Hot-electron energy-loss rate in polar semiconductors in a two-temperature model

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The rate of hot-electron energy loss to the lattice in polar semiconductors is studied within a two-temperature model. We call attention to the role of the internal thermalization of the LO-phonon system in implementing this model and in establishing the concept of electron energy loss to phonons. With the help of the Feynman diagrammatic technique for the Keldysh closed-time-path Green's function, we have derived a formula for the electron-energy-loss rate, taking account of contributions from all orders of the electron-phonon interaction and including the hot-phonon effect. This formula, which carries the internal thermalization time of the LO-phonon system and the relaxation time of the whole lattice as parameters, reduces exactly to the conventional Kogan formula in the limit of weak electron-phonon coupling. In the zero-hot-phonon-effect limit (if it is allowed) this formula is shown to be equivalent to that given by Dharma-wardana [Phys. Rev. Lett. **66**, 197 (1991)]. Comparison between our formula and that of Das Sarma and Korenman [Phys. Rev. Lett. **67**, 2916 (1991)] is also given. An interesting prediction of the present formula is the low-temperature enhancement of the electron-energy-loss rate over the conventional Kogan result. A numerical calculation for a two-dimensional GaAs system shows agreement between this theory and the experimental trend.

I. INTRODUCTION

Numerous experimental and theoretical studies have been devoted to the problem of hot-electron energy relaxation in polar semiconductors.[1–14] It is generally believed that hot electrons lose their energy by first emitting longitudinal-optic (LO) phonons due to the Fröhlich interaction, except at very-low lattice temperatures ($\leq$ 15 K), where acoustic phonons are directly responsible for the carrier energy dissipation to the lattice. Treating both the electron system and the phonon system, which are assumed weakly coupled, as in equilibrium separately at electron temperature T_e and at lattice temperature T, Kogan[4] derived the formula for the energy-loss rate (ELR) of electrons to the lattice by directly using the Fermi golden rule for one phonon process:

$$\mathrm{ELR} = 2\sum_{\mathbf{q}} \Omega_{\mathbf{q}} M_{\mathbf{q}}^2 \Pi_2(\mathbf{q},\Omega_{\mathbf{q}}) \left[n\left(\Omega_{\mathbf{q}}/T\right) - n\left(\Omega_{\mathbf{q}}/T_e\right) \right]. \tag{1}$$

Here $n(x) \equiv 1/(e^x - 1)$ is the Bose function, and $\Pi_2(\mathbf{q},\omega)$ stands for the imaginary part of the electron density-density correlation function $\Pi(\mathbf{q},\omega)$. This ELR expression is for one phonon mode which has frequency $\Omega_{\mathbf{q}}$ and is coupled to electrons with the coupling matrix element $M_{\mathbf{q}}$. The above formula can be slightly generalized and written in terms of the phonon spectral function as[12]

$$\mathrm{ELR} = -\sum_{\mathbf{q}} \int (d\Omega/\pi)\Omega M_{\mathbf{q}}^2 \Pi_2(\mathbf{q},\Omega) \times \left[n\left(\Omega/T\right) - n\left(\Omega/T_e\right) \right] \mathrm{Im}D(\mathbf{q},\Omega), \tag{2}$$

where $D(\mathbf{q},\Omega)$ is the phonon propagator, and the phonon density of states, or the spectral function, is given by

$$A(\mathbf{q},\Omega) = -\pi^{-1}\mathrm{Im}D(\mathbf{q},\Omega). \tag{3}$$

For conventional bare phonons one generally assumes

$$D(\mathbf{q},\Omega) = 2\Omega_{\mathbf{q}}/(\Omega^2 - \Omega_{\mathbf{q}}^2), \tag{4}$$

$$\mathrm{Im}D(\mathbf{q},\Omega) = -\pi[\delta(\Omega - \Omega_{\mathbf{q}}) - \delta(\Omega + \Omega_{\mathbf{q}})], \tag{5}$$

and Eq. (2) reduces to Eq. (1). Of course, in deriving formulas (1) and (2) it is implicitly assumed that LO phonons thermalize rapidly among themselves and over the whole lattice with the surrounding bath. If the relaxation time for the latter process, i.e., the time for the thermalization of the whole lattice, is finite in comparison with the scattering time associated with the electron-phonon interaction, the nonequilibrium- or hot-phonon effects (LO phonons are no longer in equilibrium at the lattice temperature) have to be taken into account. The Kogan formula, extended to include hot-phonon effects accounts for the experimental electron-energy-loss data reasonably well in the temperature region of 40–150 K for wide ranges of carrier density in bulk and low-dimensional systems.[1–3,6–9] Temperature range 15–40 K, however, is an exception. There the experimental energy-loss rate per carrier shows a trend of significant enhancement over the theoretical prediction and the dominant energy-loss mechanism is still an issue of controversy.

Jain, Jalabert, and Das Sarma[10] and Das Sarma, Jain, and Jalabert[12] proposed that the couplings of LO phonons to the plasmon excitations of carriers and to

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the quasiparticle excitations of the electron gas might be responsible for the experimentally observed trend of ELR enhancement at low temperatures. They suggested the use of the phonon propagator renormalized by the electron-phonon coupling, known as the dressed-phonon or the coupled-mode propagator, described by the Feynman diagram in Fig. 1, to replace the bare phonon propagator in the Kogan formula (2). They utilized the following form for the coupled-mode propagator:

$$D_{\rm th}(\mathbf{q},\Omega)=\frac{2\Omega_{\mathbf{q}}}{\Omega^2-\Omega_{\mathbf{q}}^2-2\Omega_{\mathbf{q}}M_{\mathbf{q}}^2\,\Pi(\mathbf{q},\Omega)}, \tag{6}$$

and found that the resulting expression produces an enhancement of ELR at low electron temperatures by many orders of magnitude over what the Kogan formula (1) predicts.

This latter suggestion was challenged by Dharma-wardana,[13] who pointed out that in discussing the energy-loss problem Das Sarma and co-workers dealt with an electron system and a phonon system, respectively, at the electron temperature and at the phonon temperature. The propagator $D_{\rm th}(\mathbf{q},\Omega)$ [Eq. (6)] used in Refs. 10 and 12, however, was deduced from the Feynman diagram Fig. 1 in a thermoequilibrium electron-phonon ensemble. Taking account of nonequilibrium statistical mechanics Dharma-wardana[13] found that, instead of an enhancement over the Kogan formula, the use of the coupled-mode propagator as given by the Feynman diagram in Fig. 1, results in a suppression of the electron-energy-loss rate.

In a paper on Ref. 13 Das Sarma and Korenman[14] argued that the reason for the ELR suppression suggested by Dharma-wardana[13] is due to "the noninclusion of the decay of the emitted LO phonons into acoustic phonons (or, the so-called phonon bottle-neck or hot-phonon lifetime), ..." This criticism, in our opinion, is not pertinent, since Ref. 13 treats the "bare" LO phonons as being at the lattice temperature T, implying that emitted LO phonons decay rapidly into acoustic phonons. Inclusion of a finite decay time from LO phonons to acoustic phonons would reduce the ELR further. In this regard one should clearly distinguish two thermalization times: the internal thermalization time within the LO-phonon system and the decay time from LO phonons to acoustic phonons (or the thermalization time of the whole lattice). The well-known ELR lowering due to hot-phonon effects is conventionally referred to the finiteness of the latter relaxation time. On the other hand, we would like to point out here that the ELR suppression suggested in Ref. 13 is related to the former relaxation time. In fact, the degree of this ELR suppression depends on the magnitude of the LO-phonon internal relaxation time, or the imaginary self-energy part of the bare LO-phonon propagator. If Eq. (4) is used as the bare LO-phonon propagator without a finite imaginary self-energy, the calculation based on the Keldysh Green's functions[15–17] in a two-temperature model yields a vanishing energy-loss rate[18] from the electron system to the phonon system by way of the coupled modes as depicted in Fig. 1. This vanishing ELR is, of course, physically incorrect because of the intrinsic contradiction of assuming that there can be a phonon temperature without including a phonon-phonon scattering rate in the propagator. On the other hand, the internal relaxation time among LO phonons and the relaxation time of the whole lattice are not fully independent. The latter, for instance, must be larger than the former. One cannot achieve a proper treatment without including both effects simultaneously.

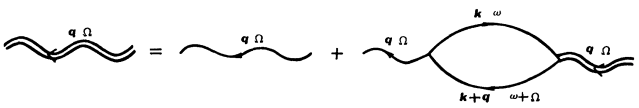

FIG. 1. Feynman diagram for the coupled-mode phonon propagator.

The purpose of this paper is to address these physical aspects of the problem: the physical implication of a two-temperature model and the related energy-loss concept, and to give a comprehensive, nonequilibrium derivation of the electron-energy-loss rate to LO phonons including all the higher-order contributions of the electron-phonon interaction within the framework of the bubble approximation (Fig. 1), i.e., the coupled modes. Both the effect of the finite internal thermalization within the LO-phonon system due to the direct and all indirect, nonelectronic (e.g., acoustic-phonon mediated) LO-phonon–LO-phonon couplings and the hot-phonon effect are taken into account. The ELR formula obtained reduces exactly to the conventional Kogan formula in the limit of weak electron-phonon coupling and is shown to be equivalent to that suggested by Dharma-wardana[13] when formally taking the zero-hot-phonon-effect limit (this limit, however, is physically appropriate only in the case of a weak electron-phonon interaction). Explicit ELR expression, which carries the LO-phonon internal scattering time and the relaxation time of the whole lattice as parameters, is given. With reasonable values of these thermalization times, this formula yields a lowering of the ELR compared to that predicted by the Kogan formula at higher electron temperatures (e.g., $T_e > 40\,\mathrm{K}$). At low electron temperatures, on the other hand, it predicts a significant ELR enhancement over the Kogan straight line in the $1/T_e$ vs ELR diagram, in agreement with the experimental trends. Comparison between our ELR formula and that of Das Sarma and Korenman[14] is also given.

II. MODEL

To understand the physical meaning of the electron energy loss we need to recall that only when two systems are *weakly coupled* can one define the energy flow from one to the other. The concept of hot-electron energy loss to LO phonons, which must be based on the distinction between an electron system and a (or a set of) phonon system(s), is physically meaningful only when the identification of the electron system and the phonon system is valid during the entire statistical process of relaxation.

By starting from the bare LO-phonon Green's function given by Eq. (4), however, the procedure described by the Feynman diagram in Fig. 1 is a *renormalization of the di-*

vergent series in powers of $M_{\mathbf{q}}^2$. This renomalization implies that what we are dealing with has become a coupled electron-phonon system, i.e., an indistinguishable entity, rather than separate electron and phonon systems, and thus the electron energy loss to LO phonons is no longer a meaningful physical quantity. The vanishing expectation of this quantity is a consequence of the renormalization. In this case the meaningful quantity is the energy loss of the coupled electron-phonon system to the surrounding bath and it is determined, as pointed by Ridley,[3] by the nonelectronic decay of these electron-phonon coupled modes.

Nevertheless, the concept of the hot-electron energy loss to LO phonons is meaningful if the internal thermalization (relaxation) in each subsystem (electron and phonon) is quick in comparison with the relaxation time due to the coupling between them, H_{ep}. In this case, a two-temperature model is applicable and H_{ep} can be treated as a perturbation. The lowest-order contribution of H_{ep} to the electron energy loss is given by the Kogan formula. Furthermore, it is possible to include higher order contributions of the electron-phonon (e-p) interaction to the electron energy loss. In fact, the Feynman diagram shown in Fig. 1 exactly represents a *sum* of the perturbative series when it *converges*. This series is indeed convergent if the bare phonon system concerned thermalizes rapidly due to internal interactions. As stated in the preceding paragraphs, a rapid (in comparison with electron-phonon coupling) thermalization among LO phonons is a requisite for identifying the LO-phonon system in the electron-energy-loss process. In order to focus our attention on the phonon system we assume that the internal thermalization of the electron system is always strong enough due to electron-electron (e-e) interactions. In the case of the electron system the rapid thermalization among electrons can be described by a finite imaginary part i/τ_{ee} in the (bare) electron retarded and advanced Green's functions,[19,20] and τ_{ee} is essentially the thermalization time of the electron system. Similarly, the internal thermalization of LO phonons should also introduce a finite imaginary part i/τ_{pp} in the (bare) phonon retarded and advanced Green's functions. This imaginary self-energy is due both to the direct (anharmonic) interaction among LO phonons (τ_{pp}^d), and to all indirect, nonelectronic (e.g., acoustic-phonon mediated) couplings among them (τ_{pp}^i): $1/\tau_{pp} = 1/\tau_{pp}^d + 1/\tau_{pp}^i$. τ_{pp} essentially represents the internal thermalization time of the LO-phonon system.

The equilibration over the whole phonon system (lattice) takes a longer time, τ_p, than that needed for thermalizing individuals or certain subsets of the phonon modes, e.g., LO phonons. Therefore, $\tau_p > \tau_{pp}$. If the thermalization over the whole lattice is determined mainly by the decay of the nonequilibrium LO phonons to, e.g., acoustic phonons, τ_p is essentially τ_{pp}^i. The idea of a finite phonon thermalizing time has been widely used in discussing hot-phonon effect in the literature. It is usually assumed that there exists a quasiequilibrium occupation number $n_{\mathbf{q}}$, or a temperature $T_{\mathbf{q}}$, for each LO phonon mode. The same model will also be used in the present paper. The relaxation time τ_p is identified sometimes in the literature as the time for the emitted nonequilibrium LO phonons to decay into acoustic phonons, which are considered always in equilibrium at the lattice temperature T. The details of the thermalization within each mode and within the whole lattice vibration system is beyond the scope of the present paper. As a preliminary analysis we will start from such a model with the above-mentioned two thermalization times included.

III. SUM OF HIGHER-ORDER TERMS IN A NONEQUILIBRIUM APPROACH

The model system is described by the Hamiltonian $H = H_e + H_p + H_{ep} + H_B$, where H_e and $H_p = \sum_{\mathbf{q}} H_{\rm ph}(\mathbf{q})$ stand for the electron and LO-phonon Hamiltonian respectively, H_{ep} is the electron–LO-phonon interaction. H_B represents the rest part of the lattice vibrations and the heat bath, and the ir coupling with LO phonons.

The energy-loss problem has been treated within the framework of the balance equation theory of Lei and Ting,[21,22] in which the coupling is assumed turned on adiabatically from the initial time $t = t_0 = 0$. To take account of hot-phonon effects we choose the initial state as that the electron subsystem is in a thermal equilibrium state at the electron temperature T_e, and LO-phonon system is in a "local" (quasi)equilibrium state with a mode-dependent temperature $T_{\mathbf{q}}$:

$$\rho_0 = \frac{1}{Z} \exp\left(-H_e/T_e\right) \exp\left(-\sum_{\mathbf{q}} H_{\rm ph}(\mathbf{q})/T_{\mathbf{q}}\right). \tag{7}$$

Since the electron-energy dissipation is through its coupling with phonons, the energy-loss rate of the electron system equals the energy increase rate of the phonon system due to electron-phonon interaction, and is given by the average

$$\mathrm{ELR} = \lim_{t\to\infty} \mathrm{Tr}[\hat{\rho}(t) W], \tag{8}$$

with the energy-transfer rate operator defined by

$$W = i[H_{ep}, H_p] = i \sum_{\mathbf{k},\mathbf{q}} M_{\mathbf{q}} \Omega_{\mathbf{q}} (b_{\mathbf{q}} - b_{-\mathbf{q}}^\dagger) c_{\mathbf{k}+\mathbf{q}}^\dagger c_{\mathbf{k}}, \tag{9}$$

where $\hat{\rho}(t)$ is the density matrix satisfying the Liouville equation $id\hat{\rho}(t)/dt = [H, \hat{\rho}(t)]$ and the initial condition $\hat{\rho}(t_0) = \hat{\rho}_0$. Note that in the balance equation theory the time evolution of the density matrix described by the Liouville equation is a virtual process, devised to minimize the time needed to reach the final steady state.[20,21] The real temporal development of the system resides in the time dependence of the velocity and temperature parameters in the initial condition.

We can write Eq. (8) in terms of Keldysh closed-time-path integration as[22]

$$\mathrm{ELR} = \mathrm{Tr}\{\hat{T}_p[\hat{\rho}_0 W(t) S_p]\}, \tag{10}$$

where $S_p = \exp[-i \int_p H_{ep}(t_1) dt_1]$ and $W(t)$ and $H_{ep}(t)$ are defined by

$$W(t) = e^{i(H_e+H_p)t} W e^{-i(H_e+H_p)t}$$

and

$$H_{ep}(t) = e^{i(H_e+H_p)t} H_{ep} e^{-i(H_e+H_p)t}.$$

The time path p is composed of a "+" branch (from $-\infty$ to $+\infty$) and a "−" branch (from $+\infty$ to $-\infty$), and $\hat{T}_p$ is the time-ordering operator on the path p.

It has been shown[23] that although two different temperatures for electrons and for phonons in the initial density matrix prevent one from directly invoking the conventional statistical Wick theorem to carry out a high-order perturbation analysis, the well-known Feynman rules and diagrammatic technique for the Keldysh closed-time-path Green's function are still valid in the balance equation theory. The only new feature that results from the two-temperature initial density matrix (7) is that all the Wick contractions associated with electron operators are taken at the electron temperature T_e, and those with phonon operators are taken at the phonon temperature $T_{\mathbf{q}}$.

The energy-loss rate can thus be expressed by appropriate Feynman diagrams in the Keldysh closed-time-path Green's function space. The Feynman rules in Keldysh space have been discussed in detail in Ref. 17. The lowest-order diagram[22] for ELR is shown in Fig. 2, in which the solid line represents the matrix electron propagator $\mathcal{G}$ and the wavy line represents the matrix phonon propagator $\mathcal{D}$:

$$\mathcal{D} = \begin{pmatrix} D^r & D^c \\ 0 & D^a \end{pmatrix}$$

and

$$\mathcal{G} = \begin{pmatrix} G^r & G^c \\ 0 & G^a \end{pmatrix}.$$

Here G^r, G^a, and G^c are the retarded, the advanced, and the correlation functions of electrons:

$$\begin{aligned} G^{r(a)}(\mathbf{k},\omega) &= (\omega - \varepsilon_{\mathbf{k}} \pm i/\tau_{ee})^{-1}, \\ G^c(\mathbf{k},\omega) &= [1-2f(\omega,T_e)]\,[G^r(\mathbf{k},\omega) - G^a(\mathbf{k},\omega)], \end{aligned} \tag{11}$$

in which $f(\omega,T_e)$ is the Fermi function of the electron subsystem at the electron temperature T_e, and the internal thermalization due to electron-electron interaction have been approximately included as an imaginary self-energy τ_{ee} in the electron retarded and advanced Green's functions. D^r, D^a, and D^c are the retarded, the advanced, and the correlation functions of LO phonons:

$$\begin{aligned} D^r(\mathbf{q},\Omega) &= (\Omega - \Omega_{\mathbf{q}} + i/\tau_{pp})^{-1} \\ &\quad -(\Omega + \Omega_{\mathbf{q}} + i/\tau_{pp})^{-1} = D^a(\mathbf{q},\Omega)^*, \end{aligned} \tag{12}$$

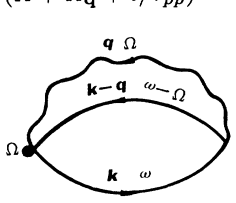

FIG. 2. Feynman diagram for the electron-energy-loss rate.

$$D^c(\mathbf{q},\Omega) = [2n(\Omega/T_{\mathbf{q}}) + 1]\,[D^r(\mathbf{q},\Omega) - D^a(\mathbf{q},\Omega)].$$

Note that the Bose function $n(\Omega/T_{\mathbf{q}}) \equiv 1/[\exp(\Omega/T_{\mathbf{q}})-1]$ is at the phonon temperature, and the internal thermalization time τ_{pp} is included as an imaginary part of the phonon self-energy.

The phonon temperature $T_{\mathbf{q}}$, or the function $n(\Omega/T_{\mathbf{q}})$, can be determined by the following consideration: Electron-phonon interaction drives LO phonons out of equilibrium with a rate of change of the phonon occupation number given by[7]

$$\left(\frac{\partial n_{\mathbf{q}}}{\partial t}\right)_{ep} = 2M_{\mathbf{q}}^2\,\Pi_2(\mathbf{q},\Omega_{\mathbf{q}})[n_{\mathbf{q}} - n(\Omega_{\mathbf{q}}/T_e)], \tag{13}$$

where $n_{\mathbf{q}} \equiv n(\Omega_{\mathbf{q}}/T_{\mathbf{q}})$ stands for the LO-phonon occupation number in state $\mathbf{q}$. At the same time there is a trend to drive these nonequilibrium phonons towards equilibrium over the whole lattice vibration system with a single lattice temperature T. Treating such an effect with a single relaxation time τ_p, we have for the steady state

$$\left(\frac{\partial n_{\mathbf{q}}}{\partial t}\right)_{ep} - \frac{1}{\tau_p}\left[n_{\mathbf{q}} - n\left(\frac{\Omega_{\mathbf{q}}}{T}\right)\right] = 0, \tag{14}$$

yielding

$$\begin{aligned} n\left(\frac{\Omega_{\mathbf{q}}}{T_{\mathbf{q}}}\right) = n\left(\frac{\Omega_{\mathbf{q}}}{T_e}\right) &+ \left[1 + \frac{\tau_p}{\tau(\mathbf{q},\Omega_{\mathbf{q}})}\right]^{-1} \\ &\times \left[n\left(\frac{\Omega_{\mathbf{q}}}{T}\right) - n\left(\frac{\Omega_{\mathbf{q}}}{T_e}\right)\right]. \end{aligned} \tag{15}$$

Here we have defined

$$1/\tau(\mathbf{q},\Omega) \equiv -2M_{\mathbf{q}}^2\,\Pi_2(\mathbf{q},\Omega)\mathrm{sgn}(\Omega) \tag{16}$$

as a wave-vector- and frequency-dependent inverse scattering time, reflecting the coupling strength between electrons and phonons. With such a definition Eq. (15) remains valid if one replaces $\Omega_{\mathbf{q}}$ by an independent parameter Ω in the whole axis:

$$\begin{aligned} n(\Omega/T_{\mathbf{q}}) = n(\Omega/T_e) &+ [1 + \tau_p/\tau(\mathbf{q},\Omega)]^{-1} \\ &\times [n(\Omega/T) - n(\Omega/T_e)]. \end{aligned} \tag{17}$$

The appearance of the imaginary self-energy, i/τ_{pp}, in the phonon retarded and advanced Green's function changes the nature of the series depicted by the Feynman diagram in Fig. 1. For weak electron-phonon interaction H_{ep}, this series converges. The Dyson equation shown in the figure exactly represents a definite sum of higher order contributions of H_{ep} in a convergent expansion. It is then possible to go beyond the Kogan formula by performing such a summation. The lowest-order contribution to the energy-loss rate shown as Fig. 2, is given by

$$\begin{aligned} \mathrm{ELR} = -\int \frac{d\omega\,d\Omega}{(2\pi)^2} \sum_{\mathbf{k},\mathbf{q}} M_{\mathbf{q}}^2\,\Omega \sum_{ll'} \mathrm{Tr}[\tilde{\gamma}^l \mathcal{G}(\mathbf{k},\omega)\tau^1\gamma^{l'} \\ \times \mathcal{G}(\mathbf{k}-\mathbf{q},\omega-\Omega)] \\ \times \mathcal{D}_{l'l}(\mathbf{q},\Omega), \end{aligned} \tag{18}$$

where the vertex functions $\tilde{\gamma}^l$ and γ^l ($l = 1,2$) are

$\tilde{\gamma}^1 = \tau_1/\sqrt{2}$, $\tilde{\gamma}^2 = \tau_0/\sqrt{2}$, $\gamma^1 = \tilde{\gamma}^2$ and $\gamma^2 = \tilde{\gamma}^1$, and τ_i (i=0, 1, 2, 3) are the conventional Pauli matrices. Note that the dot vertex in Fig. 2 carries an additional τ_1. Performing the ω integration on the right-hand side of Eq. (18) and doing the random-phase approximation summation in the electron subsystem, one gets

$$\mathrm{ELR} = \sum_{\mathbf{q}} M_{\mathbf{q}}^2 \int \frac{d\Omega}{2\pi}\,\Omega\{\tfrac{1}{2}\,\mathrm{Re}[\Pi(\mathbf{q},\Omega)\,D^c(\mathbf{q},\Omega)] + \Pi_2(\mathbf{q},\Omega)\,\mathrm{Im}D^r(\mathbf{q},\Omega) \times[2n(\Omega/T_e)+1]\}. \quad (19)$$

Here $\Pi(\mathbf{q},\Omega)$ stands for the electron retarded density-density correlation function at the electron temperature T_e with intercarrier Coulomb interaction included in the random-phase approximation:

$$\Pi(\mathbf{q},\Omega) = \Pi_0(\mathbf{q},\Omega)/[1-\nu_q\Pi_0(\mathbf{q},\Omega)], \quad (20)$$

where ν_q is the Coulomb potential and

$$\Pi_0(\mathbf{q},\Omega) = -i\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\sum_{\mathbf{k}}[f(\omega)-f(\omega+\Omega)] \times G^r(\mathbf{k}+\mathbf{q},\omega+\Omega)G^a(\mathbf{k},\omega) = 2\sum_{\mathbf{k}}\frac{f(\varepsilon_{\mathbf{k}},T_e)-f(\varepsilon_{\mathbf{k}+\mathbf{q}},T_e)}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}}+\Omega+2i/\tau_{ee}}. \quad (21)$$

Substituting the phonon Green's functions Eq. (12) into Eq. (19) and using Eq. (17), we obtain

$$\mathrm{ELR} = -\sum_{\mathbf{q}}\int\frac{d\Omega}{2\pi}\frac{|\Omega|}{\tau(\mathbf{q},\Omega)+\tau_p}\left[n\left(\frac{\Omega}{T}\right)-n\left(\frac{\Omega}{T_e}\right)\right] \times\mathrm{Im}D^r(\mathbf{q},\Omega). \quad (22)$$

This is the Kogan formula (2) modified to include finite phonon relaxation time (hot-phonon effect). If the whole lattice system thermalizes quickly ($\tau_p \ll \tau$), Eq. (22) reduces to Kogan formula (2). Experiments have shown that for low-dimensional GaAs-based heterosystems τ_p is of the same order of magnitude as τ (a few picoseconds), and the hot-phonon effects are usually important in determining the electron energy-loss rate. The function

$$\mathrm{Im}D^r(\mathbf{q},\Omega) = -\frac{4\Omega\,\Omega_{\mathbf{q}}/\tau_{pp}}{(\Omega^2-\Omega_{\mathbf{q}}^2-1/\tau_{pp}^2)^2+4\Omega^2/\tau_{pp}^2} \quad (23)$$

represents a spectrum with double peaks of finite widths. In the limit of $1/\tau_{pp} \to 0$ it becomes double δ functions given in Eq. (5), and Eq. (22) reduces to the Kogan formula (1) modified to include finite phonon relaxation time. The effect of the finite peak width on the ELR expressed in this lowest-order formula (22) is minor. There is little appreciable change in ELR when $1/\tau_{pp}$ increases from zero to the values interested in this paper.

To take account of contributions from both the lowest- and all the higher-order terms in H_{ep} to the electron-energy-loss rate, one needs only to replace the single wavy line in Fig. 2 by the double one as shown in Fig. 1. Here the double wavy line represents a propagator in the Keldysh space, i.e., a matrix Green's function of the form:

$$\mathcal{D}_{CM} = \begin{pmatrix} D_{CM}^r & D_{CM}^c \\ 0 & D_{CM}^a \end{pmatrix}.$$

The Dyson equation shown in Fig. 1 can be written as

$$\mathcal{D}_{CM}(\mathbf{q},\Omega) = \mathcal{D}(\mathbf{q},\Omega) + \mathcal{D}(\mathbf{q},\Omega)\Gamma(\mathbf{q},\Omega)\mathcal{D}_{CM}(\mathbf{q},\Omega),$$

or

$$\mathcal{D}_{CM}(\mathbf{q},\Omega) = [1-\mathcal{D}(\mathbf{q},\Omega)\Gamma(\mathbf{q},\Omega)]^{-1}\mathcal{D}(\mathbf{q},\Omega). \quad (24)$$

Here we have introduced a matrix vertex function:

$$\Gamma = \begin{pmatrix} \Gamma^r & \Gamma^c \\ 0 & \Gamma^a \end{pmatrix},$$

with the components defined by

$$\Gamma_{ll'}(\mathbf{q},\Omega) = -2iM_{\mathbf{q}}^2\int\frac{d\omega}{2\pi}\sum_{\mathbf{k}}\mathrm{Tr}[\tilde{\gamma}^l\,\mathcal{G}(\mathbf{k},\omega)\,\gamma^{l'}\,\mathcal{G}(\mathbf{k}-\mathbf{q},\omega-\Omega)],$$

or

$$\Gamma^r(\mathbf{q},\Omega) = M_{\mathbf{q}}^2\,\Pi(\mathbf{q},\Omega) = \Gamma^a(\mathbf{q},\Omega)^*, \quad \Gamma^c(\mathbf{q},\Omega) = 2iM_{\mathbf{q}}^2\,\Pi_2(\mathbf{q},\Omega)[2n(\Omega/T_e)+1]. \quad (25)$$

Equation (24) yields

$$D_{CM}^r(\mathbf{q},\Omega) = \frac{D^r(\mathbf{q},\Omega)}{1-D^r(\mathbf{q},\Omega)\Gamma^r(\mathbf{q},\Omega)} = [D_{CM}^a(\mathbf{q},\Omega)]^*, \quad (26)$$

$$D_{CM}^c(\mathbf{q},\Omega) = \frac{D^c(\mathbf{q},\Omega)+|D^r(\mathbf{q},\Omega)|^2\,\Gamma^c(\mathbf{q},\Omega)}{|1-D^r(\mathbf{q},\Omega)\Gamma^r(\mathbf{q},\Omega)|^2}. \quad (27)$$

Replacing D^c, D^r in Eq.(19) by D_{CM}^c and D_{CM}^r, we obtain the electron-energy-loss rate including the lowest-order and all the higher-order contributions in H_{ep} shown in Fig. 1:

$$\mathrm{ELR} = -\sum_{\mathbf{q}}\int\frac{d\Omega}{2\pi}\frac{|\Omega|}{\tau(\mathbf{q},\Omega)+\tau_p}\left[n\left(\frac{\Omega}{T}\right)-n\left(\frac{\Omega}{T_e}\right)\right]\frac{\mathrm{Im}D^r(\mathbf{q},\Omega)}{|1-D^r(\mathbf{q},\Omega)\Gamma^r(\mathbf{q},\Omega)|^2}. \quad (28)$$

For convenience we denote the last factor as an effective spectrum function $R(\mathbf{q},\Omega)$, which, by using (12) and (25), is explicitly written as

$$R(\mathbf{q},\Omega)=\frac{\mathrm{Im}D^r(\mathbf{q},\Omega)}{|1-D^r(\mathbf{q},\Omega)\Gamma^r(\mathbf{q},\Omega)|^2}$$
$$=\frac{-4\Omega\,\Omega_{\mathbf{q}}/\tau_{pp}}{[\Omega^2-\Omega_{\mathbf{q}}^2-1/\tau_{pp}^2+\Omega_{\mathbf{q}}/\tau_1(\mathbf{q},\Omega)]^2+[2\Omega/\tau_{pp}+\mathrm{sgn}(\Omega)\Omega_{\mathbf{q}}/\tau(\mathbf{q},\Omega)]^2}, \tag{29}$$

with

$$1/\tau_1(\mathbf{q},\Omega)\equiv -2M_{\mathbf{q}}^2\,\Pi_1(\mathbf{q},\Omega). \tag{30}$$

Equations (28) and (29) are the central results of this work.

In the limit of weak electron-phonon coupling, we can take $\Omega_{\mathbf{q}}/\tau_1\to 0$ and $\Omega_{\mathbf{q}}/\tau\to 0$ in Eq. (29) while keeping $1/\tau_{pp}$ intact, and Eq. (28) reduces exactly to the lowest-order ELR expression (22), i.e., the modified Kogan formula (2). Then, by taking the limit $1/\tau_{pp}\to 0$, we can reduce it to the modified Kogan formula (1). Such a limiting process is reasonable if the internal thermalizing trends within the LO-phonon system and within the electron system are stronger than that caused by the electron-phonon interaction. This is exactly what is required in the present model. The ELR formula (28), which represents a sum of the lowest- and all the higher-order terms, can be applied to the case of stronger electron-phonon coupling as long as the series shown in Fig. 1 remains convergent. On the other hand, if one takes the limit $1/\tau_{pp}\to 0$ first, the energy-loss rate given by Eq. (28) vanishes. It will not return to the Kogan formula for any strength of the electron-phonon interaction, however small. In this case the higher-order terms in the series are divergent, and the formal summation results in a *renormalization*. The concept of the hot-electron energy loss to LO phonons will no longer be meaningful.

IV. ELR ENHANCEMENT AT LOW TEMPERATURES

The hot-electron energy-loss rate has been calculated as a function of electron temperature T_e at lattice temperature $T=1.8\,\mathrm{K}$ from Eqs. (28) and (29) for a two-dimensional GaAs quantum-well system with well width $a=26\,\mathrm{nm}$; electron sheet density $N_s=3.9\times 10^{15}/\mathrm{m}^2$. The material parameters used in the calculation are as follows: electron effective mass $m=0.07m_e$ (m_e is the free-electron mass), LO-phonon energy $\Omega_{\mathbf{q}}=\Omega_{\mathrm{LO}}=35.4$ meV, static dielectric constant $\kappa=12.9$, and optic dielectric constant $\kappa_\infty=10.8$. In Fig. 3 we plot inverse electron temperature $1/T_e$ versus electron-energy-loss rate per carrier for three values of internal LO-phonon thermalization time $\tau_{pp}=1$, 2, and 5 ps. The hot-phonon relaxation time is assumed to be $\tau_p=7\,\mathrm{ps}$ for all the three cases. The energy-loss rate per carrier calculated from the lowest-order H_{ep} contribution, Eq. (22) (Kogan formula), is also shown for the case of $\tau_p=0$. It behaves almost like a straight line. Change in τ_{pp} value has little effect on the curve. The finite value of τ_p essentially shifts the whole $\tau_p=0$ Kogan curve down rigidly.[7,12]

The remarkable effect of the higher-order contributions of H_{ep} is the significant low-temperature enhancement of the electron-energy-loss rate over the straight-line Kogan behavior as shown in the figure. This enhancement is more pronounced at smaller τ_{pp}. In the case $\tau_{pp}=5\,\mathrm{ps}$ it begins around $T_e\simeq 40\,\mathrm{K}$. For larger τ_{pp} the enhancement appears at lower electron temperature. All the curves in Fig. 2 are obtained for fixed electron-phonon coupling (determined by the parameters specified above). Within the present two-temperature model τ_{pp} is not allowed to go to infinity. The low-temperature ELR enhancement predicted here is in agreement with the experimental data by Shah and co-workers,[1,24] which we show in the figure as closed circles.

V. COMPARISON WITH OTHER FORMULAS

The ELR formula suggested by Das Sarma and Korenman[14] was written in the same form as that suggested by Dharma-wardana:[13]

$$\mathrm{ELR}=-\sum_{\mathbf{q}}\int(d\omega/\pi)\omega M_{\mathbf{q}}^2\chi_2(\mathbf{q},\omega)\mathrm{Im}D(\mathbf{q},\omega)\Delta\bar{N}_{CM}. \tag{31}$$

Here $\chi_2(\mathbf{q},\omega)$ is our $\Pi_2(\mathbf{q},\omega)$, the imaginary part of the retarded density-density correlation function of the electron subsystem. The expressions for $\Delta\bar{N}_{CM}$ given by the

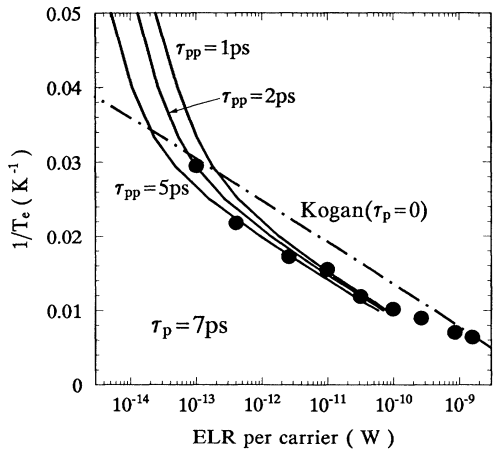

FIG. 3. Inverse electron temperature $1/T_e$ vs energy-loss rate per carrier for a two-dimensional GaAs quantum-well system at lattice temperature $T=1.8\,\mathrm{K}$. The solid curves are calculated from Eqs. (28) and (29) with $\tau_p=7\,\mathrm{ps}$, and $\tau_{pp}=1$, 2, and 5 ps, respectively. The chain line is obtained from the Kogan formula (22) with $\tau_p=0$. The closed circles are the experimental data of Shah *et al.* (Ref. 24.)

authors of Refs. 13 and 14, however, are different.

$\Delta\bar{N}_{CM}$ is given in Ref. 14 as

$$\Delta\bar{N}_{CM}=\frac{[\bar{N}(\beta_e,\omega)-\bar{N}(\beta_p,\omega)]p_2(\mathbf{q},\omega)}{p_2(\mathbf{q},\omega)+2\omega_{\mathrm{LO}}M_{\mathbf{q}}^2\chi_2(\mathbf{q},\omega)}. \tag{32}$$

If we denote

$$\omega_{\mathrm{LO}}/\tau_{\mathrm{ph}}(\mathbf{q},\omega)\equiv -p_2(\mathbf{q},\omega), \tag{33}$$

and notice definition (16) of $\tau(\mathbf{q},\omega)$, we can write the ELR formula suggested by Das Sarma and Korenman[14] in the form $[\bar{N}(\beta_e,\omega)\equiv n(\omega/T_e)$ and $\bar{N}(\beta_p,\omega)\equiv n(\omega/T)]$:

$$\mathrm{ELR}=-\sum_{\mathbf{q}}\int\frac{d\omega}{2\pi}\frac{\omega}{\tau(\mathbf{q},\omega)+\tau_{\mathrm{ph}}(\mathbf{q},\omega)}\times\left[n\left(\frac{\omega}{T}\right)-n\left(\frac{\omega}{T_e}\right)\right]\mathrm{Im}D(\mathbf{q},\omega). \tag{34}$$

Apparently, their $\tau_{\mathrm{ph}}(\mathbf{q},\omega)$, which has never been treated beyond a constant empirical parameter so far, corresponds to our relaxation time τ_p for hot LO phonons to thermalize in the whole lattice system. It is well known that this relaxation time is responsible for hot-phonon effect or bottle-neck effect. Das Sarma and Korenman[14] did not specify their expression for the "dressed phonon Green's function" $\mathrm{Im}D(\mathbf{q},\omega)$ in Ref. 14. If one uses $\mathrm{Im}D_{\mathrm{th}}(\mathbf{q},\omega)$ [Eq. (6) of this paper] for this $\mathrm{Im}D(\mathbf{q},\omega)$, i.e.,

$$\mathrm{Im}D(\mathbf{q},\omega)\rightarrow\mathrm{Im}D_{\mathrm{th}}(\mathbf{q},\omega)=\frac{-\Omega_{\mathbf{q}}^2/\tau(\mathbf{q},\omega)}{[\omega^2-\Omega_{\mathbf{q}}^2+\Omega_{\mathbf{q}}/\tau_1(\mathbf{q},\omega)]^2+[\Omega_{\mathbf{q}}/\tau(\mathbf{q},\omega)]^2}, \tag{35}$$

The resulting ELR formula does, in addition to the hot-phonon effect, lead to a large enhancement of the power loss at low temperatures, as pointed out by Das Sarma, Jain, and Jalabert.[10,12] However, this formula is still subject to the criticism by Dharma-wardana,[13] because the propagator $D_{\mathrm{th}}(\mathbf{q},\omega)$ was derived from a thermal equilibrium electron-phonon ensemble with a single temperature. Justification is needed to support this ELR formula. Another possible assignment for their dressed phonon Green's function is $\mathrm{Im}D(\mathbf{q},\omega)\rightarrow\mathrm{Im}D_{CM}^r(\mathbf{q},\omega)$, yielding another ELR formula also different from Eq. (28). This formula is of physical interest, but needs further justification.

On the other hand, the $\Delta\bar{N}_{CM}$ expression of Dharma-wardana [Eq. (8) of Ref. 13] can be written in the form

$$\Delta\bar{N}_{CM}=\frac{[\bar{N}(\beta_e,\omega)-\bar{N}(\beta_p,\omega)]\mathrm{Im}D^0(\mathbf{q},\omega)}{\mathrm{Im}D^0(\mathbf{q},\omega)+|D^0(\mathbf{q},\omega)|^2M_{\mathbf{q}}^2\chi_2(\mathbf{q},\omega)}. \tag{36}$$

If we identify their $D^0(\mathbf{q},\omega)$ as our $D^r(\mathbf{q},\omega)$, and identify their $\mathrm{Im}D(\mathbf{q},\omega)$ as our $\mathrm{Im}D_{CM}^r(\mathbf{q},\omega)$, it can be seen that the ELR formula given by Dharma-wardana[13] is equivalent to our formula (28) if the hot-phonon effect is negligible ($\tau_p\rightarrow 0$). Such a limit is meaningful only in the case of very weak electron-phonon interaction (large τ). Since τ_p is always larger than τ_{pp}, a very small τ_p implies an extremely large imaginary part in the LO-phonon propagator. This would break up the present physical picture.

VI. CONCLUSIONS

The problem of hot-electron energy loss to the lattice has been investigated using a two-temperature model. The model assumes that the internal thermalization trends in both the electron system and the (LO) phonon system are stronger than that caused by the electron-phonon interaction, which tends to thermalize among electrons and phonons. Under this circumstance, we can choose the isolated electron system and phonon system (respectively, at the electron temperature and the lattice temperature) as the unperturbed state, identify the energy flow from one to the other, and treat the electron-phonon (e-p) interaction as a perturbation. The perturbative expansion in powers of e-p coupling is a convergent series, and the sum of this series exactly takes account of the lowest order and all higher-order contributions.

On the other hand, if the trend of thermalizing among electrons and phonons (due to e-p coupling) is stronger than that within phonon system, the e-p coupling will combine electrons and phonons to form the coupled modes with a single common temperature directly, without an intermediate stage having two different temperatures. The two-temperature model is inadequate under this circumstance, and the concept of electron energy loss to phonons becomes ambiguous. The meaningful quantity should be the energy flow from the coupled electron-phonon system to the surrounding bath by some nonelectronic mechanisms.

The internal thermalization of the LO-phonon system is described in this paper by an imaginary self-energy i/τ_{pp} in the bare phonon retarded and advanced Green's functions. This self-energy includes contributions from the direct and all indirect, nonelectronic interactions among LO phonons. The ELR formula obtained by using the Feynman diagrammatic technique for the Keldysh closed-time-path Green's function takes all the orders of e-p interaction into account. This formula, which carries τ_{pp} as a parameter, gives a low-temperature enhancement of ELR over the Kogan formula, in agreement with the experimental trends.

The physical implication of two different orders of the limits has been analyzed. Taking the weak (e-p) coupling limit first and then letting $\tau_{pp}\rightarrow\infty$, our ELR formula (28) returns exactly to the Kogan formula (1). Such a limiting process keeps the two-temperature model applicable and the perturbative expansion convergent. It is inappropriate to take $\tau_{pp}\rightarrow\infty$ limit first from formula (28) while keeping e-p coupling finite. By doing so we have performed a formal summation of the divergent series and thus renormalized electron and phonon systems into a coupled one. The investigation of the energy loss from this coupled system is highly desirable, but is beyond the regime of a two-temperature model.

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