Coupled two-dimensional Fermi liquids as a model for layered superconductors: Basic equations and elementary results

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We develop a microscopic model of layered superconductors, designed specifically for the high-T_c cuprates, which is valid at all temperatures below the degeneracy temperature and for all relevant length scales. The model is based on the original idea of Bulaevskii, and consists of a stack of two-dimensional conducting planes coupled via interlayer diffusion of charge carriers. The microscopic model presented here reduces to the phenomenological Ginzburg-Landau, Lawrence-Doniach, and anisotropic London models in the appropriate limits. We derive the main equations of the interlayer diffusion model, then use the model to examine the role of dimensionality and interlayer scattering on the magnetic properties of layered superconductors.

I. INTRODUCTION

The discovery of high-temperature superconductors has revived interest in the theory of layered superconductors. The high-T_c CuO materials show strongly anisotropic properties connected with their layered structures, in both normal and superconducting phases.[1] Most theoretical investigations of layered superconductors are based on the anisotropic Ginzburg-Landau model,[2,3] the Lawrence-Doniach model,[4] or the anisotropic London theory.[5] Although these models are very important and useful, they have restricted ranges of validity. They are valid for length scales large compared to the zero-temperature coherence length (macroscopic length scales), and in the cases of the Ginzburg-Landau and Lawrence-Doniach models are restricted to temperatures near T_c.

Here we study a microscopic model of layered superconductors which is valid at all temperatures below the degeneracy temperature and all relevant length scales above atomic dimensions. Such models have been investigated by several authors.[6–15] We follow closely the original idea of Bulaevskii,[7] who modeled anisotropic superconductors as a stack of two-dimensional (2D) conducting planes (2D Fermi liquids) coupled via interlayer diffusion of charge carriers. In our microscopic model interplane diffusion originates from random (*incoherent*) interplane scattering processes, which are predominantly responsible for transport in the c direction and Josephson coupling between planes. In-plane transport, on the other hand, is of the usual Fermi liquid type, i.e., dominated by charged quasiparticles propagating (*coherently*) with a two-dimensional, in-plane Fermi velocity $\mathbf{v}_f$. The Fermi-liquid model requires a long mean free path in the planes compared to the in-plane Fermi wavelength. The interplane diffusion model should be contrasted with the model of a strongly anisotropic Fermi liquid,[16,8–15] in which charge transport perpendicular to the planes is also due to coherent quasiparticle propagation, albeit with a strongly anisotropic Fermi velocity. Such a model requires that the mean free path be larger than the Fermi wavelength of quasiparticle excitations in all directions.

The term "Fermi liquid" is used here in a more general sense than usual. In this paper a Fermi liquid is a system whose low-temperature properties are described by a Boltzmann-Landau transport equation for quasiparticles or, more generally, by the quasiclassical transport equations for the normal and superconducting states. This generalization of a Fermi liquid includes systems which might violate traditional Fermi-liquid criteria.[17] For example, we do not require the inverse lifetime of a quasiparticle excitation to be smaller than its frequency, $\epsilon/\hbar$, or that the momentum distribution have a finite jump at the Fermi surface. Thus, our notion covers, in addition to traditional Fermi liquids, borderline systems such as marginal Fermi liquids,[18,19] Fermi liquids with very strong electron-phonon interactions,[20,21] or nearly magnetic Fermi liquids.[22–30] These systems differ mainly by the form of the collision term and other self-energy terms in the Boltzmann-Landau transport equation.

We develop the interlayer diffusion model in order to calculate the magnetic properties of layered superconductors. In Sec. II we formulate the microscopic model in terms of the Fermi-liquid theory of superconductivity, and present the basic equations and microscopic parameters of the model. We examine the normal-state properties in Sec. III; specifically, we calculate the conductivity tensor in terms of the in-plane and interplane scattering rates. Then we relate the parameters of the microscopic model to the parameters of the phenomenological Ginzburg-Landau, Lawrence-Doniach, and anisotropic London models. These results are useful for determining the microscopic parameters. As a specific example, we discuss the parameters of Y-Ba-Cu-O. In particular, we examine the role of dimensionality and interplane scattering on the anisotropic penetration depth and upper critical field. In a companion paper to follow we investigate vortex structure and energetics in layered superconductors at low temperatures.

II. MICROSCOPIC MODEL

We formulate the theory of Fermi-liquid superconductivity in terms of quasiclassical transport equations, following Eilenberger,[31] Larkin and Ovchinnikov,[32] and Eliashberg.[33] This theory describes the thermodynamic and dynamical properties of the normal Fermi liquid as well as the superconducting state, and is sufficiently general to cover the full temperature and magnetic field range of interest. The theory is capable of describing the dynamics of superconductors far from equilibrium, and inhomogeneous states on length scales below the zero-temperature coherence length. We follow closely the notation of Ref. 34.

The basic elements of the quasiclassical theory are transport equations for a 2×2 matrix propagator in particle-hole space,

$$\hat{g}(s,\mathbf{R};\epsilon,t) = \begin{pmatrix} g & f \\ \bar{f} & \bar{g} \end{pmatrix} , \tag{1}$$

whose diagonal components are related to the distribution function for Bogoliubov quasiparticles, and whose off-diagonal components are identified as the Cooper-pair amplitudes. There is some redundancy in this description; the functions $\bar{f}$ and $\bar{g}$ are related to f and g by fundamental identities.[34] The central equations obeyed by the propagator are the quasiclassical transport equations

$$\left[\left(\epsilon + \frac{e}{c}\mathbf{v}_f \cdot \mathbf{A}_l\right)\hat{\tau}_3 - e\Phi_l\hat{1} - \hat{\Delta}_l - \hat{\sigma}_l\, ,\, \hat{g}_l\right]_{\otimes} + i\hbar\mathbf{v}_f \cdot \boldsymbol{\nabla}\hat{g}_l = 0 \; . \tag{2}$$

These equations generalize the Boltzmann-Landau equations describing the normal-state transport properties. We have a transport equation for each conducting layer specified by the discrete index l. The properties of a layer are described by the Fermi velocity $\mathbf{v}_f$, the in-plane vector potential $\mathbf{A}_l(\mathbf{R},t)$, the scalar potential $\Phi_l(\mathbf{R},t)$, the order parameter $\hat{\Delta}_l(\mathbf{R},t)$, and the scattering self-energy $\hat{\sigma}_l(\mathbf{R};\epsilon,t)$. The electromagnetic coupling of quasiparticles (of charge e) appears explicitly in Eq. (2), but also implicitly through the gauge-invariant self-energy $\hat{\sigma}_l$. The Fermi velocity and in-plane vector potential are two-dimensional vectors parallel to the planes. We assume the simplest model, a circular Fermi surface with radius k_f and an isotropic in-plane Fermi velocity.

In order to describe nonequilibrium properties we use Keldysh's formulation of nonequilibrium dynamics.[35] The nonequilibrium theory requires three types of propagators (retarded $\hat{g}_l^R$, advanced $\hat{g}_l^A$, and Keldysh $\hat{g}_l^K$), to describe the dynamics of the quasiparticle level spectrum, as well as the space and time dependence of the occupation of these levels. We use a single symbol $\hat{g}_l(s,\mathbf{R};\epsilon,t)$ to denote these propagators which depend on the position on the Fermi surface s, the spatial coordinate in the plane $\mathbf{R}$, the excitation energy ϵ, and time t. The $\otimes$ notation in Eq. (2) denotes the folding product in the energy-time domain and the Keldysh matrix algebra required to describe nonequilibrium phenomena. For details of this formalism see Refs. 34, 36, and 37.

The self-energy describes two scattering processes: (i) in-plane quasiparticle scattering and (ii) interplane scattering, which is an essential feature of this model. Since we neglect coherent transport along the c axis (i.e., the Fermi velocity has no c component), interplane scattering is the only mechanism for charge transport perpendicular to the planes. In the Born approximation the in-plane scattering is described by the self-energy,

$$\hat{\sigma}_l^{(ab)}(\mathbf{R};\epsilon,t) = \frac{\hbar}{2\pi\tau_0}\oint\frac{ds}{2\pi}\,\hat{g}_l(s,\mathbf{R};\epsilon,t) \; , \tag{3}$$

where $1/\tau_0$ is the in-plane scattering rate. Similarly, interplane scattering between nearest-neighbor layers is given by

$$\hat{\sigma}_l^{(c)}(\mathbf{R};\epsilon,t) = \frac{\hbar}{2\pi\tau_1}\sum_{k=l\pm1}\exp\left(-i\frac{ed}{\hbar c}A_{lk}^z\hat{\tau}_3\right) \otimes \oint\frac{ds}{2\pi}\hat{g}_k(s,\mathbf{R};\epsilon,t) \otimes \exp\left(+i\frac{ed}{\hbar c}A_{lk}^z\hat{\tau}_3\right) , \tag{4}$$

where $A_{lk}^z(\mathbf{R},t)$ is the "interplane vector potential," defined by

$$A_{l,l+1}^z = \int_{l\cdot d}^{(l+1)\cdot d}\frac{dz}{d}\,A^z(\mathbf{R},z,t) \; , \tag{5}$$

and d is the spacing between neighboring planes. The specific form of the phase factors in Eq. (4) is required by gauge invariance. To be specific, under a gauge transformation the transport equation must be invariant. For a gauge transformation generated by $\Lambda_l(\mathbf{R},t)$ the propagator transforms as

$$\hat{g}_l(s,\mathbf{R};\epsilon,t) \rightarrow \exp(i\Lambda_l\hat{\tau}_3)\otimes\hat{g}_l(s,\mathbf{R};\epsilon,t)\otimes\exp(-i\Lambda_l\hat{\tau}_3) \; , \tag{6}$$

and the potentials transform as

$$\mathbf{A}_l \rightarrow \mathbf{A}_l + \frac{\hbar c}{e}\boldsymbol{\nabla}\Lambda_l \; , \tag{7}$$

$$A_{lk}^z \rightarrow A_{lk}^z + \frac{\hbar c}{ed}(\Lambda_k - \Lambda_l) \; , \tag{8}$$

$$\Phi_l \rightarrow \Phi_l - \frac{\hbar}{e}\partial_t\Lambda_l \; . \tag{9}$$

One sees by inspection that the transport equation, with the self-energy, $\hat{\sigma}_l = \hat{\sigma}_l^{(ab)} + \hat{\sigma}_l^{(c)}$, defined in Eqs. (3) and (4), is gauge invariant.

The order parameter in layer l is defined here as the off-diagonal mean-field self-energy, which we assume obeys the weak-coupling BCS gap equation

$$\hat{\Delta}_l(\mathbf{R},t) = V \oint \frac{ds}{2\pi} \int_{-\hbar\omega_c}^{+\hbar\omega_c} \frac{d\epsilon}{4\pi i} \hat{f}_l^K(s,\mathbf{R};\epsilon,t) \ . \quad (10)$$

The dimensionless BCS interaction V and the cutoff $\hbar\omega_c$ are related to the physical transition temperature, and can always be eliminated from any measurable quantity in favor of T_c.

In addition to the transport equation, the quasiclassical propagators obey the conditions

$$\hat{g}_l^{R,A} \otimes \hat{g}_l^{R,A} = -\pi^2 \ \hat{1} \ , \quad (11)$$

$$\hat{g}_l^R \otimes \hat{g}_l^K + \hat{g}_l^K \otimes \hat{g}_l^A = 0 \ . \quad (12)$$

These normalization conditions play an important role in selecting the physical solution to the transport equations.

The charge and in-plane current densities are given by standard relations of quasiclassical theory,[34,36,37] specialized to two-dimensional Fermi liquids,

$$n_l(\mathbf{R},t) = \frac{ek_F^2}{2\pi d} - 2e^2 N_f \Phi_l(\mathbf{R},t) + eN_f \oint \frac{ds}{2\pi} \int \frac{d\epsilon}{4\pi i} \mathrm{Tr}\left[\hat{g}_l^K(s,\mathbf{R};\epsilon,t)\right] \ , \quad (13)$$

$$\mathbf{j}_l(\mathbf{R},t) = eN_f \oint \frac{ds}{2\pi} \ \mathbf{v}_f(s) \ \int \frac{d\epsilon}{4\pi i} \mathrm{Tr}\left[\hat{\tau}_3 \hat{g}_l^K(s,\mathbf{R};\epsilon,t)\right] \ , \quad (14)$$

where $N_f = k_f/2\pi\hbar v_f d$ is the single-spin density of states per unit volume at the Fermi energy. The interplane current is not given by the standard quasiclassical relation. However, it follows from the quasiclassical transport equation and the self-consistency equations [(2)–(10)] that n_l and $\mathbf{j}_l$ obey the continuity equation

$$\partial_t n_l(\mathbf{R},t) + \boldsymbol{\nabla} \cdot \mathbf{j}_l(\mathbf{R},t) = [j_{l-1,l}^z(\mathbf{R},t) - j_{l,l+1}^z(\mathbf{R},t)]/d \, , \quad (15)$$

where $j_{l-1,l}^z$ is the interlayer current density flowing from layer $l-1$ to layer l, and similarly $j_{l,l+1}^z$ is the current density flowing from layer l to $l+1$. The microscopic expressions for the interlayer current densities can be obtained by integrating the Keldysh component of the transport equation (2) over the Fermi surface and all energies, multiplying by $\hat{\tau}_3$, and taking the trace of the resulting equation. One obtains the continuity equation (15), from which we read off the interlayer currents

$$j_{l,l+1}^z(\mathbf{R},t) = \frac{1}{i} \frac{eN_f d}{2\pi\tau_1} \oint \frac{ds'}{2\pi} \oint \frac{ds}{2\pi} \int \frac{d\epsilon}{4\pi i} \ \mathrm{Tr}\left\{ \hat{\tau}_3 \left[\hat{g}_l(s,\mathbf{R};\epsilon,t), \exp\left(-i\frac{ed}{\hbar c}A_{l,l+1}^z \hat{\tau}_3\right) \otimes \hat{g}_{l+1}(s',\mathbf{R};\epsilon,t) \otimes \exp\left(+i\frac{ed}{\hbar c}A_{l,l+1}^z \hat{\tau}_3\right)\right]_{\otimes} \right\}^K \ . \quad (16)$$

This formula covers both normal currents and supercurrents in equilibrium and nonequilibrium situations. It is somewhat formal, but is more transparent in two special limits of interest. In the normal state the retarded and advanced propagators take the simple form, $\hat{g}^{R,A} = \mp i\pi\hat{\tau}_3$, and one finds, from (16) and (13),

$$j_{l,l+1}^z(\mathbf{R},t) = \sigma_\perp E_{l,l+1}(\mathbf{R},t) - D_\perp[n_{l+1}(\mathbf{R},t) - n_l(\mathbf{R},t)]/d, \quad (17)$$

which shows that the interlayer current is driven by a charge difference on adjacent layers, $n_l - n_{l+1}$, and by an electric field in the c direction, $E_{l,l+1} = -\frac{1}{c}\partial_t A_{l,l+1}^z - (\Phi_{l+1} - \Phi_l)/d$. Equation (17) describes interlayer diffusion in the normal state with an interlayer diffusion constant $D_\perp = d^2/\tau_1$, and an interlayer conductivity $\sigma_\perp = 2e^2 N_f d^2/\tau_1$.

The second special limit is a superconductor in equilibrium, in which case the interlayer supercurrent is given by

$$j_{l,l+1}^z(\mathbf{R}) = \frac{eN_f d}{2\pi^2\tau_1} \oint \frac{ds}{2\pi} \oint \frac{ds'}{2\pi} \int d\epsilon \ \tanh\left(\frac{\epsilon}{2k_B T}\right) \times \mathrm{Re}\left\{ \exp\left(2i\frac{ed}{\hbar c}A_{l,l+1}^z\right) f_l^R(s',\mathbf{R},\epsilon) f_{l+1}^A(s,\mathbf{R},\epsilon)^* - \exp\left(-2i\frac{ed}{\hbar c}A_{l,l+1}^z\right) f_{l+1}^R(s',\mathbf{R},\epsilon) f_l^A(s,\mathbf{R},\epsilon)^* \right\} \ , \quad (18)$$

which is the typical expression for the Josephson current,[38] expressed in terms of quasiclassical propagators. It is a generalization of the interlayer Josephson current in the Lawrence-Doniach model that is valid at all temperatures.

The equilibrium properties of inhomogeneous states, e.g., the Abrikosov vortex state, are most efficiently calculated with the Matsubara technique.[39] In this formulation the quasiclassical propagators depend on the Matsubara frequency, $\epsilon_n = (2n+1)\pi k_B T$, in addition to the Fermi surface and spatial coordinates. The Matsubara propagator $\hat{g}_l^M(s,\mathbf{R};\epsilon_n)$ satisfies the equilibrium transport equation,[34]

$$\left[\left(i\epsilon_n + \frac{e}{c}\mathbf{v}_f \cdot \mathbf{A}_l\right)\hat{\tau}_3 - \hat{\Delta}_l - \hat{\sigma}_l\,,\, \hat{g}_l^M(s,\mathbf{R};\epsilon_n)\right] + i\hbar\mathbf{v}_f \cdot \boldsymbol{\nabla}\hat{g}_l^M(s,\mathbf{R};\epsilon_n) = 0, \tag{19}$$

and normalization condition,

$$\hat{g}_l^M(s,\mathbf{R};\epsilon_n)^2 = -\pi^2\hat{1}\ . \tag{20}$$

This transport equation can be obtained as the stationarity condition of a free-energy functional, as originally derived by Eilenberger[31] and extended below to a stack of coupled two-dimensional Fermi liquids,

$$\mathcal{F}_s - \mathcal{F}_n \equiv \mathcal{F}[f_l, \bar{f}_l, \mathbf{A}_l, A_{lk}^z, \Delta_l, \Delta_l^*] = \sum_l \left(\mathcal{F}_l + \mathcal{V}_{l,l-1} + \mathcal{V}_{l,l+1}\right) + \mathcal{F}_{\text{mag}}\ , \tag{21}$$

where $\mathcal{F}_l$ is the free energy per unit layer associated with plane l,

$$\mathcal{F}_l = N_f d \int d^2R \left(|\Delta_l(\mathbf{R})|^2 \ln\frac{T}{T_c} + 2\pi k_B T \sum_{n=0}^{\infty}\left\{\frac{|\Delta_l(\mathbf{R})|^2}{\epsilon_n} - \frac{1}{\pi}\oint\frac{ds}{2\pi}J_l^{(0)}(s,\mathbf{R};\epsilon_n)\right\}\right)\ , \tag{22}$$

where

$$\begin{aligned} J_l^{(0)}(s,\mathbf{R};\epsilon_n) = {} & \Delta_l^* f_l - \Delta_l \bar{f}_l - \frac{1}{2}\left\{\frac{1}{2}f_l\mathcal{I}^{(0)}\bar{f}_l + \frac{1}{2}\bar{f}_l\mathcal{I}^{(0)}f_l + g_l\mathcal{I}^{(0)}g_l + \pi\hbar/\tau_0\right\} \\ & -(\pi - ig_l)\left[2\epsilon_n + \frac{1}{2}\hbar\mathbf{v}_f(s)\cdot\left\{\frac{1}{f_l}\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)f_l - \frac{1}{\bar{f}_l}\left(\boldsymbol{\nabla} + i\frac{2e}{\hbar c}\mathbf{A}_l\right)\bar{f}_l\right\}\right]\ , \end{aligned} \tag{23}$$

and the interaction energy of the planes is represented by

$$\mathcal{V}_{l,k} = N_f d k_B T \int d^2R \sum_{n=0}^{\infty}\oint\frac{ds}{2\pi}\left(\frac{1}{2}f_l\mathcal{I}^{(1)}\bar{f}_k + \frac{1}{2}\bar{f}_l\mathcal{I}^{(1)}f_k + g_l\mathcal{I}^{(1)}g_k + \pi\hbar/\tau_1\right)\ , \tag{24}$$

with

$$f_l\mathcal{I}^{(0)}\bar{f}_k = f_l\,\frac{\hbar}{\tau_0\pi}\oint\frac{ds}{2\pi}\bar{f}_k\ , \qquad f_l\mathcal{I}^{(1)}\bar{f}_k = f_l\,\frac{\hbar}{\tau_1\pi}\oint\frac{ds}{2\pi}\bar{f}_k\exp\left(+i\frac{2ed}{\hbar c}A_{lk}^z\right), \tag{25}$$

$$\bar{f}_l\mathcal{I}^{(0)}f_k = \bar{f}_l\,\frac{\hbar}{\tau_0\pi}\oint\frac{ds}{2\pi}f_k\ , \qquad \bar{f}_l\mathcal{I}^{(1)}f_k = \bar{f}_l\,\frac{\hbar}{\tau_1\pi}\oint\frac{ds}{2\pi}f_k\exp\left(-i\frac{2ed}{\hbar c}A_{lk}^z\right), \tag{26}$$

$$g_l\mathcal{I}^{(0)}g_k = g_l\,\frac{\hbar}{\tau_0\pi}\oint\frac{ds}{2\pi}g_k\ , \qquad g_l\mathcal{I}^{(1)}g_k = g_l\,\frac{\hbar}{\tau_1\pi}\oint\frac{ds}{2\pi}g_k\ . \tag{27}$$

Finally, the magnetic field energy

$$\mathcal{F}_{\text{mag}} = \int dz\; d^2R\; (\text{curl}\mathbf{A})^2/8\pi,$$

for the layered system, takes the form

$$\mathcal{F}_{\text{mag}} = \frac{d}{8\pi}\sum_l\int d^2R\{[\partial_y A_{l,l+1}^z - (A_{l+1}^y - A_l^y)/d]^2 + \left[(A_{l+1}^x - A_l^x)/d - \partial_x A_{l,l+1}^z\right]^2 + (\partial_x A_l^y - \partial_y A_l^x)^2\}\ . \tag{28}$$

This form for the field energy is a consequence of our model in which the x and y coordinates in the plane are continuous, whereas the z coordinate takes only the discrete values $l \cdot d$. The relation corresponding to $\mathbf{B} = \text{curl}\mathbf{A}$ is

$$(B_{l,l+1}^x,\ B_{l,l+1}^y,\ B_l^z) = \left(\partial_y A_{l,l+1}^z - \frac{A_{l+1}^y - A_l^y}{d},\ \frac{A_{l+1}^x - A_l^x}{d} - \partial_x A_{l,l+1}^z,\ \partial_x A_l^y - \partial_y A_l^x\right)\ . \tag{29}$$

The stationarity conditions of Eq. (21) with respect to the off-diagonal propagators f_l and $\bar{f}_l$, the in-plane vector potential $\mathbf{A}_l$, the interplane vector potential A_{lk}^z, and the order parameter Δ_l give, respectively, the transport equation (19), Ampère's equation,

$$\frac{4\pi}{c}(j_l^x,\ j_l^y,\ j_{l,l+1}^z) = \left(\partial_y B_l^z - \frac{B_{l,l+1}^y - B_{l-1,l}^y}{d},\ \frac{B_{l,l+1}^x - B_{l-1,l}^x}{d} - \partial_x B_l^z,\ \partial_x B_{l,l+1}^y - \partial_y B_{l,l+1}^x\right)\ , \tag{30}$$

with the in-plane and interplane supercurrents given by

$$\mathbf{j}_l(\mathbf{R}) = 4eN_f k_B T \sum_{n=0}^{\infty} \oint \frac{ds}{2\pi} \mathbf{v}_f(s)\; g_l(s,\mathbf{R};\epsilon_n) \;, \tag{31}$$

$$j^z_{l,l+1}(\mathbf{R}) = i\frac{eN_f d}{\tau_1 \pi} k_B T \sum_{n=0}^{\infty} \oint \frac{ds}{2\pi} \oint \frac{ds'}{2\pi} \Big\{ [\, \bar{f}_l(s,\mathbf{R};\epsilon_n) f_{l+1}(s',\mathbf{R};\epsilon_n) + \bar{f}_{l+1}(s,\mathbf{R};\epsilon_n)^* f_l(s',\mathbf{R};\epsilon_n)^*] \exp\left(-i\frac{2ed}{\hbar c} A^z_{l,l+1}\right) - \text{c.c.} \Big\} \;, \tag{32}$$

and the self-consistency equation for the order parameter,

$$\Delta_l(\mathbf{R}) \ln \frac{T_c}{T} = 2\pi k_B T \sum_{n=0}^{\infty} \left(\frac{\Delta_l(\mathbf{R})}{\epsilon_n} - \frac{1}{\pi} \oint \frac{ds}{2\pi} f_l(s,\mathbf{R};\epsilon_n) \right) \;. \tag{33}$$

This completes the summary of the basic equations of the interlayer diffusion model. The theory contains a small set (six in this simplest version) of microscopic material parameters: the density of states N_f for the conducting layers, the Fermi velocity v_f in the layer, the transition temperature T_c, the in-plane and interplane scattering rates $1/\tau_0$ and $1/\tau_1$, and the interlayer distance d.

III. SPECIAL LIMITS

The interlayer diffusion model is designed to describe layered superconducting metals at temperatures well below the degeneracy temperature. The theory is formulated in terms of nonlinear transport equations, which in their general form are difficult to solve. However, in several limits the equations simplify considerably. In particular, the normal-state quasiclassical equations reduce to Boltzmann-Landau transport equations for normal Fermi liquids. And for the equilibrium superconducting state near T_c, the full equations reduce to the Lawrence-Doniach equations for layered superconductors. In many cases, for extreme type-II superconductors, the current and magnetic field are slowly varying on the scale of the coherence length and interlayer separation. In this limit we recover the anisotropic London theory, with penetration depths $\lambda_{||}$ and $\lambda_{\perp}$, given in terms of the parameters of the microscopic theory. By relating the general equations to these well-studied standard theories we can take over results obtained from these theories and connect them with new results in temperature and field ranges which are not accessible to the standard theories. We can also relate the parameters of the interlayer diffusion model to those of the standard models. In this section we derive the standard models from the general theory and give estimates for the microscopic parameters of Y-Ba-Cu-O.

A. Normal state

In the normal state the matrix transport equations reduce to scalar equations for $g_l(s,\mathbf{R};\epsilon,t)$, the upper left matrix element of $\hat{g}^K_l$. This propagator is directly related to the distribution function for quasiparticles in layer l, and in the limit $\hbar\omega \ll E_f$ obeys the Boltzmann-Landau transport equations[33,36]

$$[\partial_t + \mathbf{v}_f(s)\cdot\boldsymbol{\nabla}] g_l(s,\mathbf{R};\epsilon,t) + \left(e\partial_t\Phi_l - \frac{e}{c}\mathbf{v}_f(s)\cdot\partial_t\mathbf{A}_l\right)\partial_\epsilon g_l(s,\mathbf{R};\epsilon,t) = I_l(s,\mathbf{R};\epsilon,t) \;, \tag{34}$$

where $I_l(s,\mathbf{R};\epsilon,t)$ is the collision integral for layer l, with

$$\begin{aligned} I_l(s,\mathbf{R};\epsilon,t) = &-\frac{1}{\tau_0}\Big\{g_l(s,\mathbf{R};\epsilon,t) - \oint \frac{ds'}{2\pi} g_l(s',\mathbf{R};\epsilon,t)\Big\} \\ &-\frac{1}{\tau_1}\Big\{2\, g_l(s,\mathbf{R};\epsilon,t) - \oint \frac{ds'}{2\pi}\,[g_{l+1}(s',\mathbf{R};\epsilon,t) + g_{l-1}(s',\mathbf{R};\epsilon,t)]\Big\} \\ &-\frac{1}{\tau_1}\frac{ed}{c}\oint \frac{ds'}{2\pi}\,\{\partial_t A^z_{l,l+1}(\mathbf{R},t)\partial_\epsilon g_{l+1}(s',\mathbf{R};\epsilon,t) - \partial_t A^z_{l-1,l}(\mathbf{R},t)\partial_\epsilon g_{l-1}(s',\mathbf{R};\epsilon,t)\} \;. \end{aligned} \tag{35}$$

This is the usual Born approximation for the collision integral, except for the terms which contain the time derivative of the vector potential. They are required for gauge invariance, and describe the effect of an electric field perpendicular to the layers on the interlayer hopping rate. The terms proportional to $1/\tau_1$ are the only source of interlayer currents in this model. In particular, the current density perpendicular to the layers is given by Eq. (16), while the current density in the plane is given by Eq. (14), which in the normal state reduces to

$$\mathbf{j}_l(\mathbf{R},t) = 2eN_f \oint \frac{ds}{2\pi}\; \mathbf{v}_f(s) \int \frac{d\epsilon}{4\pi i}\; g_l(s,\mathbf{R};\epsilon,t) \;. \tag{36}$$

The conductivity tensor is calculated from the linear response to a time-dependent vector potential. This is a straightforward calculation,[39] and one finds

$$\sigma_{\|}(\omega) = \frac{N_f e^2 v_f^2}{1/\tau - i\omega} \ , \tag{37}$$

$$\sigma_{\perp}(\omega) = \frac{2N_f e^2 d^2}{\tau_1} = 2N_f e^2 D_{\perp} \ , \tag{38}$$

where $1/\tau = 1/\tau_0 + 2/\tau_1$ is the total scattering rate from in-plane and interplane scattering, and $D_{\perp} = d^2/\tau_1$ is the interlayer diffusion constant. Note that the in-plane dc conductivity is reduced by scattering, whereas the interplane conductivity increases with increasing interlayer scattering. The Drude form for the in-plane conductivity and the frequency-independent interplane conductivity are consequences of elastic scattering. More general models which take inelastic scattering into account could be formulated,[19,20,27,29,40,41] but are outside the scope of this article.

B. Lawrence-Doniach and Ginzburg-Landau models

Ginzburg-Landau theory describes the equilibrium properties of the superconducting state near T_c. The theory assumes that the order parameter is small, $\Delta/k_BT_c \ll 1$, and in the case of an anisotropic metal, that the order parameter varies smoothly on the scales of the zero-temperature coherence lengths, $\xi_{\|}$ along the planes and $\xi_{\perp}$ perpendicular to the planes. The second condition can be quite stringent for extremely anisotropic materials, e.g., the CuO superconductors. In the case of an exceedingly small perpendicular coherence length, $\xi_{\perp} < d$, the 3D Ginzburg-Landau theory breaks down, except for a narrow region near T_c where $\xi_{\perp}(T) \sim \xi_{\perp}/\sqrt{1-T/T_c} > d$. The Lawrence-Doniach model generalizes the Ginzburg-Landau theory to include variations of the order parameter on the scale of d; thus, it stretches the temperature range of the Ginzburg-Landau theory. The Lawrence-Doniach model also assumes weak interlayer coupling.

The free energy of the Lawrence-Doniach model can be obtained from the general free-energy functional of Sec. II by expanding in the interlayer scattering rate $1/\tau_1$, the order parameter $\Delta_l(\mathbf{R})$, and the in-plane gradients $\mathbf{v}_f\cdot\boldsymbol{\nabla}$. The solution to the transport equation (19) is obtained by expanding to order $(1-T/T_c)^{3/2}$, with the assignments $\Delta_l \sim (1-T/T_c)^{1/2}$ and $\boldsymbol{\nabla} - i(2e/\hbar c)\mathbf{A}_l \sim (1-T/T_c)^{1/2}$. We first neglect interlayer coupling, and find

$$\begin{aligned} f_l = {} & \frac{\pi\Delta_l}{|\epsilon_n|} - \frac{\pi|\Delta_l|^2\Delta_l}{2|\epsilon_n|^3} - \frac{\pi\hbar}{\epsilon_n\left(2|\epsilon_n| + \hbar/\tau\right)}\mathbf{v}_f(s)\cdot\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)\Delta_l \\ & + \frac{\pi\hbar^2}{|\epsilon_n|\left(2|\epsilon_n| + \hbar/\tau\right)^2}\left\{\left[\mathbf{v}_f(s)\cdot\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)\right]^2 + \frac{\hbar v_f^2}{4|\epsilon_n|\tau}\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)^2\right\}\Delta_l \ . \end{aligned} \tag{39}$$

Interlayer coupling is now included perturbatively. We insert f_l and $\bar{f}_l$ into the free energy (21), and retain terms through first order in $1/\tau_1$ and leading order in $(T - T_c)$,

$$\begin{aligned} \mathcal{F} = \mathcal{F}_{\rm mag} + N_f d \sum_{l=-\infty}^{\infty} \int d^2R \Bigg\{ & |\Delta_l|^2 \ln\frac{T}{T_c} + \pi k_B T \sum_{n=0}^{\infty} \frac{|\Delta_l|^4}{2\epsilon_n^3} \\ & + \pi k_B T \sum_{n=0}^{\infty} \frac{1}{2\epsilon_n^2}\left[\frac{(\hbar v_f)^2}{(2\epsilon_n + \hbar/\tau)}\left|\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)\Delta_l\right|^2 \right. \\ & \left. + \frac{2\hbar}{\tau_1}\left|\Delta_l - \Delta_{l+1}\exp\left(-i\frac{2ed}{\hbar c}A^z_{l,l+1}\right)\right|^2\right]\Bigg\} \ . \end{aligned} \tag{40}$$

This is the Lawrence and Doniach free-energy functional, which can be written in the standard form,

$$\begin{aligned} \mathcal{F} = \mathcal{F}_{\rm mag} + N_f d \sum_{l=-\infty}^{\infty} \int d^2R \Bigg[& \alpha(T)|\Delta_l|^2 + \frac{\beta}{2}|\Delta_l|^4 + \xi_{\|}^2\left|\left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}_l\right)\Delta_l\right|^2 \\ & + \frac{\xi_{\perp}^2}{d^2}\left|\Delta_l - \Delta_{l+1}\exp\left(-i\frac{2ed}{\hbar c}A^z_{l,l+1}\right)\right|^2\Bigg] \ , \end{aligned} \tag{41}$$

where the parameters are given in terms of the microscopic parameters by the relations

$$\alpha(T) = \frac{T-T_c}{T_c} \ , \qquad \xi_{\|}^2 = \left(\frac{\hbar v_f}{2\pi k_B T_c}\right)^2 \frac{7\zeta(3)}{8}\chi(\rho) \ , \tag{42}$$

$$\beta = \frac{7\zeta(3)}{2\left(2\pi k_B T_c\right)^2} \ , \qquad \xi_{\perp}^2 = \frac{\pi^2}{4}\rho_1\, d^2 = \frac{\pi\hbar}{8k_BT_c}D_{\perp} \ ,$$

where ρ and ρ_1 are dimensionless scattering rates,

$$\rho = \frac{\hbar}{2\pi k_B T_c \tau} \ , \qquad \rho_1 = \frac{\hbar}{2\pi k_B T_c \tau_1} \ , \tag{43}$$

and

$$\chi(\rho) = \frac{8}{7\zeta(3)}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2\,(2n+1+\rho)} \ . \tag{44}$$

The Lawrence-Doniach free energy goes over to the Ginzburg-Landau free energy for an anisotropic system in the continuum limit, i.e., in the limit of a smoothly varying order parameter perpendicular to the layers. One obtains, from (41),

$$\mathcal{F} = \mathcal{F}_{\rm mag} + N_f \int dz\, d^2R \Bigg[\alpha(T)|\Delta|^2 + \frac{\beta}{2}|\Delta|^4 + \xi_{\parallel}^2 \left| \left(\boldsymbol{\nabla} - i\frac{2e}{\hbar c}\mathbf{A}\right)\Delta\right|^2 + \xi_{\perp}^2 \left| \left(\partial_z - i\frac{2e}{\hbar c}A^z\right)\Delta\right|^2 \Bigg] . \tag{45}$$

In order to estimate the magnitudes of the microscopic parameters we use parameters of the Lawrence-Doniach, Ginzburg-Landau, and London models (discussed below) obtained from comparisons with experiments.

C. London limit

Many of the magnetic properties of layered type-II superconductors may be adequately analyzed in terms of the anisotropic London model. This theory is not necessarily restricted to the temperature region near T_c, but requires that the magnetic field and supercurrent vary on length scales that are large compared to the coherence length. In this limit the order parameter assumes its local equilibrium value, and the microscopic theory reduces to the standard London equation for the coarse-grained vector potential, $\mathbf{A}(\mathbf{R},z) = (\mathbf{A}, A^z)$, and phase, $\chi(\mathbf{R},z)$,

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{A} = \frac{1}{\lambda_{\parallel}(T)^2}\left(\frac{\hbar c}{2e}\boldsymbol{\nabla}\chi(\mathbf{R},z) - \mathbf{A}(\mathbf{R},z)\right) + \frac{1}{\lambda_{\perp}(T)^2}\left(\frac{\hbar c}{2e}\nabla_z\chi(\mathbf{R},z) - A^z(\mathbf{R},z)\right)\hat{z} . \tag{46}$$

The length scales $\lambda_{\parallel}(T)$ and $\lambda_{\perp}(T)$ determine the decay lengths for screening currents in the Meissner and vortex phases. These parameters are obtained from the interlayer diffusion model by evaluating the current [Eqs. (31)–(32)] with the local equilibrium solution to the transport equation

$$\hat{g}_l^M = -\pi \frac{i\tilde{\epsilon}_{l,n}\hat{\tau}_3 - \hat{\tilde{\Delta}}_l}{\sqrt{(\tilde{\epsilon}_{l,n})^2 + |\tilde{\Delta}_l|^2}} , \tag{47}$$

$$\tilde{\epsilon}_{l,n} = \epsilon_n + i\mathbf{v}_f(s)\cdot\left[\frac{\hbar}{2}\boldsymbol{\nabla}\chi_l - \frac{e}{c}\mathbf{A}_l\right] + \frac{i\hbar}{2\pi\tau}\oint\frac{ds}{2\pi}\, g_l(s,\mathbf{R};\epsilon_n) , \tag{48}$$

$$\tilde{\Delta}_l = \Delta_l + \frac{\hbar}{2\pi\tau}\oint\frac{ds}{2\pi}\, f_l(s,\mathbf{R};\epsilon_n) , \tag{49}$$

where $\Delta_l(\mathbf{R})$ is the mean-field order parameter given by the self-consistent solution of Eq. (33). In zero field the homogeneous order parameter $\Delta_l(\mathbf{R}) = \Delta(T)$ satisfies the usual BCS gap equation. Note that we also have neglected corrections to the interplane self-energy of order $d/\lambda_{\perp}$. For currents flowing in the planes the screening length is easily calculated by expanding $\hat{g}_l^M$ to linear order in $\mathbf{A}_l$,[39]

$$\frac{1}{\lambda_{\parallel}(T)^2} = (8\pi e^2 N_f v_f^2/c^2)\, \pi k_B T \times \sum_{n=0}^{\infty} \frac{\Delta(T)^2}{Z(\epsilon_n,\tau)[\epsilon_n^2 + \Delta(T)^2]^{3/2}} , \tag{50}$$

where

$$Z(\epsilon_n,\tau) = 1 + \frac{\hbar}{2\tau}\frac{1}{\sqrt{\epsilon_n^2 + \Delta(T)^2}} \tag{51}$$

determines the reduction in the screening current from both in-plane and interplane scattering ($1/\tau = 1/\tau_0 + 2/\tau_1$). Note that strong interlayer scattering leads to a significant reduction of the in-plane supercurrent; for $\pi k_B T_c \ll \hbar/\tau_1$ the zero-temperature screening length is larger by the factor $1/\sqrt{\pi\Delta(0)\tau/\hbar}$ than the corresponding screening length in the clean limit.

Interlayer scattering provides a Josephson coupling between planes, and a corresponding interlayer Josephson current proportional to the interlayer scattering rate,

$$j^z_{l,l+1}(\mathbf{R}) = \frac{4eN_f d}{\tau_1}\,\pi k_B T \times \sum_{n=0}^{\infty}\frac{\Delta(T)^2}{\epsilon_n^2 + \Delta(T)^2}\,\sin[\Phi_{l,l+1}(\mathbf{R})] = \frac{\pi e N_f d}{\tau_1}\,\Delta(T)\tanh\left(\frac{\Delta(T)}{2k_BT}\right)\sin[\Phi_{l,l+1}(\mathbf{R})] , \tag{52}$$

where $\Phi_{l,l+1}$ is the gauge-invariant phase difference between layers,

$$\Phi_{l,l+1}(\mathbf{R}) = \chi_{l+1}(\mathbf{R}) - \chi_l(\mathbf{R}) - \frac{2ed}{\hbar c}A^z_{l,l+1}(\mathbf{R}) . \tag{53}$$

For slow variations from plane to plane, $\Phi_{l,l+1} \ll 1$, we obtain the continuum limit form for the supercurrent perpendicular to the planes, $j^z \propto \nabla_z\chi - (2e/\hbar c)A^z$, and a corresponding screening length given by

$$\frac{1}{\lambda_{\perp}(T)^2} = \frac{8\pi^2 e^2 N_f}{\hbar c^2}\frac{d^2}{\tau_1}\,\Delta(T)\tanh\left(\frac{\Delta(T)}{2k_BT}\right) , \tag{54}$$

which is just a Josephson penetration length.[42]

We end this section with a summary of some known results for the lower critical field. The London theory gives a reasonable estimate of H_{c1} in good type-II superconductors because the line energy is dominated by the kinetic energy of the supercurrents outside the vortex core. However, the kinetic energy must be cutoff at a length scale corresponding to the breakdown of local equilibrium of the order parameter. For flow in the planes this scale is the coherence length, whereas for currents perpendicular to the planes the scale is typically the interlayer spacing. Thus, the London theory results for the lower critical field are,[7,43]

TABLE I. Experimental data.

Quantity	Y-Ba-Cu-O ($\delta \simeq 0.1$)
T_c	92 (K)[a]
d	11.7 (Å)[b]
γ	0.20 (mJ/K^{2}cm^3)[c]
$\varrho_{\parallel}(T_c)$	50 ($\mu\Omega$ cm)[d]
$\hbar\omega_p$	1.5 (eV)[d]
$(\varrho_\perp/\varrho_\parallel)_{T_c}$	230[e]
$dH^{\perp}_{c2}/dT\vert_{T_c}$	−1.8 (T/K)[a]
$dH^{\parallel}_{c2}/dT\vert_{T_c}$	−10 (T/K)[a]
$\lambda_\parallel(0)$	1.4×10^3 (Å)[f]
$\lambda_\perp/\lambda_\parallel$	7.7[g]

[a]Reference 47.
[b]Reference 1.
[c]Reference 49.
[d]Reference 50.
[e]Reference 51.
[f]References 51–53.
[g]Reference 54.

$$H^{\perp}_{c1}(T) = \frac{\phi_0}{4\pi\lambda_\parallel(T)^2}\left[\ln\left(\frac{\lambda_\parallel(T)}{\xi_\parallel(T)}\right) + \varepsilon^{\rm core}_\perp\right] , \tag{55}$$

$$H^{\parallel}_{c1}(T) = \frac{\phi_0}{4\pi\lambda_\parallel(T)\lambda_\perp(T)}\left[\ln\left(\frac{\lambda_\parallel(T)}{d}\right) + \varepsilon^{\rm core}_\parallel\right] , \tag{56}$$

where the core contributions are of order 1 compared to the logarithms. These core energies, which are outside the scope of the London theory, are due to pair-breaking effects from the combination of the vortex supercurrents and scattering processes. The core energy contributions to the lower critical field are discussed in a forthcoming paper.[44]

D. Application to Y-Ba-Cu-O

The large anisotropy of the resistivity of CuO materials suggests that the interlayer diffusion model may be appropriate to high-temperature CuO superconductors. Here we use the available experimental data on Y-Ba-Cu-O to compare with the interlayer diffusion model, and to estimate the microscopic parameters. These parameters, particularly those associated with scattering processes, may vary from material to material. We know of no report of a complete set of relevant measurements on a single sample of crystalline Y-Ba-Cu-O; thus, the parameters obtained here are estimates with the uncertainty of having been derived from measurements on different samples. Nevertheless, the interlayer diffusion model gives a good description of the basic normal-state and superconducting properties of Y-Ba-Cu-O.

The normal-state electronic heat capacity, extrapolated to zero temperature, determines the normal-state density of states at the Fermi level,

$$C_{\rm el}/T \equiv \gamma = \tfrac{2}{3}\pi^2 k_B^2 N_f . \tag{57}$$

TABLE II. Microscopic model parameters.

T_c (K)	d (Å)	N_f 1/(eV cell-spin)	v_f (cm/sec)	$\hbar/\tau_1\vert_{T_c}$ (meV)	$\hbar/\tau_0\vert_{T_c}$ (meV)
92	11.7	3.9	1.1×10^7	0.5	14.2

The measured value of this parameter for $YBa_2Cu_3O_{6.9}$ is listed in Table I, as well as other input data that we use to determine the microscopic parameters.

Measurements of the Drude plasma frequency, combined with the density of states, allow us to infer the in-plane Fermi velocity from the formula

$$\omega_p^2 = 4\pi N_f e^2 v_f^2 . \tag{58}$$

The total scattering rate $1/\tau(T_c)$ is then obtained from the measured in-plane dc conductivity,

$$\sigma_\parallel(T) = \frac{\omega_p^2\tau(T)}{4\pi} . \tag{59}$$

In principle, the interplane scattering rate $1/\tau_1(T_c)$ can be inferred from the c-axis conductivity using Eq. (38). However, magnetic torque measurements in the mixed state, interpreted with anisotropic London theory,[45] give precise values for the anisotropy of the London penetration depths, $\lambda_\parallel(T)/\lambda_\perp(T)$. We use this parameter to determine the interlayer scattering rate from Eqs. (50) and (54). The data summarized in Table I are used to determine the microscopic parameters as described above; the results are given in Table II. Other experimental data can be used as a consistency check of the model. In particular, the c-axis conductivity is calculated from Eq. (38) and compares reasonably well with experimental values (see Table IV). Also, the slopes of $H^{\parallel}_{c2}(T)$ and $H^{\perp}_{c2}(T)$ near T_c,

$$-T_c dH^{\perp}_{c2}/dT|_{T_c} = \Phi_0/2\pi\xi_\parallel^2 ,$$
$$-T_c dH^{\parallel}_{c2}/dT|_{T_c} = \Phi_0/2\pi\xi_\perp\xi_\parallel , \tag{60}$$

are determined by the two principal coherence lengths (Table III), which in turn are related to v_f, T_c, and the pair-breaking parameters $\rho = \hbar/2\pi k_B T_c\tau$ and $\rho_1 = \hbar/2\pi k_B T_c \tau_1$ by Eq. (43). Finally, measurements of the in-plane penetration depth are in reasonable agreement with the value of $\lambda_\parallel(0)$ calculated from Eq. (50). The comparison of the experimental and calculated values for the consistency checks is summarized in Table IV. There is overall good agreement with the predictions of the interlayer diffusion model, with discrepancies of at most 50%. Properties associated with the c-axis transport show larger discrepancies than those associated with in-

TABLE III. Derived parameters.

$l_\parallel(T_c)$	$D_\perp(T_c)$	$\xi_\parallel(0)$	$\xi_\perp(0)$	$\lambda_\parallel(0)$	$\lambda_\perp(0)$
49 Å	1.0×10^{-2} cm^2/sec	14 Å	1.8 Å	1.6×10^3 Å	8.8×10^3 Å

TABLE IV. Consistency checks.

	$\varrho_\perp/\varrho_\parallel\|_{T_c}$	$\lambda_\parallel(0)$	$dH_{c2}^\parallel/dT\|_{T_c}$	$dH_{c2}^\perp/dT\|_{T_c}$
Measured	~ 230	$\sim 1.4\times10^3$ Å	$\sim$ -10 T/K	$\sim$ -1.8 T/K
Calculated	300	1.6×10^3 Å	-15 T/K	-2.0 T/K

plane transport. This is probably because the measured c-axis transport properties contain the most uncertainty, and are the most sensitive to differences in material quality. Our feeling is that the remaining discrepancies can be accounted for either by uncertainties in the input data, or by generalizations of the simplest version of the model presented here; for example, anisotropy of the Fermi surface and scattering rates, temperature-dependent lifetimes, Fermi-liquid effects, and more complicated interlayer scattering mechanisms could be incorporated. Generalizations that include unconventional pairing (cf. Ref. 46), as proposed for the CuO superconductors,[29,30] can easily be included.

IV. CONCLUSION

In this paper we have presented the interlayer diffusion model, which is specifically designed for high-T_c cuprate superconductors. The model is based on the quasiclassical theory of Fermi-liquid superconductivity. From solutions to the quasiclassical transport equations for the diffusively coupled layers we calculate the dc conductivity tensor in the normal state, and some basic superconducting properties, such as the coherence lengths parallel and perpendicular to the layers, the penetration depth tensor, and the upper critical fields near T_c. We also show that the interlayer diffusion model reduces to the anisotropic Ginzburg-Landau theory, the Lawrence-Doniach theory, and the anisotropic London theory in the appropriate limits. Our model is consistent with experimental data on Y-Ba-Cu-O.

The theory is most powerful for calculating magnetic properties of superconductors. It can be used, e.g., to calculate the structure and dynamics of the vortex core of a flux line, as well as other properties of layered superconductors which are outside the reach of most other theories. In a forthcoming paper[44] we discuss the structure and energetics of vortex lines in layered superconductors.

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