Exactly solvable model of flux creep in high-T_c superconductors

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An exactly solvable model of flux creep in high-T_c superconductors within the framework of both thermally activated and quantum-tunneling flux-creep theory is presented in this paper. The activation energy and crossover temperature between activated creep and quantum tunneling are explicitly expressed for an elastic flux line in a typical weakly periodical or quartic pinning potential involving a viscous medium. The exact analytic expression of magnetic relaxation rate is derived for the full parameter range. These expressions are somewhat better because several superconducting mechanism-sensitive parameters have been included. The present results are in reasonable agreement with the experimental data obtained in high-T_c superconductors.

The classical Anderson-Kim model of flux creep in type-II superconductors predicts a linearly vanishing magnetic relaxation rate at low temperatures and a magnetization that decays logarithmically with time. Recently, however, experimental data on high-T_c superconductors[1-8] have shown that the magnetic relaxation rate does not appear to extrapolate linearly with temperature to zero at the lowest temperature and rolls over to a plateau over a broad range of intermediate temperature. The data have also shown that the nonlinear logarithmic time decay of magnetization appears if the measuring time is long enough. On the one hand, it is well known that the Anderson-Kim model assumes thermal activation of flux lines over a net potential barrier in which the hopping rate obeys an Arrhenius law. However, in deducing magnetic relaxation expressions, most authors[4-6] took the same approximation of considering the motion of flux along the Lorentz force direction without taking into consideration the reverse motion. It is clear that this approximation is not appropriate for high-T_c superconductors; however, the expressions of this approximation have been extensively used. Although many authors[9-12] have considered the two processes of flux creep, the exact solution of the flux-creep equation has not yet been worked out. On the other hand, the essentially temperature-independent magnetic relation rate as $T\rightarrow 0$ suggests a quantum tunneling of flux lines.[2,5,7,13] It has been pointed out by Caldeira and Leggett[14] that in macroscopic systems dissipation should have a strong influence on the tunneling rate. Larkin and Ovchinnikov[15] and Ivlev, Ovchinnikov, and Thompson[16] have discussed the quantum tunneling at finite temperature in a dissipative system and in a sufficiently perfect layered high-T_c superconductor, respectively. In this paper an exactly solvable model of flux creep will be presented. The value of the activation energy is determined by the saddle-point solution for the system with the traditional periodic or quartic potentials. At low enough temperature where the thermal activation processes are frozen out, the quantum-tunneling process for flux soliton motion dominates the magnetic relaxation. The nonzero eigenvalue is identified with the crossover temperature where the crossover from the classical to the quantum regime of motion takes place. The calculated relaxation rate explicitly shows a temperature-independent plateau or at least a flat maximum over a broad range of temperature, usually with a downward trend at lower temperature, but with a nonzero value at $T=0$. The present results also show a nonlogarithmic time decay of magnetization.

It is usually assumed that the flux line sits at the bottom of a potential well of depth U_0 due to the pinning of defects which, even without external current, lead to a distortion of the flux-line lattice. Considering the Lorentz-force-induced flux motion, the effective well depth becomes $U_0(1-J/J_0)$ and $U_0(1+J/J_0)$ on both sides of the flux line, respectively, where J is the current density and J_0 is the current density without thermal activation. Based on both the Arrhenius law for thermally activated hopping and the phenomenological superconducting equation for current-density decay in time, the equation of net rate of flux creep can be written in the following form:

$$\frac{\partial J}{\partial t}=-2k\sinh\left[\frac{JU_0}{J_0k_BT}\right]\exp\left[-\frac{U_0}{k_BT}\right], \quad (0\le J\le J_c), \qquad (1)$$

where J_c is the critical current density and k is the decay coefficient which depends on the geometry of the experiment. By integrating Eq. (1) with the initial condition $J(t=t_0)=J(t_0)$, the exact solution can be obtained, which leads to an expression of the parameter-dependent current density of the system,

$$J(t)=J_0\frac{k_BT}{U_0}\ln\left[\frac{1+X}{1-X}\right], \quad X\equiv\tanh\left[\frac{J(t_0)U_0}{2J_0k_BT}\right]\exp\left[-\frac{2(t-t_0)}{\tau}\exp\left[-\frac{U_0}{k_BT}\right]\right], \tag{2}$$

where $\tau=J_0k_BT/kU_0$ is the hopping attempt time. The normalized relation rate S for the current density J can be defined as $S\equiv-\partial\ln J/\partial\ln t$. From Eq. (2) it is found that

$$S(T)=\frac{2tX\exp(-U_0/k_BT)}{\tau(1-X^2)\tanh^{-1}X}. \tag{3}$$

In particular, if $U_0 \gg k_BT$, the reverse creep is negligible. At long times,

$$t\gg\tau\exp\left[\frac{U_0}{k_BT}\left[1-\frac{J(t_0)}{J_0}\right]\right],$$

but $t\ll\tau\exp(U_0/k_BT)$, Eqs. (2) and (3) can be reduced to the usual Anderson-Kim formulas[4-6] which have been extensively used in the form

$$J(t)\approx J_0[1-(k_BT/U_0)\ln(t/\tau)], \tag{4}$$

$$S(T)\approx(k_BT/U_0)/[1-(k_BT/U_0)\ln(t/\tau)]. \tag{5}$$

In order to get the expressions of activation energy and crossover temperature and to describe the quantum-tunneling process, we consider a flux system in a pinning potential involving a viscous medium. The effective Euclidean action $\mathcal{A}$ in real-Fourier mixed space without external current takes the form[14-17]

$$\mathcal{A}=\int_{-\infty}^{\infty}dz\int_0^{h/k_BT}d\tau\{\tfrac{1}{2}\epsilon_l[\partial_z u(z,\tau)]^2+V_p[u(z,\tau)]\} + \int_{-\infty}^{\infty}dz\int_0^{k_BT/\hbar}\frac{dw}{2\pi}[\tfrac{1}{2}Mw^2+\eta|w|]|\bar{u}(z,w)|^2. \tag{6}$$

Here the two-dimensional field $u(z,\tau)$ describes a local displacement of the flux line in the z direction from its equilibrium position in the xy plane, $\bar{u}(z,w)$ is its Fourier transformation, M is the flux mass per unit length, and $\eta=\Phi_0H_{c2}/c^2\rho_N$ is the friction coefficient with H_{c2} the upper critical field and ρ_N the normal-state resistivity.[16,17] The exact expression for the elastic energy of the flux-line lattice has been obtained by Brandt and Sudbø.[18] Their results showed that the line tension in the isotopic local case given by $\epsilon_l=(\Phi_0/4\pi\lambda)^2\ln(\lambda/\xi)$ is anisotropic and weakly nonlocal, where $\Phi_0=hc/2e$, and λ and ξ are the London penetration depth and Ginzburg-Landau coherence length of the xy plane, respectively. From Refs. 15 and 16, we are familiar with the finite temperature formalism for the problem, in which the upper limits of the integrations with respect to imaginary time $\tau=-it$,[16] and its Fourier counterpart, frequency w,[17] are taken to be finite and given by h/k_BT and $k_BT/\hbar$, respectively, as indicated in action (6). The pinning potentials V_p traditionally considered are[19]

$$V_p(u)=V_0\sin^2(\pi u/L), \quad \text{periodic}, \tag{7}$$

and

$$V_p(u)=V_0[(u/L)^2-f(u/L)^4], \quad \text{quartic}, \tag{8}$$

where the quantity V_0 determines the scale of the height of the potential barrier, L is the hopping length of the flux, and f is the nonlinear coefficient. By variation of the action (6) with u and $\bar{u}$, the Euler-Lagrange equation of motion can be obtained as

$$\epsilon_l\partial_{zz}u(z,t)-V_p'[u(z,t)]=\hat{\mathcal{F}}[(Mw^2+\eta|w|)\bar{u}(z,w)], \tag{9}$$

where $\hat{\mathcal{F}}$ is the Fourier transformation operator.

The stationary function $u(z,0)\equiv u_0(z)$ satisfies the equation

$$\epsilon_l u_0''(z)-V_p'[u_0(z)]=0. \tag{10}$$

For potentials (7) and (8), the soliton solutions of Eq. (10) with the conditions $u_0(z_0)=\frac{1}{2}L$ and $L/f^{1/2}$, respectively, are

$$u_0(z)=\frac{2L}{\pi}g^{-1}\exp\left[\pi\left[\frac{2V_0}{\epsilon_l}\right]^{1/2}\left[\frac{z-z_0}{L}\right]\right] \tag{11}$$

and

$$u_0(z)=\frac{L}{f^{1/2}}\operatorname{sech}\left[\left[\frac{2V_0}{\epsilon_l}\right]^{1/2}\left[\frac{z-z_0}{L}\right]\right]. \tag{12}$$

Let $u(z,\tau)=u_0(z)+u_1(z,\tau)$ and assume $u_1(z,\tau)\ll u_0(z)$, we are able to determine $u_1(z,\tau)$ from $u_0(z)$ by perturbation. From Eq. (9) one gets the equation for $u_1(z,\tau)$,

$$\epsilon_l\partial_{zz}u_1(z,\tau)-V_p''[u_0(z)]u_1(z,\tau) = \hat{\mathcal{F}}[(Mw^2+\eta|w|)\bar{u}_1(z,w)]. \tag{13}$$

This is an eigenvalue equation. For the quartic potential, it has one eigenfunction

$$u_1(z,\tau)=-L\,\operatorname{sech}^2\left[\left[\frac{2V_0}{\epsilon_l}\right]^{1/2}\left[\frac{z-z_0}{L}\right]\right]\cos\left[2\pi\frac{T_0}{T}\right], \tag{14}$$

with eigenvalue (crossover temperature)

$$T_0=\frac{\hbar\eta}{2k_BM}\left[\left[1+\frac{24MV_0}{\eta^2L^2}\right]^{1/2}-1\right].$$

This describes the quantum-tunneling process for flux soliton motion at $0<T\le T_0$. The tunneling frequency for the barrier of height $U_0(1-J/J_0)$ can be written as[7,15,16]

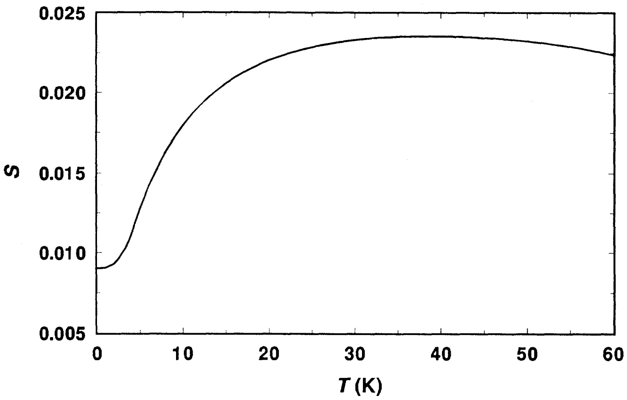

FIG. 1. Normalized time-logarithmic current-density decay rate $S \equiv -\partial \ln J(t)/\partial \ln t$ vs temperature T for high-T_c superconductor $Y_1Ba_2Cu_3O_7$ with $T_c = 90$ K.

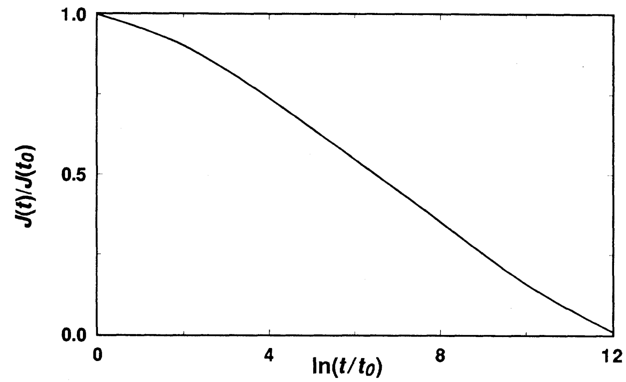

FIG. 2. Reduced current-density decay $J(t)/J(t_0)$ vs reduced logarithmic time $\ln(t/t_0)$ for high-T_c superconductors at $T=20$ K.

$$\nu \propto \exp[-U_0(1-J/J_0)/k_B T_0] \ . \qquad (15)$$

Similarly, an expression for normalized quantum-tunneling relaxation rate $S_{\rm QT}$ is obtained, which takes the same form as Eq. (3) but with $U_0/k_B T$ replaced by $U_0/k_B T_0$. Since $k_B T_0 \ll U_0$, $S_{\rm QT}$ is simplified as

$$S_{\rm QT} \approx \frac{k_B T_0}{U_0} \frac{t}{t - t_0 + J_0 k_B T_0 / k U_0} \ . \qquad (16)$$

It is obvious that $S_{\rm QT}$ does not strongly depend on temperature. Therefore at sufficiently low temperature, as long as $S_{\rm QT} \gg S$, the decay rate is temperature independent and remains finite down to zero temperature.

The value of U_0 is determined by the stationary solution. Substituting Eqs. (7) and (11) and Eqs. (8) and (12), respectively, into Eq. (6), the expressions of activation energy for the periodic and quartic potentials can be obtained, respectively, as

$$U_0(T) = \frac{2L}{\pi}(2V_0\epsilon_l)^{1/2} \qquad (17)$$

and

$$U_0(T) = \frac{2L}{3f}(2V_0\epsilon_l)^{1/2} \ . \qquad (18)$$

From $U_0(T) \propto \epsilon_l^{1/2} \propto 1/\lambda(T)$, one has[20] $U_0(T) = U_0(0)(1-T/T_c)^{1/2}$. As a numerical estimation, we choose $U_0(0) = 20$ meV,[5,6] $J_0 = J(t_0)$, $kt_0/J(t_0) = 0.025$, and $T_c = 90$ K in high-T_c superconductor $Y_1Ba_2Cu_3O_7$. At fixed time $t = 1.60 t_0$, $S(T)$ is calculated from Eq. (3) for $T \geq T_0$ (say, $T_0 = 4$ K) and from Eq. (16) for $T \leq T_0$ and the result is shown in Fig. 1. It is seen that the curve has a flat maximum in the range of $S = 0.021-0.024$ over a wide range of $T = 15-60$ K, with a nonlinear dropoff around $T = 8$ K, and, particularly, with an essential temperature-independent relaxation rate at the lowest temperatures, which are in good agreement with the experiments.[5-7] Meanwhile, Fig. 2, according to Eq. (2) at fixed temperature $T = 20$ K, gives the relationship between the reduced current density $J(t)/J(t_0)$ and reduced logarithmic time $\ln(t/t_0)$. A nonlinear logarithmic time decay of current density over both a long and a short time is clearly shown. This behavior has also been observed experimentally.[8,12]

In summary, we have presented the exactly solvable model of flux creep in high-T_c superconductors within the framework of both thermally activated and quantum-tunneling flux-creep theory by exactly treating the magnetic relaxation rate, quantum tunneling, and activation energy. The theoretical results show a flat maximum magnetic relaxation rate over a broad range of temperature where the reverse creep is not negligible, a nonlinear downward trend at low temperature, in which the Anderson-Kim creep is of major significance, and a temperature-independent nonzero value at $T = 0$ where the quantum tunneling is dominant. The results also show a nonlinear logarithmic time decay of current density at high temperature. These are all in reasonable agreement with the experimental data obtained for high-T_c superconductors.

Note added. We recently became aware of Refs. 21 and 22 which obtain the similar solution (2) of the Anderson-Kim model (1).

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