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Metallic phase of the quantum Hall system at even-denominator filling fractions

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We propose an explanation for the resistivity anomalies exhibited by the two-dimensional spin-polarized electron gas at $\nu=\frac{1}{2}$ and related even-denominator filling fractions. Within the Chern-Simon-Landau-Ginzburg theory we argue that the inhomogeneity in the electron density, caused by the impurity potential, induces a spatially random distribution of statistical flux, and that electron localization can be suppressed by this random flux. As a result, a metallic phase appears in the phase diagram of the Hall system near $\nu=\frac{1}{2}$, leading to a temperature-independent resitivity minimum.

The magnetotransport behavior observed in quantum Hall samples near filling fractions $\nu=\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{2}$ is strikingly different from the well-known signature of the fractional quantum Hall effect. Commonly referred to as "the $\nu=\frac{1}{2}$ anomaly," the unusual features in the resistivity occur in very-high-mobility heterojunctions,[1] with ρ_{xx} displaying a sharp minimum at $\nu=\frac{1}{2}$ and saturating at a nonzero value as $T\rightarrow 0$, while ρ_{xy} is not quantized and has the classical form $\rho_{xy}=B/nec$.

At present there is no successful theory of the $\nu=\frac{1}{2}$ anomaly. Unlike the case of incompressible quantum Hall liquids, numerical results for small clusters[2] at $\nu=\frac{1}{2}$ remain inconclusive. A class of incompressible states with p-wave pair correlations was proposed recently;[3] however, it predicts a quantized ρ_{xy} and vanishing ρ_{xx} and is therefore inconsistent with experiment. A more plausible scenario, based on phase separation, treats the system as a composite of electron droplets forming odd-denominator incompressible states with densities close to $\nu=\frac{1}{2}$. However, the minimum at $\nu=\frac{1}{2}$ is observed up to temperatures $T\sim 10$ K, where the features of all fractional quantum Hall states have disappeared.[1]

In this paper we develop a theory of the $\nu=\frac{1}{2}$ state and show that the $\nu=\frac{1}{2}$ anomaly can be understood in the context of this theory as a breakdown of electron localization. The Chern-Simons-Landau-Ginzburg (CSLG) theory has emerged in the last few years as a powerful tool for the description of the quantum Hall system,[4-8] leading to the construction of its global phase diagram.[9] At odd-denominator filling fractions, the CSLG theory treats electrons as bosons carrying an odd number of flux quanta. Read[10] has suggested that the analogous procedure for $\nu=\frac{1}{2}$ is to attach two flux quanta to every electron so that the transformed particles have Fermi statistics. Each of these particles, referred to as Chern-Simons fermions, is subject not only to the external magnetic field, but also to the gauge potential of the flux tubes attached to other particles. At $\nu=\frac{1}{2}$ the average of the Chern-Simons flux precisely cancels the external flux, and in the mean-field approximation the system behaves as a two-dimensional gas of fermions in zero magnetic field.

We argue that the $\nu=\frac{1}{2}$ anomaly arises because of the suppression of electron localization[11] by the random Chern-Simons flux distribution induced by the impurities. Since Chern-Simons fermions carry electric charge, they participate in the screening of the impurity potential. Therefore, a screened impurity, in addition to being a potential scatterer, binds some of the Chern-Simons flux. The latter breaks the time-reversal symmetry of the impurity scattering process which is crucial for the coherent backscattering mechanism of localization. Thus, for sufficiently weak disorder, when the backscattering argument is valid, the logarithmic correction to the conductivity is cut off and the system ground state becomes metallic. We verify this qualitative prediction analytically and by numerical simulation. Based on this evidence, we argue that the phase diagram of the Hall system contains a pocket of metallic phase at $\nu=\frac{1}{2}$, as shown in Fig. 1. The metallic phase is characterized by a positive magnetoresistance if the magnetic field is measured relative to its value at $\nu=\frac{1}{2}$. This implies a minimum in ρ_{xx} at $\nu=\frac{1}{2}$, in qualitative agreement with experiment. Furthermore, the Hall resistance is not quantized in this metallic phase. When disorder increases and the filling fraction remains fixed at $\frac{1}{2}$, the system enters a phase where the carriers are exponentially localized despite the presence of the random flux and where the magnetoresistance measured relative to $\nu=\frac{1}{2}$ is negative. Similar arguments apply to other even-denominator filling fractions related by the law of correspondence.[9]

To proceed formally, we attach an even number of flux quanta (two in the case of $\nu=\frac{1}{2}$) to the electrons and represent them as fermions ψ coupled to a gauge field a_μ. One obtains a Lagrangian density similar to that found in the Chern-Simons theory of the fractional quantum Hall effect:[4]

$$\mathcal{L}=\frac{1}{4\theta}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda+\psi^\dagger(i\partial_t-A_0-a_0)\psi-\frac{1}{2m}\left|\left(\frac{1}{i}\nabla-\mathbf{A}-\mathbf{a}\right)\psi\right|^2-\tfrac{1}{2}\int d^2y\,\delta\rho(x)V(x-y)\delta\rho(y)-V_{\text{imp}}\delta\rho\ . \quad (1)$$

Here $V(x-y)$ is the original electron-electron interac-

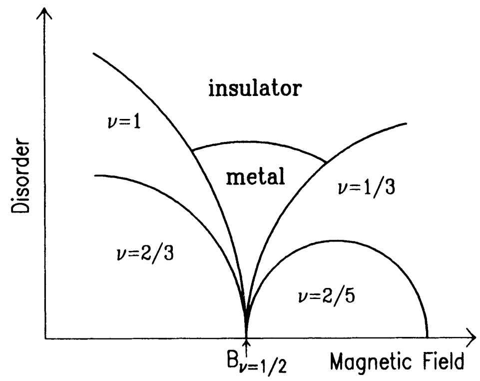

FIG. 1. Schematic phase diagram of the Hall system in the disorder–magnetic-field plane.

tion, $V_{\rm imp}$ is the random potential due to the impurities, $\delta\rho(x)=\psi^\dagger\psi-\bar{\rho}$ is the deviation from the average electron density, and θ is an even multiple of π. The resistivity $\rho_{\mu\nu}^{\rm CS}$ of the Chern-Simons (CS) fermions is defined in terms of their response to the total gauge potential $(A+a)$. The physically measured resistivity $\rho_{\mu\nu}$, expressed in units of h/e^2, is related to $\rho_{\mu\nu}^{\rm CS}$ as follows:[9]

$$\rho_{xx}(B)=\rho_{xx}^{\rm CS}(\delta B),\quad \rho_{xy}(B)=\theta/\pi+\rho_{xy}^{\rm CS}(\delta B)\ , \tag{2}$$

where B is the external magnetic field and $\delta B=B-\langle\epsilon^{ij}\partial_i a_j\rangle$ is the average magnetic field acting on the fermions.

When the filling factor is equal to $\frac{1}{2}$ exactly and $V_{\rm imp}=0$, the mean-field solution of the equations of motion which follow from (1) is given by $\psi^\dagger\psi=\bar{\rho}$ and $\epsilon^{ij}\partial_i a_j=B$. This solution corresponds to a uniform density of fermions in the uniform field $\delta B=0$. A nonzero impurity potential changes the mean-field solution by producing a local deviation $\delta\rho(x)$ from the average fermion density $\bar{\rho}$. By virtue of the equation of motion of a_0, given by $\epsilon^{ij}\partial_i a_j=2\theta\rho(x)$, this generates a local deviation from the average magnetic field acting on the fermions. A self-consistent mean-field solution in the presence of disorder must therefore treat each isolated impurity as a flux tube bound to a scattering center.

Motivated by these considerations, we examine the quantum correction $\Delta\sigma$ to the conductivity of the CS fermions subject to both a random potential and a random flux distribution. In the limit of weak disorder and zero flux, $\Delta\sigma$ acquires a negative logarithmically divergent contribution from coherent backscattering processes.[11] The effect of random flux on $\Delta\sigma$ can be expressed in the following form:[12]

$$\Delta\sigma=-\frac{e^2}{\pi\hbar}D\langle C(r,r)\rangle\ . \tag{3}$$

Here

$$\langle C(r,r)\rangle=\lim_{\omega\to 0}\int dt\,\langle C(r,r;t)\rangle e^{i\omega t}\ ,$$

the angular brackets denote an average over the probability distribution of the gauge potential, and $C(r,r';t)$ is the cooperon propagator obeying the differential equation

$$\{\partial_t-D[-i\nabla-(2e/c)\mathbf{a}(r)]^2-1/\tau_\phi\}C(r,r';t)=\delta(r-r')\delta(t)\ , \tag{4}$$

where $D=\frac{1}{2}v_F^2\tau_0$ is the diffusion constant, τ_0 is the elastic scattering time, and τ_ϕ is the phase relaxation time. Equation (4) is the Schrödinger equation for a particle with mass $(2D)^{-1}$ and charge $2e$ moving in a random magnetic field in imaginary time. Therefore, $C(r,r';t)$ has the following path-integral representation:

$$C(r,r';t)=\int_{r(0)=r}^{r(t)=r'}\mathscr{D}r\,\exp\left\{-\int_0^t d\tau\left[\frac{\dot{r}^2}{4D}+i\frac{2e}{c}\dot{\mathbf{r}}\cdot\mathbf{a}+\frac{1}{\tau_\phi}\right]\right\}\ . \tag{5}$$

The probability distribution of the gauge field is assumed to be Gaussian:

$$P[\mathbf{a}]=\exp\left\{-(n_i\alpha^2)^{-1}\int d^2r(\nabla\times\mathbf{a})^2\right\}\ , \tag{6}$$

where α can be viewed as the flux bound to an impurity (in units of the flux quantum hc/e) and n_i is the density of impurities. The average over $P[\mathbf{a}]$ can then be performed exactly, yielding

$$\langle C(r,r;t)\rangle=\int_{r(0)=r}^{r(t)=r}\mathscr{D}r\,\exp\left\{-\int_0^t d\tau\left[\frac{\dot{r}^2}{4D}+\frac{1}{\tau_\phi}\right]-(2\pi\alpha)^2 n_i S[\mathbf{r}(\tau)]\right\}\ , \tag{7}$$

where the path integral is over all loops beginning and ending at $\mathbf{r}$ and $S[\mathbf{r}(\tau)]=\frac{1}{2}|\oint d\mathbf{r}\times\mathbf{r}|$ is the (positive) area of a given loop. The average of S over all loops is finite and is given by the diffusion law: $\langle S[\mathbf{r}(\tau)]\rangle=\gamma Dt$, for $t\ll\tau_\phi$, where γ is a number of order unity. In the saddle-point approximation to (7), we replace $S[\mathbf{r}(\tau)]$ by $\langle S[\mathbf{r}(\tau)]\rangle$ and thus obtain the following result:

$$\langle C(r,r)\rangle=\int\frac{d^2q}{(2\pi)^2}\frac{1}{\tau_\phi^{-1}+\tau_a^{-1}+Dq^2}\ , \tag{8}$$

where $\tau_a^{-1}=\gamma(2\pi\alpha)^2 n_i D$.

Therefore, to first order in the weak localization correction, it is evident from (8) and (3) that the effect of random flux on the conductivity of a weakly disordered system is to cut off the logarithmic divergence in $\Delta\sigma$, i.e., to suppress localization. The cutoff scale is $(\tau_a^{-1}+\tau_\phi^{-1})$ and remains nonzero in the limit $\tau_\phi\to\infty$. Therefore, neglecting higher-order weak localization corrections, one concludes that at sufficiently low temperatures where $\tau_a^{-1}>\tau_\phi^{-1}$, the resistivity is finite and temperature independent. In view of relations (2), we argue that the Hall system at $\nu=\frac{1}{2}$ behaves like a metal.

Further evidence for the suppression of weak localization by random flux comes from a numerical study of single-particle eigenstates on a lattice. Previous calculations of the scale-dependent conductivity of a two-dimensional Fermi system with random flux[13] did not yield a definite conclusion. We have performed numerical diagonalization on square lattices of up to 10^4 sites, and have found strong evidence for the existence of delocalized states in the presence of random flux. The Hamiltonian has the following form:

$$\mathcal{H} = -\sum_{(ij)} f_i^\dagger f_j e^{ia_{ij}} + \sum_i \varepsilon_i f_i^\dagger f_i \ , \tag{9}$$

where $f_i^\dagger$ creates a spinless fermion on site i of a two-dimensional square lattice, the on-site energy ε_i is a random variable uniformly distributed in $[-W/2, W/2]$, and the phase a_{ij} corresponds to a zero-average random flux per plaquette plus a uniform magnetic field δB. The sum of a_{ij} around a plaquette is thus $a_{ij}+a_{jk}+a_{kl}+a_{li}=\phi_{ijkl}$, with ϕ_{ijkl} uniformly distributed in $[-F+\delta B, F+\delta B]$. The sum on (ij) in (9) runs over all nearest neighbors j of ever site i; boundary conditions are periodic in the horizontal direction and free in the vertical direction.

As a measure of localization of the eigenstates of $\mathcal{H}$, we study the sample size dependence of the participation ratio[14]

$$\alpha_N(E) = \left\langle \sum_{i=1}^{N} |\psi_i|^4 \right\rangle_E \ , \tag{10}$$

where ψ_i is the amplitude of the normalized eigenstate $|\psi\rangle$ at site i, N is the number of sites, and $\langle\ \rangle_E$ denotes an average over random samples as well as over all states for a given sample with energies in a small interval around E. When E is in the extended-state region, one expects $\alpha_N(E) \sim N^{-\mu}$, where μ is of order 1; on the localized side of the mobility edge, $\alpha_N(E)$ is expected to be independent of N as $N \to \infty$.

To test our numerical scheme, we first set $W=F=\delta B=0$ and obtain $\alpha_N(E) \sim N^{-1}$, as shown by open squares in Fig. 2(a). Eigenstates of a strongly disordered system without flux, $W=10$, $F=\delta B=0$, show a qualitatively different scaling behavior typical of localization, with α_N independent of N. The results for $W=0$, $\delta B=0$, $F=\pi$ indicate that random flux by itself does not lead to localization. This is true not only for the midband states ($E=0$), shown in Fig. 2(a), but across the whole spectrum.

We are specifically interested in the case of correlated on-site and flux disorder, which is relevant to the $\nu=\frac{1}{2}$ Hall problem: $\phi_{ijkl}=(F/2W)(\varepsilon_i+\varepsilon_j+\varepsilon_k+\varepsilon_l)$ and $\delta B=0$. The scaling of midband eigenstates for $W=6$, $F=\pi$, shown in Fig. 2(b), suggests that these states are extended. The delocalizing effect of random flux is evident from a comparison of these data with those for $W=6$, $F=0$. For large values of $W \geq 10$ the system is in the exponentially localized regime, where random flux has no effect.

These results support our hypothesis that the Hall system at $\nu=\frac{1}{2}$ is metallic for weak disorder, and enters an exponentially localized phase as disorder increases. In addition to different temperature dependences of the resistivity, the two phases are characterized by opposite signs of the magnetoresistance. Our numerical results reflect this tendency in the dependence of the exponent $\mu=-d\ln\alpha_N/d\ln N$ on the magnetic field δB. As shown in Table I, μ increases with increasing δB when the system is in the insulating phase ($W=6$, $F=0$). This suggests a negative magnetoresistance, typical of an Anderson insulator. In the metallic phase ($W=6$, $F=\pi$) the trend is opposite, in agreement with the minimum of ρ_{xx} observed in the clean Hall samples at $\nu=\frac{1}{2}$.

In conclusion, we have proposed a scenario for the $\nu=\frac{1}{2}$ state of the Hall system, based on the mapping to a system of Chern-Simons fermions. It has been argued that impurity scattering is modified in an essential way by the presence of the random statistical flux. This modification has been shown, both analytically to the first order in the weak localization correction and by numerical simulation on a lattice, to suppress logarithmic localization and to stabilize a metallic phase for weak disorder

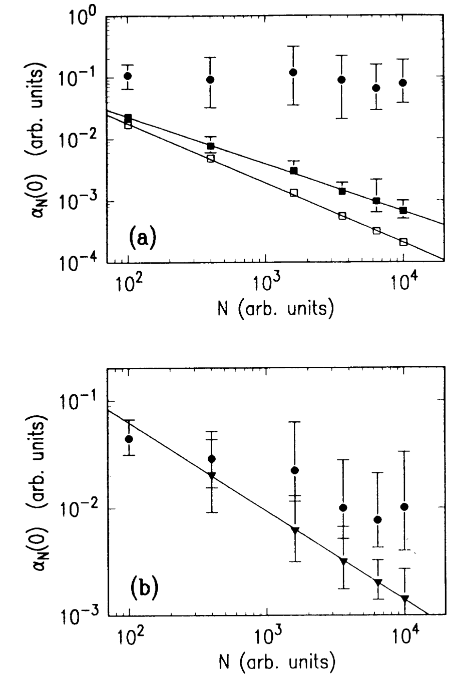

FIG. 2. (a) $\alpha_N(0)$ as a function of lattice size N for $W=F=0$ (open squares), $W=10$, $F=0$ (dots), $W=0$, $F=\pi$ (solid squares). The average in (10) is taken over five random samples and over eight eigenstates for each sample. (b) $\alpha_N(0)$ vs N for $W=6$, $F=0$ (dots) and $W=6$, $F=\pi$ (triangles).

TABLE I. The exponent μ, obtained form the plot of $\ln\alpha_N$ vs $\ln N$, as a function of δB in units of flux quanta per plaquette, for two choices of disorder parameters F and W.

	Exponent μ	
δB	$W=6, F=0$	$W=6, F=\pi$
0.0	0.378	0.713
0.1	0.640	0.700
0.2	0.558	0.651
0.3	0.568	0.571

and small deviation of the magnetic field from its value at $\nu=\frac{1}{2}$. We have presented evidence that this phase is characterized by a positive magnetoresistance, as measured relative to the field at $\nu=\frac{1}{2}$, and unquantized Hall resistance, in qualitative agreement with experiment.

The authors were recently informed of the work by Halperin, Lee, and Read on the $\nu=\frac{1}{2}$ anomaly, where similar effects were investigated.

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