Cyclotron resonance of magnetopolarons in a parabolic quantum dot in strong magnetic fields

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Cyclotron resonance of magnetopolarons in a parabolic quantum dot with a strong magnetic field normal to the plane of the quantum dot is investigated theoretically. It is shown that for a strong magnetic field ($\omega_c \gg \omega_{LO}$), the cyclotron mass in a parabolic quantum dot is split into two cyclotron masses (m_{+}^{*} and m_{-}^{*}). One (m_{+}^{*}) is lower than the bare band mass, but increases with increasing effective confinement length of the quantum dot, and approaches that of the two-dimensional case. The other (m_{-}^{*}) is greater than the bare band mass and might be a measurable effect for small quantum dots.

I. INTRODUCTION

In recent years, there has been a great deal of interest in the investigation of quasi-zero-dimensional electronic systems (quantum dots) derived from originally two-dimensional electronic systems in $Al_xGa_{1-x}As/GaAs$ heterostructures or similar systems.[1-3] The electron-energy spectrum of such quantum dots is fully quantized. These quantum dots are often referred to as artifical atoms in which the atomic potential is the place of the artifically constructed dot potential. Because of the potential device applications and the interesting physical effects in such structures, understanding the electronic properties of these systems is of particular importance.

The interaction of the electrons with longitudinal-optical (LO) phonons in quantum dots has been investigated by various authors.[4-9] Recently, Rousignol, Ricard, and Flytzanis[4] have shown experimentally and explained theoretically that phonon broadening is quite important in very small semiconductor quantum dots. Klein *et al.*[6] studied the size dependence of electron-phonon coupling in semiconductor nanospheres; they derive the expression of the vibrational LO and SO eigenfunctions for a sphere in the continuum approximation. Zhu and Gu[9] have shown that the polaron effects on quantum dots are larger than on lateral quantum wires. Recently, some theoretical work on cyclotron resonance in quantum dots has been done by several authors.[10-12] In this paper we consider the effect of an external magnetic field B (applied normal to the plane of the quantum dot), on zero-dimensional polarons in the weak-coupling limit. For the sake of analytic simplicity, we will model the relevant vibrational modes by the corresponding bulk modes, i.e., we will neglect any size quantization of the phonons. This assumption has been used by Schmitt-Rink, Miller, and Chemla[7] and Bockelmann and Bastard[8] to treat the phonon broadening of optical spectra and the phonon scattering in quantum dots. Taking into account the effect of phonon confinement, one would certainly vary the results in comparison with those of the bulk-phonon model. We will treat this effect in a forthcoming paper. In addition, for simplicity, the polaron levels are taken to be perfectly sharp; no phenomenological damping parameters are introduced into the calculations.[13]

II. THEORY

The electrons are much more strongly confined in one direction (taken as the z direction) than in the other two directions. Therefore, we will confine ourselves to consider only the motion of the electrons in the x-y plane. Kumar, Laux, and Stern[14] have shown that even if the defining cap layer is square shaped, the confining potential seen by electrons in a quantum dot has nearly circular symmetry. The energy levels are found to be insensitive to the charge in the dot at a fixed-gate voltage, and the evolution of energy levels with increasing magnetic field is similar to that for a parabolic potential. These results make the parabolic confining-potential model very appealing. In the presence of a magnetic field, this model potential offers exact analytic information on the single-particle energy states. We will use this parabolic confining-potential model in the present work.

We assume that the confining potential in a single quantum dot is parabolic,

$$V(\rho)=\tfrac{1}{2}m^{*}\omega_0^2\rho^2 , \tag{1}$$

where m^{*} is the bare band mass and ρ is the coordinate vector of a two-dimensional quantity. In the presence of a magnetic field in the z direction, the Hamiltonian of electron-phonon systems is given by

$$H=H_0+H_1 , \tag{2}$$

$$H_0=\frac{1}{2m^{*}}(\mathbf{p}+e\,\mathbf{A})^2+\tfrac{1}{2}m^{*}\omega_0^2\rho^2+\sum_{q}\hbar\omega_{LO}b_{\mathbf{q}}^{+}b_{\mathbf{q}} , \tag{3}$$

$$H_1=\sum_{\mathbf{q}}(V_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}b_{\mathbf{q}}+V_{\mathbf{q}}^{+}e^{-i\mathbf{q}\cdot\mathbf{r}}b_{\mathbf{q}}^{+}) , \tag{4}$$

where $b_{\mathbf{q}}^{+}$ creates a bulk LO phonon of wave vector $\mathbf{q}$, $\mathbf{q}=(\mathbf{q}_{\parallel},q_z)$, and $\mathbf{r}=(\rho,z)$ is the coordinate of the electron,

$$V_{\mathbf{q}}=i(\hbar\omega_{LO}/q)(\hbar/2m^{*}\omega_{LO})^{1/4}(4\pi\alpha/V)^{1/2} , \tag{5}$$

$$\alpha=(e^2/2\hbar\omega_{LO})(2m^{*}\omega_{LO}/\hbar)^{1/2}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right] , \tag{6}$$

where ϵ_∞ is the optical dielectric constant and ϵ_0 is the static dielectric constant.

In the symmetric gauge $\mathbf{A}=(-\frac{1}{2}By,\frac{1}{2}Bx)$, the energy levels of the unperturbed Hamiltonian H_0 are given by[15,16]

$$E_{nm}^{(0)}=(2n+|m|+1)\hbar\omega_c\left[\frac{1}{4}+\frac{\omega_0^2}{\omega_c^2}\right]^{1/2}+\tfrac{1}{2}m\hbar\omega_c\ , \qquad (7)$$

where $\omega_c=eB/m^*c$ is the cyclotron frequency, m is the angular quantum number, $m=0,\pm1,\pm2,\ldots$, and n is the radial quantum number, $n=0,1,2,\ldots$. The corresponding wave functions are given by

$$|n,m,0_q\rangle=\frac{1}{\sqrt{2\pi}}e^{im\theta}\left[\frac{2m^*\omega_c\left[\frac{1}{4}+\frac{\omega_0^2}{\omega_c^2}\right]^{1/2}n!}{\hbar(n+|m|)!}\right]^{1/2} \times x^{|m|}L_n^{|m|}(x^2)e^{-x^2/2}|0_q\rangle\ , \qquad (8)$$

where $x=\rho[m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}/\hbar]^{1/2}$, $L_n^{|m|}$ are associated Laguerre polynominals, and $|0_q\rangle$ is vacuum state of the phonon, which satisfies $b_q|0_q\rangle=0$.

Since the electron-phonon interaction is weak in these systems, in the sense that the Fröhlich coupling constant (α) is of the order of 0.1, we shall use the second-order Rayleigh-Schrödinger perturbation theory to obtain the electronic self-energy shift δE_{nm}, which is given by

$$\delta E_{nm}=-\alpha(\hbar\omega_{\rm LO})^2\left[\frac{\hbar}{2m^*\omega_{\rm LO}}\right]^{1/2}\sum_{n'}\sum_{m'}Q_{nmn'm'}\left\{[2(n'-n)+(|m'|-|m|)]\left[\frac{1}{4}+\frac{\omega_0^2}{\omega_c^2}\right]^{1/2}\hbar\omega_c \qquad (9)$$

$$+\frac{m'-m}{2}\hbar\omega_c+\hbar\omega_{\rm LO}\right\}^{-1}\ , \qquad (10)$$

where

$$Q_{nmn'm'}=\int_0^\infty dq_\parallel[V_{nmn'm'}(q_\parallel)]^2\ , \qquad (11)$$

$$V_{nmn'm'}(q_\parallel)=2\left[\frac{n'!n!}{(n'+|m'|)!(n+|m|)!}\right]^{1/2}\int_0^\infty dx\, x^{|m'|+|m|+1}L_{n'}^{|m'|}(x^2) \qquad (12)$$

$$\times L_n^{|m|}(x^2)J_{m-m'}(-q_\parallel lx)\ , \qquad (13)$$

where $l=[\hbar/m^*(\omega_0^2+\frac{1}{4}\omega_c^2)^{1/2}]^{1/2}$ and J_n are Bessel functions of the first kind.

For strong magnetic field ($\omega_c\gg\omega_{\rm LO}$), one can neglect all the off-diagonal ($n\neq n'$, $m\neq m'$) terms in the right-hand side of Eq. (9), yielding[17]

$$\delta E_{nm}=-\alpha\hbar\omega_{\rm LO}\left[\frac{\hbar}{2m^*\omega_{\rm LO}}\right]^{1/2}Q_{nmnm}\ . \qquad (14)$$

Therefore, the ground-state energy correction is given by

$$\delta E_{00}=-\alpha\hbar\omega_{\rm LO}\left[\frac{\hbar}{2m^*\omega_{\rm LO}}\right]^{1/2}\frac{\sqrt{2\pi}}{2l}\ . \qquad (15)$$

When only the lowest-energy level is occupied, the selection rules allow only two excitations, from the state (0,0) to (0,−1), and from (0,0) to (0,1).[18] Consequently, the correction δE_{01} to the excited level (0,1) and the correction δE_{0-1} to the excited level (0,−1) are given by

$$\delta E_{01}=\delta E_{0-1}=-\alpha\hbar\omega_{\rm LO}\left[\frac{\hbar}{2m^*\omega_{\rm LO}}\right]^{1/2}\frac{11\sqrt{2\pi}}{32l}\ . \qquad (16)$$

The relevant cyclotron resonance frequency for the (0,0)→(0,1) transition (i.e., when the Fermi level is in the lowest Landau level) is given by

$$\omega_{c-}^*=\omega_c\left[\frac{1}{4}+\frac{\omega_0^2}{\omega_c^2}\right]^{1/2}-\frac{\omega_c}{2}+\alpha\omega_{\rm LO}\left[\frac{\hbar}{2m^*\omega_{\rm LO}}\right]^{1/2}\frac{5\sqrt{2\pi}}{32l} \qquad (19)$$

$$=\omega_c\left\{\frac{1}{2}+\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/2}+\tfrac{5}{16}\pi^{1/2}\beta\alpha\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/4}\right\}\ , \qquad (18)$$

where $\beta^2=\omega_{\rm LO}/\omega_c$ and $u=l_0/r_0$, where $l_0=(\hbar/m^*\omega_0)^{1/2}$ is the effective confinement length of the quantum dot and $r_0=(\hbar/2m^*\omega_{\rm LO})^{1/2}$ is the polaron radius.

The relevant cyclotron resonance frequency for the (0,0)→(0,−1) transition (i.e., when the Fermi level is in the lowest Landau level) is given by

$$\omega_{c-}^{*}=\omega_c\left[\frac{1}{4}+\frac{\omega_0^2}{\omega_c^2}\right]^{1/2}-\frac{\omega_c}{2}+\alpha\omega_{\mathrm{LO}}\left[\frac{\hbar}{2m^{*}\omega_{\mathrm{LO}}}\right]^{1/2}\frac{5\sqrt{2\pi}}{32l} \tag{19}$$

$$=\omega_c\left\{-\frac{1}{2}+\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/2}+\tfrac{5}{16}\pi^{1/2}\beta\alpha\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/4}\right\}. \tag{20}$$

Equations (18) and (20) imply two renormalized cyclotron masses given by

$$m_{+}^{*}=\frac{2m^{*}}{1+2\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/2}+\frac{5}{16}\pi^{1/2}\beta\alpha\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/4}} \tag{21}$$

and

$$m_{-}^{*}=\frac{2m^{*}}{-1+2\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/2}+\frac{5}{16}\pi^{1/2}\beta\alpha\left[\frac{1}{4}+\frac{4\beta^4}{u^4}\right]^{1/4}}\,. \tag{22}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

The numerical results of the cyclotron mass in GaAs parabolic quantum dots are presented in Figs. 1,2, and 3. From the conditions that the cyclotron energy is much bigger than the LO-phonon energy, the magnetic-field range may be estimated to be $B>21.3$ T. Figure 1 and 2 show that for strong magnetic field, the mass renormalization is negative and the cyclotron mass (m_{+}^{*}) is lower than the bare band mass (approaching it as $\omega_c\rightarrow\infty$); this behavior is similar to the two-dimensional magnetopolaron. With increasing the effective confinement length of

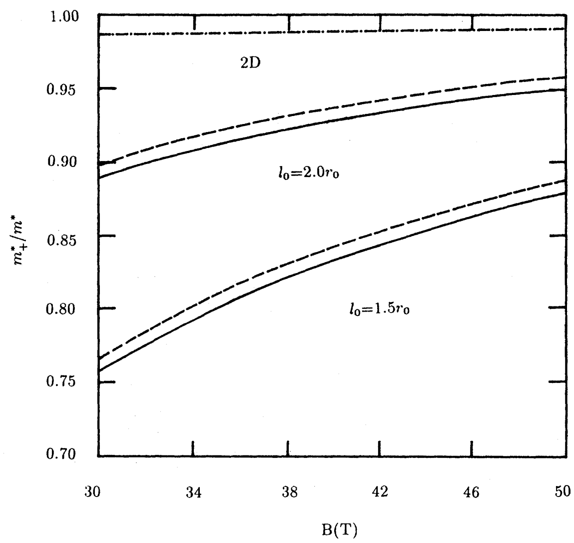

FIG. 2. The cyclotron mass (m_{+}^{*}) of GaAs parabolic as a function of magnetic-field strength (B) for two effective confinement lengths. The broken curves are the results for cyclotron mass (m_{+}^{*}) without the polaron correction and the chain curve is the result for two-dimensional systems (Ref. 17) [$l_0=(\hbar/m^{*}\omega_0)^{1/2}$ and $r_0=(\hbar/2m^{*}\omega_{\mathrm{LO}})^{1/2}$].

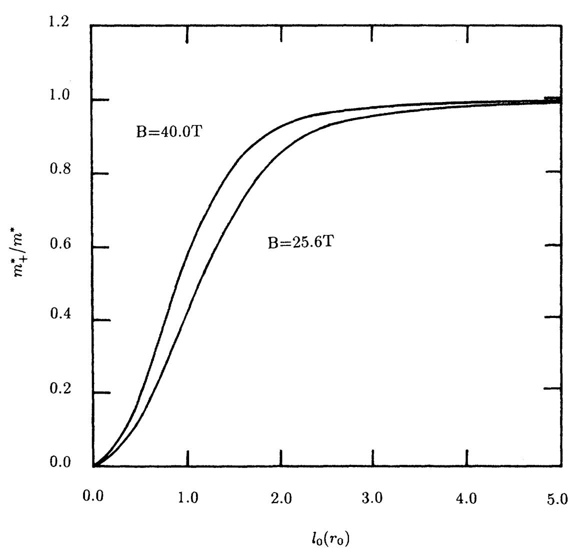

FIG. 1. The cyclotron mass (m_{+}^{*}) of GaAs parabolic quantum dots as a function of the effective confinement length of the quantum dot for two magnetic-field strengths (B) [$l_0=(\hbar/m^{*}\omega_0)^{1/2}$ and $r_0=(\hbar/2m^{*}\omega_{\mathrm{LO}})^{1/2}$].

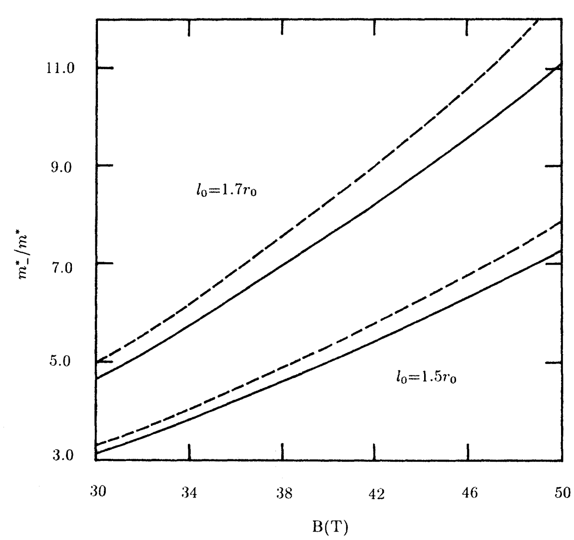

FIG. 3. The cyclotron mass (m_{-}^{*}) of GaAs parabolic quantum dots as a function of magnetic-field strength (B) for two effective confinement lengths. The broken curves are the results for cyclotron mass (m_{-}^{*}) without the polaron correction [$l_0=(\hbar/m^{*}\omega_0)^{1/2}$ and $r_0=(\hbar/2m^{*}\omega_{\mathrm{LO}})^{1/2}$].

the quantum dot, the cyclotron mass enhances and approaches that in the two-dimensional case. The broken curves in Fig. 2 show the result for the cyclotron mass without the polaron correction. It is obvious that the polaron correction to the cyclotron mass (m_+^*) is not very large. The chain curve in Fig. 2 depicts the result for cyclotron mass in two-dimensional systems obtained by Sarma.[17] The cyclotron mass (m_+^*) in quantum dots is always smaller than that in two-dimensional quantum systems. Figure 3 illustrates the cyclotron mass (m_-^*) of GaAs parabolic quantum dots as a function of the magnetic-field strength (B) for two effective confinement lengths. It is shown that for small quantum dots the cyclotron mass (m_-^*) is greater than the bare band mass, and increases with the enhancement of magnetic-field strength (B). The result is qualitatively surprising because one is accustomed to thinking that for a strong magnetic field the cyclotron mass is lower than the bare band mass. In general, in the high-magnetic-field limit, for larger quantum dots, transitions ω_{c+}^* become cyclotron resonances between Landau levels $\hbar\omega_c(n+\frac{1}{2})$ and the electron gas exhibits 2D behavior. Simultaneously, transitions ω_{c-}^* approach zero. As a result, m_-^* does not exist. However, if the quantum dot is small enough, the cyclotron mass (m_-^*) may be greater than the bare band mass and might become a measurable effect. The broken curves in Fig. 3 show the results for the cyclotron mass without the polaron correction. We can see that the polaron correction to the cyclotron mass (m_-^*) is obvious and cannot be neglected.

IV. CONCLUSIONS

In conclusion, we have investigated the problem of the electron-phonon interaction effect on the cyclotron mass of zero-dimensional electrons in GaAs parabolic quantum dots. We find that for a strong magnetic field ($\omega_c \gg \omega_{\rm LO}$), the cyclotron mass in a parabolic quantum dot is split into two cyclotron masses (m_+^* and m_-^*). One (m_+^*) is lower than the bare band mass. By increasing the effective confinement length of the quantum dot, it enhances and approaches that in the two-dimensional case. The other (m_-^*) is greater than the bare band mass and might be measurable effect for the small quantum dots. It should be emphasized that use of the 2D disk approximation for the electronic wave function is not an essential restriction of this paper; it has been done only for the sake of analytic convenience and clarity of the final results. Introduction of the finite width of the electronic wave function in the z direction into the above formalism is straightforward. Indeed, the width of the z wave function would substantially (by a factor of 2–3) reduce the effective polaronic correction.[19] The actual reduction in the effective electron-phonon coupling will obviously depend on the details of the system involved and can only be obtained numerically for specific systems. However, the qualitative features in this paper are independent of this approximation. Finally, it is hoped that this paper will stimulate more experimental work in the high-field region which will be helpful in a better understanding of the role of electron–LO-phonon interactions in quasi-zero-dimensional systems.

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