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Intrinsic integer quantum Hall effect in a quantum wire

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Self-consistent calculations are made of the electrostatic Hall potential (EHP), local chemical potential (LCP), and current density in a 100-nm-wide wire containing two-dimensional electrons in a perpendicular magnetic field B when either one or two subbands are occupied. The corresponding Hall resistances, $R_{\rm EHP}$ and $R_{\rm LCP}$, are also calculated. The former is nearly linear in B in spite of subband depopulation. The latter is quantized but the quantization steps are rounded because of overlap of the forward and backward wave functions.

Since the discovery of the integer quantum Hall effect (IQHE) by von Klitzing, Dorda, and Pepper,[1] much attention has been devoted to the transport properties of a two-dimensional electron gas (2DEG) in a magnetic field.[2] Rapid technological development has made possible the study of very-small-scale structures based on high-mobility GaAs-$\rm Al_xGa_{1-x}As$ heterostructures.[3] A number of studies of the magnetic transport properties of a 2DEG in a ballistic quantum wire have been made by both experimentalists[4–9] and theoreticians[10–17] from different points of view. Explanations of the four-terminally measured electronic transport properties have been given successfully using the Landauer-Büttiker formulas.[18,19] These formulas apply only when conducting channels are set up between the reservoirs and the detecting probes and give the terminal behavior due to all the different parts of the whole system. Here we go further and ask what the behavior of the 2DEG in a two-terminal ballistic wire actually is and how will noninvasive measurements of Hall potential differences differ from what is measured when four terminals are used. We suppose that it is possible to measure the intrinsic physical quantities of a two-terminal quantum wire by using weakly coupled probes as discussed by Engquist and Anderson[20] and Landauer.[21] Very recently, for example, Shepard, Roukes, and Van der Gaag[9] have measured quantum Hall resistance behavior in this limit.

The IQHE of an interacting 2DEG was investigated by MacDonald, Rice, and Brinkman[22] with the assumption of a slowly varying potential which is appropriate to high magnetic fields. The redistribution of charge in real space, which generates the electrostatic Hall potential (EHP), is given as well as the Hall current density distribution. The IQHE of a 2DEG confined in a quantum wire is quite different from that of its unconfined counterpart. First, there is no energy gap; and second, each Landau level (i.e., each subband) is always only partially occupied. The conductivity is finite even without impurities because of the finite width of the wire. Li and Thouless[14] study this problem for a GaAs wire in a weak magnetic field when only the lowest subband is occupied and give results for the EHP. They do not include more subbands because of a numerical instability which they argue is due to the assumed hard-wall confining potential. These calculations do not yield a quantized integer Hall resistance. This is not only because the Fermi level lies below the excited subbands. More importantly, these authors concentrate on the EHP and, as we show here, the corresponding Hall resistance is *always* nearly linear in the magnetic field.

In this paper we present numerical IQHE results for a 2DEG in a ballistic two-terminal wire subjected to a perpendicular magnetic field when either one or two subbands are occupied. Electrostatic interactions between electrons within the same subband and among different subbands are included self-consistently. Spin degeneracy is also taken into account. We distinguish two kinds of intrinsic Hall potential, EHP and local chemical potential (LCP), which depend differently on magnetic field and correspond to different measurements. Both EHP and LCP differences and the two kinds of intrinsic Hall resistance associated with them are investigated. We demonstrate that quantization occurs only for the Hall resistance connected with the LCP. We also find that the leading edges of the quantization steps are rounded off at low B because of the overlap of wave functions propagating in opposite directions along the wire. Furthermore, we show that the resistance associated with the EHP retains the classical linear dependence on the magnetic field. Distributions of EHP, LCP, and current density across the wire when one or two subbands are occupied are also calculated and shown.

Let us consider a 2DEG with an electron density n_s which is confined in a space of width W in the x-y plane by infinite potential barriers at $y = \pm W/2$. We suppose that a uniform magnetic field B is applied in the z direction and describe it in the Landau gauge by writing the vector potential as $\mathbf{A} = (-By, 0, 0)$. Following previous authors[22,14] we also introduce an EHP $V(y)$ which is induced by the external magnetic field. The normalized eigenfunctions of the Schrödinger equation are then of the form $L_x^{-1/2} e^{ik_x x} \chi_{n,k_x}(y)$, where L_x is the length of the wire. $\chi_{n,k_x}(y)$ satisfies the equation

$$\left[\frac{1}{2m^*}(\mathbf{p}+e\mathbf{A})^2 + (-e)V(y)\right]\chi_{n,k_x}(y) = \epsilon_n(k_x)\chi_{n,k_x}(y), \quad (1)$$

where m^* is the effective mass and n is the index of the subbands. The EHP, which must be determined self-consistently, can be expressed as

$$V(y) = \begin{cases} \dfrac{1}{4\pi\epsilon_0\epsilon}\displaystyle\int_{-W/2}^{W/2} dy'(-2\ln|y-y'|)\delta\sigma(y'), & -W/2 \le y \le W/2 \\ -\infty & \text{otherwise,} \end{cases} \tag{2}$$

where we consider the electrostatic interactions between electrons as homogeneous in the x direction. The redistribution of the electron charge density as a result of the external magnetic field is

$$\delta\sigma(y) = -\frac{e}{2\pi}\sum_{n,\sigma}\int_{k^{(n)}_{-x,E_F-\Delta/2}}^{k^{(n)}_{x,E_F+\Delta/2}} dk_x[\,|\chi_{n,k_x}(y)|^2 - |\chi^{(0)}_{n,k_x}(y)|^2], \tag{3}$$

where $k^{(n)}_{x,E_F+\Delta/2}$ and $k^{(n)}_{-x,E_F-\Delta/2}$ are the Fermi wave numbers of subband n for the positive and the negative x directions, respectively, Δ is the chemical potential difference between the two terminals, and σ is the spin label. We make Δ small enough to ensure that we stay in the linear transport regime. The functions $\chi^{(0)}_{n,k_x}(y)$ are the eigenfunctions of the Schrödinger equation, Eq. (1), in the absence of a magnetic field. With these definitions, we know that the EHP is related to the applied magnetic field and describes a kind of Hall effect and we have hard-wall, square-well confinement when $B=0$. Furthermore, from Eqs. (2) and (3), we can see the spin degeneracy is important. The up and down spins give the same contribution to the EHP if we ignore the Zeeman splitting which is a reasonable first approximation in a model calculation for a GaAs system. To complete the calculation, we need to constrain E_F in the Fermi wave numbers so as to yield the given electron density n_s for fixed Δ, i.e.,

$$n_s = \frac{1}{2\pi W}\sum_{n,\sigma}[k^{(n)}_{x,E_F+\Delta/2} - k^{(n)}_{-x,E_F-\Delta/2}]. \tag{4}$$

Then charge neutrality is ensured by the normalization of the wave functions.

Solving Eqs. (1)–(3) self-consistently with the constraint Eq. (4), we obtain the wave functions and the charge density redistribution as well as the self-consistent EHP. The current density distribution across the wire can also be calculated from

$$j_x(y) = -\frac{e\hbar}{2\pi m}\sum_{n,\sigma}\int_{k^{(n)}_{-x,E_F-\Delta/2}}^{k^{(n)}_{x,E_F+\Delta/2}} dk_x\left(k_x - \frac{1}{l_B^2}y\right) \times\chi^2_{n,k_x}(y), \tag{5}$$

where $l_B = (\hbar/eB)^{1/2}$ is the magnetic length. Since the total current is $I_x = \int dy\, j_x(y)$, the longitudinal resistance can be calculated straightforwardly.

In addition to the EHP we may also calculate the LCP $U(y)$ from the formula[23,24]

$$eU(y) = \frac{\displaystyle\sum_{n,\sigma}[\mu_1|\chi_{n,k_{x,E_F+\Delta/2}}(y)|^2 + \mu_2|\chi_{n,k_{-x,E_F-\Delta/2}}(y)|^2]/v_n}{\displaystyle\sum_{n,\sigma}[|\chi_{n,k_{x,E_F+\Delta/2}}(y)|^2 + |\chi_{n,k_{-x,E_F-\Delta/2}}(y)|^2]/v_n}, \tag{6}$$

where μ_1 (μ_2) is the chemical potential of the reservoir connected at the left (right) end of the wire, $\chi_{n,k_{x,E}}(y)$ and $\chi_{n,k_{-x,E}}(y)$ are the right- and left-going electron wave functions, respectively, and v_n is the velocity at Fermi level. When the external magnetic field vanishes we may use the symmetry of the electron wave functions to show that the LCP is a constant everywhere. The difference of the LCP across the wire when $B \neq 0$ gives another kind of Hall effect.

The EHP is the electrostatic potential response when the electron density redistributes to balance the Lorentz force. It describes the real-space potential which the electrons in the wire experience. On the other hand, the LCP characterizes the local electron energy distribution and is determined by the overlap of wave functions propagating in opposite directions along the wire. By using Eqs. (2), (6), and (5), we can calculate both these potential distributions and current density distribution in the wire. We can also calculate two kinds of intrinsic Hall resistance, $R_{\rm EHP}$ and $R_{\rm LCP}$ from the EHP and the LCP, respectively. Previous authors have suggested ways to simulate[11] or measure[14] intrinsic Hall potentials. We believe that contacted probes give the LCP differences in the weak-coupling limit (when the measurement does not change the detected system) while the noncontacted-probe method proposed by Li and Thouless[14] gives the EHP differences across the wire.

In our numerical calculation, we use the parameters of a GaAs wire with $W = 100$ nm and $n_s = 2\times10^{14}$ and 4×10^{14} m^{-2}. Our results are very sensitive to the accuracy of the self-consistent EHP and the Fermi wave numbers and, contrary to the experience of Li and Thouless,[14] we get stable solutions for multisubband occupancy in a hard-wall, square-well confining potential.

Figure 1(a) shows the current density distribution when $B = 0.25$ T and $n_s = 4\times10^{14}$ m^{-2} and two subbands are occupied; Fig. 1(b) is the corresponding result when $B = 1.25$ T and only one subband is occupied. Figures 1(c) and 1(d) show the distributions of the EHP when $B = 0.25$ and 1.25 T, respectively, and Figs. 1(e) and 1(f) show the corresponding distributions of the LCP. The LCP in Fig. 1(f) is shifted down for convenience. The offset is 0.458 mV as marked in the picture. Comparing Figs. 1(a) and 1(b), we can see that the current density for the case of two occupied subbands spreads in the wire more than for the case of one occupied subband since B is increased and the Fermi wave numbers of each occupied subband in the former case are both smaller than the one in the latter case. For the same reason, the amplitude of the EHP, $V(y)$, in Fig. 1(d) and the difference of the LCP across the wire in Fig. 1(f) are larger than that in Figs. 1(c) and 1(e), respectively. We note that there is a kink of the LCP in Fig. 1(e) because there are two subbands being occupied and no such kink in Fig. 1(f) for the case of one occupied subband.

Figure 2 exhibits the different behavior of the two kinds of intrinsic Hall resistance. In Fig. 2(b), the $R_{\rm LCP}$ (circles) shows the steplike behavior which is characteristic of the IQHE and the value of resistance at the Nth plateau is $h/2e^2N$ except on the "last plateau" ($0 < B < 0.5$ T). This quenching behavior of the IQHE is due to the overlap of opposite-going wave functions of the same subband and we will discuss the details of it in another paper. We note that the leading edge of the $R_{\rm LCP}$ step is not as sharp as that of the longitudinal resistance (crosses). The curvature arises from the overlap between the right- and left-going waves in our 100-nm wire. Calculations for larger values of B show that the higher quantization steps in the $R_{\rm LCP}$ are sharper because the opposite-going waves are more separated. We can easily show from Eq. (6) that, if there is no overlap, then the LCP between the two edges is equal to that between the two terminals of the wire and the quantization steps of $R_{\rm LCP}$ become identical to those of the longitudinal resistance. Overlap is significant when the flux through an area W^2 is in the order of, or less than, h/e. In Fig. 2(b), the results for $R_{\rm EHP}$ (squares) show, in complete contrast to $R_{\rm LCP}$, nearly linear dependence on B despite the subband depopulation which occurs at $B = 0.5$ T. The slope of the line delineated by these squares is less than that appropriate to an unconfined 2DEG (dashed line) because of the finite width of the wire and the electrostatic interactions between electrons. Further calculations have shown that failure to achieve self-consistency leads to spurious jumps in $R_{\rm EHP}$ associated with the subbands depopulation and a larger slope which increases when the number of occupied subbands decreases. We see that quantization of the IQHE is seen only when differences of chemical potential are measured. The longitudinal resistance is exhibited by the crosses in Fig. 2. It is exactly quantized because it is again the result of measuring differences of chemical potential.

In summary, we distinguish two kinds of potential responses to a magnetic field and find that they reveal different aspects of the intrinsic quantum Hall effect in a quantum wire. The two corresponding intrinsic Hall resistances are calculated when one or two subbands are occupied. Quantization is found for the Hall resistance associated with local chemical potential in a large magnetic field. The overlap of oppositely propagating wave functions rounds off the front edge of the quantization steps at lower fields. To a very good approximation the Hall resistance associated with the electrostatic poten-

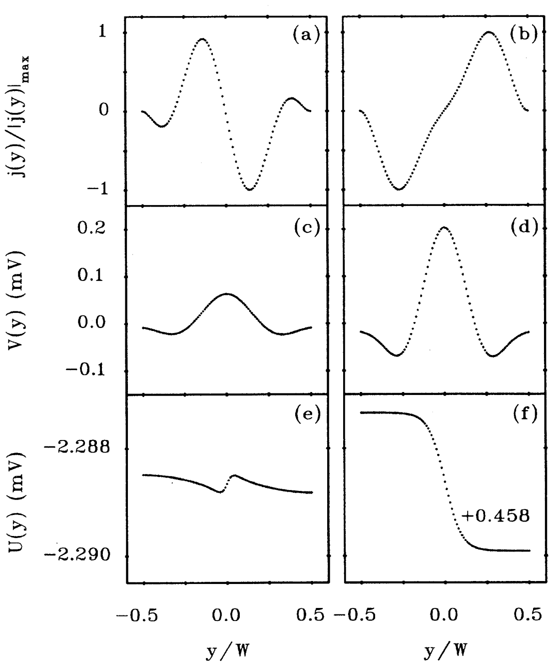

FIG. 1. Current density $j(y)$: (a) $B = 0.25$ T and $|j(y)|_{\rm max} = 0.3354$ A/m and (b) $B = 1.25$ T and $|j(y)|_{\rm max} = 1.6968$ A/m; EHP $V(y)$: (c) $B = 0.25$ T and (d) $B = 1.25$ T; and LCP $U(y)$: (e) $B = 0.25$ T and (f) $B = 1.25$ T. The offset to the LCP in (f) is 0.458 mV. Wire width is $W = 100$ nm and electron charge density is $n_s = 4 \times 10^{14}$ m^{-2}.

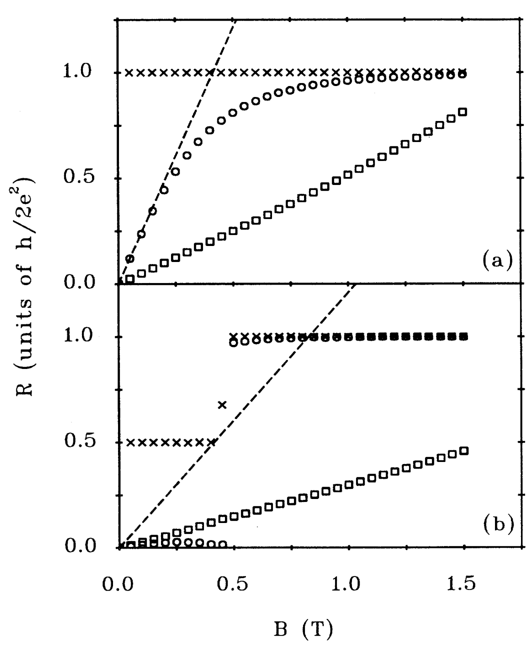

FIG. 2. Plots vs B of the two kinds of intrinsic Hall resistance, $R_{\rm EHP}$ (squares) and $R_{\rm LCP}$ (circles), and the longitudinal resistance (crosses) for a quantum wire of width 100 nm for the electron densities (a) $n_s = 2 \times 10^{14}$ m^{-2} and (b) $n_s = 4 \times 10^{14}$ m^{-2}. The dashed lines in (a) and (b) are the corresponding Hall resistances for an unconfined 2DEG.

tial is linearly proportional to the magnetic field (as in classical systems) despite the occurrence of subband depopulation. Distributions of electrostatic Hall potential, local chemical potential, and current density are given.

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