Effect of fractons and magnons on the resistivity of dilute ferromagnets

Jian-Xin Li
International Centre for Theoretical Physics, Trieste 34100, Italy
*and Department of Physics, Wuhan University, Wuhan 430072, People's Republic of China**

De-Cheng Tian
International Centre for Materials Physics, Academia Sinica, Shenyang 110015, People's Republic of China
and Department of Physics, Wuhan University, Wuhan 430072, People's Republic of China

Qing Jiang
Department of Physics, Wuhan University, Wuhan 430072, People's Republic of China
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Based on the *s-d* exchange model we investigate the temperature-dependent resistivity arising from the scattering of electrons off fractons and magnons in dilute Heisenberg ferromagnets on three-dimensional bond-percolation networks. The results indicate that the contribution of fracton scatterings to the resistivity varies from a $T^{3/2}$ to a T^2 dependence as the concentration of bonds approaches the percolation threshold, while the magnon scattering contributes a resistivity varying as $T^{3/2}$ regardless of the changes in concentration of bonds.

I. INTRODUCTION

The electrical transport properties in randomly diluted magnets are a subject of considerable interest. Spatial disorder in the arrangement of magnetic and nonmagnetic atoms leads to fluctuations of exchange energy, and consequently to a drastic change in the spectrum of magnetic excitations. In this case, one may expect contributions to the electrical resistivity from spin-dependent scattering between electrons and magnetic excitations in addition to the effects due to structural disorder.

Many attempts have been made to find the contribution to the resistivity due to the scattering of electrons off magnetic excitations in dilute ferromagnets.[1,2] The temperature dependences of the resistivity of the magnetic origin are found to fall into two categories, both in theoretical and experimental investigations: One is a T^2-dependent term,[3,4] the other was found to be a $T^{3/2}$-dependent term.[2,5,6] This difference involves the details of the interaction between electrons and magnetic excitations, which remains an unresolved problem.

For any ferromagnet at sufficiently long wavelengths we may ignore the details of the atomic interactions, and apply linear spin-wave theory at low temperature. We then should expect that the long-wavelength magnetic excitations will be conventional spin waves, and this was confirmed in the initial studies of the spin dynamics. At shorter wavelengths the inherent randomness of dilute magnets will manifest itself, and this randomness qualitatively changes the nature of the magnetic excitations in this regime. As we know, the magnetic diluted systems may be considered as the bond-percolation networks. Percolation networks appear to be homogeneous at length scales L longer than the percolation correlation length ξ, and thus support propagating phonons or magnons. For shorter length scales $a < L < \xi$ (a is the lattice constant), the random networks exhibit fractal characteristics so that one would expect localized excitations called fractons.[7,8] The theoretical investigation of the low-energy excitation spectrum for magnetic excitations on a percolation ferromagnet showed a crossover from magnons at lower frequencies to fractons at higher frequencies.[9] The experimental evidence of the magnon-fracton crossover has also been found by Uemura and Birgeneau[10,11] in the dilute antiferromagnet $Mn_xZn_{1-x}F_2$. Based on this picture, the density of states was derived for magnon and fracton excitations,[11,12] and the results have been employed to explain the deviations from the Bloch $T^{3/2}$ law which has been observed in dilute amorphous magnets[13] and the temperature dependence of normal-state resistivity in high-T_c cuprates.[14]

In this paper, we present a comparison of the contribution from the fracton and magnon scattering to the temperature-dependent resistivity in a dilute three-dimensional ferromagnet. It is our intention to take the effect of the disordered spin coupling into consideration and to understand the details of the interactions between the electrons and magnetic excitations in the randomly diluted ferromagnetic systems.

II. THEORETICAL CALCULATION

We consider a ferromagnetic system with a certain number of randomly distributive bonds. The interaction between conduction electrons and magnetic excitations (magnons and fractons) is assumed to be *s-d* exchange coupling. So, the Hamiltonian takes the following form:

$$H=\sum_{k\mu}\varepsilon_k C^{\dagger}_{k\mu}C_{k\mu}+H_d - \frac{I}{N}\sum_{l}\sum_{kk'}\sum_{\mu\mu'} e^{i(k'-k)\cdot R_l}\mathbf{S}_l\cdot\sigma_{\mu\mu'}C^{\dagger}_{k\mu}C_{k\mu}\,, \tag{1}$$

where $C_{k\mu}^{\dagger}$ and $C_{k\mu}$ are the creation and annihilation operators for electrons with wave vector $\mathbf{k}$ and spin μ, $\mathbf{S}_l$ is the localized spin operator residing on site $\mathbf{R}_l$, $\boldsymbol{\sigma}$ is the Pauli matrix, I is the *s-d* exchange parameter, and N is the number of the lattice sites.

The Heisenberg Hamiltonian H_d is given by,

$$H_d=-\sum_{\langle ij\rangle} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j \ . \tag{2}$$

The random distribution of the bonds is expressed by the effective exchange-coupling constant J_{ij}, which follows the probability density,

$$P(J_{ij})=p\,\delta(J_{ij}-J)+(1-p)\delta(J_{ij}) \ , \tag{3}$$

and the concentration of the ferromagnetic bonds is p.

The one-particle retarded Green's function for electrons is defined as follows:

$$G(\mathbf{k},E)=\langle\langle C_{k\uparrow}|C_{k\uparrow}^{\dagger}\rangle\rangle=[E-\varepsilon_k-\Sigma(\mathbf{k},E)]^{-1} \ . \tag{4}$$

For the second order in I, we obtain the self-energy $\Sigma(\mathbf{k},E)$ for electrons,

$$\Sigma(\mathbf{k},E)=-IS+\frac{I^2}{N^2}\sum_{ll'}\sum_{k'k''}\sum_{\mu\mu'} e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_l}e^{i(\mathbf{k}-\mathbf{k}'')\cdot\mathbf{R}_{l'}}\langle\langle S_l\cdot\boldsymbol{\sigma}_{\uparrow\mu'}C_{k'\mu'}|S_{l'}\cdot\boldsymbol{\sigma}_{\mu\uparrow}C_{k''\mu}^{\dagger}\rangle\rangle \ . \tag{5}$$

We introduce the Dyson-Maleev transformation for the Heisenberg ferromagnet Hamiltonian H_d,

$$\begin{aligned} &S_l^z=a_l^{\dagger}a_l-S, \quad S_l^{\dagger}=\sqrt{2S}\,a_l^{\dagger}\left[1-\frac{a_l^{\dagger}a_l}{2S}\right] , \\ &S_l^{-}=\sqrt{2S}\,a_l \ . \end{aligned} \tag{6}$$

Then, the Heisenberg Hamiltonian Eq. (2) is changed into

$$\begin{aligned} H_d=-\sum_{\langle ij\rangle} J_{ij}[&S^2+S(a_i^{\dagger}a_j+a_ia_j^{\dagger}-2a_i^{\dagger}a_i) \\ &+\tfrac{1}{2}a_i^{\dagger}(a_ia_j^{\dagger}-a_i^{\dagger}a_i)a_j \\ &+\tfrac{1}{2}(a_i^{\dagger}a_i-a_ia_j^{\dagger})a_j^{\dagger}a_j] \ . \end{aligned} \tag{7}$$

The expression for the self-energy equation (5) contains two Green's functions,

$$\langle\langle S_l^{-}C_{k'\downarrow}|\mathbf{S}_{l'}\cdot\boldsymbol{\sigma}_{\mu\uparrow}C_{k''\mu}^{\dagger}\rangle\rangle\equiv G_{ll'}(E)$$

and

$$\langle\langle S_l^{z}C_{k'\uparrow}|\mathbf{S}_{l'}\cdot\boldsymbol{\sigma}_{\mu\uparrow}C_{k''\mu}^{\dagger}\rangle\rangle\equiv P_{ll'}(E) \ .$$

Their equation of motion can be calculated as follows:

$$(E-\varepsilon_{k'})G_{ll'}(E)=\langle 2S_l^zC_{k''\downarrow}^{\dagger}C_{k'\downarrow}\delta_{ll'}+S_l^{-}S_{l'}^{\dagger}\delta_{k'k''}-S_l^{-}C_{k''\uparrow}^{\dagger}C_{k'\downarrow}\delta_{ll'}\rangle+2\sum_j J_{lj}[(S-\langle a_j^{\dagger}a_j\rangle)G_{ll'}(E)-(S-\langle a_l^{\dagger}a_l\rangle)G_{jl'}(E)] \tag{8}$$

and

$$(E-\varepsilon_{k'})P_{ll'}(E)=\langle S_l^zS_{l'}^z\delta_{k'k''}-S_l^{\dagger}C_{k''\downarrow}^{\dagger}C_{k'\uparrow}\delta_{ll'}\rangle+\sum_j J_{lj}\langle a_l^{\dagger}a_j\rangle[P_{ll'}(E)-P_{jl'}(E)] \ . \tag{9}$$

In order to obtain Eqs. (8) and (9), we have used the decoupling approximations,

$$a_l^{\dagger}a_l^{\dagger}a_la_j\approx\langle a_l^{\dagger}a_j\rangle a_l^{\dagger}a_l$$

and

$$a_j^{\dagger}a_j^{\dagger}a_ja_l\approx\langle a_j^{\dagger}a_l\rangle a_j^{\dagger}a_j \ .$$

It is obvious that we have

$$\langle a_l^{\dagger}a_j\rangle=\langle a_j^{\dagger}a_l\rangle \ .$$

In order to proceed with the calculation, we introduce the Fourier transformation for operators $a_l^{\dagger}$ and a_l,

$$\begin{aligned} a_l&=N^{-1/2}\sum_q e^{i\mathbf{q}\cdot\mathbf{r}_l}b_q \ , \\ a_l^{\dagger}&=N^{-1/2}\sum_q e^{-i\mathbf{q}\cdot\mathbf{r}_l}b_q^{\dagger} \ . \end{aligned} \tag{10}$$

Equations (8) and (9) are in the site representation, and their Fourier transforms are not simple because of the presence of the randomly distributive bonds. We treat them in the effective-medium approximation[15,16] in which J_{ij} is replaced by a uniform coupling constant $\bar{J}(E)$; $\bar{J}(E)$ is chosen in such a way that the scattering produced by one bond, for which the original coupling constant J is maintained, is zero on the average. According to Ref. 15, we obtain $\bar{J}(E)$ for a cubic lattice,

$$\bar{J}(E)=\frac{3J(p-p_c)-EG_0(\epsilon)+\{3J(p-p_c)-EG_0(\epsilon)]^2+8EG_0(\epsilon)J\}^{1/2}}{4} \ , \tag{11}$$

with

$$G_0(\epsilon)=\int_0^\infty \exp[-(Z/2+\epsilon)x][I_0(x)]^2dx \ ,$$

where $\epsilon=E/\bar{J}(E)$, p_c is the percolation threshold, and $I_0(x)$ is the modified Bessel function of order 0. Thus, we can take the Fourier transformation of Eqs. (8) and (9). Consequently, these two equations of motion are changed into

$$(E-\varepsilon_{k'})G(q,E)=\{2[F(0)-S]f_{k'}+2S(1+n_q)\}\delta_{k'k''} +2Z\bar{J}(q,E)(1-\gamma_q)[S-F(0)]G(q,E) \ , \quad (12)$$

with

$$F(0)=\langle a_l^\dagger a_l\rangle=\langle a_0^\dagger a_0\rangle=\frac{1}{N}\sum_q n_q \ ,$$

$$\gamma_q=\frac{1}{Z}\sum_\delta e^{i\mathbf{q}\cdot\delta} \ ,$$

and

$$(E-\varepsilon_{k'})P(q,E)=NS[S-2F(0)]\delta_{q,0}\delta_{k'k''} +Z\bar{J}(q,E)[I(0)-I(q)]P(q,E) \ , \quad (13)$$

with

$$I(0)=\frac{1}{N}\sum_q \gamma_q n_q, \quad I(q)=\frac{1}{N}\sum_{q_1}\gamma_{q+q_1}n_{q_1} \ .$$

From Eq. (5), through Eqs. (12) and (13), we find the self-energy for electrons,

$$\Sigma(\mathbf{k},E)=IS+\frac{I^2}{N}\sum_q \frac{\{2[F(0)-S]f_{k-q}+2S(1+n_q)\}}{E-\varepsilon_{k-q}+2Z\bar{J}(q,E)(1-\gamma_q)[F(0)-S]}+I^2S\frac{[S-2F(0)]}{E-\varepsilon_k} \ . \quad (14)$$

The relaxation rate τ_k^{-1} of a carrier with momentum $\mathbf{k}$ is given by

$$\tau_k^{-1}=-2\,\mathrm{Im}\Sigma(\mathbf{k},E+i\delta) \ . \quad (15)$$

So, we finally obtain

$$\tau_k^{-1}=\frac{2\pi I^2}{N}\sum_q\{2[F(0)-S]f_{k-q}+2S(1+n_q)\}\delta\{E-\varepsilon_{k-q}+2Z\bar{J}(q,E)(1-\gamma_q)[F(0)-S]\} +2\pi I^2S[S-2F(0)]\delta(E-\varepsilon_k) \ . \quad (16)$$

After making use of the Drude formula, we arrive at the following expression for the resistivity arising from the scattering of electrons by magnetic excitations,

$$\rho=m^*/ne^2\tau \ , \quad (17)$$

where m^* is the effective electron mass, n is the concentration of the conduction electrons, and τ^{-1} is given by

$$\tau^{-1}=\int \tau_k^{-1}\mathcal{N}(\varepsilon_F)d\varepsilon_k \ , \quad (18)$$

with $\mathcal{N}(\varepsilon_F)$ the density of states of the electrons on the Fermi surface.

The first term of the expression (16) is the relaxation rate of electrons scattering with spin flip, it is due to the coherent scattering of electrons by spin waves. In the second term of this expression, the transferred momentum is not coupled to the magnons' momentum, i.e., in the scattering process electron momentum is not conserved, so it contributes an incoherent scattering to electrons. At the same time this term does not include any factor p [the $\bar{J}(q,E)$ contains p in the first term], i.e., incoherent scattering is more essential than coherent scattering.

III. NUMERICAL STUDY

For the dilute ferromagnet, dynamics can be introduced by making an assumption concerning the length dependence of the exchange-coupling constant as follows;[17]

$$J_{ij}\sim r^{-\theta}, \quad r<\xi_p \ , \qquad \sim \text{const}, \quad r>\xi_p \ . \quad (19)$$

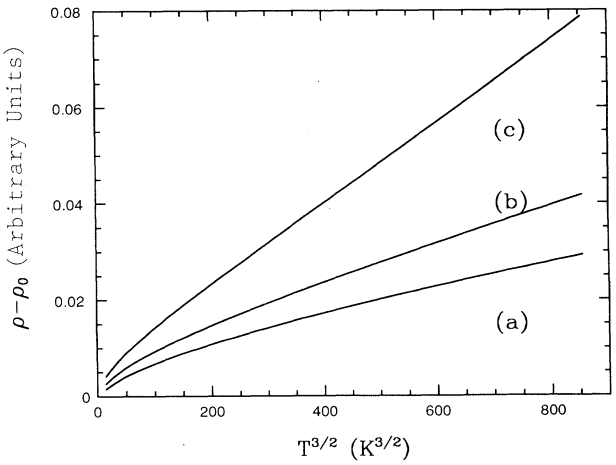

FIG. 1. Resistivity vs $\frac{3}{2}$ power of the temperature, which arises from the scattering of electrons by fractons. The Fermi energy of the conduction electrons takes the value of 5.0 eV. Curves (a), (b), and (c) correspond to $p-p_c=0.1$, 0.07, and 0.04.

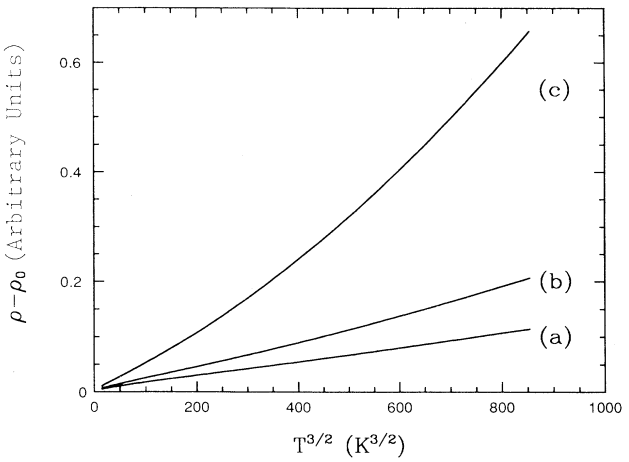

FIG. 2. Resistivity vs $\frac{3}{2}$ power of the temperature, which arises from the scattering of electrons by fractons. The Fermi energy of the conduction electrons takes the value of 5.0 eV. Curves (a), (b), and (c) correspond to $p-p_c=0.03$, 0.02, and 0.01.

The exponent θ was first introduced in the context of anomalous diffusion by Gefen, Aharony, and Alexander.[18] From the δ function of Eq. (16), we can see that the excitation energy of the magnetic excitations (fractons and magnons), which involves the scattering processes, is given by $2Z\bar{J}(q,E)(1-\gamma_q)[F(0)-S]$. Thus, the excitation energy of magnons has the form

$$\hbar\omega=2\bar{J}(E)[F(0)-S]q^2a^2 , \tag{20}$$

and the excitation energy of fractons has a different form according to Eq. (19),

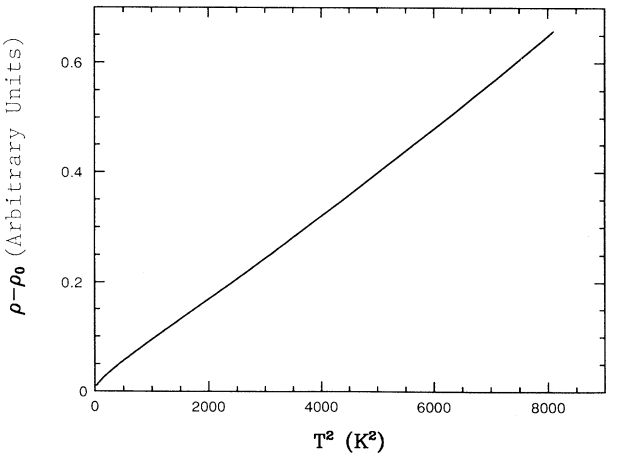

FIG. 3. Resistivity vs square temperature at $p-p_c=0.01$, which arises from the scattering of electrons by fractons. The Fermi energy of the conduction electrons takes the value of 5.0 eV.

$$\hbar\omega=2\bar{J}(E)[F(0)-S](qa)^{2+\theta} , \tag{21}$$

where we have used the approximation $1-\gamma_q\approx q^2a^2/Z$ for the cubic lattice. Following the conjecture of Alexander and Orbach[7] we define the fracton dimension $\bar{d}$ as $\bar{d}=2D/(2+\theta)$, with D the fractal dimension of the network. It is expected that $\bar{d}$ and D are equal to 1.42 and 2.5, respectively, for a three-dimensional percolation network.[7]

Correspondingly, the magnon density of states is given by

$$N(\omega)\sim\omega^{d-1}, \quad (\omega<\omega_c) , \tag{22}$$

where d is the Euclidean dimensionality, and ω_c is the crossover frequency of magnons and fractons which is related to the percolation correlation length ξ, by $\omega_c\sim\xi^{-1}$.

For frequencies above ω_c, the density of states of fractons is given by

$$N(\omega)\sim\omega^{\bar{d}-1} \ (\omega>\omega_c) . \tag{23}$$

As a result, the sum over q in Eq. (16) can be transformed into an integral according to

$$\frac{1}{N}\sum_q=\frac{a^3}{2\pi^2}\int_0^{q_m}q^2dq , \tag{24}$$

for the magnon excitation, and

$$\frac{1}{N}\sum_q=\frac{2K_{D-1}a^D}{(2\pi)^D}\int_{1/\xi}^{1/a}q^{D-1}dq\int_0^{\pi}(\sin\theta)^{D-2}d\theta , \tag{25}$$

for the fracton excitations, where $K_D=\pi^{D/2}\Gamma(D/2)$.

The percolation correlation length ξ exhibits the relation[19]

$$\xi=a|p-p_c|^{-0.80} . \tag{26}$$

By using Eqs. (16), (17), and (18), we can calculate the temperature dependences of resistivity due to the scattering of electrons by magnetic excitations (magnons and fractons). The results are presented in Figs. 1–4. In Fig. 3, the resistivity versus square temperature from fracton scattering at $p-p_c=0.01$ are plotted. It is shown in Figs. 1, 2, and 3 that as the concentration p of the ferromagnetic bonds approaches the percolation threshold p_c, the contribution from fracton scatterings to the resistivity varies from a $T^{3/2}$ to a T^2 term. However, the contribution from magnon scatterings changes very slightly, which always contributes a $T^{3/2}$ temperature dependence to the resistivity as shown in Fig. 4. Thus, in a dilute ferromagnet the resistivity, due to the scattering by magnons, is proportional to $T^{3/2}$, rather than to T^2 as in an ordered ferromagnet. It coincides with the theoretical result derived by Richter, Wolf, and Goedshe.[6]

From Eq. (26) we can see that the percolation correlation length ξ increases as p approaches p_c. As has been argued above, on length scales smaller than ξ both the finite clusters and the infinite clusters are self-similar or exhibit the fractal characteristics. So one would expect

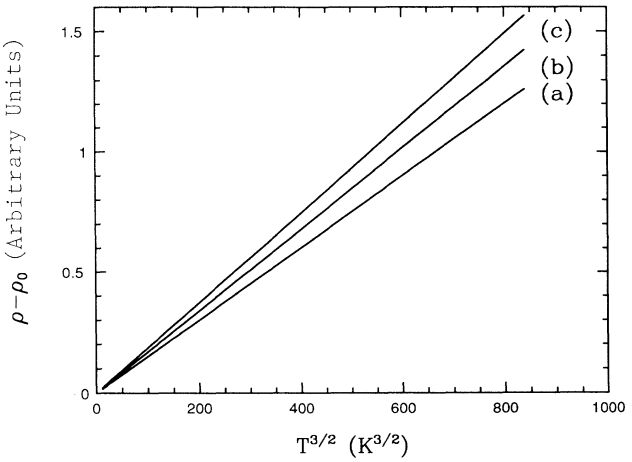

FIG. 4. Resistivity vs $\frac{3}{2}$ power of the temperature, which arises from the scattering of electrons by magnons. The Fermi energy of the conduction electrons takes the value of 5.0 eV. Curves (a), (b), and (c) correspond to $p-p_c=0.1$, 0.05, and 0.01.

that the relative weight of the scattering of electrons by fractons will become more and more large, and the T^2-dependent term in the resistivity will dominate when $p-p_c$ is smaller enough. When the concentration p deviates from the threshold p_c ($p>p_c$), the correlation length becomes shorter and shorter, and on a longer length scale, the system is homogeneous. Therefore, the weight of the scattering to electrons from magnons increases with the deviation of p from p_c. Meanwhile, the contribution from fracton scattering to resistivity also changes from T^2 dependence to $T^{3/2}$ ones. We may therefore conclude that the T^2-dependent term of resistivity only appears when the concentration of ferromagnetic bonds approaches the percolation threshold p_c, otherwise, the contribution of magnetic origin to the resistivity has $T^{3/2}$-dependent term.

We point out that at sufficiently low temperatures, a minimum is observed in the $\rho(T)$ curve for the majority of metallic glasses which can be described based on the percolation method.[20] Below this minimum, the resistivity increases as the temperature is lowered. This phenomena is believed to be dependent on the competition between several scattering mechanisms in these materials,[20,21] and is beyond the scope of this paper.

IV. CONCLUSION

We have adapted the fracton model to discuss the temperature dependence of resistivity arising from the spin-dependent scattering of electrons and magnetic excitations in a dilute three-dimensional ferromagnet based on the bond-percolation network. The dynamics is introduced when each ferromagnetic bond is assigned a scaling law [Eq. (19)]. The appearance of fractal structure at short length scale induces a crossover from magnon to fracton dynamics at the percolation correlation length ξ.

When the concentration of bond p approaches the percolation threshold p_c, we found that the contribution to resistivity from the scattering of electrons by fractons varies from a $T^{3/2}$-dependent term to a T^2-dependent term, while the scattering of electrons by magnons contributes a $T^{3/2}$-dependent term regardless of the changes of $p-p_c$. It suggests that near the percolation threshold, one would expect a T^2-dependent contribution of magnetic origin to the resistivity, which arises from the scattering of electrons off fractons. Otherwise, it would make a contribution to the resistivity proportional to $T^{3/2}$, which results from both the magnon and fracton scatterings.

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*Permanent address.
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