dielectric constant $\epsilon(\infty)=10.9-2.74x$; the Brillouin-zone-center frequencies of the LO and TO modes are, respectively, $\omega_{Lx}(0)=292.2-52.8x+14.4x^2$ (cm^{-1}) and $\omega_{Tx}(0)=268.3-5.2x-9.3x^2$ (cm^{-1}) (GaAs type). In the above calculations we have neglected the q depen-

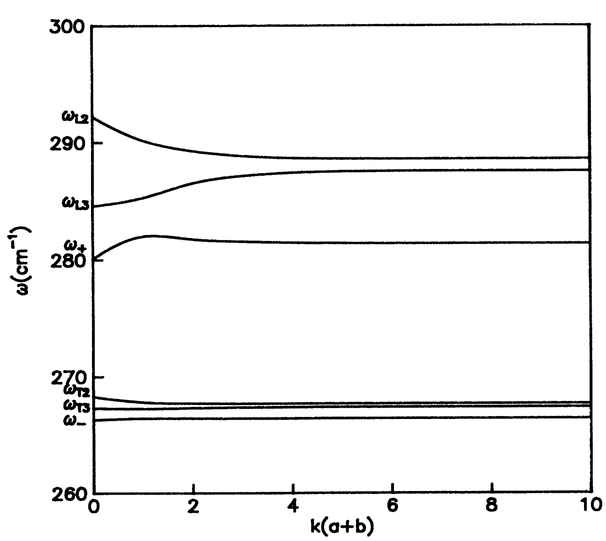

FIG. 2. Plot for the dispersion relation of the interface optical-phonon modes for a four-layer heterostructure $Al_{0.2}Ga_{0.8}As/GaAs/Al_{0.15}Ga_{0.85}As/Al_{0.3}Ga_{0.7}As$ (GaAs type) with GaAs thickness $a=60$ Å and $Al_{0.15}Ga_{0.85}As$ thickness $b=40$ Å. The abscissa is given in the dimensionless quantity $k(a+b)$ with $a+b=100$ Å.

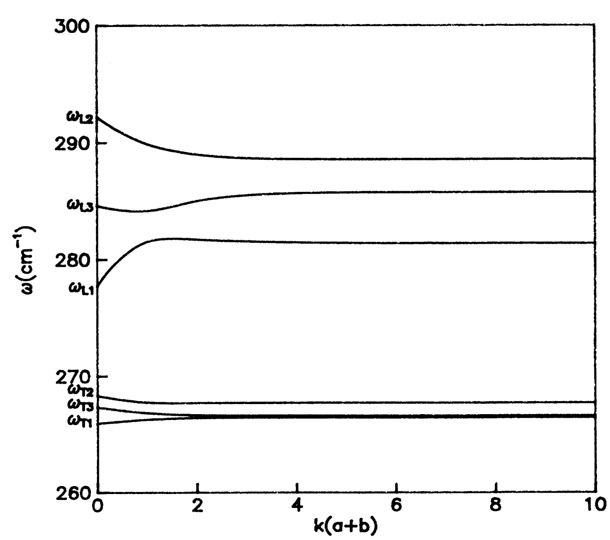

FIG. 3. As in Fig. 2, but for a step quantum well $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.15}Ga_{0.85}As/Al_{0.3}Ga_{0.7}As$ (GaAs type).

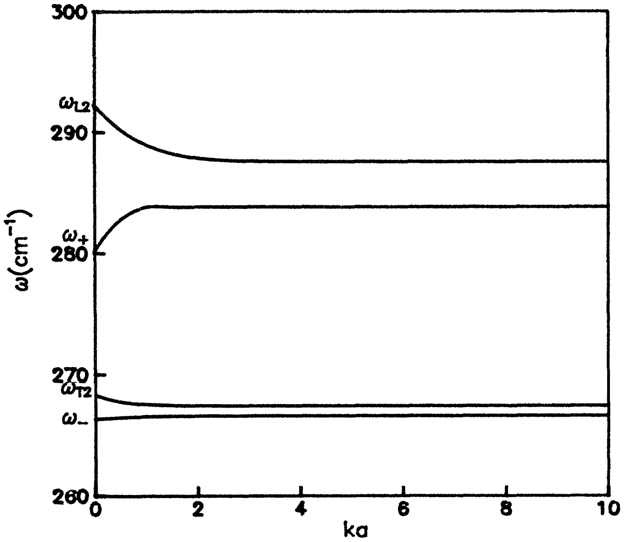

FIG. 4. As in Fig. 2, but for an asymmetric trilayer heterostructure (an asymmetric single quantum well) $Al_{0.2}Ga_{0.8}As/GaAs/Al_{0.3}Ga_{0.7}As$ (GaAs type) with GaAs thickness $a=50$ Å. The abscissa is given in the dimensionless quantity ka.

dence of the dielectric function $\epsilon_v(\omega)$ for simplicity. [Otherwise, the two-dimensional diagrams $\omega(k)$ in Figs. 2, 3, and 4 should be replaced by corresponding three-dimensional diagrams $\omega(k,q)$.] By combining Eqs. (2), (30), and (32) we can see the influence of the q dependence on the dispersion relation $\omega(k,q)$ in the long-wavelength limit.

From Figs. 2 and 3 we can see that the dispersion is obvious for small $k(a+b)$, whereas the dispersion can be neglected for $k(a+b)>6$. Furthermore, it is very interesting to note that there are only six (not eight) frequency solutions for a FLHS. This result has been justified by the above-mentioned analytical expressions (30a) and (30b) in the $k \rightarrow 0$ limit. This conclusion is of physical significance, showing that there are only six interface optical-phonon modes with different energies in a FLHS. The polaron in a FLHS may be formed by virtue of the interaction between an electron and these six interface optical-phonon modes and confined bulklike LO phonons. Figure 4 shows a similar behavior for an asymmetric trilayer heterostructure, i.e., there are only four (not six) interface optical-phonon modes with different energies in this structure.

IV. SUMMARY

We have solved exactly the equations of motion for the p-polarization field in a FLHS of polar crystals for the interface optical-phonon modes. We have obtained the eigenvectors, the dispersion relation, and the interface charge densities.

The dispersion relation is discussed for some limiting cases in detail. Our results show that in the case when $a=b$, $r_2=r_3$, $r_1=r_4=0$, and $k \rightarrow -k$, the result of Licari and Evrard[15] is obtained; when $a=b$, $r_2=r_3$, and $r_1=r_4$, the result of Refs. 18 and 19 is obtained; when $r_1=r_4$, a dispersion relation for commonly used step quantum wells[1] is obtained; in the limit $b \rightarrow 0$ with a being finite, a dispersion relation for asymmetric trilayer heterostructures[2] is obtained. The calculations for the dispersion relations of both asymmetric trilayer and four-layer heterostructures show that there are only $2(n-1)$ (not $2n$) interface optical-phonon frequency solutions for the n-layer ($n=3$ or 4) heterostructures, and that in the long-wavelength limit, the frequencies of the longitudinal and transverse modes in two side materials 1 and n (ω_{L1}, ω_{Ln}, ω_{T1}, and ω_{Tn}) are forbidden; two new frequency solutions $\omega_{\pm}$ are obtained in their stead. These results are completely due to the asymmetry of the structures.

The results obtained in this paper are useful in further investigation of the electron-phonon interaction and polaron effects in a four-layer heterostructure which can be regarded as a generalized structure of commonly used step quantum wells[1] and trilayer heterostructures.[2]

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