Tunneling conductance between parallel two-dimensional electron systems

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We derive and evaluate expressions for the low-temperature dc equilibrium tunneling conductance between parallel two-dimensional electron systems. Our theory is based on a linear-response formalism and on impurity-averaged perturbation theory. The disorder broadening of features in the dependence of tunneling conductance on sheet densities and in-plane magnetic-field strengths is influenced both by the finite lifetime of electrons within the wells and by momentum-nonconserving tunneling events. Disorder vertex corrections are important only for weak in-plane magnetic fields and strong interwell impurity-potential correlations. We comment, on the basis of our results, on the possibility of using tunneling measurements to determine the lifetime of electrons in the quantum wells.

I. INTRODUCTION

Tunneling of electrons has long been studied both as a fundamental manifestation of the quantum nature of matter and as a powerful probe of many different electronic systems.[1] It has only recently become possible to measure the tunneling conductance between two-dimensional electron systems[2–6] (2DES's) in semiconductors. The 2DES's occur in GaAs quantum wells which are separated by thin AlAs tunneling barriers.[5,6] The strong constraints imposed by energy and momentum conservation in two dimensions can potentially lead to very sharp features in the dependence of the equilibrium tunneling conductance on the sheet densities in the quantum wells and on the strength of an in-plane magnetic field. As we discuss below, tunneling conductance studies probe the properties of 2DES's in a way which is qualitatively different from conventional in-plane transport studies. In this paper we present a theory which provides a framework for the detailed interpretation of these experiments and apply it to study the influence of disorder on the tunneling conductance. We find that these experiments can provide a direct measure of the quantum lifetime of electrons in the 2DES's. When combined with measurements of the 2DES transport lifetime from in-plane transport experiments, this provides information on the nature of the disorder scatterers in the system. We also show that vertex corrections to the naive result for the impurity broadening of features in the tunneling conductance are important only for strong correlations between the disorder potentials in the separated 2DES's and for weak in-plane magnetic fields. In Sec. II we briefly review the linear-response formalism which is the basis of our theory. In Sec. III we present results for the dependence of tunneling conductance on the density difference of the 2DES's and the strength of an in-plane magnetic field in the absence of correlation between the disorder potentials in the two layers. In the ideal disorder-free limit the tunneling conductance can be evaluated analytically and diverges at certain magnetic-field strengths. We examine how these divergences are suppressed by disorder and discuss the possibility of using the resulting tunneling conductance curves to measure the quantum lifetime of electrons in the double quantum-well systems. In Sec. IV we examine vertex corrections to the naive expression for the tunneling conductance in disordered systems. We find that when the disorder in the two 2DES's is strongly correlated, features in the conductance curves may sharpen. Vertex corrections are important only at weak in-plane magnetic fields. Finally in Sec. V we briefly summarize our results.

II. LINEAR-RESPONSE FORMALISM

We restrict our attention here to the case of zero perpendicular magnetic field so that states of the 2DES's may be described in a plane-wave-state basis. Our interest here is in studying the dependence of the tunneling conductance on experimentally tunable parameters like the 2DES layer densities and the strength of an in-plane magnetic field rather than on its absolute value. We therefore describe the tunneling phenomenologically using a tunneling Hamiltonian

$$\hat{H}_T = -\sum_{\mathbf{k}',\mathbf{k},\sigma} (t_{\mathbf{k}',\mathbf{k}} c^\dagger_{\mathbf{k}',\sigma,r} c_{\mathbf{k},\sigma,l} + \text{H.c.}). \tag{1}$$

Here we use l and r to label states localized in left and right 2DES's, respectively. $\mathbf{k}$ is the in-plane mometum of electrons. The remaining terms in the Hamiltonian may include the interaction of electrons in either 2DES with a disorder potential and interactions between electrons in the same or in different 2DES's but are assumed to commute separately with the total number operator for each 2DES, $\hat{N}_{l,r}$. Following a familiar line[7] we note that the operator describing the net current flowing from the left 2DES to the right 2DES is given by

$$\hat{I} = -e\frac{d}{dt}\hat{N}_r = e\frac{d}{dt}\hat{N}_l = ie[\hat{N}_r, \hat{H}_T]/\hbar$$
$$= \frac{ie}{\hbar}\sum_{\mathbf{k}'\mathbf{k},\sigma}(t_{\mathbf{k}',\mathbf{k}}C^\dagger_{\mathbf{k}',\sigma,r}C_{\mathbf{k},\sigma,l} - \text{H.c.}). \quad (2)$$

Since the expectation value of $\hat{I}$ is zero in any eigenstate of the Hamiltonian, it follows that no current flows in equilibrium. We evaluate the current which flows in response to an electric potential difference $V(t) = V\exp(i\omega t)$ between the two wells. The perturbing term in the Hamiltonian is

$$\hat{H}_1(t) = -eV(t)(\hat{N}_l - \hat{N}_r)/2. \quad (3)$$

Using finite-temperature linear-response theory we find that in the limit $\omega \to 0$ the tunneling conductance $G \equiv \langle I\rangle/V$ is given by

$$G = \lim_{\omega\to 0}\frac{\text{Im}K(\omega + i\eta)}{\hbar\omega}, \quad (4)$$

where $K(z)$ is the analytic continuation of the Matsubara current-current correlation function:

$$K(i\omega_n) = -\int_0^{\hbar\beta} d\tau e^{i\omega_n\tau}\langle T_\tau[\hat{I}(\tau)\hat{I}(0)]\rangle. \quad (5)$$

Equation (5) is the basic expression we use to evaluate the tunneling conductance.

III. UNCORRELATED ELECTRON LAYERS

We first apply Eq. (5) to the case where the two 2DES's can be considered to be decoupled in the absence of the perturbation.[8] It then follows from the translation invariance of each well and the definition of the Matsubara Green's function[9] that

$$K(i\Omega) = \frac{-2e^2}{\hbar^2\beta}\sum_{\mathbf{k},\mathbf{k}'}\sum_{i\omega_n}|t_{\mathbf{k}',\mathbf{k}}|^2\left[\mathcal{G}_{\mathbf{k}',r}(i\omega_n)\mathcal{G}_{\mathbf{k},l}(i\omega_n + i\Omega) + \mathcal{G}_{\mathbf{k},r}(i\omega_n)\mathcal{G}_{\mathbf{k}',l}(i\omega_n - i\Omega)\right]. \quad (6)$$

(We have assumed spin degeneracy.) The frequency sum may be evaluated by contour integration methods with the result that

$$G = \frac{2e^2}{\hbar^3}\sum_{\mathbf{k}',\mathbf{k}}|t_{\mathbf{k}',\mathbf{k}}|^2\int\frac{d\epsilon}{2\pi}\frac{-\partial n_F(\epsilon)}{\partial\epsilon}A_{\mathbf{k}',r}(\epsilon/\hbar)A_{\mathbf{k},l}(\epsilon/\hbar). \quad (7)$$

Here $A_{\mathbf{k},m}(\omega) = -2\,\text{Im}G_{\mathbf{k},m}(\omega + i\eta)$ is the spectral weight of the exact finite-temperature Green's function in layer m. In the absence of interactions and disorder $A_{\mathbf{k},m} = 2\pi\hbar\delta[\epsilon - \epsilon_m(\mathbf{k})]$ and Eq. (7) reduces to the Fermi-golden-rule expression for the tunneling conductance. Deviations from this limit probe many-body or disorder effects in the 2DES's. We see from Eq. (7) that when correlation between the two layers may be ignored the tunneling experiments measure the one-body Green's function of the individual layers, i.e., they measure the spectrum of states created when an extra electron is added to the ground state of a system. In contrast in-plane transport experiments measure a two-body Green's functions of the layers, which reflect the correlations of particle-hole excitations of a system. In many-body systems the information available in these two experiments is complementary.

The numerical results we present below are all for the ideal case where the in-plane component of the momentum is conserved during tunneling, i.e., where $t_{\mathbf{k}',\mathbf{k}} = t\delta_{\mathbf{k}',\mathbf{k}}$. Physically this limit corresponds to the case where the tunneling amplitude is uniform over the area of the 2DES's and the comparison of our results with experiment[5] discussed below indicates that this uniformity is achieved to a remarkable degree. The results presented below also do not account for electron-electron interactions which do not appear to play any important role[10] experimentally. We will restrict our attention to $T = 0$.

When momentum is conserved during tunneling it follows from Eq. (7) that in the absence of interactions and disorder $G \equiv 0$ unless the 2DES's have identical densities. For this reason it is useful to study the dependence of G on both the layer densities of the 2DES's and on the strength of an in-plane magnetic field, $\mathbf{B} = B\hat{e}_y$. If we neglect the finite thickness of the electron layers, the effect of $\mathbf{B}$ on the electronic wave functions is just to add a gauge transformation phase factor which shifts the origin of momentum space in the two layers. For example, if we choose the gauge in which the vector potential is $\mathbf{A} = Bz\hat{e}_x$, it will shift the origin of momentum space by $-z_m\hat{e}_x/\ell^2$ in a layer located in the plane $z = z_m$. [Here $\ell = (\hbar c/eB)^{1/2}$ is a magnetic length.] With these shifts the single-particle energies in layer m are given by

$$\epsilon_m(\mathbf{k}) = E_m + \frac{\hbar^2}{2m^*}[(k_x + z_m/\ell^2)^2 + k_y^2], \quad (8)$$

where E_m is the subband energy in layer m. [The layer density difference is related to the subband energy difference by $n_r - n_l = (E_l - E_r)/\nu_0$, where $\nu_0 = m/\pi\hbar^2$ is the 2DES density of states.] In Fig. 1 we show the tunneling conductance versus in plane-field strength for the situation studied experimentally in Ref. 5. The dashed

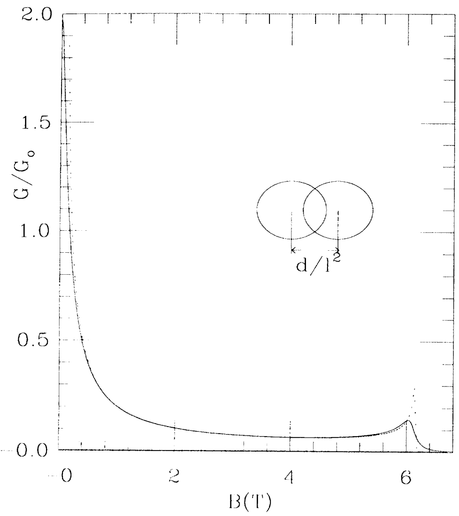

FIG. 1. Tunneling conductance vs in-plane magnetic-field strength at equal layer density, $n_l = n_r = 1.5 \times 10^{11}$ cm^{-2}. The inset shows the displaced Fermi circles in the two layers. Momentum-conserving tunneling events at the Fermi energy are allowed at the intersections of the two Fermi circles. The dashed line shows the result in the absence of disorder while the solid line was evaluated for a quantum lifetime $\tau = 15\hbar/\epsilon_F$. $G_0/A = 1.5 \times 1.0^2(m^{*2}e^2t^2/\pi^4\hbar^5 n_l)$.

line shows the result without any disorder which can be obtained analytically:

$$\frac{G}{A} = \frac{4e^2t^2m^{*2}}{\pi\hbar^5[(k_{Fl}+k_{Fr})^2 - K^2]^{1/2}[K^2-(k_{Fl}-k_{Fr})^2]^{1/2}} \tag{9}$$

when both terms in square brackets are positive and is zero otherwise. Here $K = (z_r - z_l)/\ell^2 \sim B$ is the amount by which the two Fermi circles are displaced as illustrated by the inset of Fig. 1, A is the area of the 2DES's, and k_{Fm} is the Fermi radius in layer m. Note that for equal densities in the two layers ($k_{Fl} = k_{Fr}$) G diverges as B^{-1} for $B \to 0$ but only as $(B_c - B)^{-1/2}$ for $B \to B_c \equiv \pi|z_r - z_l|/k_F\Phi_0$. ($B_c$ is the maximum field for which tunneling can occur and Φ_0 is the magnetic flux quantum.)

These divergences of G are suppressed by disorder. The solid line in Fig. 1 shows results obtained using the Born approximation expression for the disorder-averaged Green's function:[9,11]

$$G_{\mathbf{k},m}(\omega \pm i\eta) = \left(\omega - \hbar^{-1}\epsilon_m(\mathbf{k}) \pm \frac{i}{2\tau}\right)^{-1}. \tag{10}$$

Here τ is the quantum lifetime of the electrons. The results in Fig. 1 were calculated with $\tau\epsilon_F/\hbar = 15$. This level of disorder is important in suppressing the divergences at $K = 2k_F$ and especially at $K = 0$ but has relatively little effect on G at other values of K. For equal-density 2DES's the maximum conductance reached at zero in-plane magnetic field ($K = 0$) is proportional to τ:

$$\frac{G(K=0)}{A} = \frac{e^2t^2\nu_0}{\hbar^2\pi}\int d\omega A_B^2(\omega) = \frac{2e^2t^2\nu_0\tau}{\hbar^2}. \tag{11}$$

Here $A_B(\omega) = (1/\tau)/[\omega^2+(2/\tau)^2]$ is the Born approximation Lorentzian spectral function and we have assumed that $\tau\epsilon_F \gg 1$. This suggests the possibility of using the shape of the tunneling conductance curve to measure the electron lifetime. For example, assuming the $G(K = k_F)$ is nearly independent of disorder gives

$$\frac{G(K=0)}{G(K=1)} = \sqrt{3}\tau\epsilon_F/\hbar. \tag{12}$$

Equation (12) provides a lower bound on τ since the divergence of $G(K = 0)$ will also be suppressed by finite temperatures and also possibly by momentum-nonconserving tunneling events and macroscopic inhomogeneities. Another estimate of $\tau\epsilon_F/\hbar$ may be obtained by examining the dependence of G on the density difference between the two layers. In the Born approximation we find that at $K = 0$

$$G(K=0) = \frac{2e^2t^2\nu_0\tau}{\hbar^2\{1+4[(n_l-n_r)/(n_l+n_r)]^2(\epsilon_F\tau/\hbar)^2\}}. \tag{13}$$

In Fig. 2 we show results for the layer density dependence of the conductance at $K = 0$ and at several nonzero values of in-plane magnetic field. According to Eq. (13) the relative density difference at which $G(K = 0)$ is reduced to half its equal density value is $\epsilon_F\tau/\hbar$.

Comparing with the experimental data of Eisenstein *et al.*,[5] Eq. (12) gives the estimate $\epsilon_F\tau/\hbar \sim 5$ while the half-maximum density-difference value gives the estimate $\epsilon_F\tau/\hbar \sim 10$ for their sample. These values should be compared with the measured transport lifetime for the sample of Ref. 5, $\tau_{tr}\epsilon_F/\hbar \sim 250$. In high-mobility

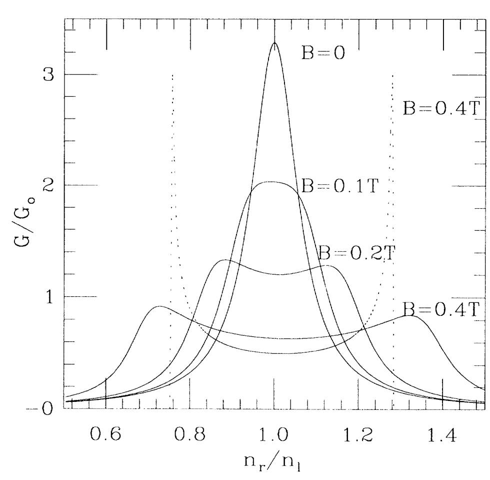

FIG. 2. Tunneling conductance vs the ratio of electron densities in the two layers ($n_l = 0.9 \times 10^{11}$cm^{-2}). These results were calculated with $\tau\epsilon_F/\hbar = 15$. The dashed line shows the result obtained in the absence of disorder at $B = 0.4$ T. $G_0/A = 0.9 \times 1.0^2(m^{*2}e^2t^2/\pi^4\hbar^5 n_l)$.

quantum-well structures, scattering is dominated by ionized impurities which are spatially separated from the 2DES's. These impurities produce small-angle scattering of electrons because the Coulomb potential they produce is smooth in the 2DES layers. Small-angle scattering is ineffective in limiting the transport lifetime. For disorder dominated by remote ionized donors large values of $\tau_{\rm tr}/\tau \sim (k_F d_s)^2$ are therefore expected[12] and it is widely appreciated that the transport lifetime no longer effectively characterizes the degree of disorder in the sample. (d_s is the distance between the 2DES and the ionized impurities.) The estimates of $\tau\epsilon_F/\hbar$ derived from the tunneling date above are in reasonable accord with expectations, although the value obtained from Eq. (12) is probably an underestimate. The peak value of the conductance is likely to be reduced by large length scale inhomogeneities in the samples[13] which are not adequately described in the Born approximation. Any momentum nonconservation during tunneling would also lead to further broadening of the features in these curves. The fact that the $\tau\epsilon_F/\hbar$ values inferred without accounting for momentum-nonconserving tunneling events are so close to expected values demonstrates that momentum-nonconserving tunneling events are remarkably infrequent in the experiments of Ref. 5. This property allows the tunneling experiments to probe the properties of the 2DES's in more detail than would otherwise be possible.

IV. VERTEX CORRECTIONS

Now we will turn to the discussion of the case where correlations exist between the disorder potentials in the two 2DES's. It is useful to begin with a formally exact—for the idealized system of noninteracting electrons—discussion in terms of the exact eigenstates in the presence of disorder. We will see that when the disorder potentials in the two layers are similar, $V_l(\mathbf{r}) \sim V_r(\mathbf{r})$, the characteristics of the resonant tunneling change qualitatively when no magnetic field is presented. The resonant-tunneling peaks are always sharp, no matter how strong the disorder; i.e., disorder no longer suppresses the divergence in the tunneling conductance.

To leading order in $|t|$ we may use the Fermi-golden-rule expression for the tunneling conductance ($\mathbf{B}=0$):

$$\begin{aligned} I &= \frac{2\pi e}{\hbar}\sum_{\alpha\beta}|t|^2|\langle\Psi_{\alpha l}|\Psi_{\beta r}\rangle|^2\delta(\epsilon_{\alpha l}-\epsilon_{\beta r}) \\ &\quad\times[\theta(\varepsilon_{f,l}-\varepsilon_{\alpha l})-\theta(\epsilon_{f,r}-\epsilon_{\beta r})] \\ &= \frac{2\pi e^2 V^{\rm ext}}{\hbar}\sum_{\alpha\beta}|t|^2|\langle\Psi_{\alpha l}|\Psi_{\beta r}\rangle|^2 \\ &\quad\times\delta(\epsilon_{\alpha l}-\epsilon_{\beta r})\delta(\varepsilon_f-\epsilon_{\alpha l}), \end{aligned} \tag{14}$$

where $\Psi_{\alpha l}$, $\Psi_{\beta r}$ are the *exact* eigenstates of the disordered 2DEG systems and the second equality holds to leading order in the voltage difference between the layers. This discussion can be generalized to the case of larger $|t|$. When the electrons in different layers have identical disorder potentials $V_l(\mathbf{r}) \equiv V_r(\mathbf{r})$, the eigenstates are orthonormal $\langle\Psi_{\alpha l}|\Psi_{\beta r}\rangle = \delta_{\alpha,\beta}$ and $\epsilon_{\alpha l}-\epsilon_{\alpha r} = E_{ol}-E_{or}$ (E_{ol} E_{or} are the energies at the subband edges). In this case Eq. (14) gives the tunneling conductance as

$$\begin{aligned} \frac{I}{V^{\rm ext}} &= \frac{2\pi e^2}{\hbar}\sum_{\alpha}|t|^2\delta(E_{ol}-E_{or})\delta(\epsilon_\alpha-\epsilon_F) \\ &= \begin{cases} 0 & \text{if } E_{ol}\neq E_{or} \\ \infty & \text{if } E_{ol}=E_{or}. \end{cases} \end{aligned} \tag{15}$$

A δ-function-like sharp resonant-tunneling peak occurs regardless of the amount of disorder presented, provided that $V_l(\mathbf{r}) \equiv V_r(\mathbf{r})$.

Clearly, this exact result is not captured in the discussion of Sec. V. In Feynman diagram language, we have to sum over diagrams which are important when the disorder potentials in the two layers are strongly correlated. When disorder is treated in the Born approximation the vertex correction which yields a conserving approximation is shown in Fig. 3; it is given by a sum of ladder diagrams with electron propagators of opposite 2DES's connected by the rungs of the ladder. The first diagram in Fig. 3 is the "bubble" diagram which gives the results of the preceding section.

$$S_{\rm rl}(i\omega) = \sum_{ip_n}\mathcal{G}_{\mathbf{k},r}(ip_n)\mathcal{G}_{\mathbf{k},l}(ip_n+i\omega_n). \tag{16}$$

Adding the vertex correction:

$$\begin{aligned} S_{\rm rl}^{\rm lad}(i\omega) &= \frac{1}{\beta}\sum_{ip_n}\mathcal{G}_r(\mathbf{p},ip_n)\mathcal{G}_l(\mathbf{p},ip_n+i\omega) \\ &\quad\times\Gamma_{\mathbf{p}}(ip_n,ip_n+i\omega), \end{aligned} \tag{17}$$

where Γ satisfies the inhomogeneous integral equation,

$$\begin{aligned} &\Gamma_{\mathbf{K}}(ip_n,ip_n+i\omega) \\ &\quad= 1+\frac{1}{\hbar^2\nu}\sum_{\mathbf{K}'}f_{\rm rl}(\mathbf{k}-\mathbf{k}')\mathcal{G}_r(\mathbf{K}',ip_n)\mathcal{G}_l \\ &\qquad\times(\mathbf{K}',ip_n+i\omega)\Gamma_{\mathbf{K}'}(ip_n,ip_n+i\omega). \end{aligned} \tag{18}$$

Here the impurity-averaged disorder potential is

$$\langle V_r(\mathbf{r})V_l(\mathbf{r}')\rangle = \frac{1}{\nu}\sum_{\mathbf{P}}f_{\rm lr}(\mathbf{p})e^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')}. \tag{19}$$

To illustrate the physics, we consider the following short-range model-potential calculation:

$$\begin{aligned} f_{\rm rr}(\mathbf{p}) &= f_{\rm ll}(\mathbf{p}) = U^2, \\ f_{\rm lr}(\mathbf{p}) &= \lambda U^2, \end{aligned} \tag{20}$$

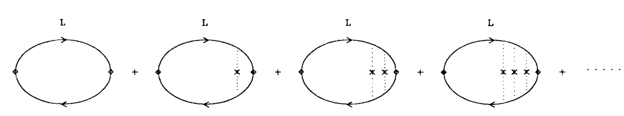

FIG. 3. Ladder diagrams for the vertex correction approximation for the current-current response function.

where $U^2 and\ \lambda$ are constants. The case of identical impurity potentials in the two 2DES's corresponds to $\lambda = 1$, and the uncorrelated layer limit in which the "bubble" approximation is appropriate corresponds to $\lambda = 0$. This simple disorder-potential model allows us to solve the integral equation [Eq. (18)] very easily, since Γ is now independent of wave vector:

$$\Gamma(ip_n, ip_n + i\omega) = \frac{1}{1 - \frac{\lambda U^2}{\hbar^2 \nu} \sum_{\mathbf{K}} \mathcal{G}_r(\mathbf{K}, ip_n) \mathcal{G}_l(\mathbf{K}, ip_n + i\omega)}. \tag{21}$$

With the above expression for Γ, we can evaluate Eq. (17) by contour integration.[9] After some standard manipulations we find that the conductance for $\omega \to 0$ and $\beta \to \infty$ (low temperatures) is

$$G = \frac{2t^2e^2}{\pi\hbar^3} \sum_{\mathbf{p}} \mathrm{Re}\{ G_r^{\mathrm{adv}}(\mathbf{p},0) G_l^{\mathrm{ret}}(\mathbf{p},0) \Gamma_{\mathbf{p}}(-i0^\dagger, i0^\dagger) - G_r^{\mathrm{adv}}(\mathbf{p},0) G_l^{\mathrm{adv}}(\mathbf{p},0) \Gamma_{\mathbf{p}}(i0^\dagger, i0^\dagger)\}. \tag{22}$$

Equation (22) is easily evaluated numerically and the results for $\mathbf{B} = 0$ are shown in Fig. 4. The second term on the right-hand side becomes negligible compared to the first for $\tau\epsilon_F/\hbar \gg 1$. We see that, for $\lambda = 1$, the resonant tunneling is singularly sharp even for a finite lifetime; this result is qualitatively different from the situation with $\lambda < 1$ and demonstrates that the ladder captures the exact result discussed above. Numerical results with finite magnetic fields are shown in Fig. 5; we see that while vertex corrections still reduce the effects of disorder, the effect is quickly suppressed with increasing field.

In typical double-layer samples the dopant layers are placed outside of each 2DES. In this case the disorder in each layer will come mainly from the closest dopant layer and we can expect the disorder potentials in the two layers to be weakly correlated. However, in some circumstances it may be desirable to place the doping layer in the middle of the barrier separating 2DES's. In this case the disorder potentials in the two layers will be strongly correlated. The calculations in this section show that, even though doping in the center of the barrier is likely to decrease τ substantially, it will not effect tunneling between the 2DES's at $B = 0$. Doping in the center of the barrier may increase the importance of momentum-nonconserving tunneling, however, and obscure the effect discussed in this section.

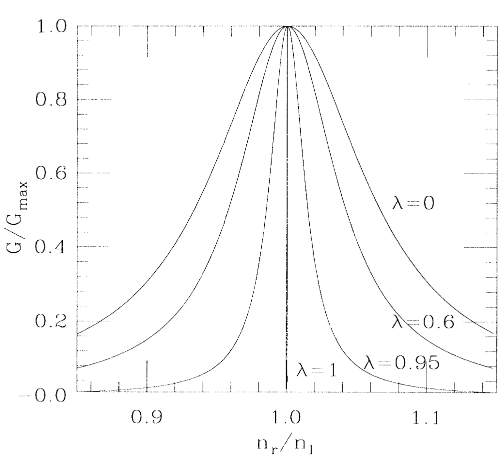

FIG. 4. Tunneling conductance vs layer density ratios at zero in-plane magnetic field for different degrees of correlation between the impurity potentials in the two 2DES's. As the correlation becomes strong ($\lambda \to 1$) tunneling occurs only at nearly equal densities even if there is substantial disorder in each layer.

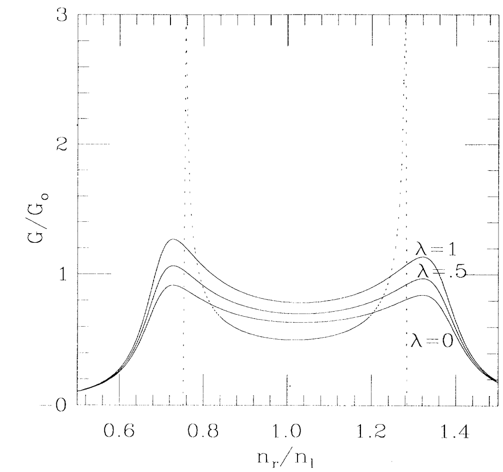

FIG. 5. Tunneling conductance vs layer density ratios at in-plane magnetic field $\mathbf{B} = 0.4$ T for different degrees of correlation between the impurity potentials in the two 2DES's. The correlation does not change the tunneling behavior qualitatively.

V. SUMMARY

In summary we have derived and evaluated expressions for the low-temperature dc equilibrium tunneling conductance between parallel two-dimensional electron systems. Analytic results were obtained for the dependence of the tunneling conductance on the strength of an in-plane magnetic field and on the layer density difference between the 2DES's. The effect of uncorrelated disorder on the tunneling conductance was discussed. Expressions relating measures of the broadening of conductance curves by disorder to $\tau\epsilon_F/\hbar$ were given and the possibility of using them to measure the electron lifetime in a 2DES was discussed. Comparisons of our theoretical results to the experimental results of Eisenstein *et al.* demonstrate that momentum-nonconserving tunneling events are remarkably infrequent in their samples. Finally we have shown that vertex corrections become important, especially at zero in-plane field, when strong correlations exist between the disorder potentials in the two 2DES's. These vertex corrections are likely to be important in practice in samples where the doping layer is in the middle of the barrier separating the 2DES's.

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[11]We make a common approximation by ignoring the contribution of disorder to the real part of the self-energy. This is justified when the disorder is sufficiently weak as is the case in high-mobility samples. In the Born approximation τ is the Fermi-golden-rule lifetime for an electron prepared in a state of definite wave vector.

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