Charge-spin separation, spin-excitation spectrum, and the normal-state properties of the copper oxide superconductors

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We show that the unusual normal-state properties of the copper oxide superconductors can be derived from the one-band t-J model with the use of a rigorously imposed constraint of single-electron occupancy and the Wigner-Jordan representation of the spin-excitation spectrum. These include the linear resistivity in the ab plane, the $1/T$ resistivity along the c direction, the anomalous spin-relaxation rate in the Cu site, the excessive absorption in the infrared conductivity over that expected from the Drude theory, the flat electronic Raman background, the broadening of the photoemission spectra, and the linear-bias dependence of the normal-metal–normal-metal tunneling conductance (along the ab plane). The zero-bias tunneling conductance along the ab plane is predicted to have a linear temperature dependence. A particularly interesting prediction is that the ratio of the linear temperature term of the zero-bias conductance to the linear-bias term of the zero-temperature conductance is $2T/|eV|$, independent of all other parameters. The static spin susceptibility at low temperature is predicted to have the form $A+B\ln(\omega_c/T)$, where $\omega_c \sim J$ is a cutoff energy. The doping (x) dependences of various physical quantities are also predicted for the range of x large enough to allow the holons to form a band and the spins to have a disordered (liquid) ground state, but not too large to destroy the antiferromagnetic spin correlation. The imaginary part of the spin susceptibility has a wave-vector-independent component with an x dependence of $x(1-x)^2$. The nuclear relaxation at the Cu site, $1/(TT_1)$, has a $1/T$ component with an x dependence of $x(1-x)^2$. The zero-bias tunneling conductance along the ab plane at zero temperature increases with x^2. The $1/T$ resistivity along the c direction at low temperature is inversely proportional to x, and the linear resistivity of the ab plane has a weak x dependence in the doping range mentioned above.

I. INTRODUCTION

The normal-state properties of the copper oxide superconductors are just as puzzling as the high-transition temperature of the superconductivity itself, if not more so. These include (a) the linear resistivity in the ab plane,[1] (b) the low-temperature resistivity in the c direction[2] (perpendicular to the ab plane) behaving roughly as $1/T$, (c) the infrared conductivity showing excessive absorption over that expected from the usual Drude theory,[3] (d) the NMR spin relaxation at the Cu site not satisfying the Korringa law,[4] (e) the Raman spectra showing a flat electronic background,[5] (f) the lifetime of the electrons near the Fermi surface measured from photoemission spectra[6] depending on energy roughly as $|E-E_F|^{-1}$, rather than $(E-E_F)^{-2}$, and (g) the normal-metal–normal-metal tunneling conductance showing a mysterious linear dependence on the bias.[7] These unusual properties put severe constraints on any successful theory, and have been taken as indications that the conduction electron system in the copper oxide superconductors is not a conventional Landau Fermi liquid. In particular, Anderson[8] has strongly argued that these unusual properties are characteristic of spin-charge separation. The possibility of a non-Fermi liquid would have enormous implications on both the ground state and the mechanism of superconductivity, since it raises fundamental questions such as what is the quasiparticle in the system, what to pair, and how the new quasiparticles respond to various interactions. The understanding of the normal-state properties is therefore a crucial first step towards the understanding of the superconducting state.

Interpretation of the normal-state properties has been a focus of theoretical studies for the last few years. Anderson and Zou[9] first suggested that the linear ab-plane resistivity, the $1/T$ c-direction resistivity, and the linear-in-bias normal-state tunneling conductance are the consequence of spin-charge separation. They used a bosonic holon, fermionic spinon representation. If the momentum conservation law is strictly applied, and a free-particle spectrum for both holon and spinon is used, however, the temperature dependence of the ab-plane resistivity would be $T^{3/2}$, and similarly the temperature dependence of the c-direction conductivity would also be $T^{3/2}$, as shown by Kallin and Berlinsky.[10] As argued by Anderson and Zou,[11] this deviation points to a strong interaction between their holons and spinons. Nagaosa and Lee[12] later developed a gauge-field theory based on the uniform resonant-valence-bond theory of Baskaran, Zou, and Anderson, and showed that the linear resistivity can be obtained. Anderson[13] later developed a Luttinger-liquid theory to interpret the normal-state properties. The normal-state properties have also been modeled

through phenomenological modes. Varma *et al.*[14] showed that many of the properties can be derived from a particular form of the spectral density of the quasiparticles, and systems with such a spectral function are termed as a marginal Fermi liquid. Calculation[15] of the spectral density for the Hubbard model for small to intermediate U has also been carried out, and a significant change from the free-particle-like spectral density is found. The marginal-Fermi-liquid-like behavior could also give rise from an electron gas with a nearly nested Fermi surface, as shown by Virosztek and Ruvalds,[16] providing an interpretation of the electronic Raman background. From investigations of the nuclear magnetic resonance experiments, Pines *et al.*[17] proposed an antiferromagnetically correlated Fermi-liquid model. In a recent work, we have shown, with the Wigner-Jordan (WJ) representation of the spin-excitation spectrum, that all the above-mentioned properties can be derived[18] from the t-J model in a unified way. The purpose of this paper is to give a more detailed derivation of our theory, and to present results which are not presented in our previous work.

The paper will be organized as follows. In Sec. II, we discuss our treatment of the t-J Hamiltonian, especially the effect of spin fluctuations on the hopping process of the electrons in the presence of doped sites. In Sec. III, we review the spin-excitation spectrum derived previously using the Wigner-Jordan representation for the *disordered* (paramagnetic) spin state of the antiferromagnetic (AFM) Heisenberg model in a square lattice. A detailed derivation of all the above-mentioned normal-state properties will be presented in Sec. IV. A brief summary and discussion will be presented in Sec. V.

II. TREATMENT OF THE t-J MODEL

As strongly argued by Anderson,[8] and by Zhang and Rice,[19] the appropriate model for describing the copper oxide materials is the one-band Hubbard model in the large-U limit. In this limit, the Hubbard model can be reduced to the t-J model,

$$H=-t\sum_{ij\sigma}c_{i\sigma}^{\dagger}c_{j\sigma}+J\sum_{\langle ij\rangle}(\tilde{\mathbf{S}}_i\cdot\tilde{\mathbf{S}}_j-\tfrac{1}{4}n_in_j)\ , \tag{1}$$

where $\tilde{\mathbf{S}}_i$ is the spin operator at site i, and the other notations are standard. The electron operators in the kinetic-energy term are understood to be already projected into the single-occupation space. Mathematically, this single-occupation restriction can be expressed as

$$\sum_{\sigma}c_{i\sigma}^{\dagger}c_{i\sigma}=1-\nu_i\ , \tag{2}$$

where ν_i is the doping index, $\nu_i=1$ if the site is doped, and $\nu_i=0$ if it is not. There have been numerous studies[20] of the t-J model since the discovery of the high-T_c superconductors, and various techniques have been developed. In this work we shall use the CP^1-boson representation[21] of the t-J model. In this representation, an electron operator is considered as a composite of a fermionic holon, and a bosonic spinon (the CP^1 boson), i.e., $c_{i\sigma}=e_i^{\dagger}b_{i\sigma}$, where $e_i^{\dagger}$ is a holon creation operator obeying Fermi statistics, $b_{i\sigma}$ is a boson annihilation operator carrying spin σ. Using this representation, we have

$$H=-t\sum_{ij\sigma}e_ie_j^{\dagger}b_{i\sigma}^{\dagger}b_{j\sigma}+J\sum_{\langle ij\rangle}(1-e_i^{\dagger}e_i)(\mathbf{S}_i\cdot\mathbf{S}_j-\tfrac{1}{4})(1-e_j^{\dagger}e_j)\ , \tag{3}$$

where $S_i^z=\frac{1}{2}(b_{i\uparrow}^{\dagger}b_{i\uparrow}-b_{i\downarrow}^{\dagger}b_{i\downarrow}^{\dagger})$, and similar definitions for S_i^+ and S_i^-. The doping index in Eq. (2) can now be identified as the holon occupation number, i.e., $\nu_i=e_i^{\dagger}e_i$. The single-occupation constraint in Eq. (2) can thus be expressed as

$$\sum_{\sigma}b_{i\sigma}^{\dagger}b_{i\sigma}=1\ . \tag{4}$$

Equation (4) states that there is one CP^1 boson at each lattice site, regardless of doping, and that these bosons are hard-core bosons. The existence of a CP^1 boson at a doped site does not contradict with the absence of the electrons at that site, since a holon is created at that site. In this sense, the CP^1 bosons at the doped sites are "ghost" bosons, only those at the undoped sites are "real." Since the CP^1-boson operators are defined to be present for every lattice site, S_i^z (and also S_i^+ and S_i^-) is also defined for every lattice site, and is related to the true spin operator, $\tilde{S}_i^z$, by $\tilde{S}_i^z=(1-e_i^{\dagger}e_i)S_i^z$. For convenience, here we still refer to S_i^z, etc., as spin operators. Finally, we notice that one could also start from the Schwinger boson representation of the t-J model, and with appropriate redefinition of the boson operators, one could derive the same expression as that given in Eq. (3).

If the spin-charge separation as suggested from the CP^1-boson method (or other methods such as the slave boson or slave fermion method) is rather robust, we would expect a mean-field separation of the holon and the boson in the kinetic-energy term in Eq. (3) to be reasonable. This leads to

$$H\approx-t\sum_{ij\sigma}e_ie_j^{\dagger}\langle b_{i\sigma}^{\dagger}b_{j\sigma}\rangle+K_b+\text{scattering term}+J\text{ term}\ , \tag{5}$$

where

$$K_b=-t\sum_{ij\sigma}\langle e_ie_j^{\dagger}\rangle[b_{i\sigma}^{\dagger}b_{j\sigma}-\langle b_{i\sigma}^{\dagger}b_{j\sigma}\rangle]\ ,$$

and the scattering term represents the scattering between the holons and the CP^1 bosons [cf. Eq. (12)]. The term K_b represents the direct hopping of the CP^1 bosons. Since each lattice site is occupied by a CP^1 boson which has a hard core, direct hopping is forbidden, and only virtual hopping (second-order process) can be considered (the latter process contributes a kinetic energy on the order of Jx^2, where x is the doping concentration). We shall neglect the contribution of K_b to the kinetic energy in this work.

The difficulty is the evaluation of quantities such as $\langle b_{i\sigma}^{\dagger}b_{j\sigma}\rangle$, since the spectrum of the CP^1 boson is not well established. It would be preferred if one could express the CP^1-boson operators in the first term of Eq. (1) in terms of the spin operators, since the spin-excitation

spectrum is relatively well established. To do this, we draw attention to the following resemblances between the CP^1-boson operators and the spin operators: $S_i^z=\frac{1}{2}(b_{i\uparrow}^\dagger b_{i\uparrow}-b_{i\downarrow}^\dagger b_{i\downarrow})$ vs $S_i^z=\frac{1}{2}(S_i^+S_i^- - S_i^-S_i^+)$, and $b_{i\uparrow}^\dagger b_{i\uparrow}+b_{i\downarrow}^\dagger b_{i\downarrow}=1$ vs $S_i^\dagger S_i^- + S_i^- S_i^+ = 1$. Clearly, the CP^1-boson operators are related to the spin operators by $b_{i\uparrow}^\dagger=(-1)^i S_i^\dagger e^{i\varphi_{i\uparrow}}$ and $b_{i\downarrow}^\dagger=(-1)^i S_i^- e^{i\varphi_{i\downarrow}}$, where $\varphi_{i\uparrow}$ and $\varphi_{i\downarrow}$ are real operators, and the reason of including the factor $(-1)^i$ has to do with the antiferromagnetic coupling in the J term. While we are not able to obtain $\varphi_{i\uparrow}$ and $\varphi_{i\downarrow}$ explicitly, we notice that $\varphi_{i\uparrow}$ can be replaced by $\varphi_{i\uparrow}-\langle\varphi_{i\uparrow}\rangle$ and still satisfy the above equations. Thus, in an appropriately chosen guage, the fluctuation in $\varphi_{i\uparrow}$ and $\varphi_{i\downarrow}$ can be suppressed, at least to first order, and we have approximately $b_{i\uparrow}^\dagger\approx(-1)^i S_i^+$ and $b_{i\downarrow}^\dagger\approx(-1)^i S_i^-$. The actual approximation required in all of the following calculations can be slightly relaxed from that stated above, since we only need to have $b_{i\uparrow}^\dagger b_{j\uparrow}\approx(-1)^{i+j}S_i^+S_j^-$, etc., where i and j are two nearest-neighbor sites. (Notice that $b_{i\uparrow}^\dagger b_{i\uparrow}=S_i^+S_i^-$ is an identity.) It is evident from this expression that the approximation is for the second-order process of the CP^1-boson hopping, and preserves the continuation of the above-stated identity. We notice that the single-occupancy constraint is automatically satisfied by the above-stated identity when the spin operators are transformed into Wigner-Jordan fermions (see the discussion in Sec. III). The approximation also gives the correct total degrees of freedom of the system: The degrees of freedom of the $2N$ electron operators now are partitioned into that for the N holon operators, and that for the N spin operators. These features show clear advantages of our approximation over mean-field treatments of the constraint. [In the mean-field treatment of the constraint, the N equations implied in Eq. (4) are replaced by the average, $\langle b_{i\sigma}^\dagger b_{i\sigma}\rangle=\frac{1}{2}$. In so doing, the constraint is satisfied only in the average sense, and the total degree of freedom of the system is that of $3N-2$ operators ($e_i, b_{i\uparrow}$, and $b_{i\downarrow}$), exceeding the correct number, that of $2N$ operators.] While it is not clear with our approximation whether higher-order fluctuations in the phase factors (more precisely, the phase differences, $\varphi_{i\sigma}-\varphi_{j\sigma}$, for neighboring sites i and j) are important, we shall not settle this question in this work by direct calculations of the fluctuations. Instead, we approach this problem by calculating all the normal-state properties mentioned in the Introduction with this approximation and compare them with experimental results.

III. REVIEW OF THE SPIN-EXCITATION SPECTRUM

In the last section, we have expressed the t-J model in terms of the holon operators and the spin operators. The spin-excitation spectrum is therefore crucial for the holon dynamics. Before discussing the spin-excitation spectrum, we first notice the following two facts. (1) The spin state of the undoped copper oxide materials is antiferromagnetically ordered at low temperature. The ordering is largely due to a small interlayer coupling. With a small amount of doping ($x\approx0.05$), however, the long-range spin order is destroyed, although strong AFM correlations remain. (2) The Mermin-Wagner theorem[22] states that, for the two-dimensional (2D) AFM Heisenberg model, the spin state has no true long-range order at any finite temperature. The relevant spin-excitation spectrum for the copper oxide superconductors, therefore, is that of the *disordered* spin state of the 2D Heisenberg model. The disordered spin state can, of course, have short-range orders within the size of its spin correlation length.

We shall first review the spin-excitation spectrum of the disordered spin state of the undoped Heisenberg model using the Wigner-Jordan fermion representation,[23] and then extend the discussion to the doped system. The spin Hamiltonian for the undoped system is

$$H_{\rm spin}=J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j\ . \tag{6}$$

The spin operators can be transformed into WJ spinless fermions[23] by $S_i^+=d_i^\dagger e^{i\varphi_i}$, and $S_i^z=d_i^\dagger d_i-\frac{1}{2}$, where $\varphi_i=\sum_{j\neq i}d_j^\dagger d_j\,\mathrm{Im}\ln(\tau_j-\tau_i)$, and τ_j is the complex spin coordinate at site j. The advantage of using the WJ transformation is that it automatically satisfies the single-occupancy restriction, since $S_i^+S_i^-+S_i^-S_i^+\equiv d_i^+d_i+d_id_i^+\equiv1$. Within the mean-field approximation of the phase, φ_i, we obtain a mean-field Hamiltonian:

$$H_{\rm MF}=J(1+2\Delta)\sum_{\langle ij\rangle}e^{i\theta_{ij}}d_i^\dagger d_j+J\sum_{\langle ij\rangle}|\Delta_{ij}|^2\ . \tag{7}$$

The resulting spin state is the in-phase flux state.[23,24] In Eq. (7), θ_{ij} is the gauge phase factor. In the gauge chosen in Fig. 1, $\theta_{ij}=\pi$ for the heavy bond, and 0 for the light bond (the π-flux state). The in-phase bonding amplitude, Δ, can be calculated self-consistently through $\langle d_id_j^\dagger\rangle=\Delta e^{i\theta_{ij}}$, and is approximately 0.24 at $T=0$. The energy spectrum of the in-phase flux state is

$$E_{\mathbf{k}}^{\pm}=\pm E_{\mathbf{k}}=\pm J(1+2\Delta)\sqrt{\sin^2k_x+\cos^2k_y}\ . \tag{8}$$

All the negative-energy states are filled by the WJ fermions, and all the positive energy states are empty. The density of states of the in-phase flux state is linear at small excitation energies, and has two van Hove singularities, one at $-J(1+2\Delta)$ for the filled states, and the other

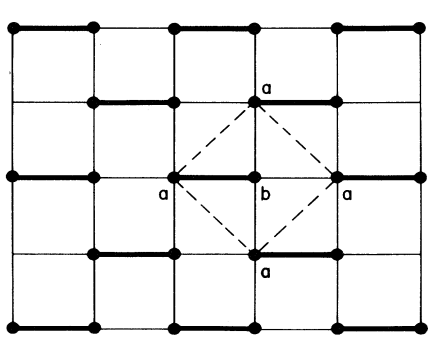

FIG. 1. The gauge used for calculating the energy spectra of the in-phase flux state.

at $J(1+2\Delta)$ for the empty states [cf. Fig. 4(a)]. It is also useful to write down the Green's functions of the WJ fermion for our later use:

$$G_{\mathbf{k}}^{aa}(\omega)=G_{\mathbf{k}}^{bb}(\omega)=\frac{1}{2}\left[\frac{1}{\omega-E_{\mathbf{k}}}+\frac{1}{\omega+E_{\mathbf{k}}}\right], \quad (9a)$$

$$G_{\mathbf{k}}^{ab}(\omega)=\tfrac{1}{2}e^{i\theta_k}\left[\frac{1}{\omega-E_{\mathbf{k}}}-\frac{1}{\omega+E_{\mathbf{k}}}\right], \quad (9b)$$

where a and b are two sublattices of the square lattice, and

$$e^{i\theta_k}=(i\sin k_x+\cos k_y)/\sqrt{\sin^2 k_x+\cos^2 k_y}\ .$$

That the in-phase flux state is a good description of the disordered spin state of the quantum AFM Heisenberg model in a square lattice can be inferred from several results. The mean-field energy at $T=0$ of the in-phase flux state is $-\Delta(1+\Delta)J\approx -0.3J$ per bond,[24] reasonably close to that from numerical calculations ($-0.334J$). Higher-order corrections from fluctuations around the mean-field state are likely to lower the energy further. The specific heat[24] of the in-phase flux state is in excellent agreement with numerical calculations, as shown in Fig. 2. In particular, the temperature dependence is correct over the entire temperature range, namely, T^2 at low temperature, a peak at $T/J\approx 0.6$, and a $1/T^2$ decay at high temperature. The spin Raman spectra[24,25] of the in-phase flux state are in excellent agreement with the experimental spectra of the undoped copper oxide materials, as shown in Fig. 3, and shows considerable improvement over that calculated from the spin-wave theories. Finally, the temperature dependence of the staggered and the uniform spin susceptibility of the in-phase flux state is also qualitatively correct over the entire temperature range.[25]

We now extend our calculation to the doped system. The spin Hamiltonian is the J term in Eq. (3). We assume that a holon can stay in a doped site longer than the spin fluctuation time, $\sim 1/J$, so that from the spins' point of view, the holon is localized. This assumption is consistent with our approximation described in the previous section. According to our approximation, a holon can hop to its nearest neighbor only if the spins at both sites are reversed. The time required to reverse a spin, in an antiferromagnetically correlated background, is on the order of $1/J$. We thus believe that for small doping and a reasonably large value of J, this assumption is valid. The spin Hamiltonian then becomes the "defected" Heisenberg model[26] in the sense that the bonds connected with the "spins" in the doped sites are statically cut, as one can see by setting $1-e_i^{\dagger}e_i=0$ in the J term in Eq. (3).

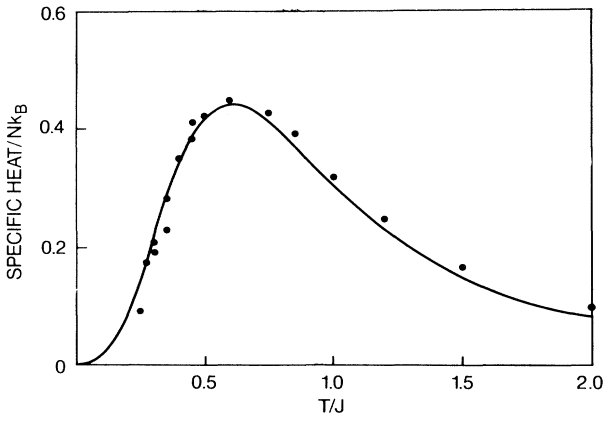

FIG. 2. The specific heat of the in-phase flux state. The solid circles are numerically calculated by Makivic and Ding [Phys. Rev. B **43**, 3562 (1991)] for the Heisenberg model in Eq. (6).

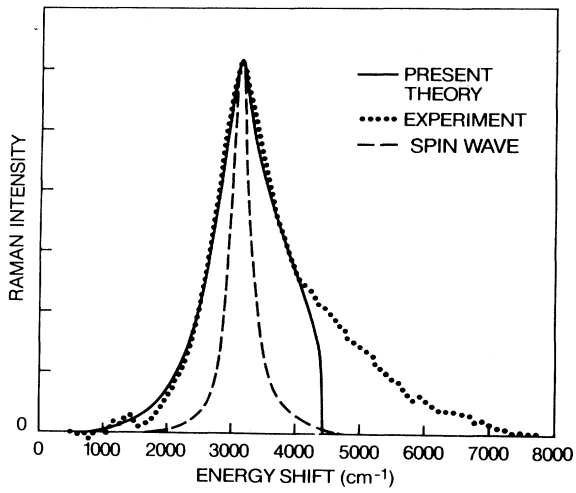

FIG. 3. The Raman spectra of the in-phase flux state. The experimental data are measured for La_2CuO_4 [R.R.P. Singh *et al.*, Phys. Rev. Lett. **62**, 2736 (1989)].

Before discussing the detailed energy spectrum of the WJ fermions for the doped Heisenberg model, we notice two intrinsic properties[24,26] of the WJ representation of the AFM Heisenberg model in an even numbered lattice with $N\to\infty$. The first is that the WJ fermions always fill half of the total available states at $T=0$, and the second is that the excitation spectrum of the WJ fermions has a particle-hole symmetry. The two properties are the direct consequence of the fermionic nature of the transformation, and the relation $d_i^{\dagger}d_i=\frac{1}{2}+S_i^z$. The energy spectrum and its filling of the in-phase flux state of the undoped Heisenberg model are the examples of the two properties. We also notice that since we have defined "spin" operators to be present in the doped sites in the CP^1-boson formalism, the WJ fermions are also defined for every lattice site. This means that the flux state resulting from the mean-field treatment of the WJ phase factor will always be π phased, i.e., we always have half-flux quanta per elementary plaquette. There is *no* PT violation from our treatment of the doped system. With these properties, we expect that the structure of the Green's function of the WJ fermions as given in Eqs. (9a) and (9b) does not alter significantly at small doping. [The π-flux state may become incommensurate upon doping. For the physical properties described in Sec. IV, however, the calculations are rather insensitive to the phase $e^{i\theta_k}$ in Eq. (9b). We therefore shall neglect the complications

from the incommensurability.]

Within the same mean-field approximation as that for the in-phase flux state of the undoped Heisenberg model, the defected Heisenberg model can be reduced to that of the doped in-phase flux state:[26]

$$H_{\rm spin}=J(1+2\Delta)\sum_{\langle ij\rangle}(a_i^\dagger b_j e^{-i\theta_{ij}}+\text{H.c.}) -J(1+2\Delta)\sum_{\langle l\delta\rangle}(a_l^\dagger b_{l+\delta}e^{-i\theta_{l,l+\delta}}+\text{H.c.})+\text{const} , \tag{10}$$

where δ is the nearest-neighbor vector and l the doped sites. This Hamiltonian can be exactly diagonalized numerically.[26] The density of states of the WJ fermions is shown in Fig. 4(b) for the case of one hole for each 20 lattice sites. Comparing with the undoped density of states, the density of states of the defected Heisenberg model has a finite value at small excitation energies together with a δ function at zero excitation energy, whereas at large excitation energy, the overall spectrum does not alter significantly. The δ-function density of states comes from the doped sites, since the "spins" at the doped sites are isolated from the rest of the system. The finite density of states at small excitation energy comes from the undoped sites.

For most of low-energy phenomena, we only need to use the density of states of the WJ fermions at small excitation energies,

$$\rho_{\rm WJ}(E)=x\delta(E)+N_{\rm WJ} , \tag{11}$$

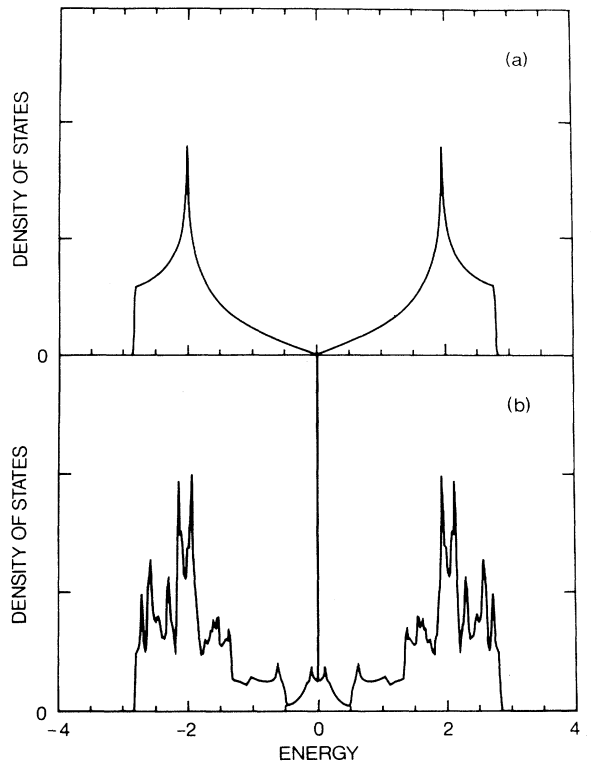

FIG. 4. Density of states of the in-phase flux state. (a) Undoped. (b) Doped with one hole per 20 lattice sites. The energy is measured in units of $J(1+2\Delta)/2$.

where x is the doping concentration, and $N_{\rm WJ}$ is a constant.

Using the WJ representation of the spin operators, the Hamiltonian in Eq. (3) now can be written as

$$H\approx\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}e_{\mathbf{k}}^\dagger e_{\mathbf{k}} +\frac{1}{N/2}\sum_{\mathbf{kpq}}{}'[V(\mathbf{k},\mathbf{p},\mathbf{q})e_{\mathbf{k}+\mathbf{q}}^\dagger e_{\mathbf{k}}b_{\mathbf{p}-\mathbf{q}}^\dagger a_{\mathbf{p}}+\text{H.c.}] +J\text{ term} , \tag{12}$$

where $\varepsilon_{\mathbf{k}}=2t(2\gamma)(\cos k_x+\cos k_y)-\mu$ is the holon band energy,

$$\gamma=\langle b_{i\sigma}^\dagger b_{i+\delta\sigma}\rangle\approx-\langle S_i^+ S_{i+\delta}^-\rangle=\Delta$$

is the nearest-neighbor bonding amplitude of the in-phase flux state. $\Delta\approx0.24$ for the undoped system and

$$V(\mathbf{kpq})=-t[\cos(k_y+q_y-p_y)+\cos(k_y+p_y) +i\sin(k_x+p_x)-i\sin(k_x+q_x-p_x)] .$$

From $\varepsilon_{\mathbf{k}}$ we see that the holon bandwidth is reduced by roughly a factor of 2 at small doping because the holon has to carry spin fluctuations when it hops from site to site. Equivalently, the holon band mass is increased by a factor of 2 compared to the "free" holon band mass. For a large value of doping, we expect the effective mass of the holons to be close to the free-holon band mass. The prime in the summation in the second term means that the $\mathbf{q}=0$ term is excluded. Finally, we notice that the Hamiltonian in Eq. (12) has precisely the form suggested by Wang, Rice, and Choi previously.[26]

IV. NORMAL-STATE PROPERTIES

We now report our calculation of various physical quantities. We shall restrict ourselves in the doping region where the doping concentration is small so that the spin-charge separation method is valid (equivalently, the strong AFM correlation between the electron spins is not destroyed), but is also large enough to allow the holons to form a delocalized band and the spins to have a disordered (i.e., liquid) state even at $T=0$. In this region, the spin excitation spectrum can be described by that of the in-phase flux state, and all the derivations of the normal-state properties can be done within one-loop expansion.

A. Spin susceptibility and nuclear spin-relaxation rate

Experimental measurements[4] of the in-plane Cu nuclear relaxation rate, T_1^{-1}, reveal that it can be described approximately as $T_1^{-1}\approx AT+B$. The surprise is the temperature-independent term B, which is absent in a conventional Fermi liquid. T_1^{-1} is related to the spin susceptibility, $\chi(\mathbf{q},\omega)$, by

$$T_1^{-1}\sim\lim_{\omega\to0}(T/\omega)(1/N)\sum_{\mathbf{q}}\text{Im}\chi(\mathbf{q},\omega) .$$

The spin susceptibility can be calculated from the correlation function $\langle T_\tau\tilde{S}_i^z(\tau)\tilde{S}_j^z(0)\rangle$ by taking the Fourier transformation of both space and time. Define

$$F^{+}(\mathbf{q},\tau)=\frac{1}{N/2}\sum_{\mathbf{p}}[G^{aa}_{\mathbf{p}+\mathbf{q}}(\tau)G^{aa}_{\mathbf{p}}(-\tau)+G^{ab}_{\mathbf{p}+\mathbf{q}}(\tau)G^{ba}_{\mathbf{p}}(-\tau)]\ , \tag{13}$$

where τ is the imaginary time. We find $\chi(\mathbf{q},\omega)=\chi_1(\mathbf{q},\omega)+\chi_2(\mathbf{q},\omega)$, where

$$\chi_1(\mathbf{q},i\omega)=-(1-x)^2\int_0^{\beta}d\tau\, e^{i\omega\tau}F^{+}(\mathbf{q},\tau)\ , \tag{14a}$$

$$\chi_2(\mathbf{q},i\omega)=\frac{1}{N^2}\sum_{\mathbf{k}\mathbf{q}'}\int_0^{\beta}d\tau\, e^{i\omega\tau}G^{e}_{\mathbf{k}+\mathbf{q}'}(\tau)G^{e}_{\mathbf{k}}(-\tau)\times F^{+}(\mathbf{q}-\mathbf{q}',\tau)\ . \tag{14b}$$

In Eq. (14b), $G^{e}_{\mathbf{k}}(\tau)$ is the holon Green's function. Carrying out the calculation, we obtain,

$$\mathrm{Im}\chi_1(\mathbf{q},\omega)=\frac{\pi(1-x)^2}{4(N/2)}\sum_{\mathbf{k}}\{g^{+}(\mathbf{k},\mathbf{q})[n_F(E_{\mathbf{k}+\mathbf{q}})-n_F(E_{\mathbf{k}})][\delta(\omega+E_{\mathbf{k}+\mathbf{q}}-E_{\mathbf{k}})-\delta(\omega-E_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{k}})] +g^{-}(\mathbf{k},\mathbf{q})[n_F(-E_{\mathbf{k}})-n_F(E_{\mathbf{k}+\mathbf{q}})][\delta(\omega-E_{\mathbf{k}}-E_{\mathbf{k}+\mathbf{q}})-\delta(\omega+E_{\mathbf{k}}+E_{\mathbf{k}+\mathbf{q}})]\}\ , \tag{15}$$

where we have defined $g^{\pm}(\mathbf{k},\mathbf{q})=1\pm\cos(\theta_{\mathbf{k}+\mathbf{q}}-\theta_{\mathbf{k}})$. We next carry out the summation over the δ-function density of states of the WJ fermions. For them we can set either $E_{\mathbf{k}}=0$ or $E_{\mathbf{k}+\mathbf{q}}=0$ (the term with $E_{\mathbf{k}}=0$ and $E_{\mathbf{k}+\mathbf{q}}=0$ simultaneously is zero). The geometry factors, $\cos(\theta_{\mathbf{k}+\mathbf{q}}-\theta_{\mathbf{k}})$, are averaged to zero by considering all $\mathbf{k}$ in the Brillouin zone (BZ). We then have

$$\mathrm{Im}\chi_1(\mathbf{q},\omega)=\frac{\pi}{2}x(1-x)^2\tanh(\omega/2T)\rho_1(\omega)+\mathrm{Im}\chi_{12}(\mathbf{q},\omega)\ , \tag{16}$$

where $\mathrm{Im}\chi_{12}(\mathbf{q},\omega)$ is given by Eq. (15) with the δ-function density of states of the WJ fermions excluded, and

$$\rho_1(\omega)=\frac{1}{N/2}\sum_{\mathbf{k}}\delta(\omega-E_{\mathbf{k}}) \tag{17}$$

is the density of states of WJ fermions in the undoped sites ($\mathbf{E}_{\mathbf{k}}\neq 0$). For small ω, $\rho_1(\omega)=N_{\mathrm{WJ}}$. The imaginary part of the spin susceptibility therefore has a $\mathbf{q}$-independent component. This is in accordance with the marginal-Fermi-liquid suggestion of Varma *et al.*,[14] and also in agreement with neutron-scattering experiments.[27] $\chi_{12}(\mathbf{q},\omega)$ is peaked near $\mathbf{q}=(\pm\pi,\pm\pi)$, a feature required in the previous phenomenological models.[17]

The summation of $\mathrm{Im}\chi_1(\mathbf{q},\omega)$ over $\mathbf{q}$ in the $\omega\to 0$ limit can be easily done. We obtain, for $T\ll J$,

$$\frac{1}{N}\sum_{\mathbf{q}}\mathrm{Im}\chi_1(\mathbf{q},\omega\to 0)=\frac{\pi}{4}(1-x)^2N_{\mathrm{WJ}}\omega(N_{\mathrm{WJ}}+x/T)\ . \tag{18}$$

The experimental T_1^{-1} follows from this result. The relative magnitude of order of A and B is also correct since N_{WJ} is scaled by $1/J$.

$\chi_2(\mathbf{q},\omega)$ is derived from the terms in which the holons are dynamically coupled to the WJ fermions,

$$\chi_2(\mathbf{q},i\omega)\frac{1}{4N^2(N/2)}\sum_{\mathbf{k}\mathbf{p}\mathbf{q}'}[n_F(\varepsilon_{\mathbf{k}})-n_F(\varepsilon_{\mathbf{k}+\mathbf{q}'})]L(\mathbf{q},i\omega;\mathbf{k},\mathbf{p},\mathbf{q}')\ , \tag{19a}$$

where

$$L(\mathbf{q},i\omega;\mathbf{k},\mathbf{p},\mathbf{q}')=g^{+}(\mathbf{p},\mathbf{q}-\mathbf{q}')[n_F(E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'})-n_F(E_{\mathbf{p}})]\left[\frac{n_B(E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'}-E_{\mathbf{p}})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}'}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}'}+E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'}-E_{\mathbf{p}}}-\frac{n_B(E_{p}-E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}'}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}'}-E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'}+E_{\mathbf{p}}}\right] +g^{-}(\mathbf{p},\mathbf{q}-\mathbf{q}')[n_F(-E_{\mathbf{p}})-n_F(E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'})]\left[\frac{n_B(-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}'}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}'}-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'}}-\frac{n_B(E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}'}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}'}+E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}-\mathbf{q}'}}\right]\ . \tag{19b}$$

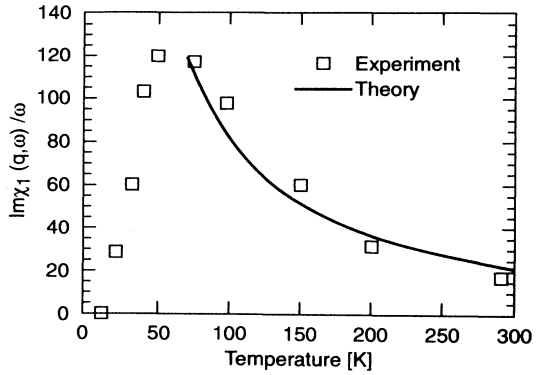

FIG. 5. Temperature dependence of the spin susceptibility. The sharp drop below about 50 K in the experimental data is due to the formation of the superconducting state.

The contribution of χ_2 to T_1^{-1} at low temperature is small,

$$\frac{1}{N}\sum_{\mathbf{q}} \mathrm{Im}\chi_2(\mathbf{q},\omega\to 0)=\frac{\pi}{4}\omega x N_e^2(x+\pi^2 N_{\mathrm{WJ}}T/2) . \tag{19c}$$

Figure 5 shows the temperature dependence of $\mathrm{Im}\chi_1(\mathbf{q},\omega)$ for $\mathbf{q}=(3\pi/10,3\pi/10)$ and $\omega=2$ meV with the parameters $J=1300$ K and $x=0.1$. The experimental data are from Ref. 28.

The real part of $\chi_1(\mathbf{q},\omega)$ can be obtained from Eq. (15) by replacing all the δ functions by their corresponding energy denominators. Alternatively, it can be obtained from Eq. (16) by the Kramers-Kronig transformation. In particular, the static (or Pauli) susceptibility, χ_{1P}, is obtained by taking $\omega\to 0$ first and then taking the $\mathbf{q}\to 0$ limit. For $T\ll J$,

$$\chi_{1P}=\tfrac{1}{2}(1-x)^2 N_{\mathrm{WJ}}[1+2x\ \ln(1.14\omega_c/T)] , \tag{20}$$

where ω_c is a high energy cutoff of the WJ spectrum ($\omega_c\sim J$). The static susceptibility therefore will increase logarithmically at low temperature. It would be interesting to see if this prediction can be verified experimentally.

B. Normal-state tunneling conductance and photoemission

The experimental tunneling conductance[7] of a copper oxide superconductor in the normal state is found to be $g=g_0+g_1[V]$, for the applied bias $|V|\le 0.1$ eV. This linear dependence of the conductance in bias is very unusual. The tunneling conductance between two metals in the normal state, one conventional and one copper oxide, is

$$g(V)=2e^2|T_g|^2 N_R \sum_{\mathbf{k}}[-2\,\mathrm{Im}G(\mathbf{k},eV)] ,$$

where T_g is the tunneling matrix element assumed to be a constant, N_R the density of states of the conventional metal, and $G(\mathbf{k},\omega)$ the Green's function of the electrons in the copper oxide metal. Since

$$\sum_{\mathbf{k}}[-2\,\mathrm{Im}G(\mathbf{k},eV)]=\sum_i[-2\,\mathrm{Im}G_{ii}(eV)] ,$$

the calculation only requires the use of the on-site electron Green's function, $G_{ii}(\omega)$. Within our approximation, the on-site Green's function of the electrons is

$$G_{ii\sigma}(\tau)=\langle T_\tau e_i^+(\tau)e_i(0)\rangle\langle T_\tau S_i^+(\tau)S_i^-(0)\rangle .$$

For a disordered spin system, one can replace $\langle T_\tau S_i^+(\tau)S_i^-(0)\rangle$ by $2\langle T_\tau S_i^z(\tau)S_i^z(0)\rangle$. With the help of the spectral density representation of the holon Green's function, we obtain,

$$G_{ii}(i\omega)=\frac{1}{2N(N/2)^2}\sum_{\mathbf{kpq}}\int_{-\infty}^{+\infty}\frac{d\varepsilon}{2\pi}A_{\mathbf{k}}^e(\varepsilon)\left\{[n_F(E_{\mathbf{q}})-n_F(E_{\mathbf{p}})]\left[\frac{n_B(E_{\mathbf{p}}-E_{\mathbf{q}})+n_F(\varepsilon)}{i\omega+\varepsilon+E_{\mathbf{q}}-E_{\mathbf{p}}}-\frac{n_B(E_{\mathbf{q}}-E_{\mathbf{p}})+n_F(\varepsilon)}{i\omega+\varepsilon-E_{\mathbf{q}}+E_{\mathbf{p}}}\right]\right.$$
$$\left.+[n_F(-E_{\mathbf{p}})-n_F(E_{\mathbf{q}})]\left[\frac{n_B(E_{\mathbf{p}}+E_{\mathbf{q}})+n_F(\varepsilon)}{i\omega+\varepsilon-E_{\mathbf{p}}-E_{\mathbf{q}}}-\frac{n_B(-E_{\mathbf{p}}-E_{\mathbf{q}})+n_F(\varepsilon)}{i\omega+\varepsilon+E_{\mathbf{p}}+E_{\mathbf{q}}}\right]\right\} , \tag{21}$$

where $A_{\mathbf{k}}^e(\varepsilon)$ is the spectral density of the holons. For a doping concentration x not too small, the condition $E_F^e\gg J$ is valid, where E_F^e is the holon Fermi energy. Under this condition,

$$\frac{1}{N}\sum_{\mathbf{k}}\frac{1}{2\pi}A_{\mathbf{k}}^e(\varepsilon)\approx N_e . \tag{22}$$

Most of the calculations in this work are done in this doping region. The summation over $\mathbf{p}$ and $\mathbf{q}$ in Eq. (21) can be done easily, using the density of states of the WJ fermions in Eq. (11). We obtain, at $T=0$ and to the linear order in V,

$$(1/N)\sum_{\mathbf{k}}[-2\,\mathrm{Im}G(\mathbf{k},eV)]=x\pi N_e(x+2N_{\mathrm{WJ}}|eV|) , \tag{23}$$

where N_e is the density of states of the holons. The experimental observation follows from Eq. (23). We notice that a similar result has been derived by Anderson and Zou[9] using the slave-boson method. Since N_e can be taken approximately as a constant for a wide range of doping concentrations, Eq. (23) predicts that the zero-bias conductance at zero temperature is proportional to x^2. It is interesting also to calculate the temperature depen-

dence of the zero-bias conductance. We find

$$g(V=0)=2\pi x e^2|T_g|^2 N_R N_e(x+4N_{\rm WJ}T) . \quad (24)$$

In deriving Eqs. (23) and (24), we have used

$$[n_F(E_{\rm k})-n_F(E_{\rm p})][n_B(E_{\rm p}-E_{\rm k})-n_B(E_{\rm k}-E_{\rm p})]\rightarrow\tfrac{1}{2} , \quad (25)$$

when $E_{\rm p}\rightarrow 0$ and $E_{\rm k}\rightarrow 0$. Unlike conventional metals, the zero-bias conductance of the copper oxide materials has a strong (linear) temperature dependence at low temperature. The ratio of this linear temperature term of the zero-bias conductance to the linear-bias term of the zero-temperature conductance is predicted to be $2T/|eV|$, independent of all other parameters. It would be interesting to see if these predictions can be experimentally verified.

It is also appropriate to comment here that our approximation of the on-site electron Green's function rigorously conserves the total number of electrons, i.e., $\langle c_{i\uparrow}^\dagger c_{i\uparrow}\rangle=\langle c_{i\downarrow}^\dagger c_{i\downarrow}\rangle=(1-x)/2$, since

$$\lim_{\tau\rightarrow 0}\langle T_\tau S_i^+(\tau)S_i^-(0)\rangle=\lim_{\tau\rightarrow 0}\langle T_\tau S_i^z(\tau)S_i^z(0)\rangle=\tfrac{1}{2} ,$$

using Eq. (9a). In terms of this aspect, our approximation is better than other approximations with mean-field treatment of the single-occupancy constraint.

The result in Eq. (23) can also be used to interpret the broadening of the experimental photoemission peak, since the term linear in $|eV|$ implies a broadening linear in energy. Detailed calculation of the photoemission spectra will be discussed in a forthcoming paper.

C. Resistivity in the *c* direction

The temperature dependence of the resistivity in the c direction[2] is experimentally found to be approximately $1/T$ at low temperature. Following the suggestion of Zou and Anderson,[9] we assume that the resistivity is derived from electron tunneling between adjacent Cu-O planes. The tunneling Hamiltonian can be written as

$$H_T=t_c\sum_{i\sigma}[c_{i\sigma}^\dagger(1)c_{i\sigma}(2)+\text{H.c.}] ,$$

where t_c is the tunneling matrix element which is assumed to be a constant, and the sum is over all the ab-plane lattice sites. Using our approximation, we have

$$c_{i\uparrow}^\dagger(1)c_{i\uparrow}(2)\approx e_i(1)e_i^\dagger(2)d_i^\dagger(1)e^{i[\varphi_i(1)-\varphi_i(2)]}d_i(2) .$$

Since the WJ phase difference between two layers, $\varphi_i(1)-\varphi_i(2)$, is static and has the same plane site index i, it can be taken to be zero. The tunneling Hamiltonian can be written as

$$H_T\approx t_c\sum_i\{e_i(1)e_i^\dagger(2)[d_i^\dagger(1)d_i(2)+d_i(1)d_i^\dagger(2)]+\text{H.c.}\} .$$

The tunneling current, I, is calculated[29] by $I=2e\,\text{Im}X(eV)$, where

$$X(i\omega)=-|t_c|^2\sum_{ij}\int_0^\beta d\tau\, e^{i\omega\tau}\langle T_\tau e_i(1,\tau)e_i^\dagger(2,\tau)e_j(2,0)e_j^\dagger(1,0)\rangle\langle T_\tau d_i^\dagger(1,\tau)d_i(2,\tau)d_j^\dagger(2,0)d_j(1,0)\rangle . \quad (26)$$

This can be written as

$$X(i\omega)=\frac{-2}{N^2}\sum_{\rm kq}\int_0^\beta d\tau\, e^{i\omega\tau}G_{\rm k+q}^e(1,\tau)G_{\rm k}^e(2,-\tau)F^+({\rm q},\tau) . \quad (27)$$

Taking the $V\rightarrow 0$ limit, we obtain the c-direction conductivity at low temperature:

$$\sigma_c(T)=e^2\pi\frac{1}{N^2}\sum_{\rm kk'}\frac{1}{(2\pi)^2}A_{\rm k}^e(0)A_{\rm k'}^e(0)Q({\rm k}-{\rm k}') , \quad (28)$$

where

$$Q({\rm q})=\frac{1}{N/2}\sum_{\rm p}g^+({\rm p},{\rm q})[n_F(E_{\rm p+q})-n_F(E_{\rm p})](E_{\rm p}-E_{\rm p+q})\Lambda(E_{\rm p}-E_{\rm p+q})+g^-({\rm p},{\rm q})[n_F(-E_{\rm p})-n_F(E_{\rm p+q})](E_{\rm p}+E_{\rm p+q})\Lambda(E_{\rm p}+E_{\rm p+q}) . \quad (29)$$

In Eq. (29),

$$\Lambda(y)=\frac{\partial n_B(y)}{\partial y}+\frac{\partial n_B(-y)}{\partial(-y)} . \quad (30)$$

We have assumed $E_F^e\gg J$ in deriving Eq. (28). Since $A_{\rm k}^e(0)$ is strongly peaked at the holon Fermi energy, we shall first integrate k and k′ in Eq. (28), thus giving a factor N_e^2. The wave vector ${\rm q}={\rm k}-{\rm k}'$ in the function $Q({\rm q})$ becomes the difference between two Fermi wave vectors of the holons, and can vary from $|{\rm q}|=0$ to $2k_F^e$. We next consider the contribution from the δ function in the density of states of the WJ fermions in performing the p integral of the function $Q({\rm q})$. Again, for this contribution,

the geometry factor $\cos(\theta_{\mathbf{p}+\mathbf{q}}-\theta_{\mathbf{p}})$ is averaged to zero, and we can set either $E_{\mathbf{p}}$ or $E_{\mathbf{p}+\mathbf{q}}$ to be zero. When $E_{\mathbf{p}}=0$, the momentum $\mathbf{p}+\mathbf{q}$ in $E_{\mathbf{p}+\mathbf{q}}$ can still vary over the entire BZ. Similarly, when $E_{\mathbf{p}+\mathbf{q}}=0$, $\mathbf{p}$ can vary over the entire BZ. Integrating over these momenta, we obtain

$$\sigma_c(T)\approx x\frac{\pi^3}{2}e^2|t_c|^2N_e^2N_{\mathrm{WJ}}T\ , \tag{31}$$

where we have used

$$\int_0^\infty \frac{y\tanh(y/2)}{\sinh^2(y/2)}dy=\frac{\pi^2}{2}\ . \tag{32}$$

The temperature dependence of the resistivity in the c direction at low temperature is therefore $1/T$. It should be appreciated that this temperature dependence is the result of spin excitations *inevitably* accompanying the tunneling process. The importance of the temperature dependence of the c-direction resistivity is even more clear when compared with the ab-plane resistivity (see the discussion at the end of Sec. IV D). The x dependence of the c-direction resistivity is also interesting. Since we expect N_{WJ} to be not very sensitive to the doping concentration for not too small doping concentrations, Eq. (31) predicts that in this doping region the c-direction resistivity at low temperature is inversely proportional to the doping concentration.

The part of integration in Eq. (29) with $E_{\mathbf{p}}\neq 0$ and $E_{\mathbf{p}+\mathbf{q}}\neq 0$ can also be done. It is proportional to T^2 for $T<0.15J$. For $0.15J<T<J$, we find its contribution to $\sigma_c(T)$ is approximately $(\pi/2)e^2|t_c|^2C_1N_e^2N_{\mathrm{WJ}}T$, where the constant $C_1\approx 0.73$.

D. Resistivity in the *ab* plane and the infrared conductivity

Since we start from a one-band Hubbard model, the current operator is[29] $\mathbf{j}=iet\sum_{i\delta\sigma}\delta c^\dagger_{i+\delta,\sigma}c_{i\sigma}$. In our representation, we have

$$j_x=\frac{iet}{N/2}\sum_{\mathbf{kpq}}U(\mathbf{kpq})e_{\mathbf{k}+\mathbf{q}}e^\dagger_{\mathbf{k}}(b^\dagger_{\mathbf{p}+\mathbf{q}}a_{\mathbf{p}}-a^\dagger_{\mathbf{p}+\mathbf{q}}b_{\mathbf{p}})\ , \tag{33}$$

where $U(\mathbf{kpq})=\cos(k_x-p_x)-\cos(k_x+p_x+q_x)$, and we have used the gauge shown in Fig. 2. The conductivity can be evaluated from the standard Kubo formula,[29] $\sigma(\omega)=-\mathrm{Im}\pi(\omega)/\omega$, where $\pi(\omega)$ is the current-current correlation function. We obtain, $\pi(\omega,T)=\pi_1(\omega,T)+\pi_2(\omega,T)$, where π_1 comes from the holon conductivity carrying static spin fluctuations. Using

$$\sum_\sigma\langle b^\dagger_{i\sigma}b_{i+\delta\sigma}\rangle=2\gamma\approx 2\Delta=\frac{-i}{N/2}\sum_{\mathbf{p}}\langle b^\dagger_{\mathbf{p}}a_{\mathbf{p}}-a^\dagger_{\mathbf{p}}b_{\mathbf{p}}\rangle\sin p_x \tag{34}$$

from the definition of the in-phase flux state, we arrive at

$$\pi_1(\omega)=-t^2(4\gamma)^2\sum_{\mathbf{k}}\int_0^\beta d\tau\, e^{i\omega\tau}G^e_{\mathbf{k}}(\tau)G^e_{\mathbf{k}}(-\tau)\sin^2k_x\ . \tag{35}$$

We next follow the standard derivation of the dc conductivity ($\omega\to 0$) from the Kubo formula, and use

$$\int_{-E_F^e}^{E_F^e}[A^e_{\mathbf{k}}(0)]^2d\varepsilon_{\mathbf{k}}\approx 2\pi/\Sigma_2(\mathbf{k}^e_F,T)\ , \tag{36}$$

where $\Sigma_2(\mathbf{k},T)$ is the imaginary part of the holon self-energy. This gives

$$\begin{aligned}\sigma_1(T)&=-\lim_{\omega\to 0}\mathrm{Im}\pi_1(\omega)/\omega\\&=\tfrac{1}{2}t^2e^2(4\gamma)^2N_e\langle\sin^2k_x/\Sigma_2(\mathbf{k},T)\rangle\ .\end{aligned} \tag{37}$$

The average is taken over the holon Fermi surface with proper vertex correction. (The vertex correction, as usual, is to convert $1/\Sigma_2$ into the transport lifetime.) Σ_2 could be contributed from various scattering mechanisms. One contribution is from electron-phonon scattering. (The electron-phonon interaction only depends on the charge density, which is equal to the holon density.) For $T\geq 0.2\Theta_D$, where Θ_D is the Debye temperature, the phonon contribution is linear in T. We shall not exclude such contributions here. Instead, we show that the contribution from scatterings between holons and WJ fermions as given in Eq. (12) is also linear in T, and that this linear dependence can be extended to $T=0$.

The evaluation of Σ_2 can be done using the standard procedure by including a WJ fermion bubble in the holon self-energy. This gives

$$\Sigma(\mathbf{k},i\omega)=\frac{-1}{N\beta/2}\sum_{\mathbf{q},iq_n}G^e_{\mathbf{k}+\mathbf{q}}(i\omega+iq_n)B(\mathbf{q},iq_n)\ , \tag{38}$$

where

$$B(\mathbf{q},iq_n)=\frac{2}{N\beta/2}\sum_{\mathbf{p},ip_n}\{|V(\mathbf{k},\mathbf{p},\mathbf{q})|^2G^{aa}_{\mathbf{p}+\mathbf{q}}(ip_n+iq_n)G^{aa}_{\mathbf{p}}(ip_n)+\mathrm{Re}[V^2(\mathbf{k},\mathbf{p},\mathbf{q})G^{ab}_{\mathbf{p}+\mathbf{q}}(ip_n+iq_n)G^{ab}_{\mathbf{p}}(ip_n)]\}\ . \tag{39}$$

The Re[$\cdots$] term contains a geometry factor $e^{i(\theta_{\mathbf{p}+\mathbf{q}}+\theta_{\mathbf{p}})}$, which can be taken as zero when we consider the δ-function part of the WJ fermion density of states. We therefore shall neglect it below for simplicity. With the same reason, we can further simplify the expression by replacing $|V(\mathbf{k},\mathbf{p},\mathbf{q})|^2$ with $\langle|V(\mathbf{k},\mathbf{p},\mathbf{q})|^2\rangle$, where the average is done on $\mathbf{p}$ and $\mathbf{q}$. With these simplifications, we obtain

$$\Sigma(\mathbf{k},i\omega)=\tfrac{1}{2}\langle|V(\mathbf{kpq})|^2\rangle\frac{1}{(N/2)^2}\sum_{\mathbf{pq}}\left\{[n_F(E_{\mathbf{p}+\mathbf{q}})-n_F(E_{\mathbf{p}})]\left[\frac{n_B(E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}})+n_F(\varepsilon_{\mathbf{k}+\mathbf{q}})}{i\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}}}-\frac{n_B(E_{\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}})+n_F(\varepsilon_{\mathbf{k}+\mathbf{q}})}{i\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}}}\right]\right.$$
$$\left.+[n_F(-E_{\mathbf{p}})-n_F(E_{\mathbf{p}+\mathbf{q}})]\left[\frac{n_B(E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}})+n_F(\varepsilon_{\mathbf{k}+\mathbf{q}})}{i\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}}}-\frac{n_B(-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}})+n_F(\varepsilon_{\mathbf{k}+\mathbf{q}})}{i\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}}}\right]\right\}. \tag{40}$$

Taking the imaginary part, integrating out the holon momentum $\mathbf{k}+\mathbf{q}=\mathbf{k}'$ first, and then following the same procedure of integration for the δ-function part of the WJ fermion density of states as we outlined in obtaining the c-direction resistivity, we arrive at, for $\omega=0$,

$$\Sigma_2(\mathbf{k},T)\approx 4\pi x\langle|V(\mathbf{kpq})|^2\rangle N_e N_{\mathrm{WJ}} T\ . \tag{41a}$$

Thus, the ab-plane dc resistivity is linear in T. The linear temperature dependence of the resistivity can be understood quite easily. The scattering between the holons and the WJ fermions is similar to that between electrons in an electron gas. The latter is known to cause a T^2 dependence in resistivity, providing that the electron gas has a finite density of states at its Fermi surface. The linear temperature dependence of the resistivity in our case follows immediately since the density of states of the WJ fermions is sharply peaked at zero excitation energy as represented by the δ function in Eq. (11).

At $T=0$, we find $\Sigma(\mathbf{k},\omega)$ to be linear in $|\omega|$ for $|\omega|\le J$:

$$\Sigma_2(\mathbf{k},\omega)\approx 2\pi x\langle|V(\mathbf{kpq})|^2\rangle N_e N_{\mathrm{WJ}}|\omega|\ . \tag{41b}$$

The holons therefore exhibit marginal-Fermi-liquid behavior. Overall, $\sigma_1(\omega,T)$ can be described by the Drude formula with marginal-Fermi-liquid behavior of the holon lifetime (cf. Refs. 14 and 26). We notice that the ratio of Eq. (41a) to Eq. (41b) is $2T/|\omega|$, independent of all other parameters. In the doping region we are considering $\langle\sin^2 k_x\rangle$ is approximately proportional to x, and N_{WJ} is only weakly dependent on x. Equation (37) together with Eq. (41a) then predicts that the linear temperature resistivity mainly depends on γ, which we expect to be also weakly dependent on x is the region we are considering.

The scattering of the holons by the other part of the WJ fermion spectrum (i.e., $E_{\mathbf{p}}\neq 0$ and $E_{\mathbf{p}+\mathbf{q}}\neq 0$) can also be estimated. For $T<0.15J$, this part of the contribution to Σ_2 is proportional to T^2, but at higher temperature, the contribution is also linear.

We now turn to the discussion of $\sigma_2(\omega)$. This part of the conductivity corresponds to the current correlation where the holons are dynamically coupled to the spin fluctuations:

$$\pi_2(i\omega)=\frac{-2}{(N/2)^2}\sum_{\mathbf{kpq}}|V(\mathbf{kpq})|^2\int_0^\beta e^{i\omega\tau}G^e_{\mathbf{k}+\mathbf{q}}(\tau)G^e_{\mathbf{k}}(-\tau)F^-(\mathbf{p},\mathbf{q},\tau)\ , \tag{42}$$

where

$$F^-(\mathbf{p},\mathbf{q},\tau)=G^{aa}_{\mathbf{p}}(\tau)G^{aa}_{\mathbf{p}+\mathbf{q}}(-\tau)-\mathrm{Re}[G^{ab}_{\mathbf{p}}(\tau)G^{ab}_{\mathbf{p}+\mathbf{q}}(-\tau)]\ . \tag{43}$$

This gives

$$\pi_2(i\omega)=\frac{1}{2}\frac{1}{(N/2)^2}\sum_{\mathbf{kpq}}|V(\mathbf{kpq})|^2[n_F(\varepsilon_{\mathbf{k}})-n_F(\varepsilon_{\mathbf{k}+\mathbf{q}})]\Gamma(i\omega;\mathbf{kpq})\ , \tag{44}$$

where

$$\Gamma(i\omega;\mathbf{kpq})=[1-\cos(\theta_{\mathbf{p}}+\theta_{\mathbf{p}+\mathbf{q}})][n_F(E_{\mathbf{p}+\mathbf{q}})-n_F(E_{\mathbf{p}})]\left[\frac{n_B(E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}}}-\frac{n_B(E_{\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}}-E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}}}\right]$$
$$+[1+\cos(\theta_{\mathbf{p}}+\theta_{\mathbf{p}+\mathbf{q}})][n_F(-E_{\mathbf{p}})-n_F(E_{\mathbf{p}+\mathbf{q}})]$$
$$\times\left[\frac{n_B(E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}}+E_{\mathbf{p}}+E_{\mathbf{p}+\mathbf{q}}}-\frac{n_B(-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}})-n_B(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})}{i\omega+\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}+\mathbf{q}}-E_{\mathbf{p}}-E_{\mathbf{p}+\mathbf{q}}}\right]. \tag{45}$$

We have derived the following results for σ_2 from the above expression: (1) In the $\omega \to 0$ limit, σ_2 is proportional to T, thus giving negligible contribution to the in-plane resistivity at low temperature. (2) For large ω, σ_2 decreases as $1/\omega$, thus giving rise to a long tail in the conductivity. (3) The largest contribution to σ_2 comes from excitation of WJ fermions from filled states to unoccupied states. The density of states of the WJ fermions in the undoped square lattice has two Van Hove singularities, one at $E=-J(1+2\Delta)$ for the filled states, and the other at $E=J(1+2\Delta)$ for the unoccupied states [cf. Fig. 5(a)]. The excitation energy between these two singularities is $2J(1+2\Delta)\sim 3000$ cm^{-1}, for $J\sim 1000$ cm^{-1}. Therefore, σ_2 has a mid-infrared peak. The peak position shifts towards lower frequencies upon doping, since the effective exchange parameter is reduced by doping approximately as $(1-2x)J$. σ_2 contributes the "excessive" oscillator strength observed in the experiments.[30]

We are not able to make an accurate numerical evaluation of σ_2 because of the six-dimensional integration implied in the wave-vector summation. Since we are mainly concerned with $\sigma_2(\omega)$ at $\omega > J$, we make the following approximations: (1) taking $|V(\mathbf{kpq})|^2$ to be a constant, and (2) $\mathrm{Im}\Gamma(\omega;\mathbf{kpq}) \approx \mathrm{Im}\Gamma(\omega;\mathbf{kp}0)$. These approximations enable us to reduce the six-dimensional integral to a four-dimensional integral. Figure 6 illustrates $\sigma_2(\omega)$ computed using these approximations. The parameters used are $J=1300$ K, $t=0.2$ eV, $|V(\mathbf{kpq})|^2=4t^2$, and $T=100$ K. Together with $\sigma_2(\omega)$, the total conductivity $\sigma(\omega)=\sigma_1(\omega)+\sigma_2(\omega)$ is also illustrated in Fig. 6, where $\sigma_1(\omega)$ is modeled by the Drude formula, $\sigma_1(\omega)=(ne^2\tau/m^*)/(1+\omega^2\tau^2)$. The parameters used for $\sigma_1(\omega)$ are $n=10^{21}/\mathrm{cm}^3$, $1/\tau=2\sqrt{\omega^2+(2T)^2}$, and $m^*=2m_0$.

It is interesting to compare the *ab*-plane resistivity with the resistivity along the *c* direction. The tunneling Hamiltonian between two layers, as discussed in Sec. IV C, can be thought of as that for electrons hopping in the *c* direction. One might naively expect that the conductivity along the *c* direction would exhibit qualitatively similar behavior to that of the *ab*-plane conductivity. Indeed, this would be true if there were no spin-charge separation. A close examination reveals the origin of the seemingly puzzling temperature dependence of the two resistivities: For the *ab* plane, $\langle S_i^+ S_j^- \rangle = -\Delta$, whereas, for the *c* direction, $\langle S_i^+(1) S_i^-(2) \rangle = 0$. The holons therefore can hop to their nearest neighbors in the *ab* plane with a static spin configuration, whereas along the *c* direction, the holons can hop to a nearest-neighbor layer only by dynamically exciting the spin background. The conductivity along the *c* direction is rather similar to σ_2 of the *ab*-plane. The qualitatively different behavior of the two resistivities is therefore the result of spin-charge separation and the confinement of the WJ fermions in the *ab* plane. The importance of the different temperature dependence of the two resistivities has been stressed by Anderson.[8]

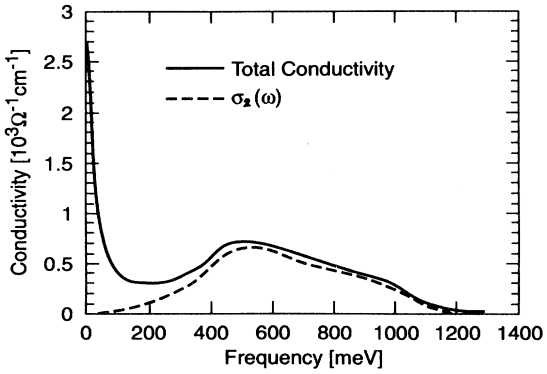

FIG. 6. Conductivity calculated from Eqs. (37) and (44).

E. Electronic Raman spectra

The electronic Raman intensity for, say, the *xx* component, is proportional to $[1-e^{-\omega/T}]^{-1}\mathrm{Im}R(\omega)$, where $R(\omega)$ is the correlation function of the electronic stress tensor,

$$R(i\omega) = -\int_0^\beta d\tau\, e^{i\omega\tau} \langle T_\tau \tau_{xx}(\tau)\tau_{xx}(0)\rangle \ ,$$

where

$$\tau_{xx} = 2t \sum_{\mathbf{k}\sigma} \cos k_x c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} = t \sum_{i\delta_x\sigma} c^\dagger_{i+\delta_x,\sigma} c_{i,\sigma} \ .$$

This result is derived for the one-band Hubbard model.[31] Had one had a free-electron metal, the $\cos k_x$ factor in τ_{xx} would be replaced by 1, and $R(\omega)$ would be strictly zero. The presence of the electronic Raman background is therefore a consequence of the hopping motion of the electrons. In our representation, τ_{xx} is

$$\tau_{xx} = \frac{-it}{N/2} \sum_{\mathbf{kpq}} U_R(\mathbf{kpq}) e_{\mathbf{k}+\mathbf{q}} e^\dagger_{\mathbf{k}} (a^\dagger_{\mathbf{p}+\mathbf{q}} b_{\mathbf{p}} - b^\dagger_{\mathbf{p}+\mathbf{q}} a_{\mathbf{p}}) \ , \tag{46}$$

where $U_R(\mathbf{kpq}) = \sin(k_x - p_x) - \sin(k_x + p_x + q_x)$. Apart from the matrix element U_R, τ_{xx} is similar to the current operator. $R(\omega)$ therefore can be discussed in parallel to the conductivity. We obtain $R(\omega) = R_1(\omega) + R_2(\omega) + R_3(\omega)$, where

$$R_1(\omega) \approx \tfrac{1}{2} t^2 (4\gamma)^2 N_e \omega \langle \cos^2 k_x / \Sigma_2(\mathbf{k},\omega)\rangle \ . \tag{47}$$

The marginal-Fermi-liquid behavior of the holons manifested in $\Sigma_2(\mathbf{k},\omega)$ implies a flat Raman spectrum down to energies $\omega \sim T$, as experimentally observed. $R_2(\omega)$ is the correspondent part of $\sigma_2(\omega)$. If the matrix element, $U_R^2(\mathbf{kpq})$, is approximated by $U^2(\mathbf{kpq})$, we would obtain $R_2(\omega) \sim \omega\sigma_2(\omega)$. $R_2(\omega)$ has the following limiting behaviors. For small ω and at low temperature, $R_2(\omega)$ is proportional to ωT, and for $\omega \gg 2J$ and at low temperature, $R_2(\omega)$ is proportional to T. $R_3(\omega)$ is contributed from spin excitations only, and is relatively small (proportional to x^2).

V. SUMMARY AND DISCUSSION

We have shown that the anomalous properties of the normal-state copper oxide superconductors, which are non-Fermi-liquid like, can be qualitatively derived from the t-J model. There are two essential ingredients involved in our calculations. (1) Each electron is considered as a composite of its charge degree of freedom and its spin degree of freedom, and (2) the spin excitations are represented as WJ fermions. The first has been advocated by the Anderson school since the discovery of the copper oxide superconductors,[8,13] and the second has been discussed extensively by one of us for the undoped two-dimensional Heisenberg antiferromagnet in the disordered spin states.[23–26] We also notice that many of our results are the realization of the phenomenology proposed by Varma *et al.*[14] This is because the holons behave as a marginal Fermi liquid. There are, however, a number of properties which cannot be explained by the marginal-Fermi-liquid behavior of the holons alone. One example is the resistivity along the c direction. It is therefore important to notice that both the holons and the spin fluctuations are responsible for the unusual properties, and that all the physical properties must be derived through the electron operators which are gauge invariant.

The theory presented in this paper is a first step towards a more complete understanding of the normal-state properties of the copper oxide superconductors. Further development and refinement of the theory will be our future research focus. In particular, a rigorous justification of our approximation, $b_{i\uparrow}^{\dagger}b_{j\uparrow}\approx(-1)^{i+j}S_i^{+}S_j^{-}$, requires calculations of higher-order fluctuations in the phase difference for the nearest-neighbor sites i and j. It would be interesting to see if the approximation can be extended to the sites i and j beyond nearest neighbors. We speculate, however, that the approximation is valid only if the two sites, i and j, are within the spin correlation length. If the two sites are separated beyond the spin correlation length, we expect the spins at these sites to behave rather independently. Clearly, the question of how the electrons propagate beyond the spin correlation length needs to be further studied. The δ-function density of states of the WJ fermions in Eq. (11) is a simplification of the true density of states. What is implied is a density of states of the WJ fermions sharply peaked near the zero excitation energy. While in the static approximation of the doped sites, the δ-function density of states of the WJ fermions clearly arises, its modification by the holon motion requires further study. Finally, the prediction of the Hall coefficient from our theory will be presented in a forthcoming paper.

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