Fractional quantum numbers and electron correlations

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It will be shown that sequences of fractional quantum numbers (in this case occupation numbers) appear to be important for the instabilities of a one-dimensional Hubbard model in the strong-correlation limit ($\Delta/U \ll 1$, but finite) if only the highest commensurability is observed and besides the $2k_F$ instability the $2lk_F$ instabilities are also taken into consideration. These sequences obey an odd-denominator rule. Further, it will be shown that an interacting two-dimensional electron gas in a strong external magnetic field and under the influence of a weak sinusoidal substrate potential may be described by a one-dimensional Hubbard model which also gives fractional quantum numbers (in this case fractional filling factors) again exhibiting an odd-denominator rule.

I. INTRODUCTION

Fractional quantum numbers have become interesting to solid-state physicists since the discovery of the fractional quantum Hall effect (FQHE) by Tsui, Störmer, and Gossard[1] in 1982. The FQHE consists of the formation of plateaus in the Hall resistivity ρ_{xx} at fractional Landau-level occupancies $\nu = n_e hc/eB = p/q$ for integers p and q, where q is odd (an exception is $\nu = \frac{5}{2}$) and n_e is the electron density per unit area. It is this odd-denominator rule which has aroused considerable general interest and has given rise to fascinating theoretical developments, from which Laughlin's theory[2] and its connection to fractional statistics[3] should be mentioned.

It is the aim of the present paper to contribute to the discussion of the sequences of fractional quantum numbers with an odd denominator. To this purpose, we study in Sec. II an approximate solution of the one-dimensional paramagnetic Hubbard model in the strong-correlation limit and we shall obtain odd-denominator sequences of fractional occupation numbers n (n is the number of electrons per lattice site in this case) if we assume that not only $2k_F$ but also $2lk_F$ instabilities are allowed and if only the highest commensurability is considered. In Sec. III it will be shown that a two-dimensional interacting electron gas in a strong magnetic field B and under the influence of a weak sinusoidal substrate potential may be described by a one-dimensional Hubbard model. Instabilities may then be discussed under conditions similar to those of Sec. II and will be found to appear at fractional quantum numbers ν (ν being the usual Landau-level filling factor). Section IV contains concluding remarks.

II. FRACTIONAL OCCUPATION NUMBERS IN THE ONE-DIMENSIONAL HUBBARD MODEL

Using the Wannier representation, the Hubbard model is defined by the Hamiltonian[4]

$$H = \sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{U}{2} \sum_{i\sigma} n_{i\sigma} n_{i-\sigma} , \tag{1}$$

where the matrix element t_{ij} (hopping integral) for the transfer of an electron from the lattice site i to the site j is nonvanishing only for pairs of sites which are nearest neigbors. t_{ij} is related to the electron dispersion relation $\epsilon(k) = -(\Delta/2)\cos ka$ by a Fourier transform, where the bandwidth is given by $\Delta = -4t$. $c_{i\sigma}$ ($c_{i\sigma}^{\dagger}$) is the annihilation (creation) operator of an electron in the Wannier state $|i\sigma\rangle$. U is the Coulomb interaction matrix element for electrons on the same lattice site.

The pseudoparticle energies of the system described by (1) may be obtained from the Green's function,

$$G_{ij}^{\sigma}(\omega) = \langle\langle c_{i\sigma}; c_{j\sigma}^{\dagger} \rangle\rangle_{\omega} , \tag{2}$$

where the symbol $\langle\langle A;B \rangle\rangle_{\omega}$ stands for the Fourier transform with respect to the time of the retarded anticommutator Green's function,

$$\langle\langle A(t);B(t') \rangle\rangle = -i\Theta(t-t')\langle [A(t),B(t')]_{+} \rangle , \tag{3}$$

where $\langle\ \rangle$ denotes the mean value for the grand canonical ensemble.

In the strong-correlation limit ($\Delta/U \ll 1$, but finite), an approximate solution for the Green's function (2) may easily be obtained by the decoupling procedure for the sequence of Green's functions introduced by Hubbard.[4(a),4(b)] This theory is essentially an expansion about the atomic limit[4(c)] of the model (1). The result for the Fourier transform of the Green's function (2) may be written as

$$G_{k\sigma}(\omega) = \frac{1}{F_{\sigma}(\omega+\mu) - \epsilon(k)} , \tag{4}$$

where μ is the chemical potential and the function $F_{\sigma}(\omega)$ is given by

$$F_{\sigma}(\omega)^{-1} = \frac{1-n_{-\sigma}}{\omega} + \frac{n_{-\sigma}}{\omega - U} . \tag{5}$$

Furthermore, it is $n_{\sigma} \equiv \langle n_{i\sigma} \rangle$ due to translational symmetry. After a simple expansion, the Green's function (4) may be represented in the form

$$G_{k\sigma}(\omega)=\frac{1-n_{-\sigma}}{\omega-[(1-n_{-\sigma})\epsilon(k)-\mu]}+\frac{n_{-\sigma}}{\omega-[n_{-\sigma}\epsilon(k)+U-\mu]}\ , \tag{6}$$

which has the following well-known interpretation: in the limit $\Delta/U\to 0$, the pseudoparticle spectrum consists of two bands separated from each other by a large correlation gap of the order of the Coulomb interaction matrix element U. In the paramagnetic case, $n_\sigma=n_{-\sigma}=n/2$, one concludes from (6) that the Fermi vector for the lower correlation band ($n\leq 1$) is given by[5]

$$k_F^L a=\frac{\pi n}{2-n} \tag{7}$$

and by

$$k_F^U a=\frac{2\pi(n-1)}{n} \tag{8}$$

for the upper correlation band ($n\geq 1$). It should be interesting to compare the solutions (6)–(8) with exact results. Lieb and Wu[6] succeeded in exactly solving the one-dimensional Hubbard model for the half-filled band case, i.e., $n=1$. Unfortunately, this value of n is not interesting in the context of the present paper. Later, Shiba[7] and Carmelo and Baeriswyl[8] extended the Lieb and Wu theory for general electron densities n. Using numerical methods, Shiba obtained a relation between the cutoff parameter Q, which corresponds to the Fermi wave number, and the electron concentration n. The results are graphically represented in Fig. 1 of his paper.[7] Carmelo and Baeriswyl[8] derived an expansion in powers of t/U which gives to lowest order,

$$Q=\pi n-\frac{4t}{U}n\ \ln 2\ \sin\pi n\ . \tag{9}$$

The convergence is best for small n. Likewise, our expression (7) is a result of an expansion

$$k_F^L a=\frac{\pi n}{2-n}-\frac{2tn}{U}\sin\frac{\pi n}{2-n}+O(t^2/U^2)\ . \tag{10}$$

For $n\to 1$ and $(t/u)\to 0$ (the hopping matrix element t of Carmelo and Baeriswyl is twice the size of ours), Q and $k_F^L a$ are in agreement, whereas they disagree for $n\approx 0$. Similarly one may conclude from the numerical work of Shiba that our results for $k_F^L a$ apply for densities $n\gtrsim 0.25$.

The Fermi vectors (7) and (8) are connected to an instability of the wave vectors,[9,10]

$$k_p=\begin{cases}2k_F^{U,L} & \text{if } k_F^{U,L}\leq\pi/2a\\ \dfrac{2\pi}{a}-2k_F^{U,L} & \text{if } k_F^{U,L}\geq\pi/2a\ ,\end{cases} \tag{11}$$

which may lead to a periodic lattice distortion of the wavelength $\lambda_p=2\pi/k_p$ and to the formation of a corresponding charge-density wave. The physical origin for these instabilities are intraband single-particle transitions within the Hubbard correlation bands defined by Eq. (6). Among all wave vectors k_p, the most distinguished one is the wave vector which gives the highest commensurability (according to the formation of a $\times 2$ superlattice). This case is realized by the occupation number $n=\frac{2}{3}$ for the lower Hubbard correlation band and by $n=\frac{4}{3}$ for the upper one, respectively. If not only the $2k_F$ instability, but also $2lk_F$ instabilities (l integer), are assumed to occur as an effect of multipair excitations,[11–13] Eq. (11) must be generalized as follows:

$$k_p^{U,L}=\begin{cases}2lk_F^{U,L}-2N\dfrac{\pi}{a} & \text{if } 2N\dfrac{\pi}{a}\leq 2lk_F^{U,L}\leq(2N+1)\dfrac{\pi}{a}\\ & (N\geq 0)\\ 2N\dfrac{\pi}{a}-2lk_F^{U,L} & \text{if } (2N-1)\dfrac{\pi}{a}\leq 2lk_F^{U,L}\leq 2N\dfrac{\pi}{a}\\ & (N\geq 1).\end{cases} \tag{12}$$

Obviously, for $l=1$ and $N=0$, the first line of (12) coincides with the first line of (11). The second lines of (11) and (12) coincide with each other for $l=1$ and $N=1$. The condition of highest commensurability (i.e., $k_p=\pi/a$) gives for both parts of (10),

$$k_F^{U,L}=\frac{(2N+1)\pi}{2la}\ . \tag{13}$$

This equation is to be combined with Eqs. (7) and (8) leading to the distinguished occupation numbers at which the instability of highest commensurability appears. One easily obtains for the lower correlation band,

$$n=\frac{4N+2}{2l+2N+1}\ , \tag{14}$$

for $N\geq 0$, $l\geq 1$ under the condition $0<n<1$. For the upper correlation band,

$$n=\frac{4l}{4l-(2N+1)}\ , \tag{15}$$

for $N\geq 0$, $l\geq 1$ under the condition $1<n<2$. Equation (14) gives the following sequence of occupation numbers n of the lower Hubbard correlation band:

$$\begin{aligned}&\tfrac{2}{3},\tfrac{2}{5},\tfrac{2}{7},\tfrac{2}{9},\tfrac{2}{11},\tfrac{2}{13},\ldots,\\ &\tfrac{6}{7},\tfrac{6}{9},\tfrac{6}{11},\tfrac{6}{13},\tfrac{6}{15},\tfrac{6}{17},\ldots,\\ &\tfrac{10}{11},\tfrac{10}{13},\tfrac{2}{3},\tfrac{10}{17},\tfrac{10}{19},\tfrac{10}{21},\ldots.\end{aligned} \tag{16}$$

From (15), one obtains for the upper Hubbard correlation band,

$$\begin{aligned}&\tfrac{4}{3},\tfrac{8}{7},\tfrac{12}{11},\tfrac{16}{15},\tfrac{20}{19},\ldots,\\ &\tfrac{8}{5},\tfrac{4}{3},\tfrac{16}{13},\tfrac{20}{17},\tfrac{8}{7},\ldots,\\ &\tfrac{12}{7},\tfrac{16}{11},\tfrac{4}{3},\tfrac{24}{19},\tfrac{28}{23},\ldots,\end{aligned} \tag{17}$$

etc. The sequence (17) represents the value of the total charge per atom. If only the net charge of the upper band ($n'=n-1$) is considered, one obtains instead of (17)

$$\begin{aligned}&\tfrac{1}{3},\tfrac{1}{7},\tfrac{1}{11},\tfrac{1}{15},\tfrac{1}{19},\ldots,\\&\tfrac{3}{5},\tfrac{1}{3},\tfrac{3}{13},\tfrac{3}{17},\tfrac{1}{7},\ldots,\\&\tfrac{5}{7},\tfrac{5}{11},\tfrac{1}{3},\tfrac{5}{19},\tfrac{5}{23},\ldots,\\&\tfrac{7}{9},\tfrac{7}{13},\tfrac{7}{17},\tfrac{1}{3},\ldots.\end{aligned} \tag{18}$$

The obtained sequences of occupation numbers (16)–(18) not only exhibit an odd-denominator rule, they also contain many values which are known for the fractional quantum Hall effect.[13–15] It should be noted that some fractions (e.g., $\frac{2}{3}$ or $\frac{8}{7}$) are repeated: this is originated by the periodicity of the Hubbard model. Other values do not appear because of the selection of only those wave numbers which correspond to the highest commensurability. Fractions near zero are not very reliable and should be excluded from further discussions, since they are connected to Fermi wave vectors which are not explained by the more exact theory of Carmelo and Baeriswyl.[8] Nevertheless, the fractions described by (16)–(18) suggest the following question: is there a connection between the one-dimensional Hubbard model and the two-dimensional interacting electron gas in an applied strong magnetic field? The next section will give a positive answer to this question.

III. A MAGNETO-HUBBARD MODEL

A model may be constructed starting from a two-dimensional electron system in an applied external magnetic field $\mathbf{B}=(0,0,B)$ and in an additional weak external sinusoidal potential $V(x)=V_0\cos[(2\pi/\lambda_0)x]$ as has been discussed in a somewhat different way earlier by other authors.[16–18] In the following, we use the complete set of eigenfunctions of the two-dimensional Hamiltonian,

$$H_0=\frac{1}{2m^*}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}(\mathbf{x})\right]^2-g^*\mu_B\sigma B\ , \tag{19}$$

where $\mathbf{A}(\mathbf{x})$ is the vector potential $\mathbf{A}(\mathbf{x})=B(0,x,0)$, m^* is the effective mass, and g^* is the effective Landé factor.

The one-particle part H_1 of the second-quantized Hamiltonian will then be given by,

$$\begin{aligned}H_1=&\sum_{nk_y\sigma}\epsilon(n,\sigma)c^\dagger_{nk_y\sigma}c_{nk_y\sigma}\\&+\sum_{\substack{n,n'\\k_y\sigma}}\langle n,k_y,\sigma|V(x)|n',k_y,\sigma\rangle c^\dagger_{nk_y\sigma}c_{n'k_y\sigma}\ .\end{aligned} \tag{20}$$

In the limit of a high magnetic field B, which will be considered in the following, the matrix elements in the second term of (20) assume a very simple form,[19]

$$\begin{aligned}\lim_{B\to\infty}&\langle n,k_y,\sigma|V(x)|n',k_y,\sigma\rangle\\&=\delta_{n,n'}V_0e^{-(\pi a/\lambda_0)^2}\cos(2\pi k_y a^2/\lambda_0)\ ,\end{aligned} \tag{21}$$

where $a^2=\hbar c/eB$ is the square of the magnetic length. The right-hand side of Eq. (21) defines a "magnetolattice" constant,

$$c\equiv 2\pi a^2/\lambda_0\ . \tag{22}$$

Another characteristic length of the system is defined by

$$b\equiv L_y/N_y\ , \tag{23}$$

where L_y is the dimension of the system in the y direction and N_y is the degeneracy of the Landau one-particle energy level in the case $V(x)\equiv 0$,

$$N_y=\frac{\Omega_2}{2\pi a^2}\equiv\frac{L_xL_y}{2\pi a^2}\ . \tag{24}$$

From this last equation, it follows immediately that b may also be expressed by $b\equiv L_y/N_y=2\pi a^2/L_x$. Furthermore, one should note that $b/c=\lambda_0/L_x=1/n$, where n is an integer.

The Hamiltonian (20) essentially describes one-dimensional motion. Hence, it is consistent to introduce a Wannier representation based on the characteristic length b defined by Eq. (23),

$$c_{0k_y\sigma}=\frac{1}{\sqrt{N_y}}\sum_l e^{ik_yR_l}c_{0l\sigma}\ , \tag{25}$$

which will be substituted into the Hamiltonian (20). We note that (25) is a formal transformation, which is applied in order to obtain an expression of the Hamiltonian, which is more appropriate for the purpose of this paper. The underlying single-particle wave functions are those of the Hamiltonian (19) as before. Furthermore, in Eq. (25) and in the following, only the lowest-lying Landau level $n=0$ is considered, as it may be justified in the case of very high magnetic fields. Taking into account Eq. (21), one obtains

$$H_1=\sum_{lm\sigma}\left\{\left[\frac{\hbar\omega_c^*}{2}-g^*\mu_B\sigma B\right]\delta_{l,m}+t_{lm}\right\}c^\dagger_{0l\sigma}c_{0m\sigma}\ , \tag{26}$$

where the "hopping" matrix element is given by

$$t_{lm}=\tfrac{1}{2}V_0e^{-(\pi a/\lambda_0)^2}\delta_{l-m,n}\ . \tag{27}$$

This last result means that electron "hopping" is allowed only if the sites l and m are separated by the "magneto-lattice" constant $c=2\pi a^2/\lambda_0$. Furthermore, the effective cyclotron frequency ω_c^* is defined by $\omega_c^*=eB/m^*c$.

The electron-electron interaction may be described by the Hamiltonian,[20,21]

$$H_2=\frac{U}{2}\sum_{i\neq j}\delta(\mathbf{x}_i-\mathbf{x}_j)\ . \tag{28}$$

Again, using the wave functions of the one-particle Hamiltonian (19), the Hamiltonian (28) is expressed in the second quantization representation by

$$\begin{aligned}H_2=&\frac{U}{2\sqrt{2\pi}aL_y}\sum_{\substack{k_yk_y'p_y\\\sigma\sigma'}}e^{-(a^2/2)[(k_y-k_y'-p_y)^2+p_y^2]}\\&\times c^\dagger_{0k_y\sigma}c^\dagger_{0k_y'\sigma'}c_{0k_y'+p_y\sigma'}c_{0k_y-p_y\sigma}\ ,\end{aligned} \tag{29}$$

where only the lowest-lying Landau level has been taken into consideration. The limit of a high magnetic field B, which will be considered further, is delicate. So, the exponential function in (29) may not simply be assumed to be equal to one for $B\to\infty$ (or $a\to 0$), since the wave numbers approach infinitely high values in this limit. Nevertheless, the transformation of (29) according to (25) yields a "localized" representation with matrix elements,

$$W_{ijlm}=\frac{U}{2\sqrt{2\pi}aL_x}\Delta(i+j-l-m)e^{-(1/2a^2)[R_{il}^2+R_{jl}^2]}\ , \tag{30}$$

where $\Delta(n-m)$ is the Kronecker symbol. These matrix elements are to be compared for different "sites" i,j, etc., at a fixed value of the magnetic length $a\equiv\sqrt{\hbar c/eB}$. Obviously, the on-site matrix element W_{iiii} is much greater in magnitude than any other element of W_{ijlm}. Hence, we may approximate the interaction Hamiltonian H_2 by taking into account only the contribution of the intrasite interaction W_{iiii}. We mention that this approximation is in close analogy to the original derivation of the Hubbard model.[4(a)] Consideration of further matrix elements would yield corrections to the solution below (34) which, however, are small. So we obtain the approximate interaction part of the Hamiltonian,

$$H_2=\frac{\tilde{U}}{2}\sum_{l,\sigma}n_{0l\sigma}n_{0l-\sigma}\ , \tag{31}$$

which is of the usual Hubbard form. In (31), the magnetocorrelation matrix element $\tilde{U}$ has been introduced as

$$\tilde{U}=\frac{U}{\sqrt{2\pi}aL_x}\equiv\frac{Ub}{(\sqrt{2\pi}a)^3}\ . \tag{32}$$

Combining (26) and (31), the complete Hamiltonian is obtained,

$$\begin{aligned}H&=H_1+H_2\\&=\sum_{lm\sigma}\left\{\left[\frac{\hbar\omega_c^*}{2}-g^*\mu_B\sigma B\right]\delta_{lm}+t_{lm}\right\}c_{0l\sigma}^{\dagger}c_{0m\sigma}\\&\quad+\frac{\tilde{U}}{2}\sum_{l,\sigma}n_{0l\sigma}n_{0l-\sigma}\ .\end{aligned} \tag{33}$$

It is important to note that in many substances, such as GaAs/$Ga_xAl_{1-x}As$ systems, the first term of the diagonal part of (26) is large in comparison with the second one,

$$\frac{\hbar\omega_c^*}{2}\gg g^*\mu_B\sigma B\ , \tag{34}$$

so that spin-split effects may be neglected in the following. Furthermore, the hopping matrix element t_{lm} is of the order of the amplitude of the substrate potential V_0, such that we have the situation of the strong-correlation limit of the Hubbard model, since

$$\tilde{U}\equiv\frac{Ub}{(\sqrt{2\pi}a)^3}\gg V_0 \tag{35}$$

for $B\to\infty$. In other words, the system described by the Hamiltonian (33) is in complete analogy to that described by the usual Hubbard model (1) in one dimension. The only difference is the term $\hbar\omega_c^*/2$ (the "atomic level") which has no influence on the approximations. Hence, one obtains the solution as an expansion about the "atomic limit" [here $(U/\sqrt{2\pi}aL_x)\to\infty$ for $B\to\infty$] for the Green's one-particle function,

$$\begin{aligned}G_{k_y\sigma}(\omega)=&\frac{\nu/2}{\hbar\omega+\mu-\left[\frac{\hbar\omega_c^*}{2}+\tilde{U}+\frac{\nu}{2}t(k_y)\right]}\\&+\frac{1-\nu/2}{\hbar\omega+\mu-\left[\frac{\hbar\omega_c^*}{2}+\left[1-\frac{\nu}{2}\right]t(k_y)\right]}\ ,\end{aligned} \tag{36}$$

where the dispersion relation $t(k_y)$ is given by

$$t(k_y)=V_0e^{-(\pi a/\lambda_0)^2}\cos(2\pi k_y a^2/\lambda_0)\ , \tag{37}$$

in agreement with Eqs. (20) and (21). The solution (36) is exact in the limit $V_0=0$, giving the levels $\hbar\omega_c^*/2$ and $\hbar\omega_c^*/2+\tilde{U}$, containing the $(1-\nu/2)$ and $\nu/2$ states, respectively. The term $t(k_y)$ causes a broadening of these two levels. μ represents the chemical potential and the factor ν, which arises from the Hubbard decoupling procedure, is defined by the mean value

$$\frac{\nu}{2}=\langle n_{0i\sigma}\rangle\ , \tag{38}$$

which is supposed to be independent of the lattice site i. Hence, one obtains

$$\nu=2\langle n_{0i\sigma}\rangle=\frac{1}{N_y}\sum_{i\sigma}\langle n_{0i\sigma}\rangle=\frac{N_e}{N_y}=n_e\frac{hc}{eB}\ , \tag{39}$$

where n_e is the electron density in two dimensions. The last equality shows that ν in the context of the presented magneto-Hubbard model (33) is identical with the usual filling factor of the Landau levels.

From Eq. (36), it follows easily that Fermi wave numbers are given by

$$k_F^L b=\frac{\pi\nu}{2-\nu} \tag{40}$$

for the lower Hubbard magnetosplit band and by

$$k_F^{\mu}b=\frac{2\pi(\nu-1)}{\nu} \tag{41}$$

for the upper one. The results (40) and (41) correspond to Eqs. (7) and (8) for the one-dimensional Hubbard model, so that all conclusions concerning the instabilities (16)–(18) may also be drawn in the present case. The main difference is the replacement of the electron number per lattice site n by the filling factor ν of the Landau levels.

It should be noticed that the solutions of the Hubbard model (6) and of the magneto-Hubbard model (36) had been based on the assumption of the spin-unpolarized state. This is not always the case for the FQHE, but it is correct at least for a number of special filling factors ν.

So, Halperin[22] was the first who proposed for $\nu=\frac{2}{5}$ a spin-unpolarized ground-state wave function. Chakraborty discusses in recent papers[15] that the filling factors $\nu=\frac{2}{3}$, $\frac{4}{3}$, $\frac{2}{5}$, and $\frac{8}{5}$ should correspond to spin-unpolarized states and found this to be in agreement with experimental works.[23,24] Even these filling factors $\frac{2}{3}$, $\frac{4}{3}$, $\frac{2}{5}$, and $\frac{8}{5}$ are fundamental numbers in our approach since $\frac{2}{3}$ and $\frac{4}{3}$ correspond to simple $2k_F$ instabilities of the model which are followed by $\frac{2}{5}$ and $\frac{8}{5}$.

IV. CONCLUDING REMARKS

We note the following.

(1) The instabilities of the one-dimensional Hubbard model have been discussed, generalizing the usual $2k_F$ instability to a $2lk_F$ instability and considering the possibility of adding reciprocal-lattice vectors. Further assuming a maximum commensurability, an odd-denominator rule for the electron number per lattice site has been obtained.

(2) A two-dimensional electron system in a weak external sinusoidal potential $V(x)$ and under the influence of an applied strong external magnetic field B has been investigated. In the limit $B\rightarrow\infty$, this system may be described approximately by a one-dimensional Hubbard model. If spin splitting may be neglected (as it is justified, e.g., in the case of a very low effective mass $m^*\ll m$), sequences $\nu=p/q$ obeying an odd-denominator rule are obtained again, where ν now represents the filling factor of the Landau levels. The special values $\nu=\frac{2}{3},\frac{4}{3},\frac{2}{5},\frac{8}{5}$ are in agreement with Chakraborty's works.

(3) About half of the fractions $\nu=p/q$ obtained in this paper agree with the observed fractions of the FQHE.[14,15] It may be interesting that even-denominator fractions might be determined (together with odd-denominator fractions), when electron-electron interactions are neglected.

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