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Correspondance between correlation functions and enhanced backscattering peak for scattering from smooth random surfaces

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We study theoretically the scattering of light from a smooth, two-dimensional, random metal surface, using perturbation theory. We find that there is a good correspondence between the enhanced backscattering intensity function and the cross-polarized amplitude-amplitude correlation function (or the resultant intensity-intensity correlation function). We argue on physical grounds that this mapping should also be valid in the rough surface limit. Such a mapping enables one to have another independent way to measure the information contained in the enhanced backscattering peak about the random rough surface, using correlations of speckle intensities away from the backscattering direction.

Following the first experimental observation and theoretical understanding of the enhanced backscattering peak for light scattering from volume multiple scattering colloidal suspension systems,[1,2] and from two-dimensional random rough surfaces,[3] the enhanced backscattering peaks present in scattering from random surfaces have been intensively investigated theoretically and experimentally.[4] A particularly attractive feature of this phenomenon from the point of view of potential applications is its easy access from the incident direction where one can locate both the source of the wave as well as detectors. As is by now well established (at least for random volume scattering systems), the enhanced backscattering peak is due to the constructive interference between the various random multiply scattered light paths and their "time-reversed" partners. Such a special constructive interference can only happen near the backscattering direction, since only in this (far-field) direction do all the partial waves that arrive at the sample surface and are eventually reflected off after several scattering events, and their time-reversed partners, have exactly the same phase factor, independent of the arrival point and the exit point of a given partial wave (see Fig. 1). In fact, since the typical spatial separation between the arrival point and the exit point of any scattered partial wave for random volume scattering is a couple of the transport scattering mean free path l^*, this immediately gives the angular width of the enhanced backscattering peak, namely, $\delta\theta_{\rm ebs} \approx 1/kl^*$, where $k = \omega/c$ is the light wave number. In volume multiple scattering systems, it is possible to use the diffusion approximation to estimate the intensity function (line shape) $I_{\rm ebs}(\delta\theta)$, where $\delta\theta$ is the difference between the angles of observation and incidence, at least for $\delta\theta < \delta\theta_{\rm ebs}$, since in this limit the peak intensity comes from rather long scattering paths whose path lengths $s \gg l^*$, so that the order of multiple scattering, $n \approx s/l^*$, can be taken as $n \gg 1$, which is the required condition for using the diffusion approximation.[2]

For random surface scattering systems, the situation is quite a bit more complex. Here, as the scattered partial waves can easily propagate into the bulk on either side of the random surface, the typical order of multiple scattering partial waves is only two or three; thus it is not possible to use the simple diffusion propagators, which only depend on l^* as its parameter, to account for the intensity function $I_{\rm ebs}(\delta\theta)$. In fact, for surface scattering problems the enhanced backscattering function depends on the details of the random surface profile and the scattering properties of the bulk materials involved. This fact makes the problem much more difficult, but it certainly gives rise to a rich variety of possibilities, and opens the prospect for using the enhanced backscattering peak intensity function to characterize the geometrical and scattering properties of the random surface.

The existence of intensity-intensity correlations in the multiple-scattering regime is another interesting phenomenon, originally predicted for the case of volume scattering.[5] Recent numerical studies[6] confirm the presence of multiple-scattering correlations for scattering from random, perfectly conducting surfaces.

The purpose of this paper is to establish analytically that there is a direct one-to-one correspondence between the enhanced backscattering intensity function, $I_{\rm ebs}(\delta\theta)$, and the first-order angular intensity-intensity correlation function,

$$C^{(1)}(\delta\theta) \equiv \langle \delta I(\theta_{\rm in},\theta_{\rm out})\delta I(\theta_{\rm in}+\delta\theta,\theta_{\rm out}+\delta\theta)\rangle^{(1)}, \quad (1)$$

for a cross-polarized configuration between the incident

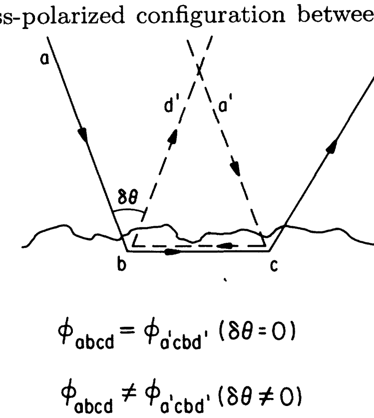

FIG. 1. Illustration of the physical origin of the enhanced backscattering peak as a time-reversed interference effect.

and the measured beams. The $C^{(1)}(\delta\theta)$ angular correlation is also known in the volume-scattering problem as the "memory effect" correlation, because of the particular angular conservation condition that it obeys (see below). Thus in some sense we have found another way to measure the information contained in the enhanced backscattering peak intensity function, by looking at speckle correlations at different incident and outgoing directions. This correspondence principle may prove useful in real applications if measurement near the backscattering direction is somehow not convenient or when it is necessary to confirm the enhanced backscattering intensity function from an independent measurement.

We believe that this correspondence principle is quite general, and should hold for scattering from *any* given random surface, be it a very rough one or a relatively smooth one. But as it is impossible to calculate analytically either the enhanced backscattering intensity function or the correlation function for an arbitrarily rough random surface, we will choose to work with relatively smooth two-dimensional, Gaussian, random surfaces so that perturbation theory can be applied. (Numerical calculations of a similar nature have been performed for rough one-dimensional surfaces in Ref. 6.) Thus even though we limit ourselves to the smooth surface limit, we at least deal with the more realistic two-dimensional (2D) surface geometry. We will later on give intuitive arguments to suggest that this correspondence should be valid for surfaces with arbitrary roughness.

The model smooth Gaussian surface is characterized as follows. We choose to work with a 2D metal surface, with dielectric function

$$\epsilon(\omega) = \epsilon_1 + i\epsilon_2, \tag{2}$$

where $\epsilon_1 < -1$, and $|\epsilon_2| \ll |\epsilon_1|$. The roughness of the surface is characterized by a height function $z = \eta(\mathbf{R})$, with $\mathbf{R} \equiv (x, y)$, such that

$$\langle \eta(\mathbf{R}) \rangle = 0 \tag{3}$$

and

$$\langle \eta(\mathbf{R})\eta(\mathbf{R}') \rangle = \sigma^2 e^{-|\mathbf{R}-\mathbf{R}'|^2/a^2}, \tag{4}$$

where σ is the rms height fluctuation of the random surface at $z = 0$, and a is the correlation length of the roughness in the plane of the surface. We assume the incident light is in a direction characterized by the wave vector $\mathbf{K}_0 = (\mathbf{k}_0, K_{0z})$ where $\mathbf{k}_0$ is the projection of $\mathbf{K}_0$ onto the x-y plane, and the light intensity is measured in a direction $\mathbf{K}_f = (\mathbf{k}_f, K_{fz})$. The angle of incidence $\theta_{\rm inc}$ is measured in the plane of incidence ($\mathbf{K}_0, \hat{z}$ plane) counterclockwise from the z axis, and the angle of scattering θ_f is measured in the plane of scattering [($\mathbf{K}_f, \hat{z}$) plane] clockwise from the z axis. For a given surface before ensemble averaging, the reflected intensity $I(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0) = |A(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)|^2$ is a random-looking complex interference pattern that fluctuates strongly as the measurement direction $\mathbf{K}_f$ is varied. The amplitude function $A(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)$ is the matrix element of a unitary and reciprocal S matrix introduced in the vector theory of light scattering from random rough surfaces.[8–10] This is the familiar speckle pattern which is omnipresent in any random scattering problems. Here β_0 denotes the polarization state of the incident beam, and β_f denotes that of the measurement beam.

Traditionally, people regarded such random-looking interference patterns as a pure nuisance, and usually tried to average it away. They also thought that the pattern is some kind of stochastic function that can be described in purely statistical terms.[7] Recent progress in understanding the correlations in speckle patterns in volume multiple scattering systems have taught us that a speckle pattern is not really random, and its correlation properties contain much information about the scattering medium itself, and can be used to extract useful information about the scattering properties of the medium. In fact, we can regard the speckle pattern as a kind of "fingerprint" of a given sample of random scattering. This is the philosophy which has led us to study the present problem in random surface scattering.

Following Brown *et al.*,[8] we can write for the nonspecular (diffuse) contribution to the average intensity in the form

$$\langle |A(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)|^2 \rangle = 4K_{0z}K_{fz}\langle |G(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)|^2 \rangle_D. \tag{5}$$

Here G is the surface polariton propagator for which the exact Dyson equation holds before averaging

$$G = G_0 + G_0VG, \tag{6}$$

where G_0 is the surface polariton Green's function for $\eta(\mathbf{R}) = 0$, and V is the effective scattering potential. The expansion of V in powers of η has been obtained by Brown *et al.*[9] In this work we shall use only the lowest-order approximation for V, which is linear in η. Using these relations, and treating the roughness parameter σ as a small parameter, it is then possible to generate a well-controlled perturbation series for G. It is convenient to express the perturbation series as a set of Feynman diagrams.

First we use the formalism of Brown *et al.*[8] for

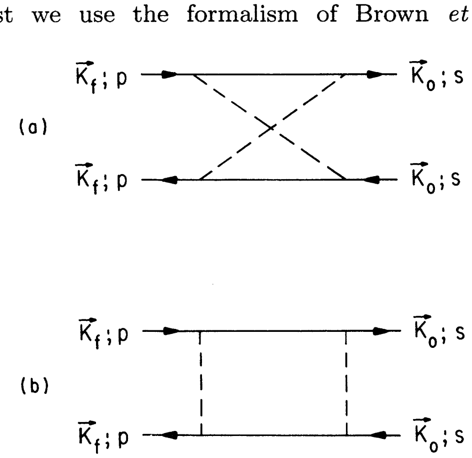

FIG. 2. Feynman diagram for (a) cross-polarized enhanced backscattering intensity function $\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; p, s) \rangle$; (b) amplitude-amplitude correlation function $\langle \delta A(\mathbf{k}_f, \mathbf{k}_0; p, s) \times \delta A^*(\mathbf{k}'_f, \mathbf{k}'_0; p, s) \rangle$.

the calculation of the leading-order contribution to the enhanced backscattering peak intensity function $\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)\rangle$. As we will see later, the correspondence between $\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; \beta_f, \beta_0)\rangle$ and the correlation function is valid only for cross-polarized configurations. Thus we will concentrate our attention from now on only to this case. So we take β_0 to be the s polarization, i.e., the magnetic field vector in the incident light is polarized in the plane of incidence (x-z), with $\mathbf{H}_{\rm inc} = (H_x, 0, H_z)$. We take β_f to be the p polarization state, i.e., the magnetic-field vector in the measurement beam is polarized normal the plane of scattering $(\mathbf{K}_f, \hat{z})$. The cross-polarized enhanced backscattering intensity function $\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; p, s)\rangle$ can be computed, to second order in the scattering potential V, using the diagram of Fig. 2(a):

$$\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; p, s)\rangle = 4K_{0z}K_{fz}|G_s(\mathbf{k}_0)|^2|G_p(\mathbf{k}_f)|^2 \times \sum_{\mathbf{k},\mathbf{k}'}\langle V_{ps}(\mathbf{k},\mathbf{k}_0)V^*_{pp}(\mathbf{k}',\mathbf{k}_f)\rangle\langle V_{pp}(\mathbf{k}_f,\mathbf{k})V^*_{ps}(\mathbf{k}',k_0)\rangle G_p(\mathbf{k})G_p(\mathbf{k}'), \tag{7}$$

where

$$\begin{aligned} V_{ij}(\mathbf{k},\mathbf{k}') &= \eta_{\mathbf{k}-\mathbf{k}'}\frac{\epsilon-1}{\epsilon}v_{ij}(\mathbf{k},\mathbf{k}'), \\ v_{pp}(\mathbf{k},\mathbf{k}') &= \frac{\epsilon k k' - (\hat{k}\cdot\hat{k}')\alpha(k)\alpha(k')}{\epsilon}, \\ v_{ps}(\mathbf{k},\mathbf{k}') &= -(\omega/c)\alpha(k)[\hat{k}\times\hat{k}']_z, \end{aligned} \tag{8}$$

$\eta(\mathbf{k}-\mathbf{k}')$ is the Fourier transform of $\eta(\mathbf{R})$, $\alpha_0(q) = (w^2/c^2 - q^2)^{1/2}$, $\alpha(q) = [\epsilon(\omega)w^2/c^2 - q^2]^{1/2}$ with $\mathrm{Re}[\alpha(q)] > 0$, $\mathrm{Im}[\alpha(q)] > 0$, and $G_p(\mathbf{k}_f)$ and $G_s(\mathbf{k}_0)$ are the average Green functions. It has been shown by Brown *et al.*[8] that only the surface polariton propagator for the p-polarized modes contributes to the summation over intermediate states in Eq. (7). The dominant contribution to $G_p(\mathbf{k})$ is through the resonant scattering from the propagating polariton modes along the mean surface, such that the Green function can be well approximated by a simple pole form. In the notations of Ref. 8,

$$G_p(|\mathbf{k}|) = \frac{C}{|\mathbf{k}| - k_{sp} - i\Delta_{\rm tot}}, \tag{9}$$

where $k_{sp} = (\omega/c)\sqrt{\epsilon_1/(\epsilon_1+1)}$ is the polariton wave vector, and the constant C is given approximately by

$$C = \frac{\epsilon_1\sqrt{-\epsilon_1}}{1-\epsilon_1^2}. \tag{10}$$

The self-energy term $\Delta_{\rm tot} = \Delta_\epsilon + \Delta_{sp}$, where Δ_ϵ and Δ_{sp} describe the polariton damping due to the loss mechanisms in the dielectric medium and due to the surface roughness, respectively. Upon substituting Eqs. (8) and (9) into the equation for $\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; p, s)\rangle$, Eq. (7), we obtain the result

$$\begin{aligned} &\langle I_{\rm ebs}(\mathbf{k}_f, \mathbf{k}_0; p, s)\rangle \\ &= D\frac{2\Delta_{\rm tot}}{(k_0-k_f)^2 + 4\Delta_{\rm tot}^2}\exp\left[-\frac{a^2}{2}(k_{sp}^2 + k_0^2)\right] \\ &\quad\times\{\tfrac{1}{4}|\alpha(k_{sp})\alpha(k_0)| - |\epsilon|k_{sp}^2k_0^2\}. \end{aligned} \tag{11}$$

Here D is a constant given by

$$D = 16\pi^2C^2K_{fz}K_{0z}|G_p(k_f)|^2|G_s(k_0)|^2 \times\frac{|\epsilon-1|^4}{|\epsilon|^6}\left(\frac{\pi a\sigma^2}{\lambda}\right)^2 k_{sp}|\alpha(k_{sp})|^2\,. \tag{12}$$

We now consider the correlation function $C^{(1)}$. Similar to the volume scattering case, $C^{(1)}$ is related simply to the square of the amplitude-amplitude correlation function, i.e.,

$$\begin{aligned} C^{(1)}(\mathbf{k}_f, \mathbf{k}_0; p, s) &\equiv \langle\delta I(\mathbf{k}_f, \mathbf{k}_0; p, s)\delta I(\mathbf{k}'_f, \mathbf{k}'_0; p, s)\rangle^{(1)} \\ &= |\langle\delta A(\mathbf{k}_f, \mathbf{k}_0; p, s)\delta A^*(\mathbf{k}'_f, \mathbf{k}'_0; p, s)\rangle|^2\,. \end{aligned} \tag{13}$$

The leading term of the amplitude-amplitude correlation function for the cross-polarized configuration is given by the diagram in Fig. 2(b). We point out that precisely because the polarizations of the incident and the scattered beams are different, the double-scattering diagram dominates this correlation function. It can be readily seen from Eq. (8) that when the plane of incidence coincides with the plane of scattering, that is, when $\mathbf{k}_0||\mathbf{k}_f$, the single-scattering diagram vanishes identically for the amptitude-amptitude correlation function in the cross-polarization configuration, because it is proportional to the vector product $[\hat{k}_0\times\hat{k}_f]$. In this situation the amplitude-amplitude correlation function is dominated by a diagram very similar to that which dominates the above-considered enhanced backscattering intensity function [Fig. 2(a)].

Upon evaluating this correlation function, we obtain the expression

$$\langle\delta A(\mathbf{k}_f, \mathbf{k}_0; p, s)\delta A^*(\mathbf{k}'_f, \mathbf{k}'_0; p, s)\rangle = \delta_{\mathbf{k}_f-\mathbf{k}'_f,\mathbf{k}_0-\mathbf{k}'_0}D\frac{2\Delta_{\rm tot}}{(k_0-k'_0)^2+4\Delta_{\rm tot}^2}\exp\left[-\frac{a^2}{4}(2k_{sp}^2+k_0^2+k_f^2)\right] \times\phi(k_0, k_f, k_{sp}, |\epsilon|, \epsilon_1, \epsilon_2), \tag{14}$$

where ϕ is a smooth function whose expression we omit here as it does not have important features.

The δ function in Eq. (14) symbolizes the "memory effect," which is by now familiar in volume scattering problems.[5] It represents the fact that the amplitude-amplitude correlation function will be rigorously zero unless the shift in the incoming beam's angle is the same as the shift in the outgoing beam's angle. This means physically that for small angular shifts of the incoming beam, the entire scattered far-field speckle pattern simply shifts as if the sample is a kind of "mirror." For large enough angular shift of the incoming beam, this shift will eventually give way to random fluctuations, which are represented as a decay of the amplitude-amplitude correlation function.

It is quite obvious now that the amplitude-amplitude correlation function and the enhanced backscattering intensity function map onto each other approximately for the cross-polarized configuration of measurement, and when the plane of incidence coincides with the plane of scattering. This can be already seen from the similarities in the corresponding Feynman diagrams for these two quantities (see Fig. 2). In Fig. 3, we have plotted these two functions as a function of $\delta\theta$ which for the enhanced backscattering is the angle from the exact backscattering direction, whereas for the correlation function it is the angle between the shifted two incident beam directions (which due to the memory effect must be the same as that in the outgoing direction). We observe clearly that these two functions essentially map onto one another.

We believe that the approximate mapping between the enhanced backscattering intensity function and the $C^{(1)}$ correlation function is a general one, beyond the validity of the lowest-order perturbation theory that we have performed here. The way to see this point is to examine the diagrams in a high-order perturbation theory for the two quantities. Going back to Fig. 2, the two diagrams are essentially the same if one were to "flip" one of the Green-function lines in the backscattering diagram, which is tantamount to making a "time-reversal" operation. Thus we expect that even when higher-order perturbation terms are included, the correspondence between the two quantities will remain, provided the correlation function does not contain a single scattering contribution, which is guaranteed by the choice of cross polarization and making the planes of incidence and scattering to be the same.

To summarize, we have shown, using perturbation theories for scattering from a smooth random metal surface, that there is a good correspondance between the enhanced backscattering intensity function and the cross-polarized amplitude-amplitude correlation function (or the resultant intensity-intensity correlation function), for the cross-polarized configuration. This correspondence should in principle be also valid in the rough surface limit. This hypothesis remains to be tested both numerically and experimentally. Such a mapping enables one to have another, independent way to measure the information contained in the enhanced backscattering peak about the random rough surface, using correlations of speckle intensities away from the backscattering direction.

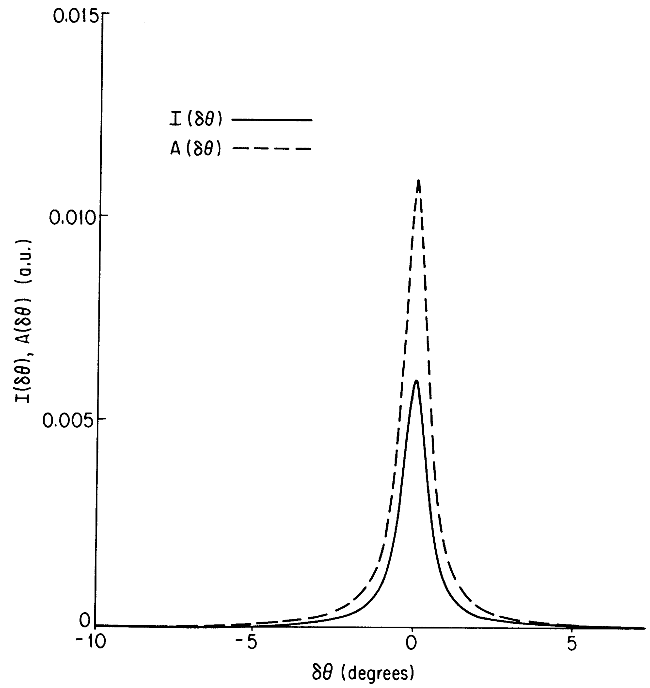

FIG. 3. Computed comparison between the cross-polarized enhanced backscattering peak intensity function $I(\delta\theta) \equiv \langle I_{\rm ebs}(\mathbf{k}_f,\mathbf{k}_0;p,s)\rangle$ and the amplitude-amplitude correlation function $A(\delta\theta) \equiv \langle \delta A(\mathbf{k}_f,\mathbf{k}_0;p,s)\delta A^*(\mathbf{k}'_f,\mathbf{k}'_0;p,s)\rangle$. The various parameters used are $\lambda = 4579$ Å, $a = 1000$ Å, $\sigma = 50$ Å, $\epsilon_1 = -7.5$, $\epsilon_2 = 0.24$, $\theta_0 = 20°$, and $\theta_f = -10°$, and the plane of incidence and the plane of scattering are assumed to coincide. $I(\delta\theta)$ and $A(\delta\theta)$ are shown in the same scale of arbitrary units.

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