One-dimensional t-J and Hubbard models in a staggered field

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The influence of the staggered magnetic field h on properties of the one-dimensional t-J and Hubbard models with nearly half-filled bands are studied. Using the exact diagonalization method for small systems as well as analytical approaches possible in the anisotropic model and for $h \gg t$, we determine the phase diagram of the t-J-h model at low doping, which displays three different regimes: the phase with repelling holes for $J < J_c$, the phase with paired holes for $J_c < J < J_s$, and the phase separation for $J > J_s$. Employing for $h \neq 0$ the analogy with the universal behavior of one-dimensional interacting fermions within the Luther-Emery regime, we calculate the charge exponent K_ρ, which can be $K_\rho > 1$ in the pairing regime, indicating dominant superconducting fluctuations. An analogous analysis, in particular the perturbation expansion in t/h, performed for the Hubbard t-U-h model shows a phase of stable hole pairs for $U < U_c$. Again K_ρ seems to approach a nonuniversal value at half-filling.

I. INTRODUCTION

In recent years there has been an intensive search for purely electronic models with repulsive electron-electron interactions, which would exhibit the superconductivity (SC) or at least strong SC pairing fluctuations within the ground state. These investigations have been motivated by the discovery of high-temperature SC and unusual normal state properties in layered copper oxides, which are representatives of such strongly correlated systems.[1]

Although one-dimensional (1D) models cannot allow for a long-range SC ordering, they are very instructive in the way to find scenarios for the possible SC in higher dimensions (in particular 2D) systems. Moreover 1D systems of interacting fermions are much better understood,[2] since they can be classified into some universal classes and characterized with few parameters. Of interest in particular is the charge exponent K_ρ,[2-4] which in the 1D system with dominant SC fluctuations should be $K_\rho > 1$. So far only few repulsive correlated models with the latter regime have been found. It has been shown that within the prototype t-J model $K_\rho > 1$ appears only at large $J > 2t$ in the narrow region before the onset of the phase separation (PS).[5,6] The regime of dominant SC correlations can be moved to $J < t$ by allowing for longer-range spin exchange,[7] still the qualitative behavior is expected to remain that of the Luttinger liquid[2,3] with $K_\rho = \frac{1}{2}$ approaching half-filling. On the other hand, a much more pronounced SC regime has been found in the dimerized t-J model and in the extended Hubbard model,[8] which can be in certain regimes mapped onto the attractive Hubbard model.

In a recent Letter[9] the present authors have introduced and analyzed 1D t-J and Hubbard models in an external staggered field h, which also shows a possibility of SC pairing near half-filling. The motivation to study such models comes clearly from 2D (or higher dimensional) correlated systems, which exhibit long-range antiferromagnetic (AFM) ordering at half-filled band, but also quite long-range AFM correlations away from half-filling. Staggered field h induces such ordering also in 1D and thus simulates some 2D effects. It, e.g., introduces in 1D effects known from the studies of 2D (or higher dimensional) systems, e.g., the spin-string phenomenon,[10] a substantial mass renormalization,[11] hole pair formation,[12] while the main difference remains in an entirely 1D motion of charge carriers. It has been demonstrated that the finite field $h \neq 0$ has a dramatic effect for few holes added to the reference AFM insulator, in particular it leads to the appearance of bound states of two holes in a broad range of model parameters. This can be demonstrated analytically for the simplest anisotropic t-J_z model, as well as for the t-J model and the Hubbard model at large $h \gg t$, while for the more challenging $h < t$ regime conclusions are based mainly on the exact diagonalization results for small systems.

The aim of this paper is to present in more detail and extend the analysis for 1D t-J-h and t-U-h models. One evident question is the phase diagram for the stability of bound hole pairs in both models. Since in the t-J-h model no bound states can appear at $J < J_c(h)$, while at $J > J_s(h)$ hole binding coexists with the PS into a hole-rich and a spin-rich phase, hole pairing is restricted to the intermediate regime $J_c < J < J_s$. Boundaries of the latter could be determined mainly from the study of small systems. For the t-U-h model there is no indication for

PS, so by analogy bound pairs are expected for $U < U_c(h)$. This is confirmed by a recent Hartree-Fock analysis of the problem,[13] where also a simple explanation for pairing in terms of h-induced bound soliton-antisoliton excitations has been given.

Another goal is to classify properly the behavior of models at finite doping and determine the character of charge density and SC pairing correlations.[2] Since $h \neq 0$ introduces a spin gap and destroys the spin rotation invariance we claim that the universal behavior should be that of 1D interacting fermions within the Luther-Emery regime,[2] with a single, i.e., charge exponent K_ρ which determines the long-distance correlations. Here $K_\rho > 1$ would indicate dominant SC fluctuations, so it is of importance to study also the dependence $K_\rho(n)$ on the electron concentration n at fixed h and J (or U). Note that at $h=0$ both models have the universal value $K_\rho = \frac{1}{2}$ (characteristic for spinless fermions) when approaching both half-filled, $n=1$, as well as the empty band, $n=0$.[4,5] We show that $h \neq 0$ can lead to a nonuniversal value $K_\rho \neq \frac{1}{2}$ at half-filling.

The organization of the paper is as follows. In Sec. II we analyze the stability of hole pairs and hole clusters in the anisotropic (Ising) and the isotropic t-J-h model. Both analytical approaches, mainly for the anisotropic model, and results of the exact diagonalization of small systems are used to establish thresholds of the hole pair stability $J_c(h)$ as well as of the PS separation $J_s(h)$. Section III is devoted to the study of scaling properties of the t-J-h model at general doping, which belongs to the universality class of the Luther-Emery model. The single relevant (charge) exponent K_ρ is calculated and shown to have a nonuniversal value on approaching half-filling, allowing also for dominant SC fluctuations. In Sec. IV we study the pair stability within the t-U-h model by using the degenerate perturbation theory in $h \gg t$ regime. Numerical results within the t-U-h model for the pair stability and K_ρ at general n are presented in Sec. V.

II. HOLE PAIRS AND CLUSTERS IN THE 1D t-J-h MODEL

In the following we study the 1D t-J model in a staggered external field h:[9]

$$H = -t\sum_{is}(c^\dagger_{i+1,s}c_{i,s} + \text{H.c.}) + J\sum_i [S^z_{i+1}S^z_i + \tfrac{1}{2}\gamma(S^\dagger_{i+1}S^-_i + S^-_{i+1}S^+_i) - \tfrac{1}{4}n_{i+1}n_i] - h\sum_i (-1)^i S^z_i \,, \quad (1)$$

where $c^\dagger_{i,s}(c_{i,s})$ are projected fermionic operators, taking into account that the double occupancy of sites is not allowed. n_i and $\mathbf{S}_i$ are corresponding local fermion number and spin operators, respectively. By introducing γ as a separate parameter we allow for the generalization of the usual ($\gamma=1$) isotropic t-J model to an anisotropic $\gamma \neq 1$ model. Although in the above model h has been introduced as a fictitious external staggered (AFM) field, the same effect can be as well obtained by performing a mean-field-type decoupling of magnetic exchange between chains forming a 2D or higher-dimensional correlated system. Here, the main underlying assumption is the existence of the AFM long-range order even in the presence of mobile holes. On the other hand, for main physical implications a milder requirement, that the interhole separation is smaller than the AFM correlation length, seems to be sufficient. Still the correspondence with 2D (or $D > 2$) is far from evident since the particle motion remains entirely 1D.

The most transparent case, although not trivial at finite doping, is the anisotropic Ising (t-J_z-h) model with $\gamma = 0$. Holes introduced by doping the reference AFM (in this case the Néel spin configuration) behave as spinless fermions (sf)—holons, with their motion only shifting the background spins. The effective model for holons can be written in general as

$$H_h = -t\sum_i (a^\dagger_{i+1}a_i + \text{H.c.}) + W \,, \quad (2)$$

where now $a^\dagger_i(a_i)$ are holon (sf) operators and the potential term W still depends on the spin background configuration. To avoid unnecessary complications in W, in the following we restrict ourselves to long chains with open boundary conditions (bc).

For a single hole $N_h = 1$ introduced far from boundaries in a Néel spin state (with initial total $S_z = 0$), the system has $S_z = \pm\frac{1}{2}$ and contains one spinon, which explicitly shows up as an AFM domain wall (DW) if the hole moves from the origin i_0. Since spins, shifted by the hole motion, are opposite to the staggered field, the energy W can be represented in terms of a spin string potential,

$$W = E_0 + \tfrac{1}{2}(J+h) + \sum_i V(i-i_0)\tilde{n}_i \,, \qquad V(r) = h|r| + \tfrac{1}{2}J(1-\delta_r) \,, \quad (3)$$

where E_0 is the reference (AFM) energy and $\tilde{n}_i = a^\dagger_i a_i = 1 - n_i$ is the hole density operator. The additional term $\frac{1}{2}J$ outside the origin is the spinon (DW) energy. Note that in Eq. (3) and further on we assume $h > 0$. The potential problem for $N_h = 1$ can be easily solved and leads for $h > 0$ always to a localized holon-spinon bound state. The solution is particularly simple in the case $h, J \ll t$, where one can introduce the continuum approximation ($i \to x$) for the wave function (wf) $\Psi(x)$, with the corresponding Schrödinger equation

$$-t\frac{d^2}{dx^2}\Psi + h|r|\Psi = \epsilon_1\Psi \,, \quad \epsilon_1 = E_1 - \tfrac{1}{2}J + 2t \,. \quad (4)$$

The lowest solution is symmetric $\Psi(-x) = \Psi(x)$ and can be represented in terms of Airy functions, so that

$$E_1 = -2t + \tfrac{1}{2}J + 1.102(th^2)^{1/3} \,, \quad (5)$$

and the characteristic radius of a bound state (polaron) is $\xi \propto (t/h)^{1/3}$.

For two holes, $N_h = 2$, or in general for an even number of holes $N_h = 2N_p$ introduced into a Néel state, the formation of spinons is not necessary. Remaining within the

$S_z=0$ manifold these configurations can be reached from the initial one, where holes are introduced as tight-bound nearest-neighbor (NN) hole pairs. In this case W can be written as

$$W=E_0+N_p(J+h)+\sum_{p=1}^{N_p}\sum_{i_p,j_p}\tilde{V}(j_p-i_p)\tilde{n}_{j_p}\tilde{n}_{i_p} -\tfrac{1}{2}J\sum_{p=1}^{N_p-1}\sum_{i_{p+1},j_p}\delta_{i_{p+1}-j_p-1}\tilde{n}_{i_{p+1}}\tilde{n}_{j_p} , \quad (6)$$

provided that proper ordering of the hole positions is satisfied $i_1<j_1\cdots<i_p<j_p\cdots<i_{N_p}<j_{N_p}$. W in Eq. (6) consists of the intrapair potential contribution with

$$\tilde{V}(r)=h(r-1)-\tfrac{1}{2}J\delta_{r-1} , \quad r>0 , \quad (7)$$

and the interpair contact attraction, due to the fact that two adjacent holes break one exchange bond less than two separate holes. The same effect results in the second term in Eq. (7). It is also evident from Eq. (6) that due to the ordering requirement and due to the long-range string potential the problem of more pairs $N_p>1$ becomes nontrivial and is not tractable by usual analytical methods for interacting fermion systems.

Analyzing the ground state for $N_h=2$ we notice that bound hole (holon-holon) pairs, as determined by the potential equation (7) for $N_p=1$, are well mobile, in contrast to the localized solution for $N_h=1$ in Eq. (4), and their ground-state wf in an infinite system is delocalized with a well-defined pair total momentum $q=0$. Hence the $N_h=2$ problem again reduces to a single particle problem. The quantity determining the stability of bound hole pairs is the pair-binding energy

$$\epsilon_b=E_2-2E_1+E_0 , \quad (8)$$

where we compare the excitation energy E_2-E_0 of a holon-holon pair with energy $2(E_1-E_0)$ of two separate holes, i.e., two holon-spinon pairs. Within the continuum approximation we get for the pair wf (with respect to the relative coordinate) $\tilde{\Psi}(x>0)$ an equation analogous to Eq. (4) by replacing $t\to 2t$, $\epsilon_1\to\epsilon_2=E_2=4t$. Due to the hard-core hole repulsion we require $\tilde{\Psi}(0)=0$, so that $\epsilon_2=2.945(th^2)^{1/3}$ and

$$\epsilon_b=0.909(\sqrt{th^2})^{1/3}-J . \quad (9)$$

The bound hole pair is characterized by $\epsilon_b<0$, hence the binding appears for $J>J_c=0.909(\sqrt{th^2})^{1/3}$ in the $h\ll t$ regime. Note however that the above analysis applies only for $h>0$. $h=0$ is a singular point, since here $\xi=\infty$ and the concept of a hole pair is meaningless, at least for $J<t$. Nevertheless, the binding appears also for $h=0$ due to the contact attractive term in Eq. (7), however this instability coincides with the onset of PS at $J=4t$.[6] From the above results the origin of hole pair stability, which remains qualitatively valid even for the isotropic $\gamma=1$ case, can be explained as follows. Finite $h>0$ allows the hole pair to form a bound state (with the characteristic size ξ), while the pair becomes energetically favorable relative to two separate holes (holon-spinon pairs) at $J>J_c$, due to the energy cost J, Eq. (9), which arises from the two additional spinons for separate holes.

Explicit expressions for ϵ_b can be obtained also for $h\gg t$, where the perturbation expansion of E_1, E_2 (Ref. 14) in powers of t/h can be performed, starting with the localized hole and the NN local hole pair in a Néel state, respectively. Apart from the broken bond term the lowest nontrivial contribution to ϵ_b comes only in the t^4 order, i.e.,

$$\epsilon_b=-\frac{J}{2}+\frac{2(2h-J)t^4}{(h+\frac{1}{2}J)^3(2h+\frac{1}{2}J)} , \quad (10)$$

so that $J_c/t\sim 4(t/h)^3\ll 1$ in this regime.

The complete critical line $J_c(t)$ interpolating between both regimes, is presented in the phase diagram, Fig. 1. Note that we get $J_c<0.24t$ for arbitrary $h>0$, J_c being far below the appearance of the PS.

To distinguish the pairing phase with separate hole pairs from the PS where pairs collapse into a cluster, dividing the system into a hole-rich and a spin-rich phase, we have to study the ground state with at least $N_h=4$. The calculation of the ground state, reduced to Eq. (6) for few bodies bound by strings, still cannot be treated analytically. We solve it numerically by the exact diagonalization on a system of $N=30$ sites. The criterion for the onset of PS is the clustering energy

$$\Delta=E_4-2E_2+E_0 , \quad (11)$$

being $\Delta\geq 0$ for two separate pairs, and $\Delta<0$ for a bound cluster. The PS line $J_s(h)$, as determined by $\Delta=0$, is also presented in Fig. 1. We notice that at $h<t$ the numerical curve $J_c(h)$ joins smoothly the point $J_s(h=0)=4t$, known exactly by mapping the t-J_z model onto the XXZ spin model.[6]

One can again treat analytically the $h\gg t$ regime. For $J>J_c$ pairs become very tightly bound, due to the steep increase of the potential in Eq. (7). They appear as hole dimers hopping by one site with an effective $\tilde{t}=t^2/(h+\frac{1}{2}J)$ and interacting with a contact attraction, so that an effective dimer model can be written

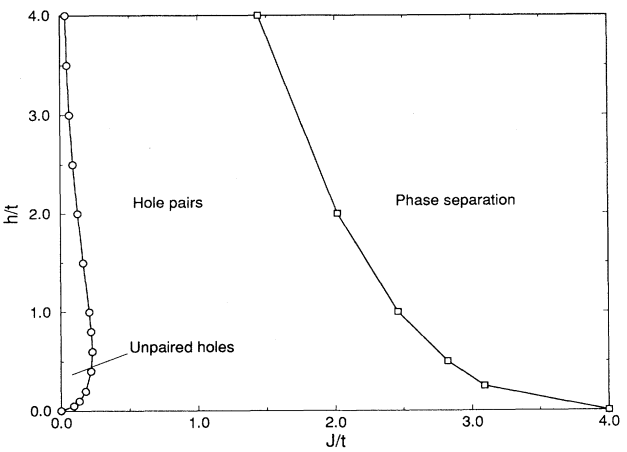

FIG. 1. Phase diagram of the t-J_z-h model at low doping.

$$H_d=-\tilde{t}\sum_i(b_{i+1}^{\dagger}b_i+\text{H.c.})+\tfrac{1}{2}\sum_{i,j}v(i-j)\rho_i\rho_j\ ,$$
$$v(i-j)=\begin{cases}\infty & \text{for } |i-j|=1\ ,\\ -\frac{1}{2}J & \text{for } |i-j|=2\ ,\\ 0 & \text{for } |i-j|>2\ ,\end{cases} \tag{12}$$

where $b_i^{\dagger}(b_i)$ and ρ_i now refer to dimer creation (annihilation) and number operators, respectively, and the form of $v(i-j)$ accounts for the fact that dimers may not overlap. Pairs thus behave as bosons (or equivalently sf) with extended hard core, so again the model can be mapped onto an anisotropic spin chain model. As a consequence the PS appears at $J_s=4\tilde{t}\propto 4t^2/h\gg J_c$ for $h\gg t$, consistent with numerical results for J_s in Fig. 1.

The isotropic ($\gamma=1$) t-J-h model becomes much more difficult for any analytical treatments and for few holes cannot be reduced to a few-body problem, as for $\gamma=0$ in Eqs. (2) and (6). Still one cannot expect essential qualitative changes, except at $h\to 0$, since the spin rotation symmetry is anyhow lost due to $h\neq 0$. On the other hand, spin flips allowed at $\gamma>0$ connect configurations, e.g., describing two separate holes and a bound hole pair, respectively, which are disconnected in the Ising case. Here no abrupt transitions can be expected in the ground state, at least in finite systems.

It is then plausible that for $h\gg t$ there should be no difference between the Heisenberg and the Ising cases, since spin flips become irrelevant under a strong field. In this regime we can also estimate the influence of the spin-flip processes by performing a perturbation expansion in $\gamma>0$ as well. Restricting ourselves to the corrections being first order in γ, we note that they appear only in E_1. Namely, for $N_h=1$ spin flips introduce the possibility of hole propagation by a double hop followed by a spin flip[11,14] erasing the remaining spin string, or of a reversed process. This leads to

$$\delta E_1=\frac{2\gamma Jt^2}{(h+\frac{1}{2}J)(2h+\frac{1}{2}J)}\cos 2q\ , \tag{13}$$

with the minimum at the wave vector $q=\pi/2$. E_0 and E_2 do not get linear contributions in γ, so the leading corrections in the $h\gg t$ regime appear to be

$$\delta\epsilon_b\sim\frac{2\gamma Jt^2}{h^2}\ ,\quad \delta J_c\sim\frac{8\gamma t^6}{h^5}\ . \tag{14}$$

The spin flips are thus expected to increase J_c, still the change should remain small even at $\gamma=1$, i.e., $\delta J_c\ll J_c(h=0)$ in the investigated regime.

Entering the most interesting $h<t$ regime we have to rely on numerical results obtained by the exact diagonalization of small systems, where we are able to study chains with up to $N=20$ sites for $N_h=2$ and $N=18$ sites for $N_h=4$. To define the onset of binding J_c in a finite system with $N_h=2$ we can again use as a criterion $\epsilon_b=0$, Eq. (8), comparing the energies of the systems with $N_h=0,1,2$. This procedure can be misleading sometimes, in particular for 1D systems. For example, in a t-J_z model ($h=0$) on an even chain with periodic bc we would typically get $\epsilon_b<0$, although two holes would be nearly free sf for $J_z<t$. These effects on ϵ_b are expected to becomes less pronounced for $h>0$, since $h>0$ prevents the existence of free spinons and holons and binds them in pairs of spinon-spinon, holon-spinon, or holon-holon types. In Fig. 2 we present the variation of ϵ_b with J at fixed h/t. One can notice a qualitative similarity with the Ising case, Eq. (9). Namely, for $J>J_c$ ϵ_b varies nearly linearly with J. The slope is only weakly dependent on h/t, still it is substantially reduced from the Ising case, Eq. (9). In Fig. 2 we present also the result along the $J=2h$ line, where one can establish an appealing relation with the 2D t-J model[9] and the variation of ϵ_b in Fig. 2 indeed follows the one observed in 2D studies.[12] $\epsilon_b>0$ found for $h=0$ is typical 1D finite-size (or bc) effect.

An alternative criterion for binding can be obtained from the hole density-density correlations within the ground-state wf $|\Psi_0\rangle$

$$g(r)=\langle\Psi_0|\tilde{n}_i\tilde{n}_{i+r}|\Psi_0\rangle\ . \tag{15}$$

In contrast to ϵ_b, $g(r)$ is calculated at fixed N_h and is usually less sensitive to size and boundary effects. We expect for $N_h=2$ a rather simple structure of $g(r)$ with a single maximum at $r=r_m$. Hence a bound state should be characterized by $r_m<N/2$, while for unbound holes the maximum should be at $r_m=N/2$. Note that the latter case applies to $h=0$, where holes behave as sf and $g(r)\propto\sin^2(\pi r/N)$. A slight complication arises at $h>t$ due to even-odd oscillations in $g(r)$, since holes prefer to be on the opposite sublattices, i.e., odd r become preferred.

As an example we present in Fig. 3 correlations $g(r)$ for increasing J/t at fixed $h/t=0.5$. It is easy to notice that at $J=0$ holes are repelling each other, in fact we are in this case (at $J=0$ charge degrees are insensitive to γ) dealing with two separate polarons (holon-spinon pairs), as given by Eqs. (3)–(5). On increasing J we are moving through an intermediate regime with $g(r)$ as for two sf into the regime of pronounced binding appearing for

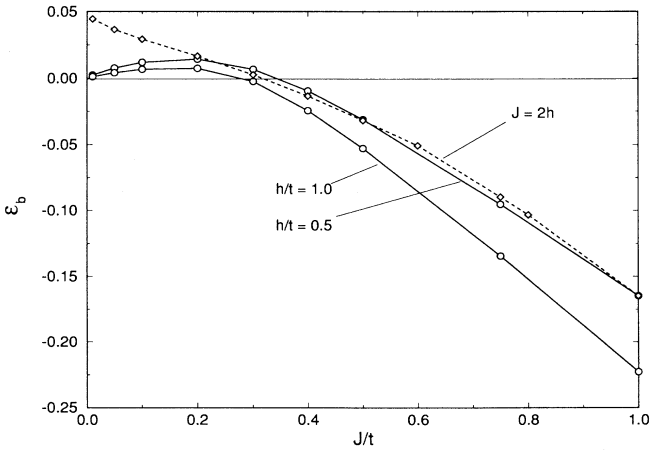

FIG. 2. The pair-binding energy ϵ_b for $N_h=2$ vs J/t in the isotropic t-J-h model at two different values of h/t (○) and along the line $J=2h$ (◇), as obtained for an $N=16$ chain.

$J > J_c \sim 0.3t$, where $r_m < N/2$. An analogous behavior of $g(r)$ is observed along the $J = 2h$ line, as presented in Fig. 1 of Ref. 9.

The characteristic radius ξ of the pair depends mainly on h, as in the Ising case. This is evident from Fig. 3, where the maximum r_m remains nearly constant in a broad regime of J for $J > J_c$, except in the vicinity of PS at $J > 1.5t$. The explicit dependence of ξ is even more evident in Fig. 4, where we present results obtained at fixed $J = 0.5t$, but increasing $h > 0$. Here we define ξ as the first moment of $g(r)$,

$$\xi = \sum_r |\bar{r}| g(r) \Big/ \sum_r g(r) , \tag{16}$$

and $\bar{r}$ is taken as the closest distance between holes on a chain with periodic bc. At $h = 0$ ξ is the same as for sf (for given chain size). It is understandable that no changes are seen for finite $h \ll t$, since the size of the bound pair (ξ, r_m) exceeds the available chain length. On the other hand, for $h > 0.2t$ the pair radius ξ as well as r_m steadily fall off, in qualitative agreement with the Ising case. In Fig. 4 we present also the variation of the total spin $S^2 = \langle \mathbf{S} \cdot \mathbf{S} \rangle$ in the ground state. Due to $h > 0$, S^2 is not a good quantum number, still it is instructive to follow S^2 for a pair. Figure 4 indicates that a bound pair steadily evolves from a singlet $S = 0$ state at $h = 0$ and is thus expected to possess a rather well-defined $S \gtrsim 0$ even for $h < t$. Clearly S^2 loses its meaning for $h > t$.

In Fig. 5 we compile results for the critical line $J_c(h)$, as obtained mainly from the criterion based on $g(r)$. Points are calculated for chains with up to $N = 20$. This large size allows us to trace the binding down to $h \sim 0.1t$, where we find $r_m \sim 7$. For lower h the characteristic pair size reaches and exceeds $N/2$, still we expect that eventually in a large system $J_c \to 0$ for $h \to 0$, as in the Ising case in Fig. 1. An indication for the latter is also a weakly pronounced maximum $J_c \sim 0.33t$ at finite $h \sim 0.5t$. Still the question of the $h \to 0$ limit remains a rather subtle one.

In order to determine the PS line $J_s(h)$, results for

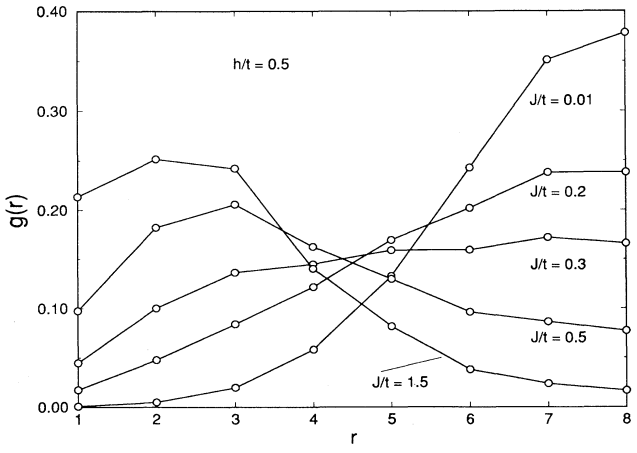

FIG. 3. Hole-density correlation function $g(r)$ vs distance r for $N_h = 2$ in the t-J-h model, presented for different values J/t at $h/t = 0.5$.

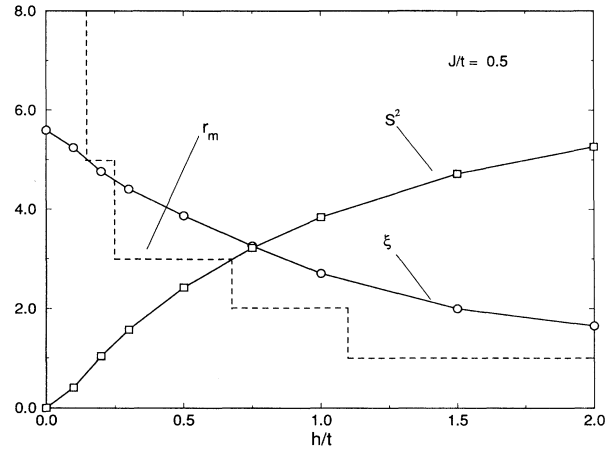

FIG. 4. h dependence of data extracted from the two hole $g(r)$ at fixed $J/t = 0.5$: (a) r_m locating the maximum of $g(r)$; (b) the effective pair radius ξ, defined by Eq. (16); and (c) S^2 of the ground state.

$N_h = 4$ are needed. Two criteria are again possible: (a) the vanishing of the clustering energy $\Delta = 0$, Eq. (11) and (b) the fall-off of $g(r)$, now for $N_h = 4$, at large distances $r_m \ll r \ll N/2$ in the PS regime. Consistent with the picture of two separate pairs, we find in the intermediate regime $J_c < J < J_s$ two maxima in $g(r)$: one at $r = r_m$ corresponding to the intrapair correlations, and another at $r = N/2$ related to the interpair correlations. The shift and the disappearance of the second maximum is taken as the sign of cluster formation and hence PS. Results of both methods are shown in Fig. 5, with calculations performed on an $N = 14$ chain for Δ and an $N = 18$ chain for $g(r)$, respectively. Both results merge with the known value $J_s(h = 0) \sim 3.5t$.[5] Still there is a substantial discrepancy among results at intermediate values

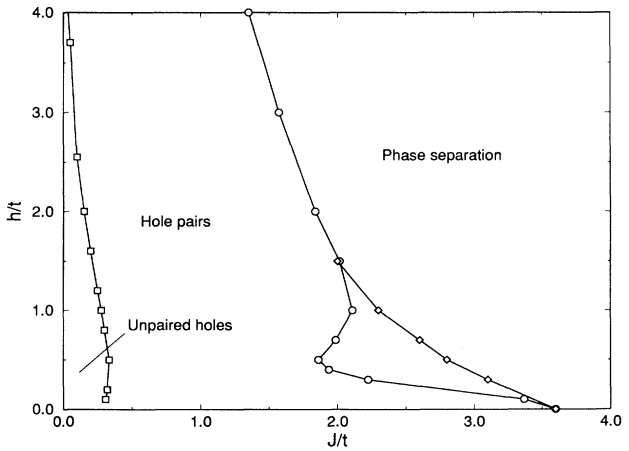

FIG. 5. Phase diagram of the isotropic t-J-h model at low doping. The PS line is determined either from $\Delta = 0$ (○), or from the disappearance of the second maximum in $g(r)$ (◇). Both lines coincide for $h > 1.5t$.

$0.2 < h/t < 1.0$. The latter can be to some extent reconciled by the fact, that in rather short chains we are working with a substantial doping at $N_h = 4$, and that the variation with doping remains strong. Namely, using $\Delta = 0$ as a condition we are effectively calculating the inverse compressibility ($\kappa^{-1} \sim \Delta$) at the doping $n_h = N_h/N = \frac{2}{14}$, whereas the $g(r)$ analysis belongs to $n_h = \frac{4}{18}$. The correctness of this interpretation is supported by the calculation of $\kappa^{-1} \sim \Delta_4 = E_6 - 2E_4 + E_2 = 0$ at the effective doping $n_h = \frac{4}{14}$, where the resulting $J_s(h)$ agrees quantitatively with the one in Fig. 5 obtained from $g(r)$. It is still not easy to predict from Fig. 5 a detailed behavior of J_s for $h < t$ for an infinite system.

Let us comment here on the relation of the obtained phase diagram to a more challenging 2D t-J model at low hole doping. The correspondence can be established by assuming the mean-field $h = 2J\langle S_z \rangle \sim J/2$ emerging from the exchange with AFM-ordered neighboring chains. From Fig. 5 we get along the $J = 2h$ line $J_c \sim 0.3t$ and $J_s \sim 2t$. These values are quite consistent, respectively, with the onset of binding[12] and clustering (PS).[15–17] Although for the PS transition in 2D smaller thresholds $J_s \sim 1.2t$ are obtained by various methods,[16] it has been recently pointed out that possibly the lowest instability is not toward the clustering,[17] which we are considering here in 1D system. The main novelty of the 1D t-J-h model is however in the fact that it allows for quite small binding onset $J_c \ll t$, being well distinct from $J_s > t$, the scenario possibly valid also for 2D t-J model.

III. t-J-h MODEL AT FINITE DOPING

The question of the macroscopic charge density and SC pairing correlations can be discussed only in connection with the finite hole doping, where the fermion density $n = N_e/N = 1 - n_h$ becomes an additional parameter. Since in 1D systems of interacting fermions the large-distance asymptotics of correlation functions as well as several macroscopic physical properties are determined by few interaction-dependent exponents,[2,4] it is important to find out the proper universality class to which the t-J-h and the t-U-h models belong at finite doping $n < 1$.

At $h = 0$ the 1D t-J model as well as the 1D Hubbard model are characterized by gapless spin and charge excitations. It has been shown that in the whole parameter regime, except for the PS regime $J > J_s$, both models belong to the universality class of Luttinger liquids,[2–4] with a nontrivial charge exponent K_ρ. The latter divides the Luttinger liquids into two qualitatively different regimes: $K_\rho < 1$ induces dominant (slowly decaying) charge density wave (CDW) fluctuations, while $K_\rho > 1$ indicates dominant SC correlations. For the t-J model outside the PS regime,[5] in particular for the symmetric $J = 2t$ case,[18] it has been shown that $K_\rho > \frac{1}{2}$, where the limiting value $K_\rho = \frac{1}{2}$ is manifested at limiting densities $n \to 0, 1$ for all J due to the similarity with sf.

Finite $h > 0$ introduces a gap in the spin excitation spectra, while the charge excitations remain gapless. The universal behavior should be in this case that of the 1D interacting fermionic system within the Luther-Emery (LE) regime.[2] While spin correlations decay exponentially with distance, the CDW and SC large-distance correlations should exhibit a power-law behavior, characterized by the charge exponent K_ρ,[2] i.e.,

$$C_{\mathrm{CDW}}(r) = \langle n_i n_{i+r} \rangle - n^2 \sim \frac{a}{r^2} + br^{-K_\rho} \cos 2k_F r \ ,$$
$$C_{\mathrm{SC}}(r) = \langle O_i^\dagger O_{i+r} \rangle \sim cr^{-1/K_\rho} \ , \tag{17}$$

where O_i is a (singlet) pairing operator

$$O_i = \frac{1}{\sqrt{2}} (c_{i\uparrow} c_{i+1\downarrow} - c_{i\downarrow} c_{i+1\uparrow}) \ . \tag{18}$$

It should be noted however, that due to the doubling of the unit cell at $h > 0$, one should regard separately correlations, Eq. (18), for the even and the odd sublattice, respectively. Still due to the symmetry this merely leads to the possibility of either equal or opposite coefficients on both sublattices.

Again, SC correlations become dominant for $K_\rho > 1$. In order to calculate K_ρ and the renormalized charge velocity u_ρ for a finite system, we use their relation to macroscopic quantities, i.e., the compressibility κ and the charge stiffness D,[4]

$$\frac{1}{n^2 \kappa} = \frac{1}{N} \frac{\partial^2 E_0}{\partial n^2} = \frac{\pi u_\rho}{2K_\rho} \ ,$$
$$D = \frac{1}{2N} \frac{\partial^2 E_0}{\partial \theta^2} \bigg|_{\theta = \theta_0} = \frac{u_\rho K_\rho}{\pi} \ , \tag{19}$$

i.e., D can be obtained by studying the dependence $E_0(\theta)$ [around the minimum $E_0(\theta_0)$] on the phase θ, entering the kinetic term in Eq. (1) as $t \to t \exp(\pm i\theta)$.[19,4] On the other hand, κ can be in a finite system calculated as

$$\frac{1}{n^2 \kappa} \sim \frac{N}{4} [E_0(N_e + 2) - 2E_0(N_e) + E_0(N_e - 2)] \ , \tag{20}$$

taking into account only states with even N_e to avoid even-odd finite-size effects.

The evaluation of K_ρ in a finite system through κ and D has the advantage, that we rely on properties of the ground state. A more direct evaluation through κ and u_ρ, used by other authors,[5,8] would require the analysis of excited states, which is often more delicate.

In Fig. 6 we present as an example results for K_ρ, obtained for $N = 14$ chain, as a function of electron density n for various J/t at fixed $h = t$. As expected, for $n \to 0$ all results converge to the sf value $K_\rho = \frac{1}{2}$, since $h > 0$ does not introduce any essential changes in this case. On the contrary, $h > 0$ dramatically changes the behavior at $n \to 1$. In contrast to $K_\rho = \frac{1}{2}$, observed at $h = 0$, the limiting $K_\rho(n = 1)$ becomes nonuniversal, depending on parameters h, J. This is evident from Fig. 7, where we show K_ρ, calculated close to half-filling, i.e., at $n_h = \frac{2}{14}$, as a function of J/t for various h/t.

At fixed $h > 0$ we can clearly establish two limiting cases for K_ρ.

(a) For $J \to 0$ the model equation (1) reduces to the

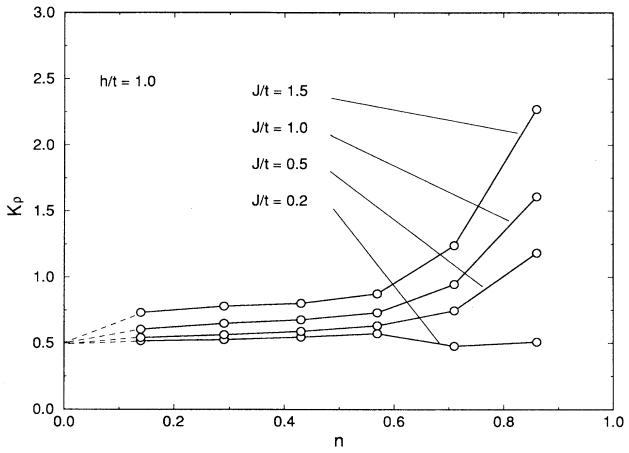

FIG. 6. The charge exponent K_ρ vs electron density n at fixed $h/t=1.0$ and varying J/t, obtained on a chain with $N=14$.

much simpler t-h model, which is equivalent to the t-J_z-h model with $J_z=0$, discussed in Sec. II. For $n\to 1$ we are dealing with a low concentration of unbound holes, as described by Eqs. (3)–(5). Such holes are entirely localized, hence $D=0$ and $K_\rho=0$. In this limit one obtains not just dominant CDW correlations, but rather a long-range CDW ordering.

(b) Approaching PS ($J\to J_s$) κ diverges and this leads to $K_\rho\to\infty$. This is the regime of extreme SC fluctuations.

While the vicinity of PS induces dominant SC correlations also in the 1D t-J model,[5] there is an essential difference in the width of the SC regime. Namely, the t-J model $K_\rho>1$ is restricted to a narrow regime above $J>2t$,[5] whereas $h>0$ induces a $K_\rho>1$ in a regime starting well below $J=t$.

In particular for larger fields $h>t$ one obtains a very wide region with $K_\rho\lesssim 2$. We can present for $h\gg t$ also a simple analytical argument in favor of SC correlations. If we are at $h\gg t$ in the regime of bound pairs $J_c<J<J_s$, pairs are well localized and we can use at low doping the representation in terms of spinless fermions Eq. (12). Since sf in Eq. (12) have charge 2, we can simply relate the quantities of the original model to the ones of sf, i.e.,

$$D=4D_{\rm sf}\ ,\quad n^2\kappa=4n_{\rm sf}^2\kappa_{\rm sf}\ ,\quad K_\rho=4K_\rho^{\rm sf}\ . \tag{21}$$

Model equation (12) with a finite NN interaction only, has been well studied through the mapping onto the anisotropic 1D XXZ spin model.[3,4,6] For low doping with $n_{\rm sf}=\frac{1}{2}(1-n)\ll 1$ one gets $K_\rho^{\rm sf}=\frac{1}{2}$ independent of the particular form of v, as long as the PS is not reached. Hence we expect $K_\rho\sim 2$ in a broad regime $J_c<J<J_s$ for $h\gg t$.

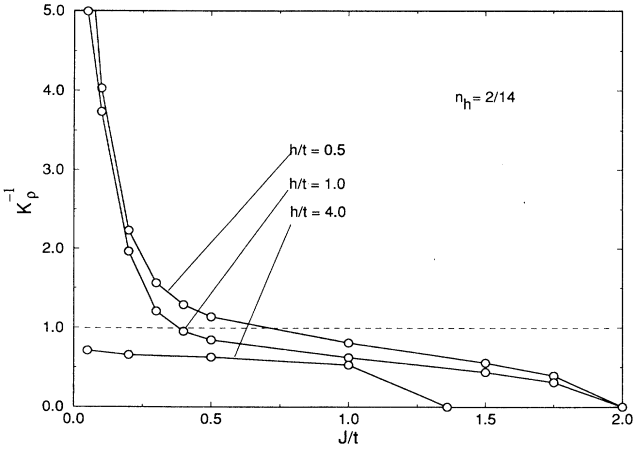

FIG. 7. The inverse charge exponent $1/K_\rho$ vs J/t for different fields h/t at the hole density $n_h=\frac{2}{14}$.

IV. PERTURBATION EXPANSION FOR THE t-U-h MODEL

In the next two sections we consider the Hubbard model in a staggered field (t-U-h model), defined as

$$H=-t\sum_{is}(c_{i,s}^\dagger c_{i+1,s}+\text{H.c.})+U\sum_i n_{i\uparrow}n_{i\downarrow}$$
$$-\tfrac{1}{2}h\sum_i(-1)^i(n_{i\uparrow}-n_{i\downarrow})\ . \tag{22}$$

In the strong-correlation limit $U\gg t$ the above model is expected to become equivalent to the t-J-h model equation (1) with $J\sim 4t^2/U$, if we neglect NNN hopping terms which are of the same order as J. This holds however only for $h\ll t$. Outside the strong-correlation regime as well as for moderate fields ($h\sim t,U$), the equivalence no longer holds, so one has to investigate both models separately.

At $h=0$ the 1D Hubbard model is exactly solvable,[20] it shows the Luttinger-liquid behavior[2,3] in the whole phase diagram, and there is no pair formation at arbitrary U/t. For 1D t-U-h model with $h>0$, the existence of bound pairs is a rather subtle problem, both analytically and numerically. Besides the numerical evidence for bound pairs, presented in Ref. 9, recently a simple interpretation for the origin of binding in the t-U-h model has been given in the framework of the Hartree-Fock approximation.[13] Unfortunately the latter analysis is not exact in any limit, so one needs alternative methods.

One approach is the analytical evaluation of the ground state for $N_h=2$ via the perturbation series in t/h (Ref. 9)—hence limited to the region $t/h\ll 1$— but for arbitrary U. However, the calculation is complicated by the degeneracy of the zeroth-order ground state. Moreover, the second-order correction does not lead to a bound state, so that the fourth-order correction needs to be considered and consequently calculations become quite tedious. We are presenting the procedure in some detail below.

At $h\gg t$ one starts with the unperturbed Hamiltonian $H_0=H(t=0)$ which is diagonal in the site representation. For $N_h=2$ the ground state is highly degenerate, allowing for any distance between two holes. All other sites are singly occupied with spins in a perfect Néel or-

der. Thus degenerate perturbation theory (DPT) must be used and we take version of the secular determinant as presented in Ref. 21, which up to the fourth order in perturbation H' reads

$$(E-E_0)\delta_{\alpha\beta} = \left\langle \alpha \left| H' \frac{Q_0}{\eta_0} H' \right| \beta \right\rangle + \left\langle \alpha \left| H' \frac{Q_0}{\eta_0} H' \frac{Q_0}{\eta_0} H' \frac{Q_0}{\eta_0} H' \right| \beta \right\rangle - \sum_\gamma \left\langle \alpha \left| H' \frac{Q_0}{\eta_0} H' \right| \gamma \right\rangle \left\langle \gamma \left| H' \frac{Q_0}{\eta_0^2} H' \right| \beta \right\rangle + \cdots . \quad (23)$$

The (bra)kets denote the set of degenerate states with the unperturbed energy E_0, P_0, and Q_0 are the projectors onto this subspace of states and its complement, respectively. H' is taken to be the hopping part in Eq. (22), while η_0 stands for the energy denominator E_0-H_0.

There are two classes of configurations in the subspace $\{|\alpha\rangle\}$ which do not mix in any (finite) order of PT, i.e., configurations with holes separated by an even, respectively, odd, number of lattice sites (including zero), which we denote as $|0\rangle$, $|2\rangle$, etc., and $|1\rangle$, $|3\rangle$, etc. Evidently, the "even" class has total $S^z=0$, whereas the "odd" class belongs to the $S^z=\pm 1$ sector.

We consider here only the "even" class, since it in general yields lower ground-state energy. Within the second order there are at most four distinct contributions, of which one is nondiagonal and results from the transition to the state with holes separated by two more (less) lattice sites, i.e., to $|2j\pm 2\rangle$, and the rest are diagonal contributions corresponding either to charge fluctuations or to a "caterpillar" translation of the hole pair (present for $j=0$ only). Thus, from Eq. (23) we have for the right side the following nonvanishing matrix elements:

$$M_{jj}^{(2)} = -\frac{4t^2}{h} + 2k_j \frac{t^2}{h+U} , \quad M_{j,j+2}^{(2)} = 2\frac{t^2}{h+U} , \quad (24)$$

where $k_{j=0}=3$ and $k_{j\neq 0}=4$. The energy is measured with respect to the $N_h=0$ case, so the second-order correction $E_0^{(2)}=2Nt^2/(h+U)$ has already been subtracted from $M_{jj}^{(2)}$.

$E_2^{(2)}$ is obtained as the lowest eigenvalue of the secular determinant, which has the tridiagonal form with diagonal elements ($a_0=M_{00}^{(2)}$, $a=M_{22}^{(2)}$, a, . . .) and with a constant subdiagonal ($b=M_{02}^{(2)}$, b, . . .). For the given form of the tridiagonal matrix, a bound state would appear below the continuum band for $a_0<a-b$. In our case we get from Eq. (24) $a_0=a-b$, hence we are in the marginal case with no states below the bottom of the band. To establish the question of the existence of a bound state one has to go to the next nontrivial, i.e., fourth, order in DPT.

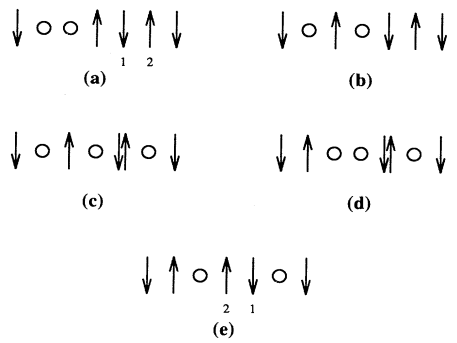

FIG. 8. The sequence of connected processes involved in the contribution m_{02}, Eq. (25), to the fourth-order matrix elements.

In the fourth order there are many processes to be considered. The contribution of a particular process to the matrix element in the secular determinant may be conveniently labeled according to its energy denominator η and to its multiplicity w. For example, for the process in Fig. 8 which connects states $|0\rangle$ and $|2\rangle$ one has $\eta=h(2h+U)(h+U)$ and $w=12$ giving the contribution

$$m_{02} = -(-1)w\frac{t^4}{\eta} = 12\frac{t^4}{h(2h+U)(h+U)} . \quad (25)$$

The extra "minus" sign comes from step (d) in Fig. 8 where fermions on sites 1 and 2 interchange. (η_n, w_n) for all the different processes $m_{ij}(n)$ are given in Table I from which the total contribution $M_{ij}^{(4)}$ is obtained by summation

$$M_{ij}^{(4)} = \sum_n m_{ij}(n) = -(-1)^k \sum_n w_n \frac{t^4}{\eta_n} , \quad k=\frac{|i-j|}{2} . \quad (26)$$

In Table I the last column refers to the chain with no

TABLE I. Values for w_n and corresponding energy denominators η_n in Eq. (26) for two-hole processes, representing the fourth order of DPT. In the last column no-hole corrections are given. Note that also contributions from disconnected processes are included.

n	η_n	m_{00}	m_{02}	m_{04}	m_{22}	m_{24}	
1	$2h^3$	16			20		
2	$2h^2(h+U)$	16	12		8	8	
3	$h^2(2h+U)$	$8(N-4)$	4		$8(N-5)$	4	
4	$h(2h+U)(h+U)$	$16(N-4)$	12		$16(N-5)$	16	
5	$2h(h+U)^2$	$4(N-2)$	8		$4(N-3)$	8	$4N$
6	$(h+U)^2(2h+U)$	$16(N-4)$	8		$8(2N-11)$	12	$8N$
7	$2(h+U)^3$	$8(N-4)(N-5)$	$16(N-5)$	12	$8(N^2-11N+33)$	$16(N-6)$	$8N(N-3)$

holes and its entries should be subtracted from the diagonal matrix elements $m_{jj}(n)$. The resulting secular determinant in the fourth order is a five-diagonal matrix with the following structure: the main diagonal has the form $(a_0,a,a,\ldots)$, the first subdiagonal is $(b_0=M_{02}^{(4)}, b=M_{24}^{(4)}, b,\ldots)$ and the second subdiagonal is $(c=M_{04}^{(4)}, c,\ldots)$.

Given the structure of the secular determinant we are now ready to calculate the ground-state energy corrections. The linear system of equations may be solved by the ansatz

$$\phi_j = A(-1)^j e^{-\mu_1 j} + Be^{-\mu_2 j} , \tag{27}$$

and $\mu_{1,2}$ are solutions to the equation

$$\begin{vmatrix} a_0-E-b_0x+cx^2 & a_0-E+b_0y+cy^2 \\ b_0-b+c/x & b_0-b-c/y \end{vmatrix} = 0 ,$$

$$x,y = e^{-\mu_{1,2}} , \tag{28}$$

which can be solved perturbatively to the necessary order. Analyzing Eq. (28) we find that $y \sim -c/b$ whereas $x \sim 1$, i.e., the second term in Eq. (27) dies off very fast and consequently the first term is more important. After some algebra we find

$$b^{(2)}\mu_1 = 2b_0 - 3b - a_0 + a + 2c . \tag{29}$$

Note that only fourth-order corrections enter the right-hand side (RHS) of Eq. (29), since within the second order $a=a_0+b$, $b_0=b$, and $c=0$. An important check of the calculation is also that $\mu=0$ for $U=0$, where no localized states exist.

Explicit calculation of μ_1, which has the meaning of the inverse pair radius, gives

$$\mu_1 = \frac{Ut^2(24h^2-U^2)}{h^3(h+U)(2h+U)} , \tag{30}$$

where Eq. (30) naturally applies only to the regime $\mu_1 > 0$, where the bound state exists. In this regime the pair-binding energy can be expressed as

$$\epsilon_b = -\frac{2t^2}{h+U}\mu_1^2 . \tag{31}$$

Equation (30) has an interesting consequence that for $h \gg t$ any finite repulsion $U>0$ induces pair binding and pairs remain stable in the regime $0 < U < U_c = \sqrt{24}\,h$.

V. NUMERICAL RESULTS FOR THE t-U-h MODEL

The t-U-h model equation (22) proves to be much more challenging for the numerical exact diagonalization studies. Due to larger basis in the Hubbard model we are only able to study chains of lengths $N \le 14$, smaller than for the t-J-h model. $h \neq 0$ also doubles the unit cell and oscillations of period 2 in $g(r)$ appear more pronounced in the t-U-h model at larger $h > t$, effectively reducing the number of available data. In addition, we observe quite disturbing size effects. In particular, results for fixed number of holes, e.g., for most studied case $N_h=2$, depend substantially on whether we use the lattices with $N=4m+2$ or $N=4m$ sites. All the above drawbacks make results, concerning hole binding and possible dominant SC pairing in the t-U-h much less reliable.

To determine the threshold for the hole binding in the t-U-h model we could again use as criteria either the binding energy ϵ_b or $g(r)$ correlations. ϵ_b, however, does not become negative in most of the studied systems. The failure to establish $\epsilon_b < 0$ can be traced back to the usual $h=0$ Hubbard model, where we find quite large $\epsilon_b > 0$ (for a long chain one would expect $\epsilon_b \sim 0$). $h > 0$ does reduce substantially ϵ_b, still it does not make it negative. Also $g(r)$ do not yield a unique answer. If we take $N=14$ chain, or similarly $N=10$, we get on increasing h more pronounced maximum in $g(r)$ at smaller $r=r_m \ll N/2$, as presented in Fig. 3 of Ref. 9, indicating a tightly bound pair. This can also be concluded from the average radius ξ which decreases substantially with U at fixed h, as shown in Fig. 9. At finite $U=U_c$ an abrupt crossover happens to an unbound state of two holes, hence a jump in Fig. 9. Since at $U > U_c$ holes behave approximately as two (possibly repelling) sf we observe that $\xi > \xi(U=0)$. The crossover shows that the bound-pair and the unbound-pair solutions have different symmetries (parity) in the $N=4m+2$ chains, leading to an overemphasized discontinuity at $U=U_c$. On the other hand, there is no crossover for $N=12$, since both types of solutions belong to the same symmetry as in the case of the t-J-h model. However the transition between both situations is less pronounced in the latter case, so we use as results for the binding transition at U_c the crossover obtained for $N=14$.

The calculated transition line $U_c(h)$ in the t-U-h model is presented in Fig. 10. We notice that at $h \gg t$, U_c increases nearly with h, in qualitative agreement with the analytical result equation (30). U_c increases also at small $h \to 0$, found also within the Hartree-Fock approximation.[13] If we assume the correspondence $J=4t^2/U$ we observe an overall qualitative similarity with the behavior of the $J_c(h)$ in the t-J-h model. However, one should not

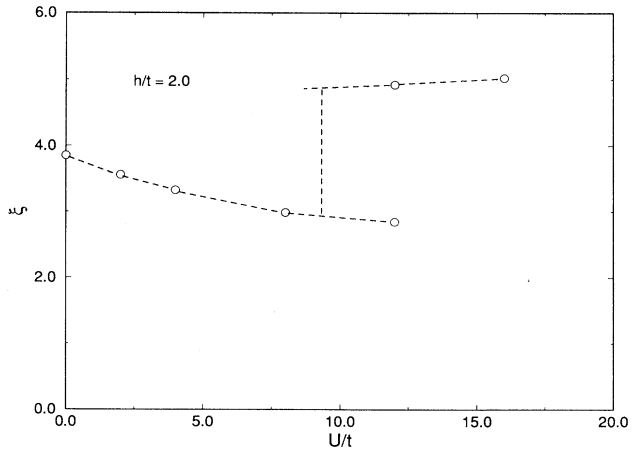

FIG. 9. The pair radius ξ vs U/t for the t-U-h model on an $N=14$ chain for $h/t=2.0$. The crossover between the paired and the unpaired ground state occurs abruptly (vertical dashed line).

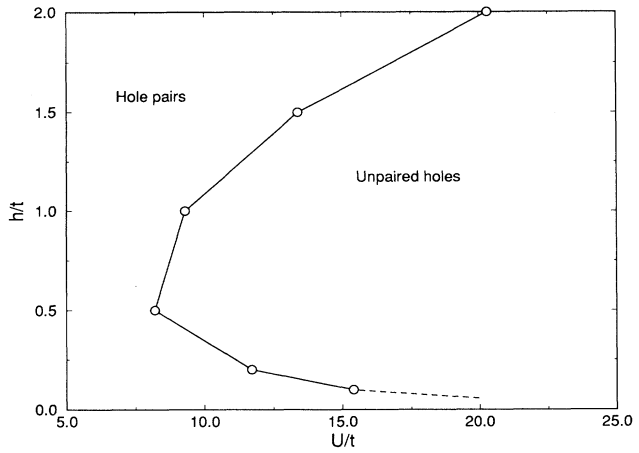

FIG. 10. The phase diagram of the t-U-h model at low doping, as obtained for $N=14$ chain.

forget quite pronounced uncertainties in the above results for the t-U-h model. There exists also a possibility for another binding threshold, so that the bound pair would again become unstable for $U<U_c^*(h)$. Namely, it seems plausible that for $U \ll h \ll t$ we enter the regime where U presents a small perturbative effect, which is not enough for binding. Clearly this regime has no mapping onto the t-J-h model.

We study the behavior of the t-U-h model at general fermion concentration $n<1$ in the same way as performed in Sec. III for the t-J-h model. Results for K_ρ, as calculated for $N=12$ chain and presented in Fig. 11, are subject to well-pronounced oscillations between $N_h=4m$ and $N_h=4m+2$ cases. These have the same origin as the differences observed between $N=4m$ and $N=4m+2$ chains at fixed N_h. From Fig. 11 we see that again $K_\rho \sim \frac{1}{2}$ for $n \to 0$ independent of h. On the other hand K_ρ

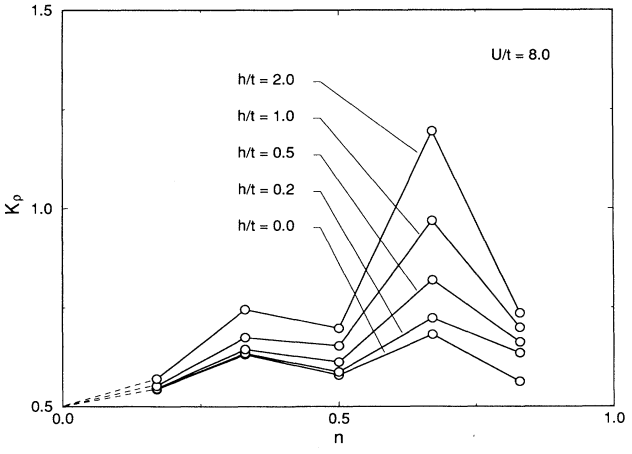

FIG. 11. The charge exponent K_ρ vs electron density n for the t-U-h model at fixed U/t and different fields h/t, calculated for $N=12$ chain. The oscillations can be traced back to the well known $N_h \equiv 0$ or 2 mod(4) effects.

seems to be nonuniversal in the $n \to 1$ limit, i.e., results indicate a systematic increase of $\tilde{K}_\rho = K_\rho(n \to 1)$ with h at fixed U and a decrease with U at fixed h. Due to finite-size effects it is hard to make a more definite statement on the values $\tilde{K}_\rho$. At fixed $h \neq 0$ and $U \to \infty$ the model reduces to the t-h model, so that we expect again $\tilde{K}_\rho = 0$. For $U \ll h$ we are approaching the limit of noninteracting fermions, hence $K_\rho = 1$, consistent with the result in Fig. 11. Still it is not clear whether the variation between both extremes is a monotonous function of U, h and whether values $K_\rho > 1$ are possible.

VI. CONCLUSIONS

In this paper we have considered properties of 1D t-J-h and t-U-h models near half-filling, where one can expect a novel qualitative behavior of holes in the system. We have shown that $h \neq 0$ can lead to the appearance of stable hole pairs, in particular in the regime $J_c(h) < J < J_s(h)$ for the t-J-h model and for $U < U_c(h)$ for the t-U-h model. In the t-J_z-h model we were able to show that the effect of $h>0$ is nonperturbative, since any weak $h>0$ induces binding. It seems plausible that the same applies for the isotropic t-J-h model, although in this case the regime $h \ll t$ remains so far inaccessible to analytical as well as numerical methods. There are several open questions concerning the pair formation in the t-U-h model. While numerical results for the critical line $U_c(h)$ are less conclusive due to finite-size effects and the analytical result is restricted to the extreme region $h \gg t$, one could also speculate on the basis of perturbation theory arguments that the binding could disappear at small $U \ll h$.

Within presented 1D models we partly answer also the important question, whether pair formation induced by $h \neq 0$ leads also to dominant SC fluctuations at finite doping. We show that $h \neq 0$ qualitatively changes CDW and SC correlations near half-filling. Due to the finite spin gap, both models belong to the Luther-Emery regime of interacting fermions with a nontrivial charge exponent K_ρ. In contrast to usual 1D t-J or Hubbard model with $\tilde{K}_\rho = \frac{1}{2}$, $h \neq 0$ introduces a nonuniversal $\tilde{K}_\rho$. For the t-J-h model we have shown that $\tilde{K}_\rho$ increases steadily at fixed h from $\tilde{K}_\rho = 0$ at $J=0$ to $\tilde{K}_\rho = \infty$ at the PS onset, $J = J_s(h)$. Although we cannot give a definite conclusion on the relation between $\tilde{K}_\rho$ and the pair binding, our results indicate that $\tilde{K}_\rho \sim \frac{1}{2}$ at the onset of binding, $J = J_c(h)$. Nevertheless, the regime with dominant SC fluctuations $K_\rho > 1$ appears to be very broad within hole-pairing phase $J_c(h) < J < J_s(h)$ and extends to $J < t$ for not too weak fields, e.g., $h > 0.3t$. Again, only some conclusions can be made for the t-U-h model. Still $\tilde{K}_\rho$ seems to be nonuniversal, it is however unclear whether $\tilde{K}_\rho > 1$ can be reached in some parameter regime of the t-U-h model.

Finally let us comment on the relevance of obtained results to even more challenging 2D t-J or Hubbard models. It is obvious that there exist several common features between, e.g., the 1D t-J-h model and the 2D t-J model at low doping. If we assume $h \sim J/2$ (Ref. 9) we can

simulate within the 1D model even quantitatively the 2D AFM order, and consequently the 2D string potential, the coherent mass enhancement, etc. It is promising that along the $J=2h$ line we get clearly different onsets for the pair formation $J_c \sim 0.3t$ and for the PS $J_s \sim 2t$. We cannot establish a direct relation of 1D dominant SC fluctuations with 2D SC, still the intermediate regime with low concentration of formed hole pairs remains a candidate for possible SC in 2D model systems.

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