RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 16 15 APRIL 1993-II

Mesoscopic fluctuations in high magnetic fields: Change in behavior due to boundary diffusion

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(Received 26 January 1993)

We have studied universal conductance fluctuations in the magnetoresistance of n^+-type GaAs submicrometer wires, which represent an orthodox mesoscopic system but also allow us to reach the high-magnetic-field regime, $\omega_c\tau>1$, where ω_c is the cyclotron frequency and τ the electron scattering time. The Lee-Stone correlation field B_c increases by more than an order of magnitude as the magnetic field increases from 0 to 18 T, but the amplitude of the fluctuations remains unchanged. This implies that the universal scaling of conductance fluctuations is not valid in high magnetic fields, in strong disagreement with theoretical predictions. We show that this behavior is *not* specific to the nonlocal geometry of measurements, where the breakdown has been reported earlier, but that it also occurs in the local magnetoresistance and rectification fluctuations. The violation of universal scaling is attributed to the appearance of a *second* phase-breaking length, in the regime $\omega_c\tau>1$, due to extended electron diffusion near the sample boundaries.

Universal conductance fluctuations (UCF) are a random quantum contribution to electrical conduction in mesoscopic systems and are due to the interference of electron waves. The fluctuations have been termed "universal" because their behavior in each part of the sample volume where the phase coherence is preserved is independent of the material of the conductor. As the magnetic field B is changed the conductance of any phase-coherent unit fluctuates with rms amplitude $\Delta G \cong e^2/h$ and a characteristic period B_c, which corresponds to changing the magnetic flux through the phase-coherent volume by about one quantum $\phi_0=h/e$. For larger samples, the UCF are no longer sample independent but they are still universal in the sense that it is possible to scale them and predict ΔG and B_c for any mesoscopic system if a single scaling length, the phase-breaking length of electrons L_ϕ, is known.

UCF have been intensively studied over the last decade and good agreement found between experiment and the theory of Altshuler and of Lee and Stone[1] (ALS) (for a review, see Ref. 2). However, most experiments have been confined to the low-magnetic-field limit, $\omega_c\tau \ll 1$, where the magnetic field changes only the phase of electrons but does not affect the electron trajectories. The question is as follows: do the fluctuations remain universal when a strong magnetic field changes the electron trajectories? Recently, there have been several experiments[3–9] in which UCF were investigated for $\omega_c\tau>1$ using ballistic submicrometer wires fabricated from a high-mobility two-dimensional electron gas. In these structures the electron mean free path l is much larger than the sample width w and it is boundary scattering that gives rise to the diffusivelike motion of electrons which is essential for the appearance of mesoscopic fluctuations. It has been found that the period of the fluctuations increases substantially as the magnetic field increases. However, a quantitative study of the amplitude ΔG has not been reported for these high-mobility systems, apparently due to the presence of Shubnikov–de Haas oscillations (SdHO) which superimpose on the conductance fluctuations and make analysis of ΔG very difficult. We emphasize that in the ballistic wires mesoscopic fluctuations are *not* predicted to be universal for $\omega_c\tau>1$.[10] In contrast, for orthodox mesoscopic systems, with $l \ll w$ ("dirty metal" regime), the ALS theory is expected to be valid in high, even quantizing, magnetic fields.[11] The magnetic field changes the diffusion constant D and hence $L_\phi=(D\tau_\phi)^{1/2}$, but the theory still assumes the universal scaling between ΔG and B_c.

Recently, we have reported mesoscopic conductance fluctuations in "dirty" wires for $\omega_c\tau>1$.[12] The Lee-Stone correlation field was found to increase dramatically as magnetic field increased, in good agreement with the expected changes in D and with the theory of Xiong and Stone (XS).[11] However, the major result was that the amplitude ΔG remained unchanged, indicating that universal scaling is not valid in high magnetic fields and the ALS and XS theories need further modification. In order to show the breakdown of universal scaling we employed nonlocal geometry[1,2,12] which allowed us to eliminate the rapidly varying background of the average resistivity due to the SdHO. Furthermore, in the nonlocal geometry the rms amplitude depends exponentially on L_ϕ and the observation of a constant value of ΔG, while B_c changes, provided the strongest evidence for nonuniversal scaling. However, it has been suggested[13] that the breakdown might be an inherent feature of this particular geometry and may not be the case in the conventional local geometry for which the XS theory has been developed. In this paper we present experimental data which show that the violation of universal scaling for $\omega_c\tau>1$ occurs in the local geometry as well. The clearest evidence for the breakdown has been seen in the Hall geometry which also allows us to avoid the obscuring background due to the SdHO. Measurements of the mesoscopic rectification fluctuations confirm the result.

In addition, we have extended our previous nonlocal measurements to much higher magnetic fields.

The structures used in the experiment are similar to those described in our previous papers.[12,14,15] Briefly, they are n^+-type GaAs submicrometer wires (see inset to Fig. 1) with electron concentration $n \cong 1\times10^{24}$ m^{-3} and conducting thickness of $\cong 30$ nm with four electrically quantized two-dimensional subbands occupied. Adjacent pairs of probes (for example, *ab* and *cd*) are separated by 1 μm and each contact probe (a–g) has the same thickness and width as the main wire. For consistency, we refer only to results for a wire with conducting width of 350 nm. At 4.2 K the sheet resistance of the material is $\rho = 650$ Ω per square with an electron mobility $\mu \cong 0.185$ $m^2\,V^{-1}\,s^{-1}$ corresponding to $l \cong 40$ nm and $\omega_c\tau = 3.3$ at the highest magnetic field of 18 T. Standard low-frequency lock-in techniques were employed in all resistance measurements. The measuring currents were chosen small enough (<50 nA) so as not to affect the temperature-dependent amplitude of the fluctuations. We use the convention $R_{ijkl} = V_{kl}/I_{ij}$ where R_{ijkl} is the voltage difference between contacts k and l due to a current I_{ij} between i and j. The rectified signal was measured by a high-impedance dc nanovoltmeter using the nearest longitudinal voltage probes, e.g., *ce*, with low-frequency ac current applied through, say, *ag*.

Typical behavior of the mesoscopic fluctuations at 4.2 K for four different types of measurements is shown in Fig. 1. The trace in Fig. 1(a) corresponds to the local longitudinal magnetoresistance $R_{xx} = R_{agce}$. At low magnetic field ($B < 1$ T) conductance fluctuations coexist with the negative magnetoresistance due to weak localization and for $B > 8$ T they are superimposed on the SdHO. Analysis of the weak localization magnetoresistance allows us to determine values of $L_\phi[\mu m] \cong 0.6/T\,[K]^{1/2}$. To avoid confusion we emphasize that the notation L_ϕ is used below only for the value of the phase-breaking length at low magnetic fields. Subsequent traces [Figs. 1(b) and 1(c)] show the conductance fluctuations obtained in the Hall geometry and the rectification fluctuations, respectively. For the rectification measurements we have used alternating currents of 200 nA which cause some heating of the electron system. The estimated temperature is about 7 K. Note that in Fig. 1(b) we have removed a linear contribution due to the classical Hall effect so that $\delta R_{agcd} = R_{agcd} - R_{xy}$ where $R_{xy} = B/ne \cong 120$ Ω/T. Finally, the nonlocal fluctuations in R_{abcd} are shown in Fig. 1(d). All the measurements show a substantial increase in B_c but there are no signs of quenching of the fluctuation amplitude. To emphasize this behavior further we plot in Fig. 2 the data from the local Hall geometry [Fig. 1(b)] for magnetic field intervals of 0–2 and 14–16 T.

For a quantitative analysis we calculate the Lee-Stone correlation field[1,2] and the rms amplitude. In low magnetic fields, their values are found to be in good agreement with theory for all our measurements. For the sake of brevity and since in this paper we are interested particularly in the field dependence of UCF, we refer a detailed consideration of the low-field behavior to our previous papers[12,15] and to Ref. 16 where the rectification fluctuations have been studied at the low fields in the same n^+-type GaAs structures. Figures 3 and 4 show the variation of B_c and ΔG with magnetic field for the Hall and nonlocal geometries where the data are not obscured by the SdHO. The values of B_c and ΔG are normalized to their values in the low-field limit $\omega_c\tau \ll 1$. The correlation field increases by a factor of 12 as the magnetic field increases up to 18 T ($\omega_c\tau \cong 3.3$) and results for both

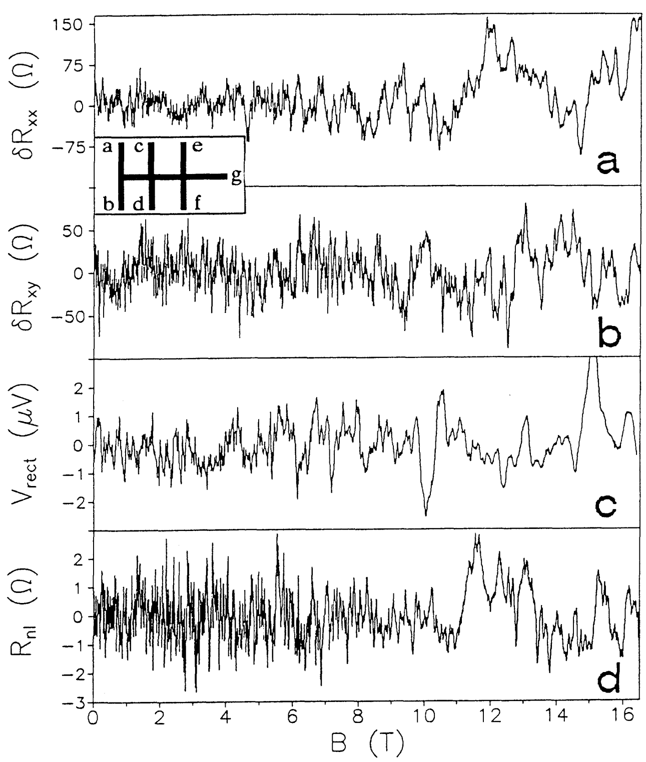

FIG. 1. Universal conductance fluctuations in different geometries: the local fluctuations in the (a) longitudinal and (b) Hall magnetoresistances and the nonlocal fluctuations in R_{abcd} (d). (c) The rectification fluctuations.

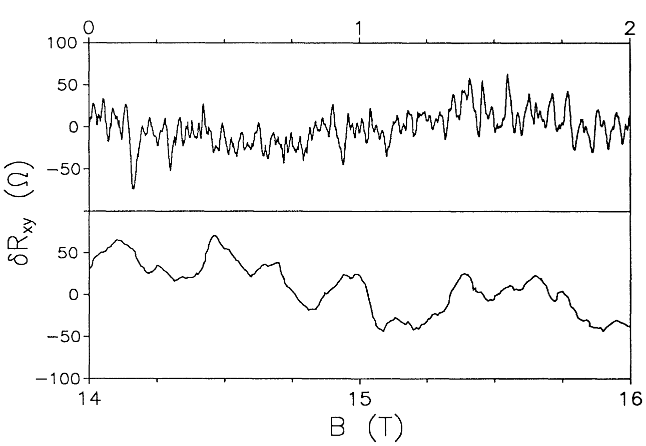

FIG. 2. Conductance fluctuations for the Hall geometry in low (upper curve) and high (lower) magnetic fields.

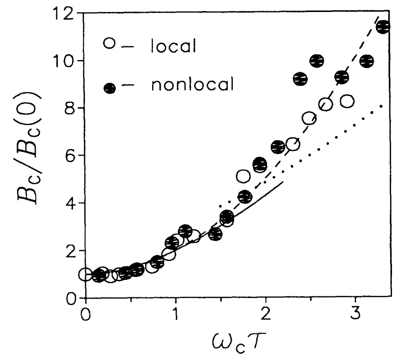

FIG. 3. Variation of the Lee-Stone correlation field B_c with magnetic field. Lines are the expected behavior due to the variation of the diffusion coefficient. Shown are cases of the semiclassical diffusion (dashed) and the diffusion in the quantizing fields. The solid line is a numerical result from Ref. 11 for the shown field interval and the dotted line is the high-field asymptote $\propto B^{3/2}$.

geometries demonstrate the same behavior within the random scatter of the experimental values. We wish to mention that some of our measurements demonstrate clear oscillations of B_c as a function of the magnetic field with a period which corresponds to the Landau quantization.[17]

The variation of B_c with magnetic field has been interpreted as a reduction of the diffusion constant D in strong magnetic fields which causes "localization" of electron trajectories and hence a decrease of the phase-coherent volume.[11,12] In fact, it is possible to explain the field dependence of B_c by the simple assumption that D varies as its asymptote in nonquantizing magnetic fields,

$$D(B)=D(0)[1+(\omega_c\tau)^2]^{-1}, \qquad (1)$$

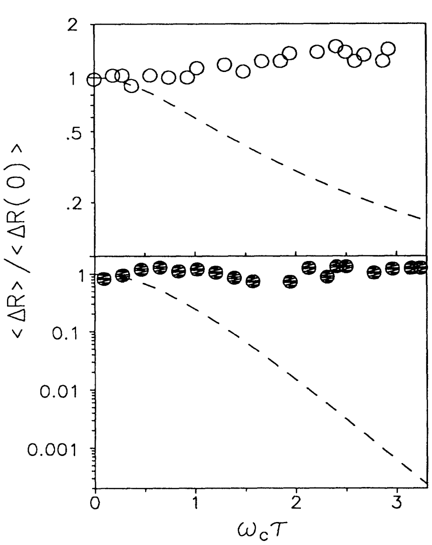

FIG. 4. Field dependences of the rms amplitude ΔR for local (upper) and nonlocal (lower) geometries. Dashed lines are the expected decay of ΔR as magnetic field increases if the fluctuations maintain the scaling with the single phase-breaking length.

where $D(0)=l^2/2\tau$ is the diffusion constant in zero field. In our case of $L_\phi < w$, $B_c \propto \phi_0/L_\phi^2$ (Refs. 1 and 2) so that $B_c(B)=B_c(0)[1+(\omega_c\tau)^2]$. This dependence is shown by the dashed line in Fig. 3. Note that we do not use any fitting parameters. The solid line shows the field dependence calculated numerically by Xiong and Stone and the dotted one is the expected asymptotic behavior $B_c \propto B^{3/2}$ for $\omega_c\tau \gg 1$.[11] As can be seen the simple theory describes the increase of B_c very well.

Universal scaling of mesoscopic fluctuations implies that B_c and ΔG are connected by a single scaling length. The increase of B_c, corresponding to a decrease of the phase-coherent volume, must be accompanied by a rapid decrease of ΔG. The observed field dependences of ΔG (Fig. 4) do not show any decrease, indicating that mesoscopic fluctuations for $\omega_c\tau > 1$ cannot be interpreted in terms of a single scaling length. In order to illustrate the degree of the violation of the universal scaling, we plot in Fig. 4 the field dependence of ΔG which could be expected from the observed variation of B_c assuming universal scaling (dashed curves). To calculate these curves we assume, without loss of generality, that the phase-breaking length varies in magnetic field according to Eq. (1) and also that $\Delta R \propto \exp(-L/L_\phi)$ for the nonlocal geometry (L is the distance between the current and voltage probes) and $\Delta G \propto \Delta R \propto L_\phi^{3/2}$ for the local one.[12,18] The discrepancy reaches one order of magnitude in the local geometry and four orders in the nonlocal one.

The experiment indicates existence of two different scaling lengths for $\omega_c\tau > 1$. One of them, which describes the UCF period, varies in reasonable agreement with the expected modification of electron motion in high magnetic fields. The other length, responsible for the amplitude, remains constant and equal to the low-field value of L_ϕ. Since there is still no theory which would explain even qualitatively this extraordinary behavior, we present below some simple considerations which can give, at least, a guide for understanding the experiment. Note that the theory has taken into account only changes in electron motion in the bulk of the sample whereas there are a number of electron trajectories which intersect with the sample boundaries and are *not* "localized" by the strong magnetic field. To illustrate the presence of such trajectories we perform a numerical simulation of electron diffusion in our samples.

Figure 5 shows several semiclassical trajectories which are randomly injected by one of the contacts then captured by the sample boundary and transmitted through the whole conductor. Trajectories transmitted in the opposite direction along the other boundary and the reflected trajectories are not shown. This picture allows us to speculate that in high fields there are two kinds of phase-coherent volumes. One of them is in the bulk and is determined by the bulk diffusion coefficient. The corresponding volume shrinks as B^{-2} in the semiclassical approximation [Eq. (1)] and as $B^{-3/2}$ in the presence of Landau quantization.[11] The other volume is defined by extended trajectories near the boundaries. A simplistic way to consider UCF in the latter volume is to assume that the fluctuations are determined by one-dimensional (1D) boundary diffusion due to the semicircular skipping

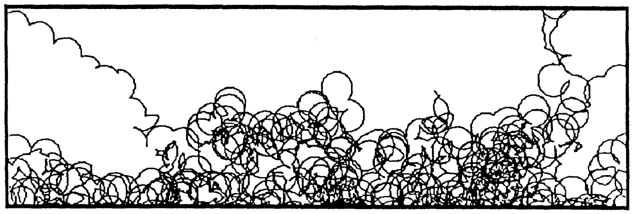

FIG. 5. Numerical simulation of semiclassical electron trajectories responsible for the enhanced near-boundary diffusion ($\omega_c\tau=2$). The relationship between w and l is the same as in the experiment.

trajectories. Then, the boundary diffusion constant $D_B=v_x^2\tau=(4/\pi^2)v_F^2\tau=(8/\pi^2)D(0)$ where v_F is the Fermi velocity. It follows that the corresponding 1D phase-breaking length $L_B=(D_B\tau_\phi)^{1/2}\cong 0.9L_\phi$ and is field independent. In fact, our numerical calculations show that an average diffusion coefficient given by all possible boundary trajectories is a smooth function of $\omega_c\tau$ and, in the high-field limit, the length of the boundary phase-coherent volume is just $\cong 0.95L_\phi$. On the other hand, the width of the boundary volume also shrinks in high fields since only trajectories at a distance of the order of the cyclotron diameter d_c are captured by the edges. In high magnetic fields, the UCF from the small phase-coherent units in the bulk are rapidly averaged out and the conductance fluctuations caused by the extended trajectories can dominate.[19] These fluctuations are expected to give a nearly constant ΔG and an increase of B_c which is rather slow for $\omega_c\tau<1$ but tending to the linear field dependence $B_c\simeq B_c(0)/(L_\phi/l)\omega_c\tau$ in higher fields. The experimental dependence of B_c is fairly close to this behavior, although we do not expect quantitative agreement with our simplistic model.

In conclusion, breakdown of universal scaling of the UCF occurs in high magnetic fields, independently of either the geometry or the type of measurements. We attribute the breakdown to the presence of the extended electron diffusion near sample boundaries.

This work was supported by the SERC. We are grateful to D. Maslov, L. Eaves, and D. I. Khmelnitskii for discussions and P. H. Beton for sample fabrication at SERC Nanolithography Facilities in the University of Glasgow. A.K.G. gratefully acknowledges SERC for financial support.

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*On sabbatical from Institute of Microelectronics Technology, Russian Academy of Sciences, Chernogolovka, 142432, Russia.

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[17]SdH oscillations of B_c have also been seen in submicrometer metal-oxide-semiconductor field-effect transistor wires by D. H. Cobden *et al.* (private communication).

[18]Both expressions have been tested experimentally using the temperature dependence of ΔR and the known temperature dependence of L_ϕ in low magnetic fields. One might expect the dependence $\Delta G \propto L_\phi$ (Ref. 2) for the 2D case, where $L_\phi < w$. The more rapid decay is presumably due to the wide voltage probes used in the experiment [see V. Chandrasekhar, P. Santhanam, and D. E. Prober, Phys. Rev. B **44**, 11 203 (1991)].

[19]It is important to note that the backscattering in the stretched volume is not totally suppressed. The boundary trajectories are strongly coupled with the bulk and can eventually be transmitted to the opposite edge as shown for two trajectories in Fig. 5.