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# Universal behavior in heavy fermions

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We analyze several experimental results, obtained at different pressures, on three heavy-fermion systems using a quantum scaling approach. In spite of the distinct ground states attained by $CeRu_2Si_2$, $UPt_3$, and $CeAl_3$, the crossover into the dense Kondo regime and the renormalized Fermi-liquid state of these materials in this regime, below the coherence line, is shown to exhibit universal behavior. This is associated with the proximity of these systems to a zero-temperature instability.

The materials which are known as heavy fermions have as a distinct feature the presence of elements with unstable $f$ shells such as Ce and U. These systems present a rich variety of physical behavior as evidenced by the different ground states they can attain. Superconductor, magnetic, or simply a Fermi liquid are possible low-temperature states for these materials.[1] Recently an insulating ground state has been found as still another possibility.[2] In spite of this diversity, it is the purpose of this paper to show the existence of an underlying universality in the physics of heavy fermions. This is done by analyzing several sets of experimental data, obtained as a function of pressure, on three different materials: the superconductor $UPt_3$, the Fermi liquid $CeRu_2Si_2$, and the antiferromagnetic[3] system $CeAl_3$. The analysis is carried out within a scaling theory recently proposed.[4,5] The theory relies on the existence of a zero-temperature phase transition associated with an unstable fixed point which controls the physics of heavy fermions and gives rise to the universal behavior our study exposes.

The scaling theory has been formulated within the context of the Kondo lattice model which has as an essential ingredient the competition between long-range order and Kondo effect.[6] The parameters of this model are the exchange interaction $J$ between the magnetic moments of the $f$ and conduction electrons and the bandwidth $W$ of the latter. At zero temperature, above a lower critical dimension[6] (probably $d_L=2$), for a critical ratio $(J/W)_c$ of these parameters, there is a quantum phase transition between a state with long-range magnetic order $[J/W<(J/W)_c]$ and a nonmagnetic state where the Kondo effect prevails.[4]

The scaling expressions of the relevant physical quantities close to the zero temperature unstable fixed point at $(J/W)_c$ are given by[5]

$$\begin{aligned} &f \propto |j|^{2-\alpha} f_F[T/T_c, H/H_c]\ , \\ &\chi_s \propto |j|^{-\gamma} f_s[T/T_c, H/H_c]\ , \\ &m_T \propto \gamma \propto C/T \propto |j|^{2-\alpha-2\nu z} f_c[T/T_c, H/H_c]\ , \\ &\tau \propto |j|^{-\nu z} f_t[T/T_c, H/H_c]\ , \\ &\xi \propto |j|^{-\nu} f_L[T/T_c, H/H_c]\ , \end{aligned} \tag{1}$$

where $f, \chi_s, m_T$ stand for the singular part of the free-energy density, the order-parameter susceptibility, and the thermal mass obtained from the linear term of the specific heat respectively. $\tau$ is the characteristic relaxation time which governs the critical slowing down and $\xi$ the correlation length measured by neutron scattering. Also $T_c=|j|^{\nu z}$, $H_c=|j|^{\Delta}$, and $j=(J/W)-(J/W)_c$ measures the distance to the critical point in parameter space. Since heavy fermions are generally close to an antiferromagnetic instability,[1] the order parameter is the staggered magnetization $m_s$ which at zero temperature scales as $m_s \propto |j|^{\beta}$. Consequently in the equations above, $H$ is a staggered field conjugated to the order parameter and $\chi_s$ a staggered susceptibility. The exponents $\alpha$, $\Delta=\beta+\gamma$, $\nu$ and $z$ are associated with the zero temperature fixed point and obey standard scaling relations like $\alpha+2\beta+\gamma=2$. However because we are dealing with a zero-temperature instability the hyperscaling relation is modified[4,5] and is given by $2-\alpha=\nu(d+z)$.

In the argument of the scaling functions temperature appears scaled by the characteristic temperature $T_c=|j|^{\nu z}$. Since the scaling functions have in general different asymptotic behavior for $(T/T_c)\gg 1$ and $(T/T_c)\ll 1$, the line $T_c=|j|^{\nu z}$, in the noncritical part of the $T/W$ vs $J/W$ phase diagram, represents a crossover line between different regimes. This line has been identified[4,5] as the coherence line which marks the onset with decreasing temperature of the *dense Kondo regime* in heavy fermions. This dense Kondo state is a renormalized Fermi liquid with enhanced susceptibility and thermal mass. The coherence temperature $T_c$ defines the characteristic energy scale of the lattice problem and is much lower than the single-ion Kondo temperature $T_K$ (see Fig. 1). In addition, the collective nature of the coherence line is clearly indicated by the fact that, according to the generalized scaling hypothesis,[7] the critical Néel line in the critical region of the phase diagram $(J<J_c)$ is governed by the exponent of the crossover line, i.e., $T_N \propto |j|^{\nu z}$. The Néel line in Fig. 1 gives the Néel temperature $T_N$ for a given ratio $(J/W)$ smaller than the critical one.

Since for $T\ll T_c$, i.e., below the coherence line, the system attains a Fermi-liquid regime as evidenced by experiments, the scaling functions $f(T/T_c)$ in Eq. (1) have Sommerfeld-like expansions in this region of the phase diagram, i.e., $f(T/T_c\ll 1)\cong 1+a(T/T_c)^2+b(T/T_c)^4+\dots$ Using such an expansion for the free energy, we have obtained[4,6] the scaling expression of the thermal mass $m_T$, defined as the coefficient of the linear term of the specific heat, which is given in Eq. (1). This thermal mass is meaningful only for $T\ll T_c$ when the system has reached a truly Fermi-liquid regime. Notice that if

 

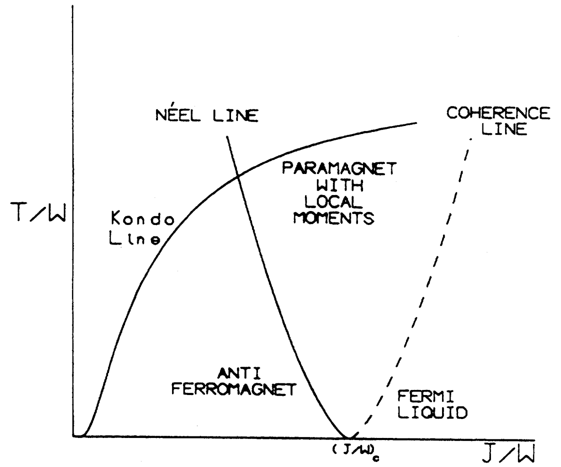


FIG. 1. The phase diagram of the Kondo lattice. The coherence line provides a new energy scale for the lattice which is lower than the single-ion Kondo temperature.

$2-\alpha-2\nu z<0$, the thermal mass will be enhanced as a consequence of the proximity to the magnetic instability. A similar enhancement occurs for the limiting Pauli-like uniform susceptibility as we discuss below.

The scaling form of the uniform magnetic-field-dependent free-energy density is given by[5]

$$f \propto |j|^{2-\alpha} f_h[(h/J)/|j|^{\phi_h}] , \tag{2}$$

where $\phi_h=\nu(\sigma+z)$ and $\sigma$ the exponent which renormalizes the uniform magnetic field $h$ close to the $T=0$ fixed point at $(J/W)_c(h'=b^{\sigma}h)$. The uniform susceptibility $\chi_h$ is given by

$$\chi_h=-\partial^2 f/\partial h^2 \propto |j|^{2-\alpha-2\phi_h} f_c[T/T_c, h/h_c]$$

with $h_c \propto |j|^{\phi_h}$. The temperature independent or Pauli-like behavior of the uniform low-field susceptibility $\chi_0$ for $T \ll T_c$ implies that in the equation above $f_c(T/T_c \ll 1)=\text{const}$ such that $\chi_0 \propto |j|^{2-\alpha-2\phi_h}$. For $2-\alpha-2\phi_h<0$ the uniform susceptibility is enhanced as found experimentally.[1] On the other hand, local moment or Curie-Weiss behavior for $T \gg T_c$ implies the following asymptotic behavior for the susceptibility scaling function: $f_c(T/T_c \gg 1) \propto (T/T_c)^{-1}$ such that $\chi_0 \propto |j|^{2-\alpha-2\phi_h+\nu z}/T$ in this limit. A useful expression for the uniform susceptibility, which interpolates between the two regimes is $\chi_0 \propto \mu^2/(T+T_c)$ where the effective moment $\mu^2 \propto |j|^{2-\alpha-2\phi_h+\nu z}$ and $T_c=|j|^{\nu z}$.

In Ref. 5 it was shown that the logarithm of the normalized ratio of a physical quantity $X \propto |j|^{-x}$, as a function of pressure $P$, can be expanded as ($P \approx P_0$):

$$\ln[X(P)/X(P_0)] \approx -x(\kappa_0\Gamma)(P-P_0) , \tag{3}$$

where $\kappa_0$ is the compressibility and $\Gamma$ a property independent Gruneisen parameter defined by $\Gamma=|\partial \ln j/\partial \ln V|$ such that $\Gamma_x=-x\Gamma$ $(x>0)$. The reference pressure $P_0$ is generally taken as zero. Then for small applied pressures the logarithm of a given quantity $X$, normalized by its equilibrium (zero pressure) value, varies linearly with pressure, with a coefficient that is directly related to the exponent $x$ characterizing its critical behavior.[5] Furthermore, the expression above shows that by comparing the pressure ratios of different physical quantities, for the same material, we can obtain relations between the exponents governing their criticality.

In Fig. 2 we show in a semilog plot the ratio of several physical quantities measured in the system $CeRu_2Si_2$, as a function of pressure, for small pressures[5] ($P_0 \cong 0$, $P \leq 8$ kbars). The expected linear behavior given by Eq. (3) is observed in every case. Furthermore, all the straight lines have the same inclination implying that the physical quantities shown there are governed by exponents which assume the same numerical values within experimental accuracy. The quantities whose pressure variations are shown in this figure are the following: (i) $T_c(P)/T_c(0)$, where $T_c$ is the coherence temperature defined by the maximum of the uniform susceptibility[8] as a function of temperature. (ii) $[A(P)/A(0)]^{1/2}$ where $A$ is the coefficient of the $T^2$ term of the low-temperature resistivity ($T \ll T_c$) and defined by $\rho=\rho_0+AT^2$ where $\rho_0$ is the residual resistivity.[8,9] For a Fermi liquid the relation $A \propto T_c^{-2}$ is expected. (iii) $h_c(P)/h_c(0)$, where $h_c$ is the characteristic uniform magnetic field at which the uniform differential susceptibility $\chi_h=-\partial^2 f/\partial h^2=dm/dh$ has a maximum for a fixed temperature $T \ll T_c$. This maximum at $h_c$ is associated[10] with a "metamagneticlike transition" which shows the existence of strong antiferromagnetic correlations in the Fermi liquid. (iv) $\chi_h(h=h_c, T \cong 0, P=0)/\chi_h(h=h_c, T \cong 0, P)$, where $\chi_h(h=h_c, T \cong 0)$ is the value of the uniform differential susceptibility[10] $\chi_h=dm/dh$, at the characteristic field $h_c$ for $T \ll T_c$. (v) $\chi_0(h \cong 0, T=T_c, P=0)/\chi_0(h \cong 0, T=T_c, P)$, where $\chi_0(h \cong 0, T=T_c)$ is the value of the uniform low-field susceptibility at the coherence temperature[8] for $h \ll h_c$.

The fact that the data of (i) and (ii) fall on the same line confirms the Fermi-liquid nature of the state attained below $T_c$ since in this case the relation $A \propto T_c^{-2}$ is expected from the scaling form of the resistivity $\rho=\rho(T/T_c)$. The data of (i) and (iii) falling on the same line imply the important exponent equality $\phi_h=\nu z$ since the characteristic uniform field $h_c$ and the coherence temperature $T_c$ shift with pressure at the same rate.

Finally the scaling form of the uniform susceptibility $\chi_h \propto |j|^{z-\alpha-2\phi_h} f(T/T_c, h/h_c)$, the results of (iv) and (v) together with the data of (iii) and (i) falling on a unique line, imply the exponents relations $2-\alpha=\nu z$ and furthermore $\nu z=\phi_h$.

A similar analysis[11] of the data for $CeAl_3$ (Refs. 12–14) shown in Fig. 2, where now the ratio $[m_T(P_0)/m_T(P)]$ is also plotted, leads to the relation $2-\alpha=\nu z$, using the Fermi-liquid assumption $A \propto T_c^{-2}$. For this material the reference pressure was taken as $P_0=1.2$ kbars to avoid complications with a possible residual antiferromagnetism[3,12] and guarantee that the system is in the noncritical part of the phase diagram ($J>J_c$).

An inspection of the data for $UPt_3$ shown in Fig. 2 (Refs. 15–17), obtained in the Fermi-liquid regime but above the superconducting transition temperature, confirms the equality $2-\alpha=\nu z$, since $m_T(P)$ and $T_c(P)$ are now available, verifies the Fermi-liquid relation $A \propto T_c^{-2}$ and implies $\phi_h=\nu z$.

We can extract some consequences of the empirical relations $2-\alpha=\nu z=\phi_h$, implied by the results contained in Fig. 2, which are independent of the particular values of these exponents (weak universality). These are the following:

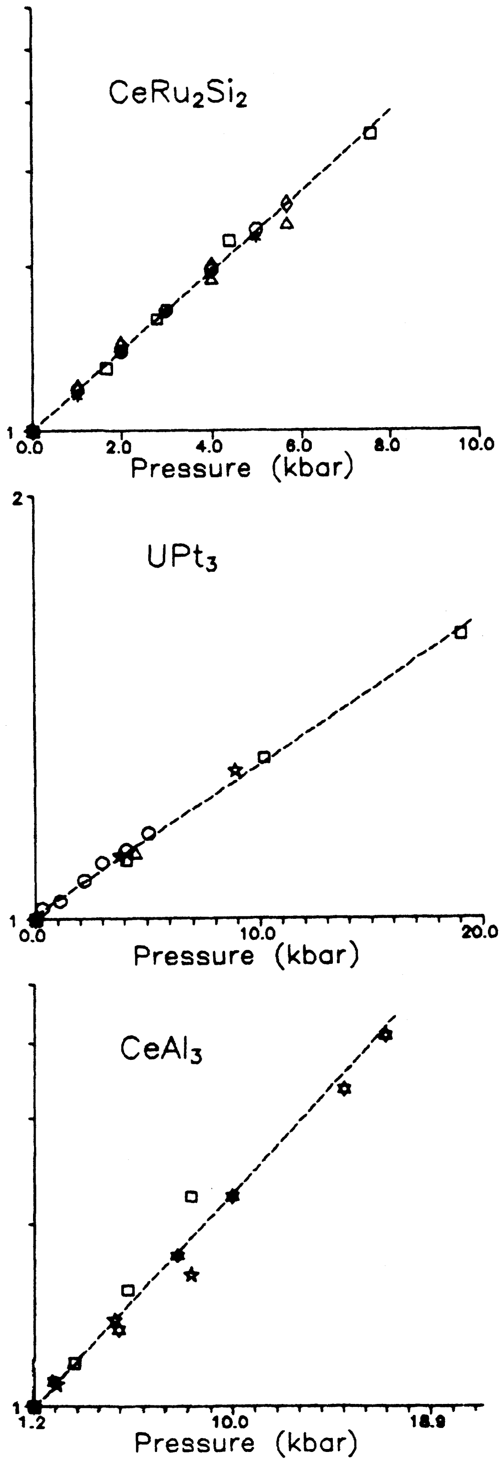


FIG. 2. Semilogarithmic plot of the pressure ratio of several physical quantities for three heavy fermion systems: (○) $h_c(P)/h_c(0)$, (✱) $\chi_h(h_c,0)/\chi_h(h_c,P)$ for $T \ll T_c$, (□) $[A(0)/A(P)]^{1/2}$, (△) $T_c(P)/T_c(0)$, (◇) $\chi_0(T_c,0)/\chi_0(T_c,P)$ for $h \ll h_c$, (★) $m_T(0)/m_T(P)$, (✡) $[b(0)/b(P)]^{3/5}$, see text. For $CeAl_3$ the reference pressure is 1.2 kbar. Dashed lines are a guide to the eye.

(i) The Wilson ratio $\chi_0(T \ll T_c, h \cong 0)/m_T$ turns out to be a constant independent of pressure (this is due to the equality $\phi_h=\nu z$). For the systems studied here the specific heat and the uniform susceptibility $\chi_0$ have not, both, been measured as a function of pressure, for the same material, to confirm this prediction. However, limited data available for $CeAl_3$ is in agreement with this result (see Refs. 11 and 14).

(ii) The empirical Kadowaki-Woods–type of relation,[18] $A \propto \gamma^2$ or $A/\gamma^2=\text{const}$, where $A$ is the coefficient of the $T^2$ term of the resistivity and $\gamma \propto m_T$ the coefficient of the linear term of the specific heat, is directly obtained from the Fermi-liquid relation $A \propto T_c^{-2}$ and the equality $2-\alpha=\nu z$.

(iii) The scaling expression for the uniform magnetization given by $m \propto \partial f/\partial h \propto |j|^{z-\alpha-\phi_h} f_m(T/T_c, h/h_c)$ and the equality $2-\alpha=\phi_h$ implies that the "metamagnetlike transition" at $h=h_c$ and $T \ll T_c$ occurs always at the same fixed value of the magnetization independent of pressure, i.e., $m=f_m(h/h_c)=\text{const}$ for $h=h_c(P)$. This has indeed been observed[8] for $CeRu_2Si_2$.

(iv) The amplitude of the magnetic moment obtained from the Curie-Weiss form of the *high temperature susceptibility* $(T \gg T_c)$ scales as $\mu^2=|j|^{2-\alpha-2\phi_h+\nu z}$ as we have shown before. Consequently the relation $2-\alpha=\nu z=\phi_h$ implies that $\mu$ does not renormalize as $|j|$ varies. This can be appreciated in Table I where in spite of the strong renormalization of $\chi_0(T \ll T_c)$ the effective moments obtained from the high-temperature susceptibilities $(T \gg T_c)$ are nearly the same for the materials shown.

The empirical relation $2-\alpha=\nu z$, which holds for the three heavy fermion systems studied above, shows that the exponents associated with the zero temperature unstable fixed point of the Kondo lattice violate the modified hyperscaling relation $2-\alpha=\nu(d+z)$. This in turn suggests that the magnetic phase transition associated with this fixed point occurs above the upper critical dimension $d_c$ for this transition.[5] Notice that $d_{\text{eff}}=d+z$ so that $d_{\text{eff}} \ge d_c$ may be satisfied for reasonable values of the dynamic exponent $z$ in three dimensions.

Although the data in Fig. 2 imposes constraints on the critical exponents they do not univocally determine these exponents. Considering the violation of hyperscaling and the fact that the uniform field acts as an additional relevant field besides the staggered field, we suggested[5] that these exponents assume classical tricritical values ($d_c=3$ for a tricritical point), i.e., $\alpha=\frac{1}{2}, \nu=\frac{1}{2}, \phi_T=\phi_h/\nu z=1$, implying $z=3$ for the

TABLE I. The effective moment obtained from the high-temperature susceptibility $[\chi=\mu^2/3k_B(T-\theta)]$ and the limiting Pauli susceptibility $\chi_0(T\to 0)$, for different heavy fermions. The effective moment for a free trivalent Ce ion is $2.54\mu_B$ (from Ref. 19 and references therein).

| System | $\mu$ ($\mu_B$) | $\chi_0$ ($10^{-3}$ emu mol$^{-1}$) |
|---|---|---|
| $CeCu_6$ | 2.69 | 34 |
| $CeRu_2Si_2$ | 2.44 | 14 |

dynamical critical exponent. This value of $z$ is generally associated with paramagnon excitations[4] and may be also related[20] to the long-range character of the Ruderman-Kittel-Kasuya-Yosida interaction decreasing as $1/r^3$.

Recently Schlott and Elschner[14] performed electron paramagnetic resonance measurements on $CeAl_3$ under pressure using as probe Gd ions in dilute proportions. The experimental very low-temperature linewidth ($T \ll T_c$) has a linear temperature dependence $\Delta H = bT$. It can be easily shown[11,14] that within the experimental conditions, $b \propto \chi_s \tau$ where $\chi_s$ is the staggered susceptibility and $\tau$ the characteristic relaxation time of the critical spin fluctuations of the host material. The scaling properties of the quantity $b$ can be obtained from the fact that $\chi_s \propto |j|^{-\gamma}$ and $\tau \propto |j|^{-\nu z}$. We obtain $b \propto |j|^{-(\gamma+\nu z)}$ and since $T_c \propto |j|^{\nu z}$, we may obtain the relation between the pressure dependence of $b$ and that of the coherence temperature $T_c$ namely[11]

$$[b(P_0)/b(P)]^{(\nu z/\gamma+\nu z)} \propto T_c(P)/T_c(P_0) . \quad (4)$$

The value of the exponents that we argued[5] are associated with the Kondo lattice fixed point imply that the logarithm of $[b(P_0)/b(P)]^{3/5}$ ($\gamma=1$ for a classical tricritical point) has the same inclination as the logarithm of $T_c(P)/T_c(P_0)$ as a function of pressure. This is explicitly verified in Fig. 2. Although this result does not yet unambiguously determine the critical exponents it imposes a further constraint, namely $\nu z=(3/2)\gamma$ and represents a step forward in determining them explicitly.[11]

We have shown that the critical exponents describing the crossover and the behavior of the renormalized Fermi liquid in the dense Kondo state of heavy fermions obey the general relations $2-\alpha=\nu z$ and $\phi_h=\nu z$ (the latter has not been explicitly checked for $CeAl_3$ due to the lack of measurements). These relations hold for the three heavy fermions studied in this paper, revealing a universal behavior of these systems in spite of the different ground states they can attain. This universality has been attributed to the existence of a zero-temperature phase transition, associated with an unstable fixed point, from a state with long-range magnetic order to a dense Kondo state as described by the Kondo lattice Hamiltonian. The exponents we have obtained violate the modified hyperscaling relation implying classical exponents for this $T=0$ phase transition. We point out that the renormalization group and the scaling theory provides the appropriate framework to unify the intense experimental investigations on heavy fermions. Finally, our results suggest that the reason $UPt_3$ becomes superconducting as the temperature is further lowered in the Fermi-liquid regime, in contrast to what happens to $CeRu_2Si_2$ where no instability is observed down to the lowest temperatures, is due to an additional interaction, probably *marginally relevant* since it does not modify the critical behavior, which is not included in the Kondo lattice Hamiltonian and is present in the former system.

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