RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 47, NUMBER 15 15 APRIL 1993-I

Collective excitations of a two-dimensional electron gas in a two-dimensional magnetic-field modulation

Xiaoguang Wu and Sergio E. Ulloa

Department of Physics and Astronomy and Condensed Matter and Surface Sciences Program, Ohio University, Athens, Ohio 45701-2979

(Received 4 November 1992)

We study the collective excitations of a two-dimensional electron gas in the presence of a spatially periodic magnetic field. Both frequency and oscillator strength of the excitations are calculated, and their dependences on the wave vector and magnetic-field strength are investigated. We focus on the regime where the uniform and modulation components of the field have the same strength, and the magnetic flux per unit cell is a few times the fundamental quantum flux hc/e. The possibility of observing the Hofstadter-type spectrum is discussed.

The motion of an electron in a magnetic field and a periodic potential is a nontrivial problem, and has been intensively studied for decades.[1] The beauty of the energy spectrum is demonstrated by the Hofstadter "butterfly," and many transport properties have been predicted.[1] However, experimental explorations of these systems have been hindered until very recently by the requirement of an extremely high magnetic field. The rapid advances in submicrometer technology have made it possible to fabricate microstructures in which one can reach the interested regime with even moderate magnetic fields.[2] Thus, one is now able to put the theoretical predictions to various experimental tests. This has attracted considerable attention in the study of transport and optical properties of these so-called laterally modulated electronic systems.[2,3]

Typically, a laterally modulated system is formed on a two-dimensional (2D) electron gas. A periodic electrostatic potential, modulating the lateral motion of electrons, is then introduced by using a grating gate or etching processes.[2] In this paper, however, we study a 2D electron gas (2DEG) in a *spatially modulated magnetic field* in both lateral directions. The experimental implementation of such a magnetically modulated system (MMS) has not been reported yet, but may be realized in the near future.[4] A MMS has an interesting feature, namely, the modulation period plays the role of controlling the effective modulation strength, a feature which should be clearly identifiable in experiments.[5] Note that this peculiar behavior is absent in an electrostatically modulated system (EMS). We also anticipate that a MMS is strongly modulated, i.e., the uniform and modulation components of the magnetic field have nearly equal strength. This comes as a natural consequence of the probable implementation of a MMS, realized by depositing a patterned superconducting metal onto the surface close to a 2DEG. In this paper, we will focus on the collective excitations of a MMS, since this type of response is readily accessible via far-infrared spectroscopy.[2] Other theoretical studies of MMS have considered transport properties of weak and unidirectional modulations.[6]

In our model MMS, a 2DEG is located on the xy plane, and described by a single-electron Hamiltonian $H=(\mathbf{p}+e\,\mathbf{A}/c)^2/2m$, where $\mathbf{p}$ is the momentum operator and $\mathbf{A}$ the vector potential. The magnetic field applied perpendicularly to the 2DEG is assumed to be $B_z=B_0+(B_1/2)[\sin(2\pi x/a)+\sin(2\pi y/a)]$, where B_0 and B_1 give the strength of the uniform and modulation components of the field, respectively. The gauge is chosen as $A_x=(B_1/2)(a/2\pi)\cos(2\pi y/a)$, $A_y=B_0 x-(B_1/2)(a/2\pi)\cos(2\pi x/a)$, and $A_z=0$. The system can be basically characterized by two dimensionless parameters, the modulation strength $s=B_1/B_0$, and the effective modulation period $p=a/l$, with $l=(\hbar/m\omega_c)^{1/2}$ the magnetic length and $\omega_c=eB_0/mc$. We use Landau levels of the 2DEG without modulation as the basis set in the calculation of the eigenstates, so that the magnetic field is limited to rational flux values, i.e., $\phi/\phi_0=n_p/n_q$, where n_p and n_q are integers, $\phi=B_0a^2$, and $\phi_0=hc/e$.[1] Since the modulation can no longer be treated as a perturbation for $s\approx 1$, we have generalized the earlier calculations to include inter-Landau-level couplings, which lead to a Harper equation in matrix form.[1]

For a system without modulation, the eigenstates are the well-known Landau levels, which are degenerate and independent from the cyclotron center coordinate. As the modulation is turned on, however, the Landau levels split and the degeneracy is partially lifted. Each Landau level will split into n_p sublevels, when the magnetic field takes a "rational" value, i.e., $B_0=(n_p/n_q)\phi_0/a^2$.[1] The energy levels now depend on the cyclotron center coordinate. This may be viewed as a broadening of the Landau level (so-called Harper broadening[1]). In Fig 1, the width of the lowest three Landau levels, including their splittings, is shown for some values of the modulation period p, with $s=1$, and $n_p\leq 6$. The energy scale is $\hbar\omega_c$. For the rational B_0, $p=(2\pi n_p/n_q)^{1/2}$. It is clear that the "band width" oscillates versus p in a nonmonotonic fashion. The resulting energy spectrum resembles the well-known Hofstadter butterfly, but with marked differences. For example, the energy spectrum here is no longer a periodic function of n_p/n_q (Ref. 1) due to interlevel couplings. For small p values (≈ 1), the broadening is small, while for larger p the broadening becomes considerable. This is a consequence of the difference between the Hamiltonian above and that without the modulation, as it is directly proportional to a and a^2. Thus, the effective modulation strength is controlled by p, as mentioned before.

We now turn to study the collective excitation spectrum of our model MMS. We wish to explore the possibility of observing the highly structured Hofstadter-type spectrum of Fig. 1. Note that the experimentally measured far-infrared spectra of a typical EMS, e.g., an array of quantum dots or antidots, have given no definite indication of a Hofstadter-type spectrum.[2] However, we believe that this is because in a typical experiment, the magnetic field is such that the magnetic flux threading a unit cell is considerably larger than ϕ_0.[2] Consequently, the energy spectrum of a strongly modulated system may be too complicated to make any definite identification. In this paper, we will focus on the regime where the magnetic flux is just a few times ϕ_0. We will study the excitation spectrum in different directions in the wave-vector (**q**) space. The random-phase approximation[7] is employed for simplicity. We also limit our results to $n_p \leq 3$, due to the large demand in computational resources.

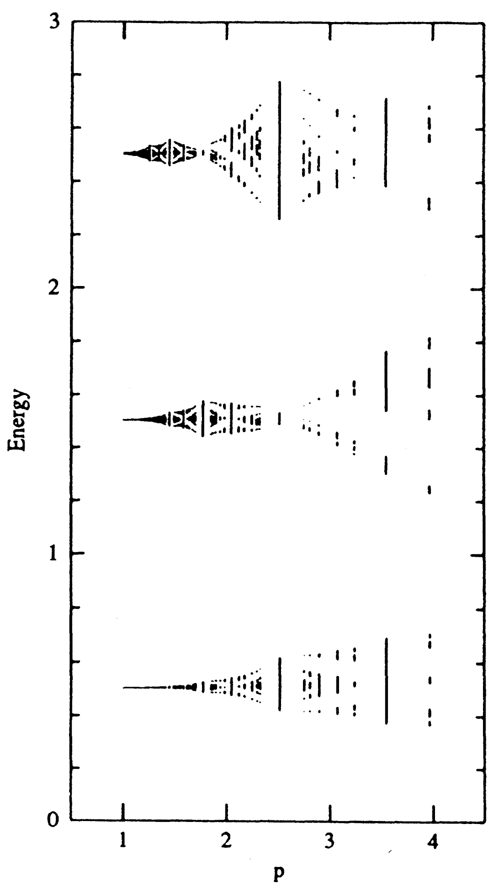

FIG. 1. The lowest three Landau levels split and broadened by the modulated magnetic field are shown as a function of the modulation period p. $s=1$, and $n_p \leq 6$.

First, let us study the wave-vector dependence of the excitations. In Fig. 2, the excitation spectra, given by the imaginary part of the electron density-density correlation function,[7] are shown for $n_p=1$, $s=1$, and at two different magnetic fields: in 2(a) and 2(b), $n_q=6$, while in 2(c) and 2(d), $n_q=1$. The peak structures in each curve give the position of the excitations, while the height of the peak gives the corresponding oscillator strength.[7] The amplitude of each curve has not been rescaled. The spectra are shown for two directions in **q** space: in 2(a) and 2(c), $q_y=0$, and in 2(b) and 2(d), $q_x=q_y$. Different curves correspond to different values of q_x scaled by $2\pi/a$. In 2(a) and 2(b), q_x starts from 0.05 (bottom curve), then ranges from 0.1 to 0.5 (top curve) in steps of 0.1. In 2(c) and 2(d), q_x varies from 0.1 (bottom curve) to 0.5 (top curve) in steps of 0.1. The origin of each curve is shifted, so the dispersion can be seen.

In Figs. 2(a) and 2(b), p is small (n_q is large), thus, the effective modulation strength is small. The spectra indeed reflect this fact: the dispersion relation closely resembles that of a 2DEG in a uniform magnetic field.[8]

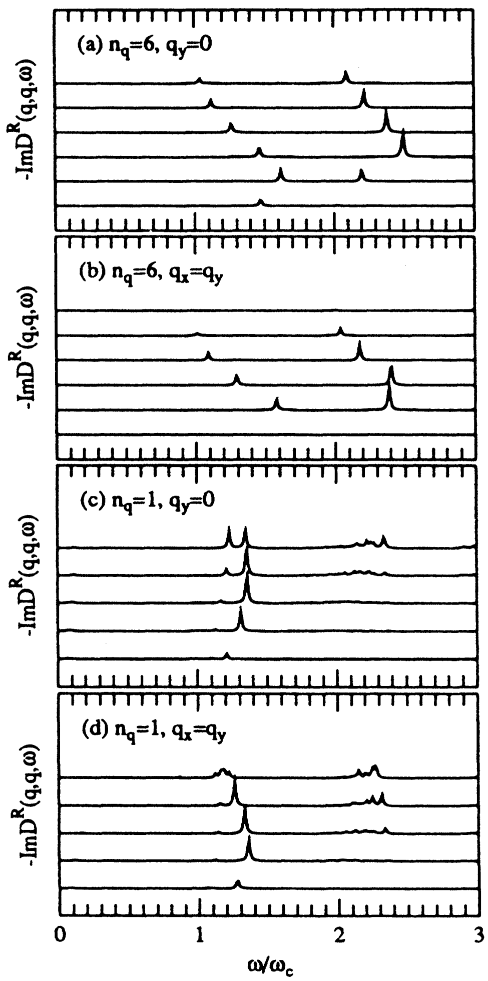

FIG. 2. The collective excitation spectrum for two different magnetic fields vs the wave vector. $n_p=1$, $s=1$ in all panels. The dispersion is obtained by tracing the peaks in each curve, while the height of peak gives the oscillator strength.

The general features of 2D magnetoplasmons (without modulation) are well known: in the long-wavelength limit, all modes start from $n\omega_c$ ($n=1,2,\ldots$), because of Kohn's theorem.[9] As the wave vector increases, they deviate away from $n\omega_c$ to higher frequencies, but for very large vectors, they reduce back to $n\omega_c$ again. The oscillator strength of these modes first increases, then decreases, and the higher-frequency modes gain strength at relatively larger wave vectors.[8] Apparently, Figs. 2(a) and 2(b) fit into this description. However, the spectra of our MMS show different behavior along different directions in **q** space. When $q_x=q_y=0.05$, the amplitude is too small to detect any structures, while a peak can be seen when $q_x=0.05$ and $q_y=0$ [bottom trace in Figs. 2(b) and 2(a), respectively]. This anisotropy is expected because the system no longer has the rotational symmetry of the unmodulated case.

Let us next examine the case of a larger p value shown in Figs. 2(c) and 2(d). The Fermi energy ($0.6\hbar\omega_c$) is chosen in such a way that it crosses energy levels, so there is an intralevel contribution to the collective excitations, which can be barely seen at low frequencies with a weak dispersion. The spectra are anisotropic, and there are multiple-peak structures, as we now have more broadened, although nonoverlapping Landau levels ($n_p=1$). In the case of an unidirectional modulation,[3,5] similar spectra are found, and are described within an intuitive zone-folding picture. As the wave vector increases across the "Brillouin"-zone edge, the mode folds back, splits at the zone edge, and forms multiple peaks.[3,5] This picture is qualitatively correct in describing the wave-vector dependence of the spectra. The exact position and strength of the excitation are, of course, given by the density-density correlation function, and the details depend not only on the energy levels, but also on the matrix elements involving the wave functions.[7] Note also that the various mode dispersions in 2(c) and 2(d) are much smaller (flatter curves) than in 2(a) and 2(b).

Next, we study the excitation spectra versus the magnetic-field strength, as shown in Fig. 3. Here the wave vector is fixed, but in two different directions: in 3(a) $q_x=2\pi/a$, $q_y=0$, while in 3(b) $q_x=q_y=2\pi/a$, $s=1$. The values of the magnetic field (n_p and n_q) are chosen in such a way that $n_p\le 3$ and $1\le p=(2\pi n_p/n_q)^{1/2}\le 4$. Twenty-three curves are plotted in order of increasing p from the bottom of each panel upwards, and the origin of each curve is shifted equally for clarity (e.g., top curve, $n_p=2$, $n_q=1$; middle curve, $n_p=3$, $n_q=10$; and bottom curve, $n_p=1$, $n_q=6$). The amplitude of all curves has been rescaled, in order to see the structures of all curves. For smaller p values (curves near the bottom of panels), the amplitude is smaller. Note that p is proportional to $B_0^{1/2}$.

In Fig. 3, we only observe single peaks around $n\omega_c$ ($n=1,2$) when p is small. This is because the effective modulation strength is small. As p increases, the single peak develops into multiple-peak structures. However, the number of peaks does not exactly correspond to the number of levels split from one Landau level by the modulation (see Fig. 1). The intralevel contribution to the excitation spectra can also been seen at low frequencies ($\omega/\omega_c\sim 0.4$), but only for larger p values (top curves). The spectra are clearly anisotropic, as expected. Note that the absence of anisotropy in the experimentally observed spectra of an EMS may be due to the coupling of incident light with the grating gate,[3] while in this paper, we have only calculated the response from the 2DEG itself. From Fig. 1, one may expect to find the excitations covering a wide range of frequencies, especially when p is large. However, one has to realize that not all resonances can be seen, as determined by the different matrix elements contributing to the oscillator strength. Note that the zone-folding picture can no longer be used to describe the magnetic-field dependence of the excitation spectra shown in Fig. 3. As p increases from a small value, the zone-folding picture predicts that the modes should deviate away from $n\omega_c$ first, and then fall back. This is not the case. The only reliable way, of course, is to examine the density-density correlation function of the system, as shown here. In a MMS with a unidirectional modulation, it is found that the system undergoes a dimensional crossover, when the modulation period p becomes very large.[5] The physical reason behind this crossover is that the electrons tend to locate in the regions

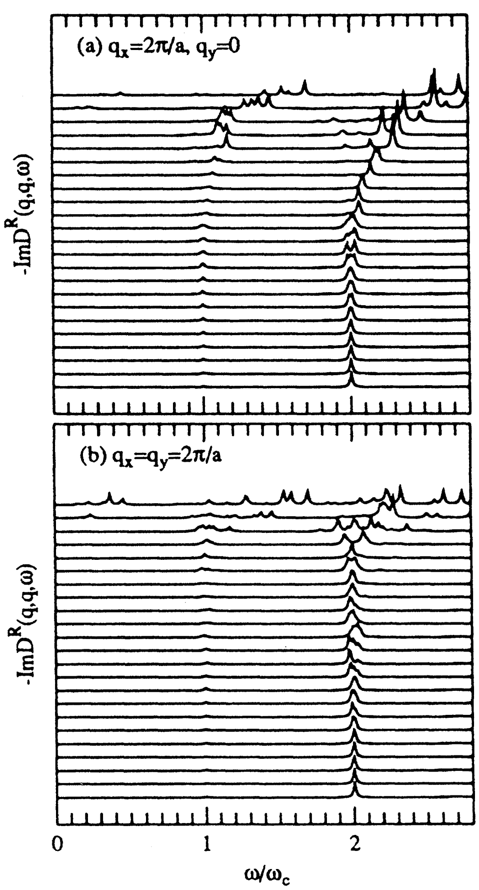

FIG. 3. The excitation spectrum vs the magnetic-field strength for two fixed wave vectors. $s=1$ in both panels. $n_p\le 3$ and $1\le p\le 4$, e.g., top curve, $n_p=2$, $n_q=1$; middle curve, $n_p=3$, $n_q=10$; and bottom curve, $n_p=1$, $n_q=6$.

where the local magnetic field is higher, as the local cyclotron radius is smaller.[5] Following the same physical reasoning, we expect a similar behavior in the MMS with a 2D modulation. This aspect will be studied in detail elsewhere.

In conclusion, we have studied the collective excitations of a 2DEG in the presence of a magnetic field spatially modulated in both lateral directions. Both frequency and oscillator strength of the excitations have been calculated, and their dependences on the wave vector and magnetic-field strength have been investigated. We find that multiple-peak structures develop either as the wave vector increases from the long-wavelength limit, or as the magnetic field increases. Although the excitation spectra do not exactly map the Hofstadter-type energy spectrum, because of complicated oscillator strength distributions, an interesting and nontrivial spectrum is found nevertheless. It is desirable to experimentally implement the MMS studied here and observe the predicted spectra.

This work was supported by the U.S. Department of Energy, Grant No. DE-FG02-91ER45334, and by the Ohio Supercomputer Center, Grant No. PHS060-3.

[1]P. G. Harper, Proc. Phys. Soc. London Sect. A **68**, 874 (1955); M. Ya. Azbel, Zh. Eksp. Teor. Fiz. **46**, 929 (1964) [Sov. Phys. JETP **19**, 634 (1964)]; D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976); G. M. Obermair and H. J. Schellnhuber, *ibid.* **23**, 5185 (1981); D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. Den Nijs, Phys. Rev. Lett. **49**, 405 (1982); A. H. MacDonald, Phys. Rev. B **28**, 6713 (1983); J. B. Sokoloff, Phys. Rep. **126**, 189 (1985); N. A. Usov, Zh. Eksp. Teor. Fiz. **94**, 305 (1988) [Sov. Phys. JETP **67**, 2565 (1988)]; J. Zak, Phys. Rev. Lett. **67**, 2565 (1991); M. Kohmoto, B. Halperin, and Y. S. Wu, Phys. Rev. B **45**, 13 488 (1992).

[2]A. Lorkes, J. P. Kotthaus, and K. Ploog, Phys. Rev. Lett. **64**, 2559 (1990); J. Alsmeier, E. Batke, and J. P. Kotthaus, Phys. Rev. B **41**, 1699 (1990); C. G. Smith, M. Pepper, R. Newbury, H. Ahmed, D. G. Hasko, D. C. Peacoak, J. E. F. Frost, D. A. Ritchie, G. A. Jones, and G. Hill, J. Phys. Condens. Matter **2**, 3405 (1990); H. Fang and P. J. Stiles, Phys. Rev. B **41**, 10 171 (1990); D. Weiss, M. L. Roukes, A. Menschig, P. Grambow, K. von Klitzing, and G. Weimann, Phys. Rev. Lett. **66**, 2790 (1991); A. Lorkes, J. P. Kotthaus, and K. Ploog, Phys. Rev. B **44**, 3447 (1991); K. Kern, D. Heitmann, P. Grambow, Y. H. Zhang, and K. Ploog, Phys. Rev. Lett. **66**, 1618 (1991); G. Berthold, J. Smoliner, V. Rosskopf, E. Gornik, G. Böhm, and G. Weimann, Phys. Rev. B **45**, 11 350 (1992); A. Lorkes, Surf. Sci. **263**, 307 (1992).

[3]G. R. Aĭzin and V. A. Volkov, Zh. Eksp. Teor. Fiz. **87**, 1469 (1984) [Sov. Phys. JETP **60**, 844 (1984)]; F. M. Peeters and P. Vasilopoulos, in *Proceedings of the 20th ICPS,* edited by E. M. Anastassakis and J. D. Joannopoulos (World Scientific, Singapore, 1990), Vol. 2, p. 1589; R. R. Gerhardts, D. Weiss, and U. Wulf, Phys. Rev. B **43**, 5192 (1991); V. Gudmundsson and R. R. Gerhardts, *ibid.* **43**, 12 098 (1991); W. Que, G. Kirczenow, and E. Castaño, *ibid.* **43**, 14 079 (1991); A. V. Chaplik and L. Ioriatti, Surf. Sci. **263**, 354 (1992); R. R. Gerhardts, Phys. Rev. B **45**, 3449 (1992); Y. Takagaki and D. K. Ferry, *ibid.* **45**, 8506 (1992); T. Deruelle, K. Ensslin, P. M. Petroff, A. L. Efros, and F. G. Pikus, *ibid.* **45**, 9082 (1992); B. L. Johnson and G. Kirczenow, Phys. Rev. Lett. **69**, 672 (1992).

[4]S. J. Bending, K. von Klitzing, and K. Ploog, Phys. Rev. B **42**, 9859 (1990); A. K. Geĭm, S. V. Dubonos, and A. V. Khaetskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 107 (1990) [JETP Lett. **51**, 121 (1990)].

[5]Xiaoguang Wu and Sergio E. Ulloa (unpublished).

[6]B. A. Dubrovin and S. P. Novikov, Zh. Eksp. Teor. Fiz. **79**, 1006 (1980) [Sov. Phys. JETP **52**, 511 (1980)]; P. Vasilopoulos and F. M. Peeters, Superlatt. Microstruct. **7**, 393 (1990); D. P. Xue and G. Xiao, Phys. Rev. B **45**, 5986 (1992).

[7]G. D. Mahan, *Many Particle Physics* (Plenum, New York, 1981).

[8]Xiaoguang Wu, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **36**, 9760 (1987), and references therein.

[9]W. Kohn, Phys. Rev. **123**, 1242 (1961).