PHYSICAL REVIEW B VOLUME 47, NUMBER 17 1 MAY 1993-I

Critical exponents for the three-dimensional superfluid–Bose-glass phase transition

Lizeng Zhang
Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996-1200 and Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

Xiao-Qian Wang
Center for Theoretical Studies of Physical Systems, Clark Atlanta University, Atlanta, Georgia 30314
(Received 10 December 1992)

The critical phenomenon of the zero-temperature superfluid–Bose-glass phase transition for hard-core bosons on a three-dimensional disordered lattice is studied using a quantum real-space renormalization-group method. The correlation-length exponent ν and the dynamic exponent z are computed. The critical exponent z is found to be 2.5 for compressible states and 1.3 for incompressible states. The exponent ν is shown to be insensitive to z as that in the two-dimensional case, and has a value roughly equal to 1.

A disordered boson system provides a prototypical example of zero-temperature quantum critical phenomena.[1,2] Experimentally, such a disordered boson system can be realized in liquid He^4 in random media,[3] or in disordered superconductors[4] where Cooper pairs can be modeled as composite bosons. As the amount of disorder is varied, these systems exhibit a continuous phase transition from the superfluid (SF) phase to a disordered [Bose-glass (BG)] phase. Understanding of this quantum critical phenomenon has been a subject of a considerable amount of recent experimental and theoretical studies.

Besides a diverging length scale (the correlation length) ξ, the SF-BG transition is also characterized by a diverging time scale τ. Denoting δ as the distance to the criticality, ξ and τ can be described by the critical exponents ν and z, defined by $\xi \propto \delta^{-\nu}$ and $\tau \propto \xi^z \propto \delta^{-\nu z}$. From a general scaling argument, Fisher *et al.*[2] concluded that for compressible states z is equal to d, the dimensionality of the system, while $z = 1$ for systems with long-range Coulumb interactions in any dimension.[5] The critical exponent ν cannot be deducted directly from the scaling theory, but a rigorous lower bound has been established, i.e., $\nu \geq 2/d$.[6] Unfortunately, a standard field-theoretical renormalization-group (RG) method, which is proven to be a powerful approach in the study of critical phenomena, has been eluded so far from being applied to this SF-BG phase transition. A major difficulty associated with this approach is due to the lack of a proper zero loop (mean field) theory describing the SF-BG transition at finite dimension,[2] upon which perturbation series, such as ϵ expansions, can be developed. For this reason, a different RG formulation, namely, the real-space RG (RSRG) approach, becomes a useful alternative for investigating the critical phenomena in disordered boson systems.

RSRG has been applied previously to one-dimensional (1D) (Refs. 7 and 8), and two-dimensional (2D) systems.[7] In this paper, we apply the RSRG method developed previously by Zhang and Ma[7] to investigate the SF-BG transition in three dimension. Such a study is important not only because of its intrinsic theoretical interest, but also due to its relevance to experiments. Three-dimensional (3D) disordered boson systems are directly realized in liquid He^4 in Vycor, aerogels, xerogels, and other random media.[3] While present experiments are mainly focused on the effect of disorder on the finite temperature superfluid phase transition, refined experiments are hopefully capable of extracting information about the zero-temperature critical point in the near future. It is clearly desirable to perform calculations for critical exponents in 3D which characterize the physical properties of the system in the vicinity of the transition.

In the following we shall briefly outline the RSRG scheme which has been presented in detail in Ref. 7. The RSRG scheme yields, when applied to the 1D systems, no (nontrivial) fixed point, indicating the instability of the superfluid phase against any amount of disorder for hard-core boson systems, in agreement with other 1D RG calculations[9] and exact results.[10] For 2D and 3D systems, it gives a nontrivial fixed point separating the superfluid phase and the disordered phase. In 2D, the critical exponent z was found to be about 1.7 for compressible states and about 0.9 for incompressible states. Since z is close to 2 and 1 in these two cases, and since it increases slowly with increasing block size,[7] this calculation can be viewed as supporting evidence of the scaling prediction by Fisher *et al.*[2] in 2D. The critical exponent ν is found to be quite insensitive to the type of states, and is roughly equal to 1.4.[7] This value of ν satisfies the rigorous lower bound of Chayes *et al.*,[6] and is also consistent with other numerical estimates.[11] The product $z\nu$ for incompressible states may be directly compared with data obtained from experiments on superconductor-insulator phase transitions in homogeneous amorphous films.[4,7] For 3D systems, on the other hand, no systematic computation has yet been performed to our knowledge. Other numerical methods used for 1D and 2D systems, such as exact diagonalization[11] or quantum Monte Carlo simulations,[12] seem quite formidable for 3D disordered systems within the present computational capacity. Thus, the RSRG method provides a unique way to probe the physical properties of the 3D SF-BG transition.

The system under consideration is a lattice model of hard-core bosons with random potential,[1]

$$\mathcal{H} = -t\sum_{\langle i,j\rangle}(b_i^\dagger b_j + \text{H.c.}) + \sum_j (W_j - \mu) b_j^\dagger b_j \ , \qquad (1)$$

where $b_j^\dagger$ and b_j denote boson creation and annihilation operators at lattice site j. $\langle i,j\rangle$ indicates nearest-neighbor summation and W_j is the random on-site potential with (independent) Gaussian distribution. The hard-core constraint is enforced by the requirement that at each site the occupation number $b_j^\dagger b_j$ equals either 0 or 1. While it is clearly a simplification to the realistic systems, this model is believed to have captured the essential physics of the zero-temperature SF-BG phase transition.[1,2]

This hard-core boson model is equivalent to a quantum spin-$\frac{1}{2}$ XY model with transverse random fields,[1]

$$\mathcal{H} = -J\sum_{\langle i,j\rangle}(S_i^x S_j^x + S_i^y S_j^y) - \sum_j h_j S_j^z \ , \qquad (2)$$

via the mapping $S_j^\dagger \leftrightarrow b_j^\dagger$, $S_j^- \leftrightarrow b_j$, $J \leftrightarrow 2t$, and $h_j \leftrightarrow \mu - W_j$. These two equivalent representations of the problem provide a convenient way to study the underlying physics of the system,[7,13] and we will use them alternatively throughout our discussions.

Our real-space RG method consists of the following steps.

(i) Break the lattice into blocks of size n_s.

(ii) Compute the block spin which is given by two low energy eigenstates of the block Hamiltonian. Since $S^z \equiv \sum_j S_j^z$ (corresponding to the particle number $N_p \equiv \sum_j b_j^\dagger b_j$ in the boson language) is a good quantum number, eigenstates of $\mathcal{H}$ are also simultaneously eigenstates of S^z (N_p). As described in Ref. 7, this can be accomplished either by (a) selecting the lowest states of two chosen subspaces of particle number q and $q+1$, or by (b) selecting the two lowest states among the ground states of the block Hamiltonian for each subspace of a definite number of particles. The two states chosen in this way are found to have adjacent particle number q' and $q'+1$. The field acting on the block spin is given by the energy difference between the two states. In general, the block fields will follow a different distribution than that of site fields. We chose to keep track of only the mean $h = \overline{h_i}$ and the variance $\tilde{h} = (\overline{h_i^2} - \overline{h_i}^2)^{1/2}$ of the renormalized field, and thus map it onto a Gaussian.

(iii) Calculate the effective couplings between the block spins, which is given by the nearest-neighbor couplings of the site spins between two adjacent blocks. Due to the presence of disorder, couplings between block spins are also randomized in the RG iterations. However, these block couplings are found to always be positive, and the renormalized system remains unfrustrated. This allows one to approximate the block couplings by its mean, and thus confine the RG iterations within the original parameter space.

(iv) Repeat the RG iteration defined above to find fixed point(s) and compute critical exponents.

Here we compute the exponents z and ν for 3D systems. For computational details, see Ref. 7. In procedure (a), fluctuations in particle number at any given region are one. Since the block size is fixed and it allows only a discrete set number of particles occupying a block, states described by such a procedure are incompressible. Procedure (b), which is valid only at the "particle-hole symmetrical" filling (half boson per site in our case), where density is conserved through statistical fluctuations in the random fields,[7] yields a compressible ground state.

In the present work we used a simple-cubic lattice with block size $n_s = 2\times 2\times 2$. Gaussian random fields were generated numerically and typically an average of 3×10^3 random configurations was performed. In a previous work,[7] calculations for 3D systems were carried out only for the incompressible state with $q = 6$. Here we complete the study for the incompressible states with other q values, and investigate the compressible states as well. Results for fixed points and critical exponents are summarized in Table I. For incompressible states with other q values, they can be obtained through the "particle-hole symmetry."[7]

As anticipated, critical exponents obtained for incompressible states at a different density have roughly the same value, indicating that the same universality class is probed. Similar to that in the 2D case, the critical exponent ν in 3D is shown to be rather insensitive to the procedures adopted, roughly equal to 1.0. The critical exponent z, on the other hand, is quite different for the compressible and incompressible states. The present RSRG calculation yields $z = 2.5$ and 1.3 for the compressible and incompressible states, respectively.

Our results for ν in both compressible and incompressible states satisfy the lower bound of Chayes *et al.*,[6] and provide the first systematic calculation on critical exponents for 3D systems. The general scaling argument suggests that $z = d$ if the system is compressible.[2] For systems with long-range (Coulomb) force, dimensional analysis indicates that $z = 1$ in any spatial dimension.[5] If one assumes that they belong to the same universality class as those incompressible states considered here, this value should be compared with our results obtained through the fixed q procedure. Our RSRG results for z in 3D deviate more from these scaling predictions than those

TABLE I. Results for fixed points and critical exponents.

State	$(h/J)^*$	ν	z
Incompressible ($q = 6$)[a]	1.6	1.0	1.2
Incompressible ($q = 5$)	2.4	1.0	1.3
Incompressible ($q = 4$)	2.8	1.0	1.3
Compressible	6.0	1.1	2.5

[a] Calculated in Ref. 7.

obtained for the 2D case.[7] The value of z deviates from the scaling result by 17% for the compressible states, and by 30% for the incompressible cases. The corresponding deviation for 2D systems is 15 and 10 %, respectively. It is worth remarking that within our RG procedure, the value of z for compressible states differs from that for incompressible states roughly by a factor of 2 in both 2D and 3D cases. While in 2D this is exactly what the scaling argument suggested, it is clearly not the case in 3D. Our computation is limited to the cubic block of size $n_s = 8$, thus we cannot address the question of size effects. The next isotropic block has the size $n_s = 3^3 = 27$ for which the calculation is computationally not feasible for the time being. Thus we are not able to test the direction in which the result would converge with increasing block size. Since the RSRG calculation is expected to approach the exact result as the block size increases,[7] such a finite-size study is highly desirable. On the other hand, from the study of 2D systems[7] one expects that the RSRG estimate for critical exponents itself is rather insensitive to small changes of the block size.

Although the RG procedure (b) for the compressible states is valid only at the particle-hole symmetric point, where particle number is conserved through the statistical fluctuation in random fields, it nevertheless allows one to probe the parameter space away from this special point by studying the flow of the RG iterations. The RG flow diagram is similar to what one gets in 2D with even block size n_s.[7] Again, the (unstable) fixed point at the $h = 0$ axis controls the critical phenomena of the SF-BG phase transition.

Before closing, we briefly comment about the RSRG approach to disordered quantum systems. Since the block states are computed through an *isolated* block Hamiltonian, one would worry about the effect of the (long-range) coherent quantum fluctuations which may not be properly taken into account. A procedure to overcome this difficulty by incorporating different boundary conditions for the block states has been recently suggested, and has been applied to noninteracting fermion systems.[14] While a direct application of such a RG method to a disordered system has not been possible within present computational facilities, we would like to remark that by averaging over random configurations one can partly achieve the goal that one wishes to accomplish by averaging over the boundary conditions. Indeed, it seems quite ironic that our RSRG procedure works precisely in the "strong" disordered regime where the effect of fluctuations can be incorporated through averaging over the randomness, and loses its validity as the pure limit is approached.[7]

In summary, we have studied a hard-core disordered boson system in a 3D cubic lattice using a quantum RSRG method. We have found that there exists a nontrivial fixed point describing the zero-temperature SF-BG phase transition. The critical exponent z for incompressible states is about 1.3, and about 2.5 for compressible states. The exponent ν is insensitive to z, and is roughly equal to 1.0. As in the 2D case, only one universality class is found, and the critical behavior of the zero-temperature SF-BG phase transition is controlled by the particle-hole symmetric fixed point.

We thank M. Ma for discussions. L. Z. acknowledges support by the National Science Foundation under Grant No. DMR-9101542 and by the U.S. Department of Energy through Contract No. DE-AC05-84OR21400 administered by Martin Marietta Energy Systems, Inc. X. Q. W. acknowledges support by the National Science Foundation under Grant No. HRD-9154077, by the Minnesota Supercomputer Institute, and by the Pittsburgh Supercomputing Center.

[1] M. Ma, B.I. Halperin, and P.A. Lee, Phys. Rev. B **34**, 3136 (1986).

[2] M.P.A. Fisher, P.B. Weichman, G. Grinstein, and D.S. Fisher, Phys. Rev. B **40**, 546 (1989).

[3] B.C. Cooker, E. Hebard, E.N. Smith, Y. Takano, and J.D. Reppy, Phys. Rev. Lett. **51**, 666 (1983); M.H.W. Chan, K.I. Blum, S.Q. Murphy, G.K.S. Wong, and J.D. Reppy, *ibid.* **61**, 1950 (1988); D. Finotello, K.A. Gillis, A. Wong, and M.H.W. Chan, *ibid.* **61**, 1954 (1988); G.K.S. Wong, P.A. Crowell, H.A. Cho, and J.D. Reppy, *ibid.* **65**, 2410 (1990); M. Larson, N. Mulders, and G. Ahlers, *ibid.* **68**, 3896 (1992).

[4] Y. Liu, K.A. McGreer, B. Nease, D.B. Haviland, G. Martinez, J.W. Halley, and A.M. Goldman, Phys. Rev. Lett. **67**, 2068 (1991); D.B. Haviland, Y. Liu, and A.M. Goldman, *ibid.* **62**, 2180 (1989).

[5] M.P.A. Fisher and G. Grinstein, Phys. Rev. Lett. **60**, 208 (1988).

[6] J.T. Chayes, L. Chayes, D.S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 2299 (1986).

[7] L. Zhang and M. Ma, Phys. Rev. B **45**, 4855 (1992).

[8] K.G. Singh and D.S. Rokhsar, Phys. Rev. B **46**, 3002 (1992).

[9] T. Giamarchi and H.J. Schulz, Phys. Rev. B **37**, 325 (1988).

[10] L. Zhang and M. Ma, Phys. Rev. A **37**, 960 (1988).

[11] K. Runge, Phys. Rev. B **45**, 13 136 (1992).

[12] W. Krauth and N. Trivedi, Europhys. Lett. **14**, 627 (1991); N. Trivedi, D.M. Ceperley, W. Krauth, and N. Trivedi, Phys. Rev. Lett. **67**, 2307 (1991); R.T. Scalettar, G.G. Batrouni, and G.T. Zimanyi, *ibid.* **66**, 3144 (1991); E.S. Sorensen, M. Wallin, S.M. Girvin, and A.P. Young, *ibid.* **69**, 828 (1992).

[13] L. Zhang and M. Ma (unpublished).

[14] S.R. White and R.M. Noack, Phys. Rev. Lett. **68**, 3487 (1992).