Multiple-quanta vortices at columnar defects

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The possibility of the formation of multiple-quanta vortices on columnar defects (CD's) produced by ion irradiation is discussed. It is shown that the upper-critical field for localized superconductivity near CD's depends nonmonotonously on the CD radius.

I. INTRODUCTION

Recently the influence of ion irradiation on the critical current of high-T_c superconductors $Bi_2Sr_2CaCu_2O_x$ (Ref. 1) and $YBa_2Cu_3O_7$ (Ref. 2) has been investigated. Along their paths ions create columnar damage tracks with thickness of the order of 50 Å and an average distance between defects, $d \cong 100-500$ Å. These columnar defects (CD) influence dramatically the vortex pinning and can increase the critical current many times. Presumably, the damage tracks are amorphous regions where superconductivity is suppressed.

In this work we consider the columnar defect as a cylindrical dielectric region of a radius R parallel to the c axis. If the vortex core coincides with the columnar defect there is no loss of superconducting condensation energy in the core and it contributes to vortex pinning.

Moreover, if the radius R is larger than the superconducting coherence length ξ, then not only is the vortex-core energy changed but the energy of the superconducting current (which gives the main contribution to the total energy) will also be different. This effect would give the leading contribution to the vortex pinning energy.

The energy (per unit length) of the Abrikosov vortex in the superconductor is[3]

$$E_v = \left[\frac{\Phi_0}{4\pi\lambda}\right]^2 \left[\ln\left[\frac{\lambda}{\xi}\right] + c\right] , \tag{1}$$

where $\Phi_0 = \pi\hbar c/e$ is the flux quantum; λ is the London penetration depth; and a cutoff in the integration of the superconducting current is performed at ξ, the normal core radius. The numerical constant c is approximately 0.12 and describes the contribution of the normal core to the total energy. In the case of the columnar defect the only difference in the vortex energy calculation is the cutoff at R rather than at ξ, and the resulting vortex energy is

$$E_v = \left[\frac{\Phi_0}{4\pi\lambda}\right]^2 \ln\left[\frac{\lambda}{R}\right] . \tag{2}$$

For $R \gg \xi$ it leads to an essential decrease of the lower critical field and vortices start to appear at columnar defects at $H > \tilde{H}_{c1}$, where

$$\tilde{H}_{c1} = \frac{\Phi_0}{4\pi\lambda^2} \ln\left[\frac{\lambda}{R}\right] , \tag{3}$$

and $\tilde{H}_{c1}$ is smaller than the bulk lower critical field H_{c1}.

When the magnetic field continues to increase, more and more columnar defects become occupied by the vortices and finally at the field $H^* = n_c\Phi_0$, (where $n_c = 1/d^2$, is the defect's concentration) there is a vortex at each defect. Further increase of the magnetic field may create vortices between CD's. However, there exists an alternative option, the appearance of two-quanta vortices at a CD.

II. THE CONDITIONS OF TWO-QUANTA VORTEX FORMATION

In order to answer the question of which option is realized, we need to compare the energy of an additional single vortex creation and the energy of the appearance of a two-quanta vortex instead of a usual one. The free energy of an arbitrary vortex configuration (see, for example Ref. 4) may be written as

$$F = \sum_i E_{v,i} + \frac{1}{2} \sum_{i,j} U_{i,j} , \tag{4}$$

where $E_{v,i}$ is the self-energy of the ith vortex which, for a single-quantum vortex, is given by (1). The interaction energy U_{ij} between the i and j vortices reads

$$U_{i,j} = \frac{\Phi_0}{4\pi} h_{i,j} , \tag{5}$$

where $h_{i,j}$ is the magnetic field created by the ith vortex at the center of the jth vortex. In the case of a triangular lattice the Fourier components of the magnetic field are[5]

$$h_{\mathbf{k}} = \frac{n\Phi_0}{1+\lambda^2 k^2} , \tag{6}$$

where $n = 2/\sqrt{3}a^2$ is the concentration of vortices and $\mathbf{k}$ are the reciprocal-lattice vectors.

When the magnetic field increases and vortices start to enter into the sample they occupy randomly positioned CD sites. The energy of a vortex on the CD is much less than elsewhere and this prevents the formation of a regular vortex lattice. If all CD's are occupied by single-quantum vortices the possibility of creation of a two-quanta vortex appears. To perform the corresponding calculations we assume that CD's form a regular triangular lattice which fits the vortex lattice and the vortex concentration coincides in this particular case with CD concentration, i.e., $d = a$. This does not affect our result obtained with logarithmic accuracy but simplifies the calculations significantly. When the first two-quanta vortex is formed at the point $i=0$, its self-energy

$$E_v=\left[\frac{2\Phi_0}{4\pi\lambda}\right]^2\ln\left[\frac{\lambda}{R}\right] \qquad (7)$$

and the energy of interaction with other vortices of the lattice is

$$U_{\rm int}=\frac{2\Phi_0}{4\pi}\sum_{j\neq 0}h_{0,j}$$
$$=\frac{2\Phi_0}{4\pi}\left[n\sum_{\mathbf{k}}\frac{\Phi_0}{1+\lambda^2k^2}-\int\frac{d\mathbf{k}\Phi_0}{(2\pi)^2 1+\lambda^2k^2}\right] . \qquad (8)$$

Note that it is necessary to subtract the integral from the sum over reciprocal-lattice vectors to exclude the field created by the $i=0$, vortex itself. Then, finally, the change in free energy due to two-quanta vortex formation may be written as

$$\Delta F_{2v}=\frac{3\Phi_0^2}{(4\pi\lambda)^2}\ln\left[\frac{\lambda}{R}\right]$$
$$+\frac{\Phi_0}{4\pi}\left[n\sum_{\mathbf{k}}\frac{\Phi_0}{1+\lambda^2k^2}-\int\frac{d\mathbf{k}}{(2\pi)^2}\frac{\Phi_0}{(1+\lambda^2k^2)}\right] . \qquad (9)$$

When an additional single-quantum vortex appears, the minimum of its energy will correspond to the center of a triangular unit cell $\boldsymbol{\rho}_0=(\mathbf{a}_1+\mathbf{a}_2)/3$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the basis vectors of the vortex lattice. The change of the energy

$$\Delta F_{1v}=\frac{\Phi_0^2}{(4\pi\lambda)^2}\ln\left[\frac{\lambda}{\xi}\right]+\frac{\Phi_0}{4\pi}n\sum_{\mathbf{k}}\frac{\Phi_0\exp(i\mathbf{k}\boldsymbol{\rho}_0)}{1+\lambda^2k^2} . \qquad (10)$$

Within logarithmic accuracy we may write that

$$\Delta F_{2v}-\Delta F_{1v}=\frac{\Phi_0^2}{(4\pi\lambda)^2}\ln\left[\frac{\xi d^2}{R^3}\right] , \qquad (11)$$

and then under the condition

$$R^3>\xi a^2 \qquad (12)$$

it turns out to be energetically more favorable to create two-quanta vortices after all CD's are occupied. Note that the assumption that CD's form a regular lattice does not affect qualitatively the physical picture and condition (12) for two-quanta vortex formation (obtained with the logarithmic accuracy calculations) remains the same for the real situation with a random CD distribution.

Our consideration is adequate for the case $\lambda \gg d$, when the CD concentration is relatively high. In the case when the opposite inequality is satisfied we may neglect the interaction between vortices and the condition of two-quanta vortex formation (when all CD's have just been occupied by single-quantum vortices) yields $R^3>\xi\lambda^2$, i.e., it is more favorable to create a two-quanta vortex at a CD then an additional vortex far away from a CD. This is because of the presence of the cutoff at R (rather than at ξ) in expression (2) for the vortex energy at the CD. For the high-field limit ($a \ll \lambda$) formula (12) starts to work again, but in this case the concentration of vortices will be higher than the concentration of the CD.

In principle, when two-quanta vortices appear, one can expect a change in the magnetization curve slope. If the external field corresponds to two-quanta vortices at a CD ($H\simeq 2\Phi_0 n_d$), its further increases in H may create three-quanta vortices, etc. This also affect the magnetization. However, the irreversibility effects can strongly influence magnetization processes in real crystals and it may be difficult to verify this prediction. The direct visualization of vortex lattices by the decoration technique seems to be the best way to observe multiple-quanta vortice formation.

III. UPPER CRITICAL FIELD IN THE PRESENCE OF CD

In the high-field regime CD's may carry multiple-quanta vortices and this circumstance strengthens their role in vortex pinning. The favorable conditions of vortex formation at CD's is also reflected in the local increase of the upper critical field near the CD—the effect that is quite similar to the surface superconductivity.[6] Indeed, in the case of a CD with a large radius $R \gg \xi$ superconductivity must appear near the interface of the superconductor-amorphous region inside the CD at $H=H_{c3}$, where the critical field for surface superconductivity $H_{c3}=1.69H_{c2}$.[6] On the other hand, for a CD with a very small radius, $R \ll \xi$, the critical field must coincide with H_{c2}.

In this section we calculate, in the framework of Ginzburg-Landau (GL) theory, the critical field H_{c3}^* for superconductivity nucleation near the columnar defect with an arbitrary radius R for parallel field, i.e., directed along the z axis. The linearized GL equation for the superconducting order parameter Ψ in the presence of a magnetic field is analogous to the equation for an electron wave function in a uniform magnetic field. In cylindrical coordinates (ρ,φ,z) with the z axis along the CD it may be written as[7]

$$-\frac{\hbar^2}{4M}\left[\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\Psi}{\partial\rho}\right]+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\varphi^2}\right]$$
$$-\frac{i\hbar\omega_H}{2}\frac{\partial\Psi}{\partial\varphi}+\frac{M\omega_H^2}{4}\rho^2\Psi=a\Psi , \qquad (13)$$

with $a=\alpha(T_{c0}-T)$ the coefficient at $|\Psi|^2$ in the GL free-energy expansion, $(1/4M)$ the usual coefficient at the gradient term, $\omega_H=eH/Mc$ the cyclotron frequency, and the vector-potential gauge $\mathbf{A}=\frac{1}{2}(\mathbf{H}\times\mathbf{r})$ is chosen. Since the region of CD's is treated as an insulator it implies that the boundary condition for Ψ at CD's is (see, for example, Ref. 4)

$$\left[\frac{\partial\Psi}{\partial\rho}\right]_{\rho=R}=0 . \qquad (14)$$

The solution of Eq. (13) can be presented in the form $\Psi=(1/\sqrt{2\pi})\exp(im\phi)F(\rho)$, where the function F satisfies the following differential equation:

$$\frac{\hbar^2}{4M}F''+\frac{1}{\rho}F'-\frac{m^2}{\rho^2}F+\left[a-\frac{\hbar\omega_H}{2}m-\frac{M\omega_H^2}{4}\rho^2\right]F=0\ . \tag{15}$$

The orbital number m corresponds to the number of flux quanta in the solution for the superconducting order parameter. In principal, the solution of (15) with the boundary condition (14) gives the complete answer. However, as has been demonstrated in Ref. 8, the variational approach for the calculation of H_{c3} is very effective, it gives the H_{c3} value within 2% accuracy. In what follows we will use the approach[8] to calculate H_{c3}^* for the CD. The solution of (15) is equivalent to the problem of searching for the minimum of the functional

$$a=\frac{\hbar\omega_H}{4}\frac{\int_0^\infty\{(m-x^2)^2F^2(x)+x^2[F'(x)]^2\}(dx/x)}{\int_0^\infty xF^2(x)dx}, \tag{16}$$

where the dimensionless coordinate $x=\rho\sqrt{(eH/c\hbar)}$ is used. The minimum of $a=\alpha(T_{c0}-T)$ corresponds to the maximum temperature at fixed magnetic field when superconductivity can appear, in other words its gives the $T_c(H)$ dependence. Choosing for $F(x)$ a trial function which satisfies the boundary condition at $x_0=R\sqrt{(eH/c\hbar)}$, dimensionless radius of CD,

$$F(x)=\exp\left[-\frac{b(x-x_0)^2}{2}\right], \tag{17}$$

we obtain for $T_c(R)$ the characteristic nonmonotonous dependence, presented in Fig. 1. The origin of this peculiar dependence is the transition to the solutions with higher orbital momenta m when the radius of the CD increases. Note that for R going to zero the critical field H_{c3}^* approaches H_{c2}, whereas for R going to infinity the result $H_{c3}^*=1.66H_{c2}$ of the variational calculation[8] is reproduced.

The temperature dependence of H_{c3}^* for a columnar defect with dimensionless thickness $R/\xi_0=10$ is presented in Fig. 2, where ξ_0 is the superconducting correlation length extrapolated to $T=0$. Since the critical field goes to zero when the temperature approaches T_{c0} this means that the dimensionless radius $x_0\to 0$ at $T\to T_{c0}$. Then, near T_{c0} the solution with $m=0$ corresponds to the highest critical field and $H\to H_{c2}$. When the temperature decreases the transition to the solution with higher orbital momentum becomes possible and this is reflected in the discontinuous change of the slope of H_{c3}^* dependence on the temperature. Note that, in principal, the predicted dependence could be observed in the experiments on superconducting films with artificial regular arrays of holes.

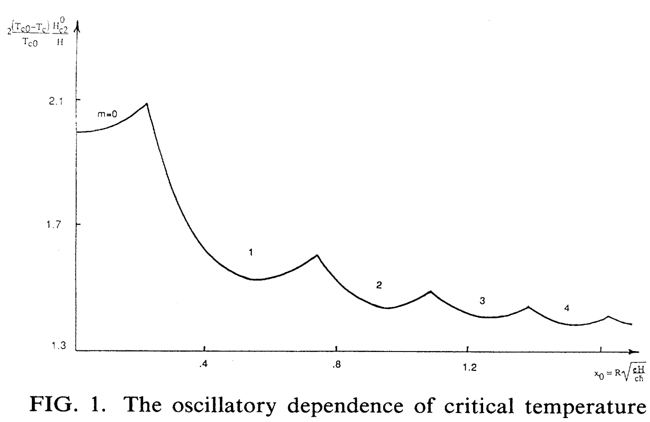

FIG. 1. The oscillatory dependence of critical temperature on the CD thickness. Different branches of the dependence correspond to solutions with different orbital momenta m, $x_0=R\sqrt{(eH/ch)}$, a dimensionless radius of the CD, H_{c2}^0 is the bulk upper critical field at $T=0$ extrapolated from linear dependence near T_{c0} ($H_{c2}(T)=H_{c2}^0[(T_{c0}-T)/T_{c0}]$).

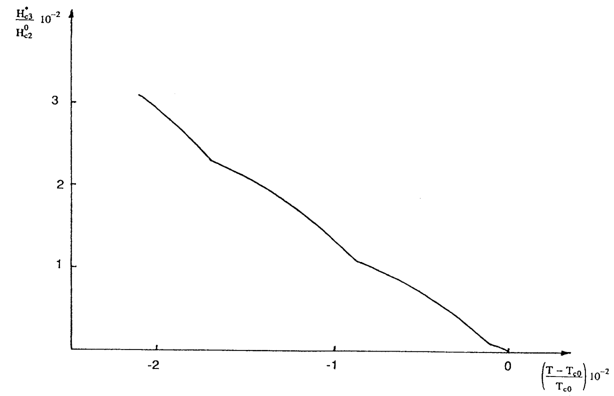

FIG. 2. Example of the temperature dependence of critical field H_{c3}^* for a columnar defect with dimensionless thickness $R/\xi_0=10$, where ξ_0 is the superconducting correlation length extrapolated to $T=0$. The discontinuities of the slope correspond to the transitions between states with different orbital momenta m.

As is known,[5] the surface superconductivity which appears at $H=H_{c3}$ does not screen the external magnetic field. This is connected with the fact that the density of superconducting current changes its sign at some distance from the surface and the total current is equal to zero. In the case of superconductivity localized near CD there is no such compensation for low-m solutions and some small magnetic moment appears.

As localized near a CD, superconductivity appears at a field $H_{c3}^*>H_{c2}$, this must be reflected in the increase of conductivity in the field range $H_{c3}^*>H>H_{c2}$, where the bulk sample is in the normal state. Naturally, this increase will be more pronounced for c-axis conductivity and if CD are continuous, c-axis resistivity will be equal to zero. Then the experimental observation of the change of anisotropy of resistivity at the field above H_{c2} could indicate the existence of localized CD superconductivity.

IV. CONCLUSIONS

We have demonstrated that the presence of rather thick columnar defects may lead to multiple-quanta vortices formation. The pinning of the vortices on CD's is determined not by a core energy change but by the decrease of the total magnetic energy of the vortex.

At high magnetic fields the superconductivity must first appear near the CD and this may result in essential resistivity decrease. The dependence of the critical field of localized CD superconductivity on CD thickness proves to be nonmonotonous.

Note that the region of the CD has been treated as an insulating one. If it is not the case and CD's should be considered as metallic regions, it does not change the conclusion about the possibility of multiple-quanta vortices formation at CD's but prevents the appearance of localized superconductivity near CD's.

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