Bunching of fluxons in a long Josephson junction with surface losses

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It is known that the sine-Gordon model of a long Josephson junction with a surface-loss term predicts a spatially oscillating trailing tail of a strongly contracted fluxon. In the present work, it is demonstrated that this should give rise to bunched (bound) states of fluxons, although the onset of the bunching drastically differs from the standard situation recently investigated in terms of the perturbed nonlinear Schrödinger model. Influence of the bunching on the I-V characteristic of an annular junction with a finite number of trapped fluxons is analyzed qualitatively. It is demonstrated that the bunching may set in with a small hysteresis, and it increases voltage at a given current. The latter effect has been observed in most recent experiments with the annular junction.

Creation of annular long Josephson junctions[1] (LJJ's) opens ways to observe new dynamical phenomena with fluxons (magnetic flux quanta, or Josephson vortices), which were not possible in traditional linear LJJ's with edges. In particular, the most recent experiments[2,3] point out a possibility of bunching (clustering) of strongly contracted fluxons moving with a velocity close to the Swihart velocity. As is mentioned in Ref. 3, the bunching should be possible in the presence of the surface losses. The aim of the present Report is to analyze this possibility briefly.

The well-known perturbed sine-Gordon (SG) equation[4] for the magnetic flux ϕ trapped in the LJJ is

$$\phi_{tt}-\phi_{xx}+\sin\phi=-\alpha\phi_t+\beta\phi_{txx}-f \ , \tag{1}$$

where α and β are small coefficients of the shunt and surface losses, and f is the current bias density. As is well known, the perturbation theory[5] gives quite a satisfactory description of the fluxon dynamics apart from the region where the fluxon's velocity V is close to one (i.e., to the Swihart velocity), so that the Lorentz contraction renders the surface-loss term in Eq. (1) comparable with the basic terms. A thorough numerical investigation of this region was performed in Ref. 6. It has been demonstrated that, at $1-V^2\lesssim\beta^{2/3}$, the trailing "tail" of the fluxon becomes oscillating. With the decrease of the ratio $(1-V^2)/\beta^{2/3}$, the oscillations grow and finally lead to an instability of the fluxon. The scenario ends with establishing the McCumber mode (the spatially homogeneous phase rotation) in the system.

In the region where the fluxons are still stable, the oscillating tails may give rise to bunching, i.e., formation of two-fluxon and multifluxon bound states. Recently, a similar phenomenon was analyzed for nonlinear Schrödinger (NS) solitons in the presence of small dissipative terms.[7] Formation of a stable bound state of two NS-like solitons has been lately observed in experiments with subcritical traveling-wave convection in a binary fluid filling a narrow *annular* channel.[8] It will be demonstrated below that in the SG system (1) the bunching is drastically different from the "standard" situation considered in Ref. 7.

Linearizing Eq. (1) far from the center of the fluxon (an exact form of which is actually unknown[9]) and looking for solutions in the form $\phi_x\sim\exp(\kappa x)$, one arrives at the well-known equation for κ,[6]

$$\beta\kappa^3+(1-V^2)\kappa^2-\alpha\kappa-1=0 \ , \tag{2}$$

in which it is implied that $\beta\ll1$, and V^2 is close to one. The parameter α competes with β if $\alpha\gtrsim\beta^{1/3}$. In real LJJ's, typical values may be $\alpha\sim0.01$, and $\beta\gtrsim0.001$. Thus, one may neglect α, and the corresponding term will be omitted in Eq. (2). Setting $\alpha=0$, one immediately sees that Eq. (2) has a pair of complex roots at [6]

$$(1-V^2)<(1-V^2)_0\equiv3(\beta/2)^{2/3} \ . \tag{3}$$

At $1-V^2=(1-V^2)_0$, the roots of Eq. (2) are $\kappa_1^{(0)}=\kappa_2^{(0)}=-(2/\beta)^{1/3}$, $\kappa_3^{(0)}=(4\beta)^{-1/3}$. At $1-V^2<(1-V^2)_0$, the pair (κ_1,κ_2) gives rise to the complex roots, the presence of which implies that the trailing edge of the fluxon is spatially oscillating. The real root κ_3 corresponds to the nonoscillating leading tail. The next step, following the lines of Ref. 7, is to calculate an effective potential of interaction of two separated fluxons, produced by overlapping of each fluxon with its mate's tail. This is how the spatial oscillations give rise to a set of bound (bunched) states of the solitons in the perturbed NS system.[7]. However, in the present case the situation is different because the full interaction potential contains two terms, produced, respectively, by overlapping the leading fluxon with the leading tail of the trailing fluxon, and by overlapping the trailing fluxon with the trailing tail of the leading fluxon. Only the latter term is oscillating, while the former one corresponds to the usual repulsion between unipolar fluxons. Next, one notices that, at the point (3) where the oscillations set in, $\kappa_3^{(0)}<|\kappa_{1,2}^{(0)}|$. Recall that the root κ_3 corresponds to the nonoscillating leading tail of the fluxon amenable for the mutual repulsion. Therefore, this inequality implies that at a large distance between the fluxons, at which bunched states may appear,[7] the nonoscillating repulsive term in the full interaction potential decays more slowly than the oscillating one; hence the bunched states are not possible at all.

Nevertheless, the bunching becomes possible at the value of $1-V^2$ at which the spatial damping rates for the leading and trailing tails, κ_3 and $-\mathrm{Re}\kappa_{1,2}$, become equal. It is easy to see that this happens at

$$I-V^2=(1-V^2)_1\equiv(\beta^2/2)^{1/3} \tag{4}$$

[cf. Eq. (3)], and at $1-V^2<(1-V^2)_1$ the bunching should be possible. At point (4), the roots of Eq. (2) are

$$\kappa_{1,2}^{(1)}=\kappa_3^{(1)}(-1\pm i)\ ,\quad \kappa_3^{(1)}=(1/2\beta)^{1/3}\ . \tag{5}$$

Let us proceed to a qualitative analysis of the bunching in the annular LJJ of a length L with n trapped fluxons. It is assumed that the fluxons are well separated from each other, i.e., $L/n \gg \sqrt{1-V^2}$. It is natural to think that, at point (4), where the bunched states appear, the distance between the bunched fluxons is infinitely large, and then it rapidly decreases with the decrease of $1-V^2$. At some value of $1-V^2$ slightly less than (4), the distance $l_{\min}$ corresponding to the nearest bunched state becomes equal to L/n. However, at still smaller values of $1-V^2$ the fluxons will not be bunched immediately. Indeed, if the distances l between $n-1$ neighboring fluxons become smaller than the mean distance L/n, $l=L/n-dl$, the distance l' between the first and the nth ones in the annulus becomes larger:

$$l'=L/n+(n-1)dl\ . \tag{6}$$

Expanding the full energy of the system, it is straightforward to see that, if the bunching sets in immediately, the change of energy is absent at order dl, and at order $(dl)^2$ the energy *increases*. This implies that the bunching in the system with a finite number of fluxons should set in with some delay. The bunching will certainly lead to a decrease of energy when the enlarged distance (6) between the first and the nth fluxon coincides with that corresponding to the *second*-nearest bound state. As we expect that this happens close to the point (5), we can estimate the corresponding value of dl as follows: $(n-1)dl\sim(\mathrm{Im}\kappa_{1,2})^{-1}\sim\beta^{1/3}$, or $dl\sim\beta^{1/3}/(n-1)$. So, the larger the n, the smaller will be the bunching delay. The delay implies that the onset of the bunching manifests itself on the *I-V* (current-voltage) characteristic of the LJJ as a small jump, and, accordingly, a small hysteresis may be possible.

Finally, let us consider influence of the bunching on the general form of the *I-V* characteristic, neglecting the delay and the corresponding jump. The effective friction coefficient λ for the bunched cluster of fluxons differs from that for the fluxons uniformly distributed along the annulus. The fluxons that are drawn closer to each other shorten their tails and thus diminish λ, while the tails between the first and the nth fluxons get longer and try to compensate this effect. Assuming that the fluxons are still sufficiently separated so that the exponential asymptotics for the tails apply, it is straightforward to see that, at the point of onset of the bunching, $d\lambda/dl=0$ (recall dl is the decrease in distance between adjacent fluxons), while $d^2\lambda/dl^2<0$. In terms of the *I-V* characteristic, this implies that the differential resistance $R\equiv dV/dI$ must be continuous at the onset, while its derivative dR/dI increases by a jump. Beyond the onset point, the resistance must be large for the same current, i.e., the voltage is larger too. An anomalous increase of the voltage at larger values of current was indeed observed for multifluxon states ($n=3$ and 4) in the experiments reported in Ref. 2, and it was interpreted as a manifestation of the bunching. A hysteresis in the transition between the usual branch of the *I-V* characteristic and the one corresponding to the bunched state has also been observed experimentally.[2,3] Thus, qualitative predictions made in the present work comply with the recent experimental findings. To achieve a better quantitative agreement, it is necessary to develop extensive numerical simulations of the fluxon-fluxon interactions in model (1). This work is in progress now.[10]

Note added in proof. As pointed out above, for a real LJJ the parameter α in Eq. (1) might be neglected in comparsion with β. Nevertheless, it could be relevant to mention that, if α is very large, it can drastically alter the situation. In particular, it follows from Eq. (2) that if α attains the value $\alpha_0=\beta^{1/3}$, one has $\kappa_3^{(0)}=|\kappa_{1,2}^{(0)}|$ at the point $1-V^2=\beta^{2/3}$, at which κ_1 and κ_2 merge to become complex. Thus, at $\alpha>\alpha_0$ the bunched states should appear immediately together with the oscillating tail.

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