Resonant reflection and transmission in a conducting channel with a single impurity

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(Received 25 September 1992)

We consider a narrow conducting channel in a two-dimensional electron gas with a single impurity in the bottleneck part of the channel. In contrast with previous works no specific assumptions concerning the shape of the channel and the impurity potential are made. It is shown that any attractive impurity potential generates sharp Breit-Wigner-type resonances—dips and peaks—in the graph of conductance versus Fermi energy. These correspond to resonant reflection and transmission due to quasibound states in the impurity potential. A simple formula for the conductance in terms of resonance energies and partial widths is found. The widths are directly related to the impurity and channel potentials. It is demonstrated that in the case of a uniform conducting channel any attractive impurity, no matter how weak, always produces zero in the conductance at a certain Fermi energy.

I. INTRODUCTION

The discovery of the quantized conductance steps in two-dimensional electron gas (2DEG) microconstrictions based on $GaAs/Al_xGa_{1-x}As$ heterostructures (see review papers[1,2]) followed an increase of interest in the study of quantum ballistic transport through narrow channels in 2DEG (see review[3]). In particular, the influence of impurities on the conductance attracted a great deal of attention since impurities inside or near the conducting channel may destroy the conductance quantization.[4–21] The effect of the impurities is especially strong near the steps, i.e., the thresholds where propagating modes are opened.

It is known from experiments as well as from theory that near the steps even a single impurity may strongly affect the conductance. For instance, measurements show[6] that when moving the position of the impurity with respect to the conducting channel, defined by a split gate, the conductance of the channel is changed drastically. Single impurity assisted resonant tunneling was observed in split-gate structures[10] and in quantum-well constrictions.[21]

The influence of a single impurity on the conductance of a 2DEG channel was studied theoretically in Refs. 12–14, 19, and 20. The theoretical treatment of this problem was based on two model potentials, of the channel and of the impurity. The simplest channel confining potential, which is an infinite uniform 2D wire (waveguide) with hard walls, was considered in papers 12–14. Actually, realistic narrow channels in split-gate devices cannot be taken as uniform wires, but rather as bottleneck constructions with expanding contact pads. The appropriate models for bottleneck constrictions are the saddle-point potential, considered in Refs. 19 and 20, and the finite wire opened to two infinite 2DEG's as in Ref. 14. As to the impurity potential, the short-range δ-type potential was used in almost all papers. There are only a few exceptions: a finite-range rectangular-shaped scatterer[13] and the scatterer of a finite range in the direction across the wire and δ type along the wire.[14]

It was shown that a single impurity produces fine-structure effects in the dependence of the conductance G on the Fermi energy E near the thresholds. For instance, an attractive impurity in an infinite uniform wire generates dips below the conductance steps.[12–14] These dips appear as a result of resonant reflection by quasibound states in the impurity potential. In some cases the value of the conductance at the threshold points is not influenced by the impurity. In a saddle-point potential and in a finite length wire attractive impurities may produce not only resonant reflection, but also resonant transmission. As a result, the conductance would show resonance peaks, as well as dips.[14,19,20] (Notice that a cavity inside the channel could also generate similar resonance dips and peaks.[22]) These sharp features in the energy dependence of the conductance near the thresholds are due to Breit-Wigner resonances.[23]

It is not clear from calculations with model potentials which features of the impurity-induced structure (dips, peaks, perfect transmission) are potential dependent and which are not.[13] Hence it is desirable to perform a general analysis of the resonant reflection and transmission due to a single impurity in a narrow channel making fewer assumptions about the confining and impurity potentials. Such general analysis which can illuminate the main physical features of the problem is the topic of the present work. We do not make any specific assumptions concerning the shape of impurity and channel potentials. Only the separation of the channel potential variables is assumed, so that the motions in the transverse direction and along the channel are factorized. We also assume that the quasibound states in the impurity potential are near the thresholds. For the sake of simplicity we consid-

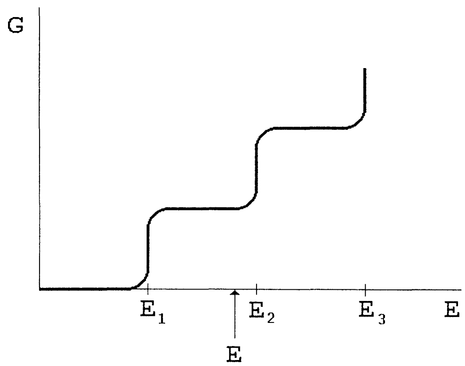

FIG. 1. Conductance steps and threshold energies.

er the region of the Fermi energies E near E_2, where the second mode is opened, Fig. 1. In this case one can take into account only the first evanescent mode, $n=2$, which is near the threshold. The higher evanescent modes, $n=3,4,\ldots$, give exponentially small contribution to the conductance, and can be neglected. (Their influence can be taken into account perturbatively.) This essentially simplifies the theory and makes the physics more transparent.

For a description of the quasibound (tunneling) states, generated by the impurity, we are following the approach developed in Ref. 24 for treatment of tunneling problems. It allows us to find a simple general formula for the conductance in terms of energy levels and partial widths of the quasibound state produced by the impurity, the levels and widths being directly connected to the impurity and channel potentials.

The outline of the paper is as follows. In Sec. II we set up the problem of a conducting channel with a single impurity in terms of a (2×2) transmission matrix. In Sec. III we consider a uniform conducting channel, and demonstrate explicitly how an arbitrarily small attractive impurity potential generates full reflection of the flux at some value of the energy. Next we discuss the case of a bottleneck confining potential. Section IV deals with a general description of quasibound (tunneling) states and the Green's functions near the resonances. The results obtained in this section are applied in Sec. V for the calculation of the transmission matrix of a bottleneck channel. Using this matrix we calculate in Sec. VI the conductance of such a channel with impurity. Some useful relations for wave and Green's functions of the one-dimensional Schrödinger equation are derived in Appendices A and B. Appendix C deals with the influence of neglected higher evanescent modes on the conductance.

II. FORMULATION OF THE PROBLEM

Consider a quasi-one-dimensional channel having the electron confined along the y direction but free to move along the x direction. Consider also an impurity inside the channel. The Schrödinger equation describing the electron motion in the (x,y) plane is

$$\left[-\frac{\hbar^2}{2m}\nabla^2+U(x,y)+V(x,y)\right]\Psi(x,y)=E\Psi(x,y)\ . \tag{2.1}$$

Here U is the confinement potential and V is the impurity potential. We assume that the channel potential allows separation of the variables,

$$U(x,y)=U(x)+W(y)\ . \tag{2.2}$$

The potential $W(y)$ provides confinement of the electron motion along the y direction. It would give rise to the channel modes $n=1,2\ldots$,

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dy^2}+W(y)\right]\Phi_n(y)=E_n\Phi_n(y)\ . \tag{2.3}$$

Here E_n is the threshold energy for the mode n and Φ_n is the corresponding eigenfunction. One can expand the wave function Ψ, Eq. (2.1), in terms of the channel-mode wave functions

$$\Psi(x,y)=\sum_n \psi_n(x)\Phi_n(y)\ . \tag{2.4}$$

Consider the case of $E_1<E<E_2$. Then only the first mode can propagate along the channel, where the others are not. However, if the electron energy E approaches E_2, the second (evanescent) mode can also contribute to the electron propagation due to coupling with the first mode by the impurity potential V and also due to tunneling, as will be seen from the following. We therefore keep only the first two terms in Eq. (2.4), $n=1,2$, and neglect all higher modes with $n>2$. Then substituting Eq. (2.4) into Eq. (2.1) one obtains

$$\begin{aligned}(K+U+V_{11})\psi_1+V_{12}\psi_2&=(E-E_1)\psi_1\ ,\\ (K+U+V_{22})\psi_2+V_{21}\psi_1&=(E-E_2)\psi_2\ ,\end{aligned} \tag{2.5}$$

where $K\equiv-(\hbar^2/2m)(d^2/dx^2)$, and

$$V_{nn'}(x)=\int dy\ \Phi_n(y)\Phi_{n'}(y)V(x,y)\ . \tag{2.6}$$

Since the impurity potential is localized, $V_{nn'}(\pm\infty)=0$. Let us consider only attractive impurities, so that $V_{11}(x), V_{22}(x)<0$. Then V_{22} always generates quasibound states near E_2 [since $V_{22}(x)$ is one-dimensional attractive potential]. We thus denote $V_{22}(x)\equiv V_b(x)$. Also we denote $V_{12}=V_{21}\equiv V_m(x)$ (since this potential is responsible for the mode mixing), and $U'(x)\equiv U(x)+V_{11}(x)$. Equations (2.5) in the new notations read

$$(\tilde{E}-K-U')\psi_1=V_m\psi_2\ , \tag{2.7a}$$

$$(\varepsilon-K-U-V_b)\psi_2=V_m\psi_1\ , \tag{2.7b}$$

where $\tilde{E}=E-E_1$ and $\varepsilon=E-E_2$. Since we consider $E\sim E_2$, then $\tilde{E}\cong E_2-E_1\gg|\varepsilon|$. Let us introduce the Green's functions

$$\begin{aligned}G_1&=(\tilde{E}-K-U')^{-1}\ ,\\ G_2&=(\varepsilon-K-U-V_b)^{-1}\ ,\end{aligned} \tag{2.8}$$

which describe one-dimensional motion in the potentials $U'(x)$ and $U(x)+V_b(x)$, shown in Fig. 2. It is assumed that $U(\pm\infty)=-U_\infty$. Notice that the Green's function $G_2(\varepsilon)$ is taken in the threshold region, and therefore it strongly depends on ε. On the other hand, $\tilde{E}$ is far away from the threshold and therefore the energy dependence of G_1 on $\tilde{E}$ is weak. Using these Green's functions one finds from Eqs. (2.7)

$$\psi_1=G_1V_m\psi_2 \,, \tag{2.9a}$$

$$\psi_2=G_2V_m\psi_1 \,. \tag{2.9b}$$

Substituting $\psi_{1,2}$ from Eqs. (2.9) into the right-hand side of Eqs. (2.7) one obtains equations for each of the modes,

$$(\tilde{E}-K-U')\psi_1=V_mG_2V_m\psi_1 \,, \tag{2.10a}$$

$$(\varepsilon-K-U-V_b)\psi_2=V_mG_1V_m\psi_2 \,. \tag{2.10b}$$

We also define wave functions $\chi^{\pm}_{1,2}(x)$ for the first and the second modes in the case of no mode mixing, $V_m=0$, and for $V_b=0$

$$(\tilde{E}-K-U')\chi_1^{\pm}=0 \,, \tag{2.11a}$$

$$(\varepsilon-K-U)\chi_2^{\pm}=0 \,. \tag{2.11b}$$

(General properties of these wave functions are described in Appendix A.) Since the potential $U(x)$ is a constant at infinity $(-U_\infty)$ the wave vectors $k_{1,2}$ which correspond to $\chi_{1,2}$ are

$$\hbar^2k_1^2/2m=\tilde{E}+U_\infty, \quad \hbar^2k_2^2/2m=\varepsilon+U_\infty \,. \tag{2.12}$$

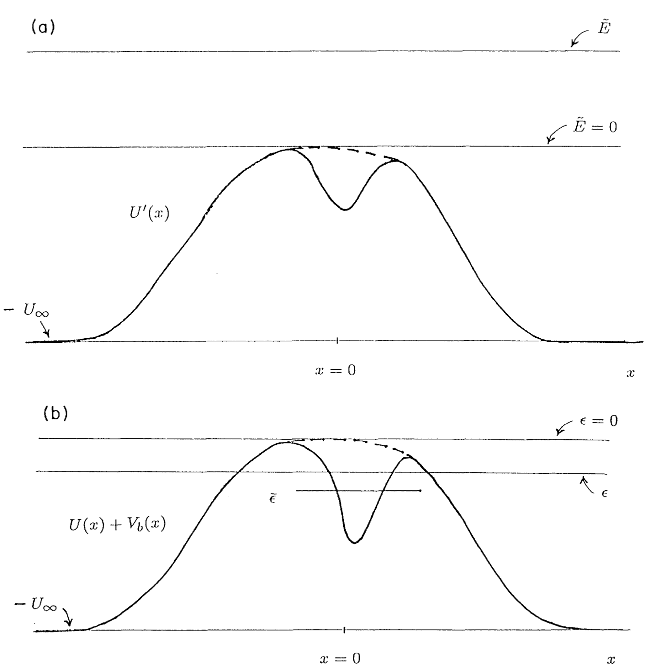

FIG. 2. (a) Effective one-dimensional potential for the propagating (above threshold) mode, $n=1$ [see Eq. (2.10a)]. (b) Effective one-dimensional potential for the threshold (tunneling) mode, $n=2$ [see Eq. (2.10b)].

In order to obtain the conductance G of a microconstriction one needs to solve the problem of wave penetration through such a structure. Consider, for instance, a wave which is arriving from $x=-\infty$. In the absence of impurities $\psi_{1,2}=\chi^+_{1,2}$ (for $U'=U$), and the conductance is expressed through the transmission coefficients $t_{1,2}$ according to the Landauer formula[25–27]

$$G=|t_1|^2+|t_2|^2 \tag{2.13}$$

(in units of $2e^2/h$). Here $t_{1,2}$ are the transmission coefficients of the waves $\chi^+_{1,2}$, defined in Appendix A. The impurity results in the mode mixing and in the appearance of the nondiagonal transmission coefficients, which are defined in the following way. Consider the solution of Eqs. (2.10) which contain only transmitted waves at $x\to+\infty$,

$$\psi_1(x)=B_1e^{ik_1x}, \quad \psi_2(x)=B_2e^{ik_2x} \,. \tag{2.14}$$

The same solution at $x\to-\infty$ is obviously given by

$$\begin{aligned}\psi_1(x)&=A_1e^{ik_1x}+C_1e^{-ik_1r} \,,\\ \psi_2(x)&=A_2e^{ik_2x}+C_2e^{-ik_2x} \,.\end{aligned} \tag{2.15}$$

The amplitudes of the scattered waves, $B_{1,2}$ and $C_{1,2}$, are linear combinations of the amplitudes of the incoming waves, $A_{1,2}$. In particular

$$\begin{bmatrix}B_1\\B_2\end{bmatrix}=\begin{bmatrix}T_{11}&T_{12}\\T_{21}&T_{22}\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix} \,. \tag{2.16}$$

The transmission matrix T allows us to obtain the conductance according to the Landauer formula

$$G=|T_{11}|^2+|T_{22}|^2+\frac{k_2}{k_1}|T_{12}|^2+\frac{k_1}{k_2}|T_{21}|^2 \,. \tag{2.17}$$

The factor k_1/k_2 takes into account that the fluxes in the modes 1 and 2 are proportional to k_1 and k_2. Notice that the absence of impurities corresponds to $T_{11}=t_1$, $T_{22}=t_2$, and $T_{12}=T_{21}=0$.

III. UNIFORM CONDUCTING CHANNEL

We start our analysis with the case of a uniform (nonexpanding) conducting channel, $U(x)=0$, in Eq. (2.5). [The motion is free along the x direction, while the confinement along the y direction is provided by the potential $W(y)$, Eq. (2.2).] For the sake of simplicity we neglect the potential $V_{11}(x)$ [i.e., $U'(x)=U(x)=0$ in Eqs. (2.10)], since $E-E_1\gg|V_{11}|$. Then the Green's functions G_1 can be written as

$$G_1(\tilde{E},x,x')=\frac{m}{ik}e^{ik|x-x'|} \,, \tag{3.1}$$

where $\hbar k\equiv\hbar k_1=\sqrt{2m\tilde{E}}$. Note that the total 2D Green's function of a nonexpanding channel in the absence of impurities, $G=(E-K-W)^{-1}$, can be written as

$$G(E;x,x';y,y')=\sum_n\frac{m}{ik_n}e^{ik_n|x-x'|}\Phi_n(y)\Phi_n^*(y') \,, \tag{3.2}$$

where $\hbar k_n=[2m(E-E_n)]^{1/2}$. It is clear from this equation that only the first mode propagates at infinity, and therefore only the transition amplitude T_{11} contributes to the total conductance, Eq. (2.17), i.e., $G=|T_{11}|^2$.

Consider Eq. (2.10a) describing the first mode in the energy region $\varepsilon\sim\varepsilon_0<0$, where ε_0 is the bound-state energy of the state φ_0 in the potential V_b, i.e.,

$$[K+V_b(x)]\varphi_0=\varepsilon_0\varphi_0 \ . \tag{3.3}$$

Then using the spectral representation of the Green's function $G_2=(\varepsilon-K-V_b)^{-1}$ for $\varepsilon\to\varepsilon_0$,

$$G_2(\varepsilon)=\frac{|\varphi_0\rangle\langle\varphi_0|}{\varepsilon-\varepsilon_0} \ , \tag{3.4}$$

one reduces Eq. (2.10a) to

$$(\tilde{E}-K)\psi_1=\hat{V}\psi_1 \ , \tag{3.5}$$

where the nonlocal potential $\hat{V}$ is energy dependent and has a separable form

$$\hat{V}(\varepsilon,x,x')=\frac{\langle x|V_m|\varphi_0\rangle\langle\varphi_0|V_m|x'\rangle}{\varepsilon-\varepsilon_0} \ . \tag{3.6}$$

The solution of Eq. (3.5) can be written straightforwardly in the form of the Born series

$$|\psi_1\rangle=|k\rangle+G_1\hat{V}|k\rangle+G_1\hat{V}G_1\hat{V}|k\rangle+\cdots \ , \tag{3.7}$$

where $\langle x|k\rangle=\exp(ikx)$. Using Eq. (3.1) for G_1 one finds that in the asymptotic region, $x\to+\infty$, Eq. (3.7) becomes

$$|\psi_1\rangle=|k\rangle+\frac{m}{ik}|k\rangle\langle k|\hat{V}|k\rangle +\frac{m}{ik}|k\rangle\langle k|\hat{V}G_1\hat{V}|k\rangle+\cdots \ . \tag{3.8}$$

Since the potential $\hat{V}$ is a separable one, the Born expansion (3.8) becomes a geometrical series and can be easily summed over. Indeed

$$\langle k|\hat{V}|k\rangle=\langle k|V_m|\varphi_0\rangle\frac{1}{\varepsilon-\varepsilon_0}\langle\varphi_0|V_m|k\rangle \ ,$$

$$\langle k|\hat{V}G_1\hat{V}|k\rangle=\langle k|V_m|\varphi_0\rangle\frac{\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle}{(\varepsilon-\varepsilon_0)^2} \times\langle\varphi_0|V_m|k\rangle \ , \tag{3.9}$$

and so on. Therefore the wave function $\psi_1(x)$ for $x\to+\infty$ is

$$\psi_1(x)=e^{ikx}+\frac{m}{ik}e^{ikx}\frac{|\langle k|V_m|\varphi_0\rangle|^2}{\varepsilon-\varepsilon_0-\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle} \ . \tag{3.10}$$

Using Eq. (3.1) one easily obtains that

$$\mathrm{Im}\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle=-\frac{m}{k}|\langle\varphi_0|V_m|k\rangle|^2\equiv-\Gamma_m \tag{3.11}$$

Therefore Eq. (3.10) can be rewritten in the form

$$\psi_1(x)=e^{ikx}\left[1-\frac{i\Gamma_m}{\varepsilon-\varepsilon_0-\Delta_m+i\Gamma_m}\right] \ , \tag{3.12}$$

where $\Delta_m=\mathrm{Re}\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle$, Δ_m and Γ_m being the shift and the width which acquire the bound state due to mode mixing. It follows from Eq. (3.12) that for $\varepsilon=\varepsilon_0+\Delta_m$ the wave function $\psi_1(x)=0$ for $x\to+\infty$, which corresponds to the total reflection of the incoming flux. Since $V_b(x)$ in Eq. (3.3) is the one-dimensional potential, generated by an attractive impurity, it always contains at least one bound state. Therefore any attractive impurity in a uniform conducting channel would produce total (resonant) reflection at certain Fermi energies, irrespective of how small the attractive impurity potential is. It is only the width of the corresponding dip, Eq. (3.11), which decreases with the impurity potential strength. In the following we demonstrate that the potential $V_{11}(x)$ and the higher evanescent modes, which were neglected in this section, do not affect the exact vanishing of the conductance, but only the position of the dip.

IV. QUASIBOUND STATES

Going to the general case of a nonuniform bottleneck-type conducting channel we first consider the states which are responsible for the singularities in the Green's function in the complex energy plane. When $U(x)\neq 0$ there are no bound states in the potential V_b+U [see Fig. 2(b)]. However, if the potential barriers separating the well $V_b(x)$ from $x=+\infty$ and $-\infty$ are wide enough, the bound state defined by Eq. (3.3) is transformed to a quasibound state (resonance) near ε_0, which acquires small width and shift, $\varepsilon_0\to\bar{\varepsilon}-i\Gamma_t$, due to tunneling through the barriers to the continuum. Similar to the previous case, Eq. (3.4), we expect that near the resonance, $\varepsilon\to\bar{\varepsilon}-i\Gamma_t$, the Green's function $G_2(\varepsilon)$ can be written as

$$G_2(\varepsilon)=\frac{|\varphi\rangle\langle\varphi|}{\varepsilon-\bar{\varepsilon}+i\Gamma_t} \ , \tag{4.1}$$

where the bound-state wave function $|\varphi_0\rangle$ is replaced by the resonance wave function $|\varphi\rangle$.

For a description of the Green's function in the resonance region we use with some modification a method developed in Ref. 24. Following this method we build up the quasibound state from the corresponding stationary state, by considering the potential U as a perturbation. To do this we rewrite the Green's function G_2, Eq. (2.8), in the form of a Born series

$$G_2=G_0+G_0UG_0+G_0UG_0UG_0+\cdots \ , \tag{4.2}$$

with

$$G_0(\varepsilon)=(\varepsilon-K-V_b)^{-1} \ . \tag{4.3}$$

From this series the equation for G_2 follows:

$$G_2=G_0+G_0UG_2 \ . \tag{4.4}$$

It is convenient to introduce the energy shift operator

$$R=U+UG_2U \; , \tag{4.5}$$

which satisfies the Lippmann-Schwinger equation

$$R=U+UG_0R \; . \tag{4.6}$$

In terms of this operator G_2 can be represented as

$$G_2=G_0+G_0RG_0 \; . \tag{4.7}$$

Now we introduce the Green's function $\tilde{G}_0=\Lambda G_0$, where $\Lambda=1-|\varphi_0\rangle\langle\varphi_0|$ is the projection operator which excludes the bound state $|\varphi_0\rangle$ from the spectral representation of $\tilde{G}_0$. Replacing G_0 in Eqs. (4.2)–(4.7) by $\tilde{G}_0$ we define a projected Green's function $\tilde{G}_2$ and energy shift operator $\tilde{R}$, which obey the following equations:

$$\tilde{G}_2=\tilde{G}_0+\tilde{G}_0U\tilde{G}_2 \; , \tag{4.8}$$

$$\tilde{R}=U+U\tilde{G}_0\tilde{R} \; , \tag{4.9}$$

with

$$\tilde{R}=U+U\tilde{G}_2U \; . \tag{4.10}$$

Notice that due to projection operator Λ, the resonance corresponding to the bound state $|\varphi_0\rangle$ is excluded from $\tilde{G}_2$ and $\tilde{R}$ (see Ref. 24).

Multiplying Eq. (4.9) by $(1+RG_0)$ and using Eq. (4.6) with $UG_0R=RG_0U$ we obtain that the operators R and $\tilde{R}$ are related through the formula

$$\tilde{R}-R=R(\tilde{G}_0-G_0)\tilde{R}=-R\frac{|\varphi_0\rangle\langle\varphi_0|}{\varepsilon-\varepsilon_0}\tilde{R} \; . \tag{4.11}$$

Multiplying this equation by $|\varphi_0\rangle$ we easily obtain

$$R|\varphi_0\rangle=\frac{(\varepsilon-\varepsilon_0)\tilde{R}|\varphi_0\rangle}{\varepsilon-\varepsilon_0-\langle\varphi_0|\tilde{R}|\varphi_0\rangle} \; . \tag{4.12}$$

Consider the energy shift operator R. Substituting Eq. (4.1) into Eq. (4.5) one obtains for $\varepsilon\to\bar{\varepsilon}-i\Gamma_t$

$$R=\frac{U|\varphi\rangle\langle\varphi|U}{\varepsilon-\bar{\varepsilon}+i\Gamma_t} \; . \tag{4.13}$$

Substituting Eq. (4.13) into Eq. (4.12) and using Eq. (4.10) for the shift operator $\tilde{R}$ one finds in the limit $\varepsilon\to\varepsilon_0+\langle\varphi_0|\tilde{R}|\varphi_0\rangle$

$$\bar{\varepsilon}-i\Gamma_t=\varepsilon_0+\langle\varphi_0|U|\varphi_0\rangle+\langle\varphi_0|U\tilde{G}_2U|\varphi_0\rangle \tag{4.14}$$

and

$$\varphi=\varphi_0+\tilde{G}_2U\varphi_0 \; . \tag{4.15}$$

Let us rewrite Eq. (4.15) explicitly as

$$\varphi(x)=\varphi_0(x)+\int dx'\tilde{G}_2(x,x')U(x')\varphi_0(x') \; . \tag{4.16}$$

Notice that $\tilde{G}_2(x,x')$ decreases exponentially whenever one of the arguments is inside the potential barrier U. One can get it by using the spectral representation of the Green's function, and taking into account that the resonance functions are excluded by the projection operator Λ (see Ref. 24). (Otherwise $\tilde{G}_2$ would exponentially increase inside the barrier.) Hence, the second term in Eq. (4.16) would be exponentially suppressed with respect to the first one in the region of small x (inside the impurity potential). On the other hand, the second term of Eq. (4.16) dominates at $x\to\pm\infty$, since the bound-state wave function $\varphi_0(x)$ decreases exponentially outside the range of $V_b(x)$. Consider therefore the second term in Eq. (4.16) at large x. Since small values of x' are suppressed in the integral by $U(x')\to 0$ when $x'\to 0$ [Fig. 2(b)], both arguments of $\tilde{G}_2(x,x')$ are effectively outside the range of $V_b(x)$. Then one can approximate

$$\tilde{G}_0=\Lambda(\varepsilon_0-K-V_b)^{-1}\cong(\varepsilon_0-K)^{-1} \; . \tag{4.17}$$

Here we replaced ε by ε_0 neglecting the terms of second order in the tunneling probability. Substituting this result into Eq. (4.8) we get

$$\tilde{G}_2\cong(\varepsilon_0-K-U)^{-1} \; . \tag{4.18}$$

[The full perturbative expansion of the exact Green's function $\tilde{G}_2$, Eq. (4.8), in terms of the approximate one, Eq. (4.18), and evaluation of the correction terms can be found in Ref. 24.] Let us express the Green's function $\tilde{G}_2(x,x')$ in terms of $\chi_2^{\pm}$, Eq. (2.11b), by use of Eqs. (A2), (A3), and (B2). Then substituting $\tilde{G}_2$ into Eq. (4.16) one finds

$$\varphi(x\to\pm\infty)=\frac{m}{ik_2}e^{\pm ik_2x}\langle\varphi_0|U|\chi_2^{\mp}\rangle \; . \tag{4.19}$$

Notice that by using the Schrödinger equation (2.11b) one can rewrite the matrix elements in Eq. (4.19) as

$$\langle\varphi_0|U|\chi_2^{\mp}\rangle=-\langle\varphi_0|K-\varepsilon_0|\chi_2^{\mp}\rangle=\langle\varphi_0|V_b|\chi_2^{\mp}\rangle \; . \tag{4.20}$$

The matrix elements in Eq. (4.19) are directly connected to the width Γ_t of the quasistationary state. Indeed, using Eqs. (4.14) and (B12) we get

$$\Gamma_t=-\,\mathrm{Im}\langle\varphi_0|U\tilde{G}_2U|\varphi_0\rangle=\Gamma_t^{+}+\Gamma_t^{-} \; , \tag{4.21}$$

with

$$\Gamma_t^{\pm}=\frac{m}{2k_2}|\langle\varphi_0|U|\chi_2^{\mp}\rangle|^2 \; , \tag{4.22}$$

where $\Gamma_t^{\pm}$ are the partial widths due to tunneling to the right and to the left.

V. TRANSMISSION MATRIX FOR A BOTTLENECK CONDUCTING CHANNEL

Using the properties of the quasibound states from the preceding section we can extend our treatment of a uniform conducting channel to a nonuniform (bottleneck) channel. Again, for calculation of the total conductance we use the Landauer formula, Eq. (2.17). However, in contrast with the case of uniform channel, the second mode is propagating through the barrier by a tunneling, Fig. 2(b), and therefore all the elements of the transition matrix T contribute to the conductance G. Since the problem is linear we can consider propagation of each of the modes separately.

A. Propagating (above threshold) mode

We begin with the case $A_1=1$ and $A_2=0$ in Eqs. (2.15), which correspond to the first mode ($n=1$) coming from $x=-\infty$. Since $\varepsilon \sim \varepsilon_0$, we can retain only the pole term, Eq. (4.1) in the Green's function G_2, (2.8). Substituting Eq. (4.1) into Eq. (2.10a) we can rewrite it as

$$(\tilde{E}-K-U')\psi_1=\frac{V_m|\varphi\rangle\langle\varphi|V_m}{\varepsilon-\bar{\varepsilon}+i\Gamma_t}\psi_1 \; . \tag{5.1}$$

Similar to the previous case of the uniform channel, the solution can be written as the Born series [cf. Eq. (3.7)]

$$|\psi_1\rangle=|\chi_1^+\rangle+G_1V_m|\varphi\rangle\frac{1}{\varepsilon-\bar{\varepsilon}+i\Gamma_t}\langle\varphi|V_m|\chi_1^+\rangle + G_1V_m|\varphi\rangle\frac{\langle\varphi|V_mG_1V_m|\varphi\rangle}{(\varepsilon-\bar{\varepsilon}+i\Gamma_t)^2}\langle\varphi|V_m|\chi_1^+\rangle+\cdots \; , \tag{5.2}$$

where χ_1^+ is the solution of Eq. (2.11a) and G_1 is given by Eq. (2.8). This expansion is a geometrical series and therefore can be summed over to

$$|\psi_1\rangle=|\chi_1^+\rangle+G_1V_m|\varphi_0\rangle\frac{\langle\varphi_0|V_m|\chi_1^+\rangle}{\varepsilon-\bar{\varepsilon}+i\Gamma_t-\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle} \; . \tag{5.3}$$

Here we replaced φ by φ_0 in all matrix elements of Eq. (5.3), since inside the range of the impurity potential $V_m(x)$ the wave function $\varphi(x)\cong\varphi_0(x)$.

Consider the wave function $\psi_1(x)$ in the asymptotic region. Using Eq. (B2) for the Green's function $G_1(x,x')$, we obtain for $x\to+\infty$

$$\psi_1(x)=\chi_1^+(x)+\chi_1^+(x)\left[\frac{m}{ik_1t_1}\right]\times\frac{\langle\varphi_0|V_m|\chi_1^-\rangle\langle\varphi_0|V_m|\chi_1^+\rangle}{\varepsilon-\bar{\varepsilon}-\Delta_m+i(\Gamma_t+\Gamma_m)} \; , \tag{5.4}$$

where

$$\Delta_m-i\Gamma_m=\langle\varphi_0|V_mG_1V_m|\varphi_0\rangle \tag{5.5}$$

[see Eq. (B12)], and t_1 is the transmission coefficient for the first mode in the absence of the mode mixing potential V_m, Eq. (A2). Δ_m and Γ_m are the shift and the width of the bound state due to mode mixing. Using Eq. (B9) we can write $\Gamma_m=\Gamma_m^++\Gamma_m^-$, where

$$\Gamma_m^{\pm}=\frac{m}{2k_1}|\langle\varphi_0|V_m|\chi_1^{\pm}\rangle|^2 \; , \tag{5.6}$$

i.e., $\Gamma_m^{\pm}$ is the decay rates due to mode mixing to the right (left) infinity.

It follows from Eqs. (B8) and (B10) that the numerator of the second term in Eq. (5.4) can be represented as

$$\frac{m}{ik_1t_1}\langle\varphi_0|V_m|\chi_1^-\rangle\langle\varphi_0|V_m|\chi_1^+\rangle=\delta_m-i\Gamma_m \; , \tag{5.7}$$

where Γ_m is given by Eq. (5.5) and

$$\delta_m=\frac{m}{k_1}\,\mathrm{Im}\left\{\frac{r_{1+}}{t_1}\langle\varphi_0|V_m|\chi_1^-\rangle\langle\chi_1^+|V_m|\varphi_0\rangle\right\} \; . \tag{5.8}$$

The diagonal transmission coefficient for the first mode, T_{11}, in the presence of impurity, is therefore

$$T_{11}=t_1\left[1+\frac{\delta_m-i\Gamma_m}{\varepsilon-\bar{\varepsilon}-\Delta_m+i(\Gamma_t+\Gamma_m)}\right] \; . \tag{5.9}$$

In particular one obtains from this expression that for the uniform conducting channel ($\Gamma_t=0$ and $\bar{\varepsilon}=\varepsilon_0$), the total transmission is zero at $\varepsilon=\varepsilon_0+\Delta_m-\delta_m$. It shows that the result of Sec. III holds even if the potential $V_{11}(x)$ is taken into account.

Let us consider the nondiagonal transmission coefficient T_{12}, which is the probability of finding the electron in the second mode at $x\to+\infty$. Substituting ψ_1 given by Eq. (5.3) into Eq. (2.9b) and using Eq. (4.1) for the Green's function G_2, one easily obtains

$$\psi_2(x)=\frac{\langle\varphi_0|V_m|\chi_1^+\rangle}{\varepsilon-\bar{\varepsilon}-\Delta_m+i(\Gamma_t+\Gamma_m)}\varphi(x) \; . \tag{5.10}$$

Going to the limit $x\to+\infty$ and using Eqs. (4.19) and (4.22) we get

$$T_{12}=\frac{m}{ik_2}\frac{\langle\varphi_0|V_m|\chi_1^+\rangle\langle\varphi_0|V_b|\chi_2^-\rangle}{\varepsilon-\bar{\varepsilon}-\Delta_m+i(\Gamma_t+\Gamma_m)} \; . \tag{5.11}$$

B. Threshold (tunneling) mode

Consider now the case where $A_1=0$ and $A_2=1$ in Eqs. (2.15), which corresponds to the second mode coming from $x=-\infty$. The solution of Eq. (2.10b) can be written as the Born series [cf. Eqs. (3.7)]

$$|\psi_2\rangle=|\psi_{2t}\rangle+G_2V_mG_1V_m|\psi_{2t}\rangle + G_2V_mG_1V_mG_2V_mG_1V_m|\psi_{2t}\rangle+\cdots \; , \tag{5.12}$$

where $\psi_{2t}(x)$ is given by

$$(\varepsilon-K-U-V_b)\psi_{2t}=0 \; . \tag{5.13}$$

It describes the resonant tunneling of the second mode through the impurity V_b without the mode mixing. We can write $\psi_{2t}(x)$ in the form

$$\psi_{2t}=\chi_2^++\psi'_{2t} \; , \tag{5.14}$$

where χ_2^+ is given by Eq. (2.11b), and describes the tunneling of the second mode through the constriction without impurity. In order to find ψ'_{2t} we substitute Eq. (5.14) into Eq. (5.13). One gets

$$(\varepsilon-K-U-V_b)\psi'_{2t}=V_b\chi_2^+ \; . \tag{5.15}$$

Using Eq. (4.1) for the Green's function G_2 near the resonance we obtain

$$\psi'_{2t}=\frac{\langle\varphi_0|V_b|\chi_2^+\rangle}{\varepsilon-\bar{\varepsilon}+i\Gamma_t}\varphi \; , \tag{5.16}$$

where we replaced $\varphi(x)$ by $\varphi_0(x)$ in the matrix element.

In the asymptotic region $x \to +\infty$ the wave function $\varphi(x)$ is given by Eq. (4.19). Then using Eqs. (4.19), (5.14), and (A2) one finds that for $x \to +\infty$, the wave function is $\psi_{2t}(x) = T_{2t} \exp(ik_2 x)$ where

$$T_{2t} = t_2 + \frac{m}{ik_2} \frac{\langle \varphi_0 | V_b | \chi_2^+ \rangle \langle \varphi_0 | V_b | \chi_2^- \rangle}{\varepsilon - \bar{\varepsilon} + i\Gamma_t} , \tag{5.17}$$

with t_2 being the (nonresonant) transmission coefficient through the constriction without impurity, which is of the order of $\Gamma_t^+ \Gamma_t^-$. If one neglects in Eq. (5.17) the nonresonant contribution t_2, and calculates $|T_{2t}|^2$ using Eqs. (4.22) one obtains the well-known formula for resonant tunneling transition probability,

$$|T_{2t}|^2 = \frac{4\Gamma_t^+ \Gamma_t^-}{(\varepsilon - \bar{\varepsilon}) + \Gamma_t^2} . \tag{5.18}$$

Let us return to Eq. (5.12) for the second mode wave function. Using Eq. (4.1) we can write

$$\begin{aligned} |\psi_2\rangle &= |\psi_{2t}\rangle + |\varphi\rangle \frac{1}{\varepsilon - \bar{\varepsilon} + i\Gamma_t} \langle \varphi_0 | V_m G_1 V_m | \psi_{2t} \rangle \\ &\quad + |\varphi\rangle \frac{\langle \varphi_0 | V_m G_1 V_m | \varphi_0 \rangle}{(\varepsilon - \bar{\varepsilon} + i\Gamma_t)^2} \langle \varphi_0 | V_m G_1 V_m | \psi_{2t} \rangle + \cdots \\ &= |\psi_{2t}\rangle + |\varphi\rangle \frac{\langle \varphi_0 | V_m G_1 V_m | \psi_{2t} \rangle}{\varepsilon - \bar{\varepsilon} + i\Gamma_t - \langle \varphi_0 | V_m G_1 V_m | \varphi_0 \rangle} . \end{aligned} \tag{5.19}$$

Using Eqs. (5.14) and (5.16) for ψ_{2t}, and Eq. (5.5) we easily obtain

$$\begin{aligned} |\psi_2\rangle &= |\chi_2^+\rangle + \frac{\langle \varphi_0 | V_b | \chi_2^+ \rangle + \langle \varphi_0 | V_m G_1 V_m | \chi_2^+ \rangle}{\varepsilon - \bar{\varepsilon} + i\Gamma_t - \langle \varphi_0 | V_m G_1 V_m | \varphi_0 \rangle} |\varphi\rangle \\ &\cong |\chi_2^+\rangle + \frac{\langle \varphi_0 | V_b | \chi_2^+ \rangle}{\varepsilon - \bar{\varepsilon} - \Delta_m + i(\Gamma_t + \Gamma_m)} |\varphi\rangle , \end{aligned} \tag{5.20}$$

where we neglected the term $\langle \varphi_0 | V_m G_1 V_m | \chi_2^+ \rangle$ since it is of second order in the impurity potential. Then using Eqs. (4.19), (4.20), and (A2) for the wave functions at $x \to +\infty$, we find

$$T_{22} = t_2 + \frac{m}{ik_2} \frac{\langle \varphi_0 | V_b | \chi_2^+ \rangle \langle \varphi_0 | V_b | \chi_2^- \rangle}{\varepsilon - \bar{\varepsilon} - \Delta_m + i(\Gamma_t + \Gamma_m)} . \tag{5.21}$$

The last transmission matrix element which we have to calculate is T_{21}. It is a probability of finding the electron in the first mode at $x \to +\infty$. (Notice that in general $T_{21} \neq T_{12}$.) Substituting ψ_2 given by Eq. (5.20) into Eq. (2.9a) we find

$$|\psi_1\rangle = G_1 V_m |\chi_2^+\rangle + G_1 V_m |\varphi\rangle \frac{\langle \varphi_0 | V_b | \chi_2^+ \rangle}{\varepsilon - \bar{\varepsilon} - \Delta_m + i(\Gamma_t + \Gamma_m)} . \tag{5.22}$$

Then using Eq. (B2) for the Green's function G_1 and considering the limit $x \to +\infty$ we obtain

$$T_{21} = \frac{m}{ik_1} \frac{\langle \varphi_0 | V_m | \chi_1^- \rangle \langle \varphi_0 | V_m | \chi_2^+ \rangle}{\varepsilon - \bar{\varepsilon} - \Delta_m + i(\Gamma_t + \Gamma_m)} , \tag{5.23}$$

where the nonresonant contribution from the first term in Eq. (5.22) is neglected.

VI. CONDUCTANCE OF A BOTTLENECK CHANNEL

The full conductance of the channel can be calculated from the Landauer formula, Eq. (2.17), introducing into it Eqs. (5.9), (5.11), (5.21), and (5.23). If we neglect the nonresonant contribution in (5.21), the conductance can be expressed in terms of decay widths

$$\begin{aligned} G &= |t_1|^2 \left| 1 + \frac{\delta_m - i\Gamma_m}{\varepsilon - \bar{\varepsilon} - \Delta_m + i\Gamma} \right|^2 \\ &\quad + 4 \frac{\Gamma_t^+ \Gamma_t^- + \Gamma_m^- \Gamma_t^+ + \Gamma_m^+ \Gamma_t^-}{(\varepsilon - \bar{\varepsilon} - \Delta_m)^2 + \Gamma^2} , \end{aligned} \tag{6.1}$$

where $\Gamma = \Gamma_t + \Gamma_m$. To make the result more transparent we assume the potential $U(x)$ to be a smooth one, which means that the effective width of the channel varies slowly. Then we can use the following approximation:

$$t_1 = 1, \quad r_{1\pm} = 0, \quad \chi_1^{\pm} = e^{\pm ik_1 x} . \tag{6.2}$$

With this assumption it follows from Eqs. (5.6) and (5.8) that

$$\Gamma_m^+ = \Gamma_m^- = \tfrac{1}{2}\Gamma_m, \quad \delta_m = 0 . \tag{6.3}$$

In the same approximation, when there is no impurity in the channel

$$T_{11} = t_1 = 1, \quad T_{22} = t_2 = 0, \quad T_{12} = T_{21} = 0 , \tag{6.4}$$

and $G_0 = 1$. Now we have the final result

$$\Delta G = G - G_0 = \frac{4\Gamma_t^+ \Gamma_t^- - \Gamma_m^2}{(\varepsilon - \bar{\varepsilon} - \Delta_m)^2 + \Gamma^2} . \tag{6.5}$$

Equations (6.1) and (6.5) are the main result of our paper. It shows that the type of Breit-Wigner resonance (dip or peak) depends on the competition between the two decay routes of the quasibound states, (i) due to intermode mixing and (ii) due to intramode tunneling. Mixing favors dips, while tunneling favors peaks. If one neglects tunneling, ΔG exhibits a dip with $\max \Delta G = -1$. If one neglects mode mixing ΔG exhibits a peak. In this case $\max \Delta G = +1$ only when the impurity is located in the channel symmetrically, i.e., $\Gamma_t^+ = \Gamma_t^-$.

It is easy to understand from our derivation why the intermode mixing generates the resonant reflection, but not the resonant transmission, as in the case of the intramode tunneling. Let us compare Eq. (3.10) describing the propagating mode ψ_1 in the uniform channel with Eq. (5.17) describing the threshold mode ψ_{2t} in the case of the resonant tunneling. In both cases the interaction with the impurity generates the same (Breit-Wigner) type of scattering amplitude. However, in the case of propagating mode the scattered wave does interfere (destructively) with the initial plane wave, whereas in the case of the threshold mode the initial wave is almost zero at $x\to+\infty$ [$t_2 \ll 1$ in Eq. (5.17)].

It can be easily shown that the result given by Eq. (6.5) is valid in the vicinity of any threshold, $n=1,2,\ldots$. (Considering the vicinity of the threshold $n=1$ one has to set $\Gamma_m=0$, see Ref. 19.) If there is more than one impurity in the bottleneck of the conducting channel one can consider $V(x,y)$ in Eq. (2.1) as the total impurity potential and consider the quasibound states as due to this total potential. Equations (6.1) and (6.5) are valid in this case too.

It follows from Eq. (2.6) that the potentials V_m and V_b in Eq. (2.10) depend on the position of the impurity with respect to the channel "walls." For instance, these potentials are strongly reduced if the impurity position in the y coordinate falls near a knot of the mode eigenfunctions $\Phi_{1,2}(y)$. This means that the resonance energies and the widths in Eq. (6.5) depend on the position of the impurity. Applying different gate voltage to the two parts of the split gate[6] one can "shift" the impurity and therefore change the position and the type of the resonance.

It is worthwhile to mention that the total reflection in the case of a uniform channel (Sec. III) is a consequence of two dimensionality of the original Schrödinger equation (2.1) (which is reduced to an effective 1D Schrödinger equation (3.5) with a *nonlocal* potential). The one-dimensional Schrödinger equation with a local potential can never generate total resonant reflection.

ACKNOWLEDGMENTS

We acknowledge P. Price for discussion and M. Büttiker for pointing out the connection of the considered problem with the phenomenological approach based on the concept of Breit-Wigner resonances. One of the authors (Y.L.) acknowledges A. Larkin and N. Sivan for some remarks relating resonance scattering, and the Weimann Institute of Science for support.

APPENDIX A: WAVE FUNCTIONS FOR ONE-DIMENSIONAL SCHRÖDINGER EQUATION

In this appendix we present for reference purposes some properties of the solutions of the 1D Schrödinger equation. Consider the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U(x)\right]\chi(x)=E\chi(x)\ , \tag{A1}$$

with a potential U constant at $x=\pm\infty$. We choose $U(\pm\infty)=0$ and $E>0$. For each $k=\sqrt{2mE}\,/\hbar$ there are two eigenfunctions $\chi_k^{\pm}(x)$, which correspond to the waves incoming from $\pm\infty$, i.e.,

$$\chi_k^+(x)=\begin{cases} t_+e^{ikx} & \text{if } x\to+\infty \\ e^{ikx}+r_+e^{-ikx} & \text{if } x\to-\infty \end{cases} \tag{A2}$$

and

$$\chi_k^-(x)=\begin{cases} e^{-ikx}+r_-e^{ikx} & \text{if } x\to+\infty \\ t_-e^{ikx} & \text{if } x\to-\infty\ . \end{cases} \tag{A3}$$

Here $t_\pm$ and $r_\pm$ are the transmission and reflection coefficient of these waves.

The Wronskian

$$W(x)\equiv W\{\chi_k^+,\chi_k^-\}=\chi_k^+\frac{d}{dx}\chi_k^- - \chi_k^-\frac{d}{dx}\chi_k^+ \tag{A4}$$

is independent of x. Calculating $W(x)$ for $x=\pm\infty$ by substituting (A2) and (A3) into (A4) one obtains $W(-\infty)=-2ikt_-$ and $W(+\infty)=-2ikt_+$. Hence

$$t_+=t_-=t\ . \tag{A5}$$

Let us make use of the time-reversal symmetry. Consider the function $\chi_k^+(x)^*$ which is a solution of Eq. (A1) with the boundary condition

$$\chi_k^+(x)^*=t^*\exp(-ikx)\quad \text{for } x\to+\infty\ . \tag{A6}$$

On the other hand, it can be written as a linear combination of two linear independent eigenfunctions $\chi_k^{\pm}(x)$,

$$\chi_k^+(x)^*=A\chi_k^+(x)+B\chi_k^-(x)\ . \tag{A7}$$

Setting here $x\to+\infty$ and using Eqs. (A2) and (A3) one obtains

$$\chi_k^+(x)^*=(At+Br_-)e^{ikx}+Be^{-ikx}\ . \tag{A8}$$

From the comparison of Eqs. (A6) and (A8) one finds

$$\begin{aligned} &At+Br_-=0\ ,\\ &B=t^*\ . \end{aligned} \tag{A9}$$

Calculating A and B from Eq. (A9) and substituting into Eq. (A7) we get

$$\chi_k^+(x)^*=-\frac{t^*}{t}r_-\chi_k^+(x)+t^*\chi_k^-(x)\ . \tag{A10}$$

In a similar way one finds

$$\chi_k^-(x)^*=-\frac{t^*}{t}\chi_k^-(x)+t^*\chi_k^+(x)\ . \tag{A11}$$

Now setting $x\to-\infty$ in Eq. (A10) and $x\to+\infty$ in Eq. (A11) and comparing these equations with Eqs. (A2) and (A3), one obtains the following relations for the transmission and reflection coefficients:

$$\begin{aligned} &tr_+^*+t^*r_-=0\ ,\\ &|t|^2+|r_+|^2=1\ ,\\ &|t|^2+|r_-|^2=1\ . \end{aligned} \tag{A12}$$

Notice that Eqs. (A12) represent the unitarity conditions for the scattering matrix

$$S=\begin{bmatrix} t & r_+ \\ r_- & t \end{bmatrix}\ . \tag{A13}$$

Using Eqs. (A12) one can derive from Eqs. (A10) and (A11) the expressions for χ_k^- in terms of χ_k^+ and vice versa,

$$\chi_k^-(x)=\frac{1}{t^*}\chi_k^+(x)^*+\frac{r_-}{t}\chi_k^+(x)\ ,$$
$$\chi_k^+(x)=\frac{1}{t^*}\chi_k^-(x)^*+\frac{r_+}{t}\chi_k^-(x)\ . \tag{A14}$$

APPENDIX B: GREEN'S FUNCTIONS OF THE ONE-DIMENSIONAL SCHRÖDINGER EQUATION

In this appendix we present for reference purposes some properties of the Green's function of the 1D Schrödinger equation. The Green's function G_E of Eq. (A1) is defined by

$$\left[E+\frac{\hbar^2}{2m}\frac{d^2}{dx^2}-U(x)\right]G_E(x,x')=\delta(x-x')\ . \tag{B1}$$

We are interested in the outgoing wave Green's function, i.e., the boundary conditions are $G_E(x,x')\sim\exp(\pm ikx)$ for $x\to\pm\infty$. Notice that the Green's function is symmetric, $G_E(x,x')=G_E(x',x)$. One can show that the Green's function can be given in terms of the solution $\chi_k^\pm$ as

$$G_E(x,x')=\frac{m}{ikt}\begin{cases}\chi_k^-(x')\chi_k^+(x) & \text{if } x>x'\\ \chi_k^+(x')\chi_k^-(x) & \text{if } x<x'\ .\end{cases} \tag{B2}$$

We use this representation to calculate the matrix element $\langle\varphi_0|VGV|\varphi_0\rangle$, where φ_0 is a localized wave function, and $V(x)$ is a localized potential. For simplicity we use the abbreviation $\varphi_0(x)V(x)\equiv f(x)$. Then we have

$$\langle\varphi_0|VGV|\varphi_0\rangle=\int\int_{x<x'}dx\,dx'\frac{m}{ikt}f(x)f(x')\chi_k^-(x)\chi_k^+(x')+\int\int_{x>x'}dx\,dx'\frac{m}{ikt}f(x)f(x')\chi_k^+(x)\chi_k^-(x')\ . \tag{B3}$$

We can add and subtract a term which is the same as the first integral in Eq. (B3) where the integration is over the region $x>x'$. Then we obtain

$$\langle\varphi_0|VGV|\varphi_0\rangle=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dx'\frac{m}{ikt}f(x)f(x')\chi_k^-(x)\chi_k^+(x')$$
$$+\int_{-\infty}^{+\infty}dx\int_{-\infty}^{x}dx'\frac{m}{ikt}f(x)f(x')[\chi_k^+(x)\chi_k^-(x')-\chi_k^-(x)\chi_k^+(x')]\ . \tag{B4}$$

The first term in (B4) can be written in the following form:

$$\frac{m}{ikt}\langle\varphi_0|V|\chi_k^+\rangle\langle\varphi_0|V|\chi_k^-\rangle \tag{B5}$$

or equivalently as

$$\frac{1}{2}\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dx'\frac{m}{ikt}f(x)f(x')[\chi_k^-(x)\chi_k^+(x')+\chi_k^+(x)\chi_k^-(x')]\ . \tag{B6}$$

Using Eqs. (A10) and (A11) the term in the brackets in this integral can be transformed to

$$\frac{m}{ikt}[\chi_k^+(x)\chi_k^-(x')-\chi_k^-(x)\chi_k^+(x')]=\frac{m}{ikt}[\chi_k^+(x)^*\chi_k^+(x')+\chi_k^-(x)\chi_k^-(x')^*]+\frac{m}{k}\,\mathrm{Im}\left[\frac{r_+}{t}\chi_k^-(x)\chi_k^+(x)^*\right]\ . \tag{B7}$$

Substituting this representation into the integral (B6) one gets

$$\frac{m}{ikt}\langle\varphi_0|V|\chi_k^+\rangle\langle\varphi_0|V|\chi_k^-\rangle=-i\Gamma+\delta\ , \tag{B8}$$

with

$$\Gamma=\Gamma^++\Gamma^-,\quad \Gamma^\pm=\frac{m}{2k}|\langle\varphi_0|V|\chi_k^\mp\rangle|^2 \tag{B9}$$

and

$$\delta=\frac{m}{2k}\int\int_{-\infty}^{+\infty}dx\,dx'\varphi_0(x)\varphi_0(x')V(x)V(x')\times\mathrm{Im}\left[\frac{r_+}{t}\chi_k^-(x)\chi_k^+(x)^*\right]\ . \tag{B10}$$

The second integral in Eq. (B4) is real. One can see it by using (A14) and representing the expression in the brackets as

$$\frac{m}{ikt}[\chi_k^+(x)\chi_k^-(x')-\chi_k^-(x)\chi_k^+(x')]$$
$$=-i\frac{m}{|t|^2}[\chi_k^+(x)\chi_k^+(x')^*-\chi_k^+(x)^*\chi_k^+(x')]\ . \tag{B11}$$

As a result we can write

$$\langle\varphi_0|VGV|\varphi_0\rangle=\Delta-i\Gamma\ , \tag{B12}$$

where Δ absorbs the real parts of both integrals in Eq. (B3) and Γ is given by Eq. (B9).

APPENDIX C: HIGHER-MODE CONTRIBUTION TO CONDUCTANCE

In this appendix we demonstrate that the total transmission of the uniform channel vanishes at certain energies even if the higher modes are taken into account. Let us keep, for instance, the third mode ($n=3$) in Eq. (2.4). Then substituting Eq. (2.4) into Eq. (2.1) we obtain

$$(E-E_1-K-V_{11})\psi_1-V_{12}\psi_2-V_{13}\psi_3=0\ , \tag{C1a}$$

$$(E-E_2-K-V_{22})\psi_2-V_{21}\psi_1-V_{23}\psi_3=0\ , \tag{C1b}$$

$$(E-E_3-K-V_{33})\psi_3-V_{31}\psi_1-V_{32}\psi_2=0\ , \tag{C1c}$$

[since we consider the uniform channel, the potential $U(x)=0$]. From Eq. (C1c) one finds

$$\psi_3=G_3V_{31}\psi_1+G_3V_{32}\psi_2\ , \tag{C2}$$

where the Green's function

$$G_3=\frac{1}{E-E_3-K-V_{33}}\ . \tag{C3}$$

Substituting Eq. (C2) into Eqs. (C1a) and (C1b) we find

$$(K+V_{11}+V_{13}G_3V_{31})\psi_1+(V_{12}+V_{13}G_3V_{32})\psi_2=(E-E_1)\psi_1\ ,$$
$$(K+V_{22}+V_{23}G_3V_{32})\psi_2+(V_{21}+V_{23}G_3V_{31})\psi_1=(E-E_2)\psi_2\ . \tag{C4}$$

Comparing Eqs. (C4) with Eqs. (2.5) one can see that taking into account the third mode corresponds effectively to replacement of the potentials V_{ij} in Eq. (2.5) by the corresponding nonlocal potentials $\tilde{V}_{ij}=V_{ij}+V_{i3}G_3V_{3j}$. The equation for the first mode is therefore [cf. with Eq. (2.10a)]

$$(\tilde{E}-K-\tilde{V}_{11})\psi_1=\tilde{V}_m\tilde{G}_2\tilde{V}_m\ , \tag{C5}$$

where $\tilde{V}_m=\tilde{V}_{12}=\tilde{V}_{21}$ is the modified wave mixing potential and $\tilde{G}_2=(\varepsilon-K-\tilde{V}_{22})^{-1}$.

Since we consider the case of $E_1<E<E_2$, and the electron energy E approaches E_2, the modified potentials $\tilde{V}_{ij}$ are very close to the potentials V_{ij}. Indeed,

$$\left|\frac{\tilde{V}_{ij}-V_{ij}}{V_{ij}}\right|=\frac{|V_{3j}G_3V_{3j}|}{V_{ij}}\sim\frac{|V_{3j}|}{|E_3-E_2|}\ll 1\ . \tag{C6}$$

Also the Green's function G_3, Eq. (C3), has no poles in this energy region, and therefore the potentials $\tilde{V}_{ij}$ are real. It means that the bound state φ_0 in the potential $V_b\equiv V_{22}$, Eq. (3.3), would appear also in the potential $\tilde{V}_{22}$, with a small energy shift $\varepsilon_0\to\tilde{\varepsilon}_0$ [where $|\tilde{\varepsilon}_0-\varepsilon_0|/\varepsilon_0\sim|V_{ij}|/(E_3-E_2)$]. As a result, the Green's function $\tilde{G}_2(\varepsilon)$ has a pole for $\varepsilon\to\tilde{\varepsilon}_0$ and it can be written in the form of Eq. (3.4). Afterwards all derivations in Secs. III and V concerning the uniform channel remain unchanged. As a result the conductance would exactly vanish at a certain energy near the second mode threshold. The inclusion of higher modes can be done in a similar way.

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